REDIMAT
Journal of Research in Mathematics Education
Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

Agents of Change in Promoting Reflective Abstraction: A Quasi-Experimental, Study on Limits in College Calculus

Robert W. Cappetta[1], Alan Zollman[2]

1) College of DuPage, United States of America
2) Northern Illinois University, United States of America

Date of publication: October 24th, 2013
Edition period: October 2013-February 2014

To cite this article: Cappetta, R.W., and Zollman, A. (2013). Agents of Change in Promoting Reflective Abstraction: A Quasi-Experimental Study on Limits in College Calculus. *REDIMAT – Journal of Research in Mathematics Education*, *2*(3), 343-357. doi: 10.4471/redimat.2013.35

To link this article: http://dx.doi.org/10.4471/redimat.2013.35

PLEASE SCROLL DOWN FOR ARTICLE

REDIMAT – Journal of Research in Mathematics Education Vol. 2 No. 3 October 24th 2013 pp. 343-357

Agents of Change in Promoting Reflective Abstraction: A Quasi-Experimental Study on Limits in College Calculus

Robert W. Cappetta
College DuPage

Alan Zollman
Northern Illinois University

(*Received: October 26th 2012; Accepted: August 27th 2013; Published: October 24th 2013)*

Abstract

We measured student performance on the concept of limit by promoting reflection through four agents of change: instructor, peer, curriculum and individual. It is based on Piaget's four constructs of reflective abstraction: interiorization, coordination, encapsulation, and generalization, and includes the notion of reversal, as refined into a construct by Dubinsky. Our quasi-experimental study examined the performance of two sections of first-semester calculus students at a midwestern community college. Scores by students in the experimental section were significantly higher than scores by students in the control traditional section on a posttest of limits. A deeper examination of a three-tiered subgroup showed the reflective abstraction section had moderate effect size on mathematical knowledge and strategic knowledge and a large effect size on explanation.

Keywords: reflective abstraction, limits, agents of change, calculus.

2013 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2013.35

REDIMAT – Journal of Research in Mathematics Education Vol. 2 No. 3 October 24th 2013 pp. 343-357

Agentes de Cambio en la Promoción de la Abstracción Reflexiva: Un Estudio Quasi-Experimental sobre los Límites en College Calculus

Robert W. Cappetta
College DuPage

Alan Zollman
Northern Illinois University

(*Recibido: 26 de Octubre 2012; Aceptado: 27 de Agosto 2013; Publicado: 24 de Octubre 2013*)

Resumen

Se midió el rendimiento de los alumnos en el concepto de límite promoviendo la reflexión a través de cuatro agentes de cambio: instructor, compañeros, currículo e individuo. Se basa en cuatro conceptos sobre la abstracción reflexiva de Piaget: interiorización, coordinación, encapsulación y generalización, e incluye la noción de inversión, definida por Dubinsky. Nuestro estudio cuasi-experimental ha estudiado el comportamiento de dos grupos de estudiantes de cálculo de primer semestre en un colegio comunitario del medio oeste. Las puntuaciones de los estudiantes del grupo experimental fueron significativamente más altas que las puntuaciones de los estudiantes del de control tradicional en un examen posterior de los límites. Un examen más profundo de un subgrupo de tres niveles mostró que la abstracción reflexiva tenía un efecto moderado en el conocimiento matemático y que el conocimiento estratégico tenía un efecto grande sobre la variable explicación.

Palabras clave: abstracción, límites, agentes de cambio, cálculo.

2013 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2013.35

Calculus plays a vital role in the undergraduate curriculum and one of the key concepts in calculus is the limit. It is the first topic that students encounter and it requires a type of subtle reasoning that is not used in algebra (Cornu, 1991). Research indicates that many students struggle with limits (Artigue, 1992; Artigue, Batanero & Kent, 2007; Cappetta, 2007; Cappetta & Zollman, 2009; Cornu, 1981, 1991; Davis & Vinner, 1986; Juter, 2005, 2007; Li & Tall, 1993; Monaghan, Sun & Tall, 1994; Oehrtman, 2009; Robert, 1982; Sierpinska, 1987; Tall 1992; Tall & Vinner, 1981; Williams, 1991). Additionally there is a significant difference between what instructors want their students to learn and what students actually understand (Hardy, 2009).

In order to improve student conceptual understanding of the concept of limit, a theoretical framework is needed to examine how and why students learn certain mathematical concepts. Kidron (2008) recognizes that multiple frameworks are necessary to understand the conceptualization of the notion of limit. In this study reflective abstraction, as defined by Piaget (Beth & Piaget, 1966) and refined by Dubinsky (1991) is used. Piaget describes four constructs of reflective abstraction in developing conceptual understanding. These are interiorization, coordination, encapsulation, and generalization. While Piaget describes reversal with these four constructs, Dubinsky adds reversal as a fifth construct. This paper investigates the question: Can these reflective abstraction constructs be initiated to increase student learning of limits, and if so, how?

First, Piaget (1967) claims that reflective abstraction is a personal activity; therefore the individual student may be capable of initiating reflective abstraction. Second, Cobb, Jaworski, and Presmeg (1996) discuss the relationship between social discourse and individual reflective abstraction, and therefore, peers may be capable of initiating reflective abstraction. Third, Cobb, Boufi, McClain and Whitenack (1997) assert the teacher can prompt shifts in the discussion that may lead to reflective abstraction, so the teacher may be capable of initiating reflective abstraction. Finally, several curricula have been developed in recent years to encourage students to reflect about their thinking in mathematics. These include the Harvard Project (Hughes-Hallett, 1997), Project CALC (Smith & Moore, 1991), Calculus and Mathematica (Davis, Porta, Uhl, 1994) among others. So it appears that a curriculum also might initiate reflective abstraction.

Based upon the writings of Piaget (Beth & Piaget, 1966) and Dubinsky (1991), reflective abstraction constructs are defined as:

- Interiorization: A student performs the steps in a procedure. The student reflects on the procedure and begins to define a concept.
- Coordination: A student examines two different processes and integrates them into a coordinated process that is used to analyze a mathematical concept.
- Encapsulation: A student encapsulates a concept by constructing individual meaning. Encapsulation is the act of personifying a concept. An abstract notion or a collection of abstract notions becomes meaningful to an individual.
- Generalization: After an individual has encapsulated a notion, it is extended and applied to a wider collection of mathematical problems.
- Reversal: A student constructs a new mathematical notion by reversing the steps of the original notion.

Similarly, based upon Cobb's discussion (Cobb, Boufi, McClain, & Whitenack, 1997) of the role of reflective discourse in the classroom community to initiate reflection, the four "agents of change" are defined as (Cappetta, 2007; Cappetta & Zollman, 2009):

- Individual: A student spontaneously engages in reflective abstraction.
- Peer: A classmate challenges or questions an individual.
- Instructor: The instructor challenges or questions an individual.
- Curricular: Activities in the curriculum are designed to challenge and question students.

This paper describes a study of a curriculum designed to initiate reflective abstraction on the concept of limit in college calculus. Specifically, this curriculum includes the four agents of change (individual, peers, instructor, and curriculum) to investigate two research questions:

- Can reflective abstraction be initiated using agents of change in the learning of limits?

- If so, does using agents of change in an experimental curriculum improve student performance?

Methodology

A quasi-experimental design was used. The traditional, a matched section of calculus students, studied a traditional curriculum. These students studied limits in a conventional manner. The experimental, a second matched section of calculus students, studied an innovative curriculum. This curriculum is designed to initiate reflective abstraction through the use of four agents of change, namely, the individual student, student peers, the instructor or the curriculum itself.

The two sections were pretested to identify any potential significant differences in concepts and knowledge of limits. After intervention, the two sections were given a posttest to identify significant differences between sections and in sections, from pre- to posttest. All pretests and posttests were scored using a two-point rubric designed by the Illinois State Board of Education (2005b): Completely correct response, including correct work shown (2 points); Partially correct response (1 point); and No response, or incorrect response (0 points). An independent grader also used the rubric to score the pretests and posttests in order to establish inter-rater reliability.

The instructor for the two sections both were experienced, dedicated calculus faculty members – committed to student learning. Both individuals regularly participate in activities designed to improve instruction, and were eager to join in the research project. Both instructors were told separately that their curriculum was designed to improve student performance and understanding on the concept of limit. Each section (traditional and experimental) viewed their section as the “experimental” group.

The traditional and experimental curricula (lesson plans, class activities, class work sets, homework sets) were scripted and covered the same topics, lasted the same amount of time, taught at the same time and days (five days per week, fifty minutes per day), and were evaluated by the same standardized examination. In order to keep external influences to a minimum (outside group work, math assistance area, tutors), students were told to complete all homework assignments independently. Both curricula were field tested for content validity. Both curricula were revised, edited and approved by a Delphi method of iterations using an expert panel of teaching-award recipient professors in a mathematics department at a research university.

The traditional curriculum used *Calculus* by Larson, Hostetler, and Edwards (2006) as a guide. The instructor demonstrated examples similar to those in the text as part of the lecture. Students completed standard text exercises in class. Students also completed standard text exercises for homework. The instructor began each subsequent day by answering student questions about the homework. The traditional curriculum was not designed to initiate reflective abstraction nor was it designed to hinder it. In order to determine how it occurs in a traditional classroom, a collection of lessons was audio taped. The transcripts of these lessons were analyzed to identify instructor as an agent of changes of reflective abstraction.

The experimental curriculum was modeled using the ACE (Activity, Classroom Discussion, Exercise) teaching cycle (Weller et al., 2003). Each day's lesson was designed to include activities that would ask students to reflect on the mathematics, so the curriculum is a potential agent of change. Students worked on activities in cooperative groups, so the student's peer group is a potential agent of change. The instructor de-briefed the activity in a classroom discussion, so the student's instructor is a potential agent of change. Each student completed a set of exercises related to the activity, so the individual student is a potential agent of change. To demonstrate that this occurred, student class work, student homework and transcripts of group work were examined.

The limit examples used in the experimental section were identical to those used in the traditional curriculum. Both instructors taught the scripted curriculum modeled on three sections of the textbook *Calculus* by Larson and Hostetler (2006). All students were given the same homework assignments from the textbook. Classes of both matched sections were observed and recorded by the researcher to validate that the prescribed curricula was followed. The only differences between the two calculus sections were the collaborative activities and the teacher scripted interactions.

Results

The researcher and an independent grader scored the pretests and posttests using the short-response rubric from the Illinois State Board of Education (2005b). There were very few discrepancies in grading. These discrepancies were remedied by constructing and consulting item-specific scoring schemes and re-grading.

Pretest Analysis. The pretests consisted of twelve computational questions. Students earned 2 points for a correct solution, 1 point for a partially correct solution, and 0 points for an incorrect solution. A total of 35 students participated. Table 1 summarizes the statistics from the pretests. Using a two-tailed t-test and a significance level of $p < .05$, the results were $t\ (32) = 0.57$ and $p = .58$. No significant difference was found between the means of the traditional section and the experimental section on the pretests.

The data was further analyzed using a standardized measure of effect size, Cohen's *d*. Effect size estimate the strength of an apparent relationship. Cohen's *d* is defined as the difference between two means divided by a standard deviation for the data. Cohen defined effect sizes ≥ 0.2 as small; ≥ 0.5 as medium and ≥ 0.8 as large (Cohen, 1988). Comparing the traditional to experimental sections on the pretest, we measured Cohen's $d = 0.192$, so there was minimal effect.

Table 1
Scores or Pretest – All Participating Students

	Section	
Measure	Experimental	Traditional
Sample Size	16	19
Mean	7.313	6.316
Standard Deviation	5.237	5.132
Effect Size – Cohen's *d*	0.192	

Posttest Analysis. The posttests consisted of 14 questions. Twelve questions were computational and two questions were essays. Again, these tests were scored using the same short response rubric from the Illinois State Board of Education (2005b) that was used in the pretest analysis. Table 2 summarizes the posttest scores for all students who participated in the study. Using a significance level of $p < .05$, a one-tailed *t*-test for equality of means was performed. The results of the t-test are $t\ (32) = 2.63$ and $p < .01$. Cohen's $d = 0.877$ so there was a large effect. The significance level and effect size show that the students in the experimental section outperformed those in the traditional section.

Table 2
Scores or Post-test – All Participating Students

	Section	
Measure	Experimental	Traditional
Sample Size	16	19
Mean	21.18	16.95
Standard Deviation	4.48	5.14
Effect Size – Cohen's *d*	0.877	

A small number of students missed at least one of the five class sessions. A second analysis of the posttests examines the performance of the students who attended all of the class sessions. Table 3 summarizes the posttest scores for the students who attended all classes. Using a significance level of $p < .05$, a one-tailed *t*-test for equality of means was performed. The results of the t-test are $t\ (25) = 1.76$ and $p = .046$. Cohen's $d = 0.66$ so there was a medium effect. Using this selection criterion, the students in the experimental section again outperformed those in the traditional section.

Table 3

Scores on Post-test Restricted to Only Students Who Attended All Classes

	Section	
Measure	Experimental	Traditional
Sample Size	13	15
Mean	21.38	18.4
Standard Deviation	4.50	4.47
Effect Size – Cohen's *d*	0.660	

In order to minimize differences that might be due to previous knowledge, an analysis of covariance is included. The posttest scores are covaried against the pretest scores. The results are F = 6.40, p = .017. Again using a significance level of $p < .05$, a significant difference is present between the experimental and traditional sections. Covariance results are summarized in Table 4.

Table 4

Post-tests versus Section Type Covaried with Pre-test Score for All Students

Analysis of Variance for Post, using Adjusted SS for Tests						
Source	*df*	Seq SS	Adj SS	Adj MS	*F*	*p*
Pre-test	1	148.25	121.03	121.03	6.01	0.020
Section Type	1	128.94	128.94	128.94	6.40	0.017
Error	32	644.36	644.36	20.14		
Total	34	921.54				

Three-Tiered Comparison Subgroup. To get additional information, a comparison subgroup was chosen from the students in the study. These students were chosen based on their level of improvement from the pretest to the posttest. Two students with greatest improvement, two students with median-level improvement, and two students with least improvement were chosen from each section. These twelve students (six from each section) had their posttests re-scored using the five-point extended response rubric from the Illinois State Board of Education (2005a).

An analysis of covariance was performed using the pretest scores as a covariate. The results of that test show that F = 2.54, p =.17; so the result was not significant at the p < .05 significance level. Results from the analysis of covariance are summarized in Table 5. With no statistical differences shown in the pretest, it may be fair to compare the six students in the experimental section to the six students in the traditional section.

Table 5

Post-tests versus Section Type Covaried with Pre-test Score for Three-Tiered Comparison Subgroup

Analysis of Variance for Post, using Adjusted SS for Tests						
Source	*df*	Seq SS	Adj SS	Adj MS	*F*	*p*
Pre-test	1	25.27	16.57	16.57	1.51	0.250
Section Type	1	24.63	24.63	24.63	2.24	0.168
Error	9	98.76	98.76	10.97		
Total	11	148.67				

The twelve posttests for the students in the three-tiered comparison subgroup were re-scored using the five-point extended response rubric from the Illinois State Board of Education (2005a). The tests were scored three on three criteria: first for mathematical knowledge, second for strategic knowledge and third for explanation. The results appear in Table 6.

Table 6
*Extended Response Results for Students in the Three-Tiered Comparison Subgroups**

	Measures					
	Mathematical Knowledge		Strategic Knowledge		Explanation	
	E	T	E	T	E	T
N	40.67	6	6	6	6	6
M	5.20	37.17	41.50	37.17	44.33	35.50
SD	0.635	5.81	6.28	5.11	7.09	8.31
d			0.756		1.14	

* Each comprised of 2 students with least; 2 with medium-level; 2 with greatest improvement; E means *Experimental* and T means *Traditional*

One-sided t-tests for equality of means of the experimental and traditional students were performed for mathematical knowledge, strategic knowledge and explanation measures. A significance level of $p < .05$ was used for each of the tests. No significant difference was identified for the mathematical knowledge measure, t (10) = 1.10, p = .15. Cohen's d = .635 so there was a medium effect. No significant difference was identified for the strategic knowledge measure, t (10) = 1.31, p = .11. Cohen's d = .756 so there was a medium effect. [According to Vernez and Zimmer (2007) these two Cohen's *d's* would be classified as large effect sizes in education interventions.] A significant difference was identified for the explanation measure, t (10) = 1.98, p = .04. Cohen's d = 1.14 indicating a large effect.

Agents of Change. The experimental curriculum was designed in to initiate reflective abstraction. To demonstrate that this occurred, student class work, student homework and transcripts of group work were examined. A sample of these protocols was collected from the three-tiered comparison subgroup. Items from these protocols were coded based on the construct of reflective abstraction (interiorization, coordination, generalization, reversal, and encapsulation) and the agent of change (curriculum, peer, instructor, individual).

The traditional curriculum was not designed to initiate reflective abstraction nor was it designed to hinder it. In order to determine how it occurs in a traditional classroom, a collection of lessons was audio taped.

The transcripts of these lessons were analyzed to identify instructor as an agent of change. Peers were not given the opportunity in the classroom to work together, so there were no opportunities to infer peer group as an agent of change. Both the experimental section students and the traditional section students completed the same textbook homework, so these protocols were collected and analyzed for individual as an agent of change.

Conclusion

The following evidence suggests that reflective abstraction is a contributing factor for improved student understanding of the limit concept. The experimental curriculum was successful in promoting reflective abstraction through individual, peer, curricular, and instructor as agents of change. The traditional curriculum was not designed to promote reflective abstraction. However, as one might expect from a good instructor, the traditional curriculum promoted reflective abstraction through instructor as an agent of change. The students in the experimental section outperformed the students in the traditional section on a test of the concept of limit. Both sections examined similar examples in class and completed the same homework exercises. For these reasons it is fair to conclude that the curriculum was a significant reason for the success of the students in the experimental section.

The artifacts from the three-tiered subgroup provide evidence that the experimental curriculum was successful in initiating reflective abstraction. They also provide evidence that the teacher in the traditional section regularly initiated reflective abstraction in his lectures. Finally, these artifacts indicate that successful calculus students engage in reflective abstraction regardless of the teacher or the curriculum. However, this research shows that a curriculum designed to promote reflective abstraction can improve student performance.

An interesting, unexpected result was that students in the experimental section were better at written explanation of mathematics than were the students in the traditional section. This indicates that opportunities to reflect on learning, together with regular writing assignments, may improve a student’s written communication skills in mathematics.

In conclusion, our study indicates that an experimental calculus curriculum can promote reflective abstraction. Furthermore, an experimental curriculum, with instructor, peer, curriculum and individual as agents of change, can show improved student performance, mathematical knowledge, strategic knowledge, and written explanation on the concept of limit.

Initiating reflective abstraction is an effective tool for improving a student's performance in mathematics. The constructs of interiorization, coordination, encapsulation, generalization and reversal should be examined in the process of mathematics curriculum development. Teachers should promote reflective abstraction using instructor, peer, and curricular as agents of change. Instructors should design problem sets that enable students to initiate reflective abstraction independently. The challenges of teaching and learning mathematics are substantial. Promoting reflective abstraction may enable teachers to help students meet this challenge.

References

Artigue, M. (1992). Functions from an algebraic and graphic point of view: Cognitive difficulties and teaching practices. In *The concept of function: Aspects of epistemology and pedagogy*. (pp. 109-132). MAA Notes No. 28.

Artigue, M., Batanero, C., & Kent, P. (2007). Mathematics teaching and learning at post-secondary level. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1011-1050). Reston, VA: NCTM.

Beth, E. and Piaget, J. (1966). *Mathematical epistemology and psychology*. Dordrecht, The Netherlands: D. Reidel.

Cappetta, R. (2007). Reflective abstraction and the concept of limit: A quasi-experimental study to improve student performance in college calculus by promoting reflective abstraction through individual, peer, instructor and curriculum initiates. Unpublished doctoral dissertation, Northern Illinois University, DeKalb, IL.

Cappetta, R., & Zollman, A. (2009). Creating a discourse-rich classroom on the concept of limits in calculus: Initiating shifts in discourse to promote reflective abstraction. In L. Knott (Ed.), *The Role of*

Mathematics Discourse in Producing Leaders of Discourse (pp. 17-39). Charlotte, NC: Information Age Publishing.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28(3)*, 258-277.

Cobb, P., Jaworski, B., & Presmeg, N. (1996). Emergent and sociocultural views of mathematical activity. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer *Theories of mathematical learning* (pp. 3-19). Mahwah, N.J.: Lawrence Erlbaum.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences (2nd ed.).* Hillsdale, NJ: Lawrence Earlbaum Associates.

Cornu B. (1981) Learning the concept of limit: Models and spontaneous modes. In *Proceedings of the fifth conference for the Psychology of Mathematics Education* (pp. 322-326). Grenoble, France: Psychology of Mathematics Education.

Cornu, B. (1991). Limits. In D. Tall (Ed.), *Advanced mathematical thinking*. (pp. 153-166). Boston: Kluwer.

Davis, B., Porta, H., & Uhl, J. (1994). *Calculus and Mathematica.* Addison-Wesley: Reading, MA.

Davis, R. & Vinner, S. (1986). The notion of limit: Some seemingly unavoidable misconception stages. *Journal of Mathematical Behavior*, *5*, 281-303.

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking.* (pp. 95-126). Boston: Kluwer.

Hardy, N. (2009). Students' perceptions of instructional practices: The case of limits and functions in college level calculus courses. *Educational Studies in Mathematics*, *72*(3), 341-358. doi: 10.1007/s10649-009-9199-8

Hughes-Hallett, D. (1997). *Calculus single variable*. New York: John Wiley and Sons.

Illinois State Board of Education (2005a). *Extended response rubric.* Retrieved May 22, 2007 from http://www.isbe.net/assessment/docs/ERMathRubric.rtf

Illinois State Board of Education (2005b). *Short response rubric.* Retrieved May 22, 2007 from http://www.isbe.net/assessment/docs/SRMathRubric.rtf

Juter, K. (2005). Limits of functions: Traces of students' concept images. *Nordic Studies in Mathematics Education, 10*(3-4), 65-82.

Juter, K. (2007). Students' conceptions of limits: high achievers versus low achievers. *The Montana Mathematics Enthusiast, 4*(1), 53-65.

Kidron, I. (2008). Abstraction and consolidation of the limit procept by means of instructional schemes: The complementary role of three different frameworks. *Educational Studies in Mathematics, 69*(3), 197–216. doi: 10.1007/s10649-008-9132-6

Larson, R., Hostetler, R. & Edwards, B. (2006). *Calculus.* Boston: Houghton Mifflin.

Li, L., & Tall, D. (1993). Constructing different concept images of sequences and limits by programming. In *Proceedings of the seventeenth conference for the Psychology of Mathematics Education* (pp. 41-48). Tsukuba, Japan: Psychology of Mathematics Education.

Monaghan, J., Sun, S. & Tall, D. (1994). Construction of the limit concept with a computer algebra system, In *Proceedings of the eighteenth conference for the Psychology of Mathematics Education* (pp. 279-286). Lisbon, Spain: Psychology of Mathematics Education.

Oehrtman, M. (2009). Collapsing dimensions, physical limitation, and other student metaphors for limit concepts. *Journal for Research in Mathematics Education, 40* (4), 396-426.

Piaget, J. (1967). *Genetic epistemology, a series of lectures delivered by Piaget at Columbia University*. New York: Columbia University Press.

Robert, A. (1982). L'Acquisition de la notion de convergence des suites numériques dans l'Enseignement Supérieur, *Reserches en didactique des mathématiques, 3* (3), 307-341.

Sierpinska, A. (1987). Humanities students and epistemological obstacles related to limits. *Educational Studies in Mathematics, 18*(4), 371-397. doi: 10.1007/BF00240986

Smith, D. & Moore, L. (1991). Project CALC: An integrated laboratory course. In L. C. Leinbach (Ed.), *The laboratory approach to teaching* *calculus* (pp. 81-92). Washington, DC: The Mathematical Association of America.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity and proof. In D. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 495-511). New York: Macmillan.

Tall, D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151-169. doi: 10.1007/BF00305619

Vernez, G. & Zimmer, R. (2007). Interpreting the effects of Title I supplemental educational services. Retrieved March 29, 2011 from http://www2.ed.gov/rschstat/eval/choice/implementation/achievementanalysis-sizes.pdf

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 65-81). Dordrecht: Kluwer Academic Publishers.

Weller, K., Clark, J., Dubinsky, E., Loch, S., McDonald, M., & Merkovsky, R. (2003). Student performance and attitudes in courses based on APOS Theory and the ACE Teaching Cycle. In A. Selden, E. Dubinsky, G. Harel, & F. Hitt (Eds.), *Research in collegiate mathematics education V* (pp. 97-131). Providence, RI: American Mathematical Society.

Williams, S. (1991). Models of limit held by college calculus students. *Journal for Research in Mathematics Education, 22*(3), 219-236.

Robert W. Cappetta is professor at the Department of Mathematics, in DuPage University, USA.

Alan Zollman is professor at the Department of Mathematical Sciences, at the Northern Illinois University, USA.

Contact Address: Direct correspondence concerning this article should be addressed to author Alan Zollman at: Department of Mathematical Sciences, Northern Illinois University. E-mail: zollman@math.niu.edu.