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A Preservice Mathematics Teacher's Beliefs about Teaching Mathematics with Technology

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Abstract

This paper analyzed a preservice mathematics teacher's beliefs about teaching mathematics with technology. The researcher used five semi-structured task-based interviews in the problematic contexts of teaching fraction multiplications with JavaBars, functions and limits, and geometric transformations with Geometer's Sketchpad, and statistical data with Excel Spreadsheet to generate data about the participant's pedagogical-technological beliefs. An additional unstructured interview was conducted with the same participant after his practice teaching. The interview data were analyzed and interpreted in two layers- the first layer portrays the participant's voice in his narratives and the second layer portrays researcher's voice in terms of his interpretation of data and interconnection to literature. The analysis and interpretation generated seven major categories of beliefs- beliefs about teaching materials, teaching strategy, bridging activities, technological tools, mathematical concepts, and meanings, activities and transformation, and issues and challenges. Finally, the author discusses implications of the study.

Key words: Teacher beliefs; Technology integration; Radical constructivist grounded theory

Introduction

At first, the author briefly introduces the literature on teacher beliefs about teaching mathematics with technology and the radical constructivist grounded theory (RCGT) as a theoretical frame for the study. He discusses methods of data generation through task-based interviews. He further discusses analysis and interpretation of interview data by applying the principles of RCGT. He highlights some key findings in the results and discussion followed by implications.

Many researchers (e.g., Chai, Wong & Teo 2011; Ertmer 2006; Leatham 2002 & 2007; Quinn 1998; Teo, Chai, Hung & Lee, 2008) highlighted challenges of technology integration in mathematics education. Quinn (1998) reported teachers' initial beliefs toward technology use in mathematics teaching as a tool to help students to discover concepts on their own with fun and greater motivation to learn. The next theme was changes in beliefs in which he reported changes in the majority of the preservice teachers' beliefs in relation to use of technology (i.e. calculators) as tools to learn concepts and basic skills, not just to calculate. He reported their educational experiences in schools being lack of technology in learning mathematics. Quinn (1998) clearly outlined how mathematics teacher education courses could enhance preservice teachers' beliefs toward the creative use of technological tools in teaching and learning mathematics. This study showed how preservice mathematics teachers' embodied technological-pedagogical beliefs could be changed with well-planned and well-focused mathematics methods course.

Leatham (2002, 2007) provided plenty of ideas about technology integration in preservice mathematics teacher education in general and mathematics methods course in particular. His analysis seemed very deep and very comprehensive in the sense that he captured essential themes and pertinent issues of technology integration in mathematics education. His analysis of preservice mathematics teachers' beliefs about teaching mathematics with technology concluded with a nonmathematical role, real-world application, visualization of mathematics, and exploration of mathematics. These roles of technology seemed to be powerful in teaching and learning of mathematics. He further discussed preservice teacher beliefs in terms of formal and informal, internal and external, subjective and inter-subjective, isolated and connected, utilitarian to developmental, temporal to spatial, to name a few.

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Next, Ertmer (2006) examined the relationship between teachers' pedagogical beliefs and their technology practices in classroom teaching and learning. She reviewed various works related to teacher beliefs in the various subject areas and examined the pedagogical beliefs in relation to technology use. She argued that the relationship between beliefs and teacher practices, teacher knowledge, and student outcomes are important aspects to be considered in educational research related to teacher beliefs. She claimed that beliefs a teacher holds may influence his or her perceptions and judgments, and finally it affects the classroom practices using technology. She reported some researches having inconsistencies between teachers' beliefs and their classroom practices. Ertmer (2006) further explained that beliefs can be quite idiosyncratic, and that is why two teachers having the similar knowledge about technology may have different beliefs about the use of technology in teaching and learning. Researchers (e.g., Peterson, Fennema, Carpenter & Loef, 1989) supposed that teachers' beliefs play a significant role in how it is adopted and implemented in classroom teaching and learning.

NCTM (2008) stated, "in a well-articulated mathematics program, students can use these (technological) tools for computation, construction, and representation as they explore problems; and the use of technology also contributes to mathematical reflection, problem identification, and decision making" (n. p.). NCTM clearly stated its position in relation to the role of technology in mathematics education that emphasized the creative use of technology in teaching and learning mathematics. Preservice or inservice teachers' beliefs toward the use of technological tools for construction can be related to efficiency of construction, learning through construction, and process of construction. One's beliefs about teaching mathematics with technology might be rooted in his or her image of mathematics, past and current anxieties associated with teaching and learning of mathematics, and his or her attitudes toward use of different tools (both manipulatives and technology) for teaching mathematics (Belbase, 2013a).

Most of the prior studies on preservice teacher beliefs are associated with either their experiences in classroom practices (as students and teachers) or surveys on what beliefs they hold about teaching and learning mathematics with technology. There is a lack of study on preservice mathematics teachers' expressed beliefs during task-situations and idiosyncratic experiences. The studies on teacher beliefs based on recalled and recurring memories of experiences as students and prospective teachers may not reflect their idiosyncratic beliefs. This study aimed to explore a preservice secondary mathematics teacher's beliefs about teaching mathematics with technology that are formed and reformed based on the problems he encountered and experienced he went through when he was asked to reflect on problems and his anticipation of teaching in classroom. In this context, the research question for this study was- What beliefs does a preservice secondary mathematics teacher hold about teaching mathematics with technology and how do those beliefs unfold with experiences within task situations and teaching experiences?

The researcher used the radical constructivist grounded theory (RCGT) as theoretical perspective to construct data, and analyze and interpret the participant's beliefs about teaching mathematics with technology.

Radical Constructivist Grounded Theory: A Theoretical Frame

This research study assumed integrated epistemology of radical constructivism with grounded theory to synthesize a radical constructivist grounded theory. There may be a variety of constructivist grounded theory based on which constructivism integrated with grounded theory- for example, trivial constructivist grounded theory, social constructivist grounded theory, and RCGT. In this study, the researcher adopted RCGT. Three principles of radical constructivism assume that knowing is an *active process in construction* of knowledge by the cognizing subject; his or her knowing is *autopoietic* (Maturana & Varela, 1980), and his or her knowing is evaluated in relation to *viability* (von Glasersfeld, 1978). These principles when integrated with the *principles of grounded theory*- coding, theoretical sampling, constant comparison, theoretical sensitivity, and memoing may result into a new hybridized tool as RCGT for the study of preservice mathematics teacher's beliefs about teaching mathematics with technology.

Glaser and Strauss (1967) founded grounded theory method as a new paradigm in qualitative research focusing on the discovery of a theory grounded in data. Many grounded theorists (e.g., Strauss, 1987; Strauss & Corbin, 1990, 1998; Corbin & Strauss, 2008; Glaser, 1978 & 1992; Charmaz, 2006; Clarke, 2005) used grounded theory from different epistemological perspectives. The marriage of RCGT and grounded theory for this study made this researcher aware of his role as a researcher, the role of the participant as a knowledge generator, and process of grounded theory as a tool for analysis and interpretation. The RCGT provided the researcher a basis to reconstruct the voice with "multiple registers and meanings" (Mazzei & Jackson, 2009, p. 5). The five principles of RCGT – researcher and participant voice, symbiosis between researcher and participant, research as cognitive function, research as an adaptive function, and praxis criteria served the guideline for the study (Belbase, 2014).

The first principle focuses on researcher and participant voice in the research process and product. The second principle focuses on the mutualism of researcher and participants through task-based interactions having cognitive gain for both. The third principle is about process of the study as a cognitive function. The research process was a journey of interaction between the researcher and the participant through which they constructed meaning of the participant's beliefs based on his experiences through the task situations. The fourth principle is about adaptation of the researcher and participants to the new situations during the interactions on task situations. The emergence or construction of new codes and categories continued until more stable categories emerge through the process. The process of equilibration or saturation of categories was achieved through adaptation to additional data, new codes, and grounding categories in the subsequent interview episodes or interactions. The fifth principle is associated with dimensions of fit and viability of the categories to the context that portray the beliefs of the participant (Belbase, 2014). The theoretical underpinning of RCGT served as a conceptual guide throughout the research process by bringing this researcher and the participant together as co-learners, teachers and research subjects.

Methodology

Recruitment of Participant

Initially, three out of a pool of twenty preservice secondary mathematics teachers enrolled in a College of Education at a Midwestern University in the U.S. in the fall of 2011 were recruited as the potential participants for the study. The selection of the three participants in the study was based on their interest to volunteer in five task-based interviews and their time available for the interviews. The research tool included five task-based interviews when they were in mathematics methods class and one post-teaching experience interview in the spring of 2012 with each of the three participants. The researcher analyzed and reported only one participant's beliefs about teaching mathematics with technology in this paper as a single-subject case analysis. The single case was selected out of the three participants based on explicit expressions of beliefs and active participation in the problem solving situations compared to other participants.

Task-based Interviews

The source of data for the study were a series of five 60-90 minute task-based interviews (Goldin, 2000) conducted with a participant during the time when he was taking mathematics methods course in the fall of 2011 and one interview after his field experience in the spring of 2012. The first five interviews were based on mathematical tasks within technological environments. The researcher designed an interview guideline with potential interview questions from the five different content domains. The first interview was on teaching fractions by using JavaBars, the second interview was on teaching functions and relations using Geometer's Sketchpad (GSP), the third interview was on teaching concepts of limit in the dynamic environment of GSP, the fourth interview was on teaching geometric transformation using GSP, and the fifth interview was on teaching of data using Microsoft Excel Spreadsheet. Each interview episode was video recorded. The interview after the post field experience was an unstructured interview focusing on his experiences of teaching mathematics using technology during his practicum.

Analysis and Interpretation

The application of RCGT as "a hybrid of radical constructivism as an epistemology and grounded theory as a methodology" (Belbase, 2013b, n. p.) helped this researcher in conceptualizing the processes of data construction, analysis, and interpretation. The principles of radical constructivism helped the researcher to form an epistemological basis to understand the nature of the data and major categories or themes. The grounded theory helped him to perform the actual analysis and interpretation through coding, categorizing, thematizing, theoretical sampling, and theoretical memoing as cognitive and adaptive function.

There are different ways to derive major categories and formulate a substantive theory from grounded theory based on different epistemological lenses. For example, the classical grounded theory of Glaser and Strauss (1967) emphasizes *discovery* of a theory grounded in data. From the grounded theory perspective of Strauss and Corbin (1990 & 1998) and Corbin and Strauss (2008), a theory is *developed out* of empirical data. According to the constructivist grounded theory of Charmaz (2006), a theory is *constructed out* of the empirical data. By applying RCGT method, key themes and a related theory are *invented by* the researcher out of the empirical data

(Belbase, 2013b). The notion of discovery, development, construction, and invention relate to how a local theory is grounded on data within different epistemological paradigms.

During the analysis and interpretation the researcher employed open, axial, and selective coding (Corbin & Strauss, 1990 & 1998) of the interview data. At first he identified some basic concepts in the data and labeled them as open codes. These open codes were then compared and contrasted among each other (i.e. constant comparison) to identify major concepts from the data. The contrasting and comparing of the open codes were helpful to identify some pivotal concepts and they were named as axial codes. He further regrouped and condensed the axial codes into final categories as selective codes. While doing this, he used the multiple interview episodes as means of theoretical sampling and theoretical memoing. The earlier categories were analyzed and interpreted across the episodes. The interview notes and coding notes were used as memos to conceptualize and reconceptualize the key categories. The process of open coding, axial coding, selective coding and memoing with theoretical sampling formed a complex of an analytic tree. Major categories emerged as the key constructs from the analytic tree. The interview data were then interpreted at two layers of interpretive accounts. The first layer was participant narrative of his beliefs about teaching mathematics with technology. The second layer was the researcher's interpretation of participant's beliefs about teaching mathematics with technology. The researcher decentered the voice of the participant and his own voice through the layered interpretive accounts in relation to beliefs on the subject of study (Pierre, 2009).

The researcher formulated the results in terms of domain specific narratives of the participant and the categorical narratives of his interpretive accounts. The first kinds of narratives were constructed from the participant's expressions of beliefs during the task-based interview episodes. These narratives portray the voice of the participant in his the first person perspective. The purpose of these narratives is to portray the participant beliefs in a direct sense without making any interpretations. The readers may have opportunity to make their own interpretations of his beliefs despite the fact that the researcher presented his categorical interpretations. The categorical narratives portray what this researcher observed in the participant's belief narratives. This process helped this researcher to portray the participant's beliefs about teaching mathematics with technology in two layers – both the first person and the second person accounts.

Results and Discussion

This section presents the outcome of this study in two layers - the first layer as the *first person perspective* of participant's beliefs in his voice, and the second layer as researcher's interpretation of participant's beliefs as the *second person perspective*. Hence, this section includes two different genres – the first-degree narrative (of participant) and the second-degree narrative (of the researcher).

First Order Description and Analysis

The first order description and analysis consisted of integrating major belief statements in the form of participant narrative. Following belief narratives in the participant's voice represented his domain specific beliefs about teaching mathematics with technology.

Beliefs about teaching fraction multiplication with JavaBars: *I like fraction multiplication of non-complex fractions that do not have mixed fraction. Cuisenaire rods, I think, would be the easiest (thing) to begin with. From what we have said fraction as a part of a whole, we can select any one (any size of a rod) as a whole unit. Then it might be easy to begin. I think the most difficult thing that students might have in multiplying a half time two thirds and get a one third. Where does that one third come from? What does it mean? I think with Cuisenaire rod we can select a larger piece that can have three different pieces of thirds (He puts three one-thirds along the large piece as one unit) and we can take two of them as two thirds (He separates two of them away.).*

Use of manipulatives in the teaching of fraction multiplication may not be necessarily helpful all the time. It depends on what the students are used to in it. If they are used to in just using Cuisenaire rods and fraction bars or whatever, I think that at that point if you are able to design may be an applet or some kind of technology and establish your own fraction bars of any size. Sometimes a meter-stick would be helpful to show fraction multiplication. It can be helpful to see smaller units like hundredths. Also, picture models can accommodate small fractions. When drawing small fraction is difficult, maybe we can use Excel Spreadsheet.
I anticipate using JavaBars for teaching fractions and fraction multiplication together with other operations. Probably I will use those before I even use fraction bars. Because it's free, and you don't have to store it. It is

always there in the computer or Internet. I think that it is something I would use depending on if students just needed a quick brush up. I may have to draw just pictures rather than doing it on the computer. I like this program (JavaBars).

I agree that technological tools like JavaBars have a positive impact on conceptual understanding of fractions compared to just use of manipulatives. I think jumping from the procedure and trying to explain the concept I would show it to them. Even better, allowing them to use this (technology) and show it themselves. Um, that's a large conceptual gap. I think moving to the computer from fraction bars is just that much, especially with JavaBars. It's got the power to present so much more than the conceptual understanding. It has the ability to increase the conceptual understanding.

Beliefs about teaching functions and limits with Geometer's Sketchpad: *When I think of a teaching activity to introduce the concept of function in a class for the first time, I consider the grade level to introduce the function concept at first. It can be probably seventh grade. I guess I would start out with some story problems. Probably the first thing the students would attempt to make a table of values that would come from the problem. Then I would begin with an equation whose values would come from the table. Probably a good place to start a function is one that just uses patterns. You may be looking at arithmetically growing patterns. The students try to find the patterns in steps one, two and three, what's the fourth step and what's the fifth step? As an example, I would start with an arithmetic pattern where each term starts with one, three, five… And, I would ask them to find next two numbers. I probably would draw the pattern in terms of blocks and ask them to draw next block. And, the next step from there would be okay to make a table and ask how many boxes are there in each step. Also, I would build the structure like in the figure with manipulatives, base ten blocks I suppose.*

So far technology in teaching function and limit is concerned; I would use technology for bridging the concept of the use of concrete objects and manipulatives to technology. I would just recommend any types of technology that help in the construction of such patterns. I really like Excel Spreadsheet because you can ask students essentially working from a written table and all they got to do is to put the table in it and graph it straight away. They can use the Excel Spreadsheet to figure out the function from the graph.

I think GSP is definitely a good for teaching function and limit compared to other tools. At the very beginning probably I would start with a paper and graph and then move toward technology. Especially, what we do with drawing a line and what the slope represents. Um, for looking at linear functions at that moment I would encourage students prepare far better for the next two steps in mathematics is do it in the paper and after understanding what the slope of the function is they can move to the technology. Probably, I would introduce function as a distance, function as a velocity, etc. and go to graphing a quadratic function introducing acceleration and just preparing students with and talk about the rate of change, and talk about slope. We can show them pretty easily here (in GSP). The great thing about it is when students ask questions or we can ask them the questions: What happens when we increase the slope and rather than asking them to draw lines of equations as functions and see the nature of slope in a paper, they can jump in GSP. At the start of our discussion, I had no question in using Excel Spreadsheet. Just my personal comfort level in using Excel Spreadsheet was very high. However, from what we have seen today I think GSP has more benefit in teaching function and limit that really we cannot show in an Excel Spreadsheet.

I imagine that for functions specifically in algebra class, that the students may not get to full understanding of concepts of functions, especially in seventh or eighth grades when they are in algebra class. I may have to go through this entire process with the students one-on-one possibly in a class project if it needs. And, I am hoping that students have this behind them the concepts. But, I imagine that these are the some of the stuff that I need to teach. I think GSP is gonna be a great way. Maybe even if we are looking at polynomials then we can show the functions of polynomials in GSP and show how different things change. Looking at how slope changes we can start talking about the rate of change preparing students to move from algebra to calculus, and maybe doing a little bit in trigonometry.

When some students are not yet ready to learn function concepts because of their weak mathematics background, for me, the easiest way to do it is a class project with some sort of constructions that the class is working on. And, during that time I would ask students to trade off who is doing what and help each other. Or, I can speak to the class- okay you guys can work on this project. And, I can go to the students who need help and work with them run through the basic processes of functions and get to the point where they need to be in terms of conceptual understanding.

To me, this (i.e. GSP) is probably the best tool I have seen for teaching the concept of (function &) limit. Again the use of the software makes it far more intuitive and easy to understand situations than drawing on the board and looking at one tangent line. This makes more sense. For next semester, I think I will not get into limits and derivatives. What we did today would definitely benefit in my future teaching and probably I will try using these activities very similar way what we went through. I will help students make that jump from limit to derivative not just teaching limit.

Beliefs about teaching geometric transformations: *At the beginning of teaching geometric transformations, I think, I would begin with any of the transformations- reflection, rotation, translation, glide, and resize. Maybe, I would begin with similar figures that are either just at different spots or they are of different size at the same place. Probably, I would ask the students what is the difference between these two bottles. Maybe, I can place one of the bottles in front of a mirror and ask the students- What's the difference between these two (object and image)? I would start from there (this) before I even talk about how to construct them and discuss what a transformation means to them. Even before the beginning of geometric transformations, I would encourage (students) just looking different objects and you can pull out many things in the class and how those are related. You can even take a paper and ask them how the two pictures are related. The students may answer whether these pictures are related with a glide or a slide or any other. What else did you see today? These might be helpful to begin with.*

I agree that I can begin from transformations we have observed from daily life to transformations in mathematics. I would bridge the informal meaning of transformation to formal one beginning from what we did today starting from transformation in everyday life and then talking about geometric transformations as one of those applying with geometric properties of lines and planes and even three dimensions. With the three dimensional example, we can go back to the water bottles (He points to the nearby water bottles on the table.). When I anticipate my future teaching of geometric transformations probably I will use GSP. If I use any manipulatives, that could be pattern blocks. Probably I can use a geo-board too. I agree that a geo-board might be easy hands-on material for students to play with for a while to learn geometric transformations. You can probably move further to use of a graph-board. Then, maybe I can move toward using GSP. I have to deal with different comfort levels of students while using GSP to learn geometric transformations. When I think of advantage of using GSP over other materials, I think it adds in conceptual understanding of geometric transformations.

I think talking through and for this turning to GSP; I learned that it is a very powerful geometry-teaching tool. We also talked last time it is a powerful tool for the algebra classroom too. I can use it in trigonometry class. So I think that I would definitely use the GSP for class projects. It was helpful talking about how we go out teaching with these tools more to students who are quite yet not conceptual. That means I will continue using these things in my future teaching, not just in the teaching practicum.

Beliefs about teaching mathematics during practicum: *In terms of basic teaching, the first couple of weeks of the past semester, spring semester of 2012, were a real shock in terms of student preparedness, student motivation, and students turning in of homework. It amazed me how the students were doing and what they were getting by with. And, throughout the semester that challenged me. I don't know if there was necessarily silence out there at a bigger city than I was used to, it was a broader problem out in the city. I hope it doesn't exist in smaller towns yet. Throughout the semester I worked and I thought the students would turn in home works. I spent 2-3 days in a week throughout the semester when I sat down and talked to the students how the study goes, taking notes, and using the book as a tool. And, I asked them, "Why don't you do homework on time?" And, we talked about getting prepared for tough learners. I think that maybe the first time a kind of push all sides, and second time talking and listening to them a little bit. The third time when I talked to them was after the spring break. The last three weeks after the spring break were what I had expected in the second semester. I had students coming to the class prepared and students turning in home works. I started with the two weeks of teaching a little slow. Now it was back to what I expected. I think having a discussion with students and from the standpoint of myself being successful, self-taught, learned, and tried to equip students with what helps them getting on, what they need to see when they come to classes, not just math. I started feeling pretty good by the end of the semester.*

I didn't know how to use GSP a lot in terms of teaching and I ended up one of the biggest usage in the past month when I was teaching quadratics, quadratic equations, and solving functions. I just built the basic quadratic formula with a, b, and c as parameters (i.e., $ax^2+bx+c=0$). We went through ten minutes of each class. We began with- okay a is one and what kind of change do you see in b and c? What happens? Do it and show what happens. Maybe I could take a and b animated and see how c changes. It was nice to show things *going up and down. I think that was one of the biggest things, probably the biggest depth of learning for my algebra students for the entire year being able to see that every day. We started factoring a quadratic equation and graphed them. We were able to see two different lines in the graphs as the factors.*

Second Order Analysis and Interpretation

The researcher analyzed and interpreted the participant's beliefs about teaching mathematics with technology from his perspective as the second-degree narrative. The following categories emerged with analytical and interpretive function of RCGT process in this study.

Manipulatives and technological tools: The participant expressed his positive beliefs toward the use of manipulatives in teaching fractions, functions and limits, and geometric transformations. Particularly, he revealed his beliefs that fraction such as 2/3 could be represented by Cuisenaire rods, fraction strips, and even a meter scale. For him, manipulation of fraction operations (e.g., multiplications) with Cuisenaire rods seemed a plausible way to clarify the meaning of such operations in a visual way. He also added that the nature of fraction multiplication determines which manipulative might be more useful in developing an understanding of the operation. Sometimes manipulatives might not be the easiest way to develop an abstract sense of complex fraction multiplication. Together with other manipulatives such as pattern blocks, base ten blocks and dices, he claimed that pictures could be helpful to use as models. These different kinds of materials are suitable in making new fraction patterns and comparing different values. He seemed to believe that moving from manipulatives to JavaBars to teach fraction operations could help students better understand fraction multiplications. After the interaction on fraction multiplication using JavaBars, he expressed that he anticipated using the fraction operations with JavaBars before even using any manipulatives.

In relation to teaching of function, he seemed to believe the story problem (with two variables) as a good start. A function could be discussed in terms of the nature of the slope when the function is plotted on a graph. He expressed his positive belief that Excel Spreadsheet as a best tool to teach a function. He preferred the use of tables showing relationship of two variables and lines in Excel Spreadsheet to teach students the nature of functions. However, after the interaction with the use of GSP he appeared to find it more useful to teach function and limit than by using an Excel Spreadsheet. He mentioned that moving back and forth between technology and manipulatives might be more helpful than just using one of them. He claimed that both manipulatives and technological tools were necessary for teaching geometric transformations. However, he was not much sure about using the geo-board in teaching geometric transformations. Only one material or technology might not be helpful in teaching geometric transformations. Sometimes worksheet, for him, seemed easier than GSP and other times GSP seemed far better.

The effectiveness of materials, tools, and technology in teaching fractions, functions, limits, geometric transformations, and statistical data may depend on how a teacher engages students in learning through the use of different teaching resources. Manipulatives provide a context for teaching mathematics for conceptual understanding. Teaching mathematics with an appropriate use of manipulatives may help in modeling situations, making the connection between abstraction of mathematics and the real or the physical world (Heddens, 1986 as cited in Dahl 2011). Dahl (2011) claims that the use of manipulatives may make children's understanding of mathematics long lasting through understanding of concepts together with procedures. However, use of manipulatives may increase children's reliance on such material to make sense of mathematical concepts. They have to think at an abstract level and solve problems using algorithms and models. Manipulatives may not help in abstract mathematical reasoning. Then, technology can be of great support in modeling abstract mathematical problems. Many scholars (e.g., Leathami 2002; Rubin, 1999) believe that the technology plays a very influential role in teaching and learning mathematics in a meaningful way. The effectiveness of technology in learning mathematics and understanding concepts and solve abstract problems depends on the pedagogical philosophy (Rubin, 1999). While using GSP in exploring the features of a quadratic function, it is imperative to engage students in dynamic interaction among the three coefficients a, b, and c (Rubin, 1999). Some teachers seem to be reluctant in using technology in the classroom teaching and learning of mathematics because they think that excessive use of technological tools may diminish the conceptual understanding and procedural fluency (Schmidt & Challahan, 1992). The participant emphasized the interface between formal and informal mathematics through the use of manipulatives and technology.

Balancing of teaching strategies: The participant expressed his beliefs about teaching fraction multiplication beginning with simple fractions and whole numbers. Then, he pointed to a move from multiplication of two simple fractions, one simple fraction and a compound fraction (mixed fraction), and two mixed fractions. He preferred using Cuisenaire rods as a starting activity while teaching fraction multiplications. He shared with the researcher that JavaBars could be used either at the beginning or after use of other manipulatives. Since it is a free downloadable application, he expressed his thought of using it even before the use of any manipulatives. It seems that what to use in teaching of mathematics as a resource may depend on how it is used, for what purpose it is used, and for how long. For him, it was always a balancing of teaching strategy.

While teaching functions and limits, he pointed to step size for different slope as an important thing to consider, as an important variable. Different step size meant, to him, different values of change in x for certain values of y, different values of y for certain x, and different values of x for different values of y. That means he positioned his strategy that a starting point in using GSP could be from a rate of change. He preferred an initial activity with paper and pencil and manipulatives while working with most of the mathematical problems including fraction multiplications and functions. He then suggested the use of technology after students have some basic skills. His beliefs on 'asking questions' while teaching about functions and slopes using technology showed that he preferred working with students and interacting with them. At this point, his beliefs seemed to be positive toward integrated teaching by bringing students of different ability groups together and using different manipulatives and technological tools. His anticipation of balancing of teaching with different approaches focused on class projects with construction activities and problem solving in groups with trading off among students and helping each other. He claimed that he preferred engaging the class in tasks and situations and helping a group of students that is struggling with the problem. While working with such students, he anticipates beginning from basics and grouping them in different ability groups with different tasks. He focused on collaborative learning from each other in the class by grouping of students who need additional help through expert groups within the same class. At this point, his beliefs seem to be aligned to differentiated instruction.
Concerning the order of teaching geometric transformations, he did not have any specific choice in such order to teach reflection, rotation, or translation. However, he clarified that he would pull out ideas from a comparison of two similar objects and their position to begin instruction of geometric transformation. He seemed to prefer differentiated instruction through discussion sessions within different ability groups. He claimed that he spent some time to talk to the students about their problems at the beginning of his teaching. This talk helped him address their problems in mathematics. After addressing students' problems, he felt comfortable to engage all students in group activities. He also seemed able to engage all the students in learning functions with the use of GSP in the school computer laboratory (during his practice teaching). For him, working with students on any problem was a balancing act between moving to and from what they already know and what they do not know.

The participant articulated a need for bridging students' prior knowledge to new areas of teaching and learning in mathematics. He emphasized different ways of bridging procedures and concepts through the use of manipulatives, diagrams, and technological tools. For example, a data-table could be useful to introduce a function. He claimed that the table could be linked with an equation and a pattern as a way to start teaching a function. For him, the activity of finding unknowns from a pattern could help one to generalize a particular concept of a function. Also, he related different examples of functions from everyday life to standard definition. While doing this, he focused on the relation of a concept of fraction, function, limit, geometric transformation and data with understanding procedures and the concepts. He expressed his strong beliefs in moving from 'algebra to pre-calculus' by the use of GSP while discussing functions and limits. He purported to begin the teaching of geometric transformation from an informal pair of objects, as one is a replica of other as object and image and differentiate them with mathematical relations and reasoning. He emphasized an order of teaching function and transformation by building foundational knowledge at first. He claimed that the use of informal transformation might help in understanding some basic features even before teaching of formal geometric transformations. He suggested a transition from manipulative, graphs, and pictures to technology (e.g., GSP); however, he seemed flexible in this order. His notion of bridging through everyday examples of various mathematical phenomena seemed extended further to an integrated and contextual teaching of mathematics.

Integrated approach has the potential for enhancing the scope and power of mathematics teaching and learning by bridging unknown to known and vice-versa. The debate still is ongoing about how to integrate. One idea of integration through unifying concepts (a concept that cuts across most branches of mathematics) and integration by merging areas of mathematics might be a good idea, but we have to pay attention toward optimal integration. The optimal integration of curriculum and pedagogy incorporates themes and builds an integrated curriculum around those themes. There are five essential roots for integration- quantitative literacy, mathematics preparation, mental development, technology, and culture (Kennedy, 2003). Other roots for integration: core curriculum root, competition root, social and political root, and cultural root are also important players in shaping school mathematics curriculum and its integration across other disciplines. Another viable approach for integration comes from Burns and Sattes's (1995) evolutionary stages of curriculum integration in terms of content, instruction, assessment, and classroom culture. Berlin (2003) emphasizes integration that should feature real-world or mathematical problem situations that meant to be engaging, interesting, appealing, and relevant to students in their targeted grade levels. However, integration should not be just putting things from different fields together, but the process of seeking connections should be a generative one – greater the number of opportunities that students have to make connections, the more likely they will be able to search for future connections themselves (Clement & Sowder, 2003).

Balancing of teaching strategy by providing all students equal learning opportunities is a great challenge to a mathematics teacher. A teacher should have knowledge of integrating technology, manipulatives, and other resources in the classroom teaching and learning environment. Integration of technology in mathematics class is a complicated one due to cost, time, and an appropriate use. "Teaching with technology is complicated further considering the challenges newer technologies present to teachers" (Koehler & Mishra, 2008). A teacher has to balance his or her teaching with technology considering various factors – nature, applicability, stability, visual and perceptual effects, both cognitive and social affordability, and flexibility. Mishra and Koehler (2006) and Koehler and Mishra (2008) introduced the framework of technological pedagogical content knowledge (TPACK) that integrates technological pedagogical knowledge, technological content knowledge, and pedagogical content knowledge. This framework could be utilized in the balancing of teaching strategies. However, having the knowledge of pedagogical knowledge, technological content knowledge, and pedagogical content knowledge does not guarantee that balance. It is more of an art of a teacher to bring the knowledge into action and maintain a balance between integration and differentiation of his or her instruction.

Working with technology: The participant's view in relation to use of technological tools in teaching mathematics seemed developmental from less use to more use and procedural to conceptual. In the beginning, he seemed not aware of any specific technological tools for teaching fraction multiplication. When he was introduced with the use of JavaBars, he mentioned that he would use this tool for teaching different fraction operations even before the use of other manipulatives. He claimed that JavaBars is a free applet and also it is easily available for download from the Internet. For him, it was an easy-to-operate tool while teaching fraction operations in general and fraction multiplication in particular. His first preference of Excel Spreadsheet for teaching function showed a naïve understanding of the broader aspect of the application of technological tools in such an abstract mathematical concept. He seemed to be aware of using Excel Spreadsheet in teaching function through tables and graphs. He revealed his beliefs about technological tools saying that any kind of technology that could help in the construction of a pattern would be good to use in teaching mathematics. His preference to Excel Spreadsheet for graph and table relation and the nature of a function from the graph explicated his prior beliefs about use of technological tools in teaching a function. When the researcher introduced the use of GSP for the concept of slope and reasoning about function with a slope in GSP, this discussion had a positive influence in the participant's idiosyncratic beliefs about the use of the tool for teaching a function. His initial belief toward Excel Spreadsheet as a most viable technological tool for teaching and learning functions changed with his experiences with GSP. Although he seemed confident in using Excel Spreadsheet for teaching a function, after the interaction between the researcher and the participant about using GSP for teaching function and some activities of plotting function graphs and observing the subtle changes of Δx and Δy in linear and non-linear functions, the participant claimed that the GSP was more useful to teach functions than the Excel Spreadsheet. He had a prior background of working with GSP; nonetheless, he seemed not having an understanding of using GSP for teaching function and limit.

The participant claimed that GSP is a powerful teaching tool that can be used not only in teaching geometry, but also in pre-calculus and trigonometry. For him, the greatest benefit to students by using GSP was conceptual learning. He also anticipated using the tool in his future teaching beyond practicum. To him, the use of GSP in teaching limit and derivative was more intuitive than without using it. He shared with the researcher that GSP helped him in making sense of limit and meaning of a derivative. His anticipation of using GSP and other tools in his future teaching related to his sense of ownership to the technology. His beliefs seemed to be consistent with earlier research findings.

Leatham (2002, 2007) highlighted the degree of ownership of technological tools in teaching mathematics by the four preservice mathematics teachers. He discussed the phases – entry, adoption, adaptation, appropriation, and invention – that his research participants moved through while they were participants to his study. Not all of the participants had these ownership levels. He found only one out of the four participants reached the level of invention with the use of computers. Entry phase was the time when they were first introduced to the technology in the context of teaching mathematics. Adoption phase is the one in which the preservice teachers begin to use technology themselves and see how it functions in the context of mathematics. Adaptation is the phase in which the preservice teachers make connections between technological tools and different mathematical contents and teaching. Next phase, appropriation is the phase in which the preservice teachers begin to make an appropriate choice of technological tools that fits with the teaching of content. In this phase, they gain more independence in the selection and use of technological tools. In the last phase, invention, they begin to use technological tools to learn mathematics. They try to explore new technologies that are suitable for different content teaching. They develop their own model to use the technological tools in teaching mathematics (Leatham, 2002). In this study, the participant seemed to be confident in the use of JavaBars, GSP, and Excel Spreadsheet in teaching fractions, functions and limits, geometric transformations, and statistical data. He appeared to be in the appropriation phase at the end of five task-based interviews and later when he used GSP in his classroom teaching of a quadratic function, he seemed to be in the invention phase. Hence, the task-based interviews provided the participant an opportunity to observe his idiosyncratic beliefs about teaching mathematics with technology and change those beliefs toward positive direction favoring toward constructivist approach of technology integration in mathematics classes.

Technology for mathematical concepts and meanings: The participant preferred some hands-on activities due to easiness to begin fraction multiplication or function or geometric transformation. For him, multiplication of a fraction by a whole number was an easy step. He appeared to think that many students perform fraction multiplication without knowing the meaning of the operation. There might be different meanings of multiplication. Multiplication of two whole numbers can produce a result that is greater than or equal to the larger of them. Multiplication of a fraction with a whole number or fraction by another fraction might be different depending on the nature of the factions involved in the process. The meaning of fraction multiplication depends on context or nature of numbers involved in the process. To clarify the meaning and operation of fraction multiplication, he preferred using Cuisenaire rods and fraction strips at the beginning. However, he seemed to consider that fraction multiplication is easy to understand the process and concept with JavaBars. A major problem in understanding a fraction operation is related to understanding of what is constructed and why it is constructed the way it is constructed either using manipulative or JavaBars. Here, construction is associated with the use of fraction multiplication as an operator to get a result.

In relation to teaching a function, he seemed to believe that it was easy to see the slope of a line in the XY-plane. He preferred introducing slopes of simple linear functions for conceptual understanding. He then claimed that GSP is helpful in making sense of a, b and c in a linear function $ax+by=c$ in the graph and a quadratic function $ax^2+bx+c = 0$. He appeared to think that students could have a hard time in understanding the concept of function in the beginning, but it might be easy with the use of GSP. One could begin with the definition of function and examples of different functions of everyday life. Then visualization of a function with the use of GSP may make a better sense than just working with algebraic functions by making implicit operations explicit with visualization. The participant also claimed that a geometric transformation could be considered as a part of the function. The reflection, rotation, translation, and dilation could be considered as different kinds of operators as functions. These operators are visible with technology such as GSP that helps in making sense of those function operations. The participant claimed that students with visual learning might have a deeper understanding of a function or limit with the use of GSP. He also accepted that procedures and concepts could be well bridged through the use of technology. Sometimes there can be a large conceptual gap between what they already learned and what they are going to learn. In such a case, for the participant, a teacher could introduce simple examples of slope and the concept of slope in a line. Then he or she could use GSP for the exploration of slope.

Leatham (2002, 2007) discussed the different roles of technological tools – motivational, procedural, and conceptual roles. Technological tools can motivate both the teacher and students to orient into the exploration and discussion. The tools such as GSP, JavaBars, and Excel Spreadsheet may enhance procedural computations, constructions, and problem solving. The greatest thing that technological tools can serve to the teaching and learning is conceptual understanding. The conceptual learning of mathematics may be achieved with visualization, demonstration, illustration, exploration, connection, and extension (Leatham, 2002 & 2007). While using GSP in teaching and learning of functions and limits, the idea of instantaneous change can be extended to the idea of derivative. The participant's beliefs seemed to be influenced by the multiple roles of GSP in relation to conceptual understanding of functions and limits and geometric transformations by making implicit mathematical operations explicit.

Transformative teaching activities: The participant seemed to believe that he could establish his own fraction bar for teaching fraction multiplication. This way he wouldn't have to rely on supplies from outside, and he might get rid of dependence on school administration to acquire teaching materials. He claimed that he could make fraction strips by paper cutting or drawing. Also, he appeared to think of constructing different kinds of charts and diagrams to model different fraction multiplications. These materials seemed to supplement the technological tools such as JavaBars. However, he anticipated using JavaBars in teaching fraction operations even before the use of manipulatives. He anticipated teaching a function at different levels of abstraction depending on the grade level.

He considered that the best place to start teaching function is to discuss it with a pattern and do some construction. The pattern of numbers and construction of different charts was valuable lessons during his teaching. He seemed to consider that the use of box and whisker plot was helpful to see many things at a time for interpretation of data. This kind of chart was helpful in conceptual learning of central tendency and dispersion at a time.

He appeared to believe that his teaching at the beginning and by the end of practicum was different because of an increased level of confidence, group work with students, and overall progress made by the students by the end of his teaching practice. He anticipated focusing on conceptual learning with a vision toward improved learning through students' active participation. He seemed very positive about the use of technological tools (e.g., GSP, JavaBars, Excel Spreadsheets and others) in future teaching of fraction, function and limit, geometric transformation, and data. His transition from limit to derivative through the use of the GSP was quite encouraging and one of the most remarkable changes in his idiosyncratic beliefs about use of technology for teaching mathematics. He anticipated creating a better teaching and learning environment. For this process, he wished to talk to the students and try to understand where they got problems. He interacted with the students in relation to their role in classroom and assignments. He mentioned that the first talk was not much successful, but the second talk was a little more successful than the first one. The third talk turned into a very successful in the sense that he was able to see the students' following his advice. He felt very contented with the transformation of students' attitudes in the class within the short period during his practice teaching in a school. The transformation of students gave him a sense of accomplishment. This change portrays that teaching as a non-linear process of dealing with content and context bringing the students into pedagogy of caring (Hackenberg, 2010).

The transformative teaching and learning of mathematics with technology can be achieved through understanding of teacher responsibility, student responsibility, and the goal of teaching and learning of mathematics with technology (Leatham, 2002). The shift in teacher's roles and responsibilities together with students' roles and responsibilities leads to a transformation. This shift should be a positive one, and it should be linked to a greater motivation, enthusiasm, and change of beliefs and actions. The preservice teachers' adoption of the technological tools as early, middle, late, interested adopters and non-adopters may influence the degree of transformation in their beliefs and practices of using the tools in their future teaching (Keren-Kolb, 2010). The participant's anticipated adoption of the technological tools (i.e. JavaBars, GSP and Excel Spreadsheet) in teaching certain contents seems to be at the level of interested adopter. He thinks that he will use technological tools in his future teaching of mathematics to transform his teaching and students' learning focusing both procedures and concepts.

Some issues and challenges: The participant considered that there were misconceptions about multiplication of fractions. General misconception may arise from students' beliefs that multiplication makes the product bigger than the multiplicands. In the case of fraction multiplication, the case might not be the same. When a whole number is multiplied by a fraction less than one, then the product is less than the number. Likewise, he seemed to believe that there might be a misconception about the use of technology. This misconception might arise from the use of technology as an alternative to manipulatives. However, he considered that a technology should not replace any manipulatives and other hands-on activities. They should complement each other. Diagnosing a misconception and correcting such misconception about fraction operations, functions and limits, transformations, and data were important considerations for teaching mathematics. He also seemed to believe that there might be some challenges associated with the use of GSP at the beginning because students might not be familiar with computer or functions of the GSP tools. Such challenges depended on contexts in the class and students' prior experiences with technology. Sometimes students might engage in off-task activities in computer while solving a problem using the GSP. Students might not pay attention to the constructive use of GSP, but they might spend time on other stuffs beyond what they were expected to do. This issue might be a problem according to the participant.

Some students even might struggle with the meaning of the function and limit. This kind of problem might arise due to the past experiences with procedural learning. The participant claimed that a teacher should be able to identify these problems in the class. These problems might be related to lack of student preparedness, lack of motivation, and lack of homework turning in on time. He faced these problems at the beginning of his teaching during practicum, which greatly surprised him when students did not work on assignments. For him, these were some of the great challenges for the preservice teachers when they go for practice teaching. He thought that there were more such problems in bigger towns than in smaller cities. More problems in the bigger cities might be due to wider cultural variations and family background of students. At the beginning of his teaching, this kind experience was a shock for him. This kind of issue was not just specific to his class. Those problems might be related to many factors including access to technology in schools and cost associated with it in terms of time and resources and student motivation in education as a whole, not just mathematics.

One of the greatest challenges to present day teaching and learning of mathematics with technology is underutilization of available technology in schools (Cuban, 2002). The participant's frustration at the beginning of teaching grows further with the problem of underutilization of the computer lab in the school. He requested the school administration for installing GSP on the computers, but it was not materialized until he bought GSP package himself and installed it in some of the computers in the lab and his classroom. Another challenge is that many preservice and inservice mathematics teachers use everyday technology for the personal use, but they do not use any of them for classroom teaching (Keren-Kolb, 2010). This problem also indicates toward the challenge of teacher resistance toward the use of technological tools in teaching mathematics. "The one challenge that is not easily addressed by the introduction of everyday *and other* technologies, however, is the most difficult: educators' reluctance to meaningfully integrate new technology tools into school learning" (Keren-Kolb, 2010, p. 12, this author added the italicized part). Although the participant did not explicitly indicate toward these challenges, his implicit beliefs about issues of technological tools and challenges of using those tools in the teaching of mathematics revealed some of these elements. More of his beliefs were related to challenges of enhancing conceptual learning, proper use of technology, and students' attitudinal concerns.

Implications of the study

Some implications of this study were synthesized from the categorical findings and discussions - interface between formal and informal mathematics, trade-off between differentiated and integrated teaching, forming beliefs with learning to use tools, making implicit operations explicit, and a nonlinear process of understanding pedagogy through practice.

Interface between formal and informal mathematics: The first category on 'manipulatives and technological tools**'** show an interface between formal and informal mathematics. Technological tools (i.e. digital tools such as JavaBars, GSP and Excel Spreadsheet) could provide a context to bridge formal and informal nature of mathematics teaching and learning. Technology integration in mathematics education may develop positive beliefs toward the use of different digital tools together with manipulatives in teaching fractions, functions and limits, and geometric transformations. Some mathematical concepts could be taught with Cuisenaire rods, fraction strips, a meter scale, pattern blocks, base ten blocks and dices and other concepts could be instructed with JavaBars, Excel Spreadsheet, and GSP. In fact, any mathematics concept could be taught using both kinds of materials (manipulatives and digital tools) because these tools function as an interface between formal and informal mathematics.

Interface between differentiated and integrated teaching: The second categorical finding on 'balancing of teaching strategies' points to possible trade-off between differentiated and integrated teaching of mathematics. Differentiated teaching is related to treating different students differently based on their ability to learn and make progress on their own pace. The teacher may form a group of students based on their learning ability in the class. Then he or she may provide them different tasks or problems to solve. Some may use manipulatives, and others may use technological tools in the process of learning any mathematical concept. The teacher can apply different approaches focusing on class projects with construction activities and problem solving in groups with trading off among students and helping each other. He or she can engage the class in tasks and situations and help a group of students that is struggling with the problem. Integrated teaching can connect students' prior knowledge to new areas of teaching and learning in mathematics. He or she may emphasize different ways of bridging procedures and concepts through the use of manipulatives, diagrams, and technological tools. The process of integration may go through unifying concepts by merging areas of mathematics with integration of curriculum and pedagogy. Hence, use of technological tools (e.g., JavaBars, Excel Spreadsheet and GSP) may provide an interface between differentiated and integrated teaching of mathematics.

Forming beliefs with learning to use tools: The third categorical finding 'working with technology' relates to forming beliefs with learning to use tools. This study showed that when the participant learned to use the tools (e.g., JavaBars and GSP) for the first time, the experience formed idiosyncratic beliefs about the integration of tools in teaching mathematics. The preservice teachers may have limited experience of using technological tools in their prior mathematics classes. When they come to experience new tools and techniques integrated with mathematics concepts that themselves struggled through before the use of technological tools, a new experience challenges their earlier conceptual understanding and subsequently may challenge their existing beliefs. Hence, positive experience of learning to use new technological tools integrated with mathematics may help the preservice or inservice teachers to form new beliefs or modify their existing beliefs about teaching mathematics. Power of technological tools to make implicit meanings explicit may empower teachers and help them in forming new beliefs.

Making implicit operations explicit: The fifth categorical finding ‘technology for mathematical concepts and meanings’ is associated with making implicit mathematical operations explicit. The use of JavaBars for fraction operations, GSP for operations of geometric transformations and functions, and Spreadsheet in the operation of central tendencies and dispersions provided the participant and the researcher to make the inherent mathematical operations explicit through constructions of models. Mathematical processes related to fraction multiplications, limits and functions, and geometric transformations had operator roles that were invisible (implicit), but the use of the digital tools (e.g. JavaBars and GSP) revealed what was happening in those operations by making them explicit. These were the ‘aha’ moments for both the researcher and the participant when they were able to experience the essences of those operations within the dynamic environment of the technological tools. Hence, making implicit operations explicit could have a great significance of technology integration in mathematics teaching.

A nonlinear process of understanding pedagogy through practice: The sixth and the seventh categorical findings ‘transformative teaching activities and some issues and challenges’ are associated with a nonlinear process of understanding pedagogy through practice. Preservice mathematics teachers have a limited opportunity for practical pedagogy of mathematics when they are in content and method courses. The content courses in mathematics provide them knowledge of mathematical contents and the courses of teaching mathematics provide them knowledge of mathematical pedagogy, technology, and theory. Their knowledge of content and pedagogy through these courses seem to be ineffective in forming and changing their beliefs. However, when they go for practice teaching, they have an opportunity to implement their knowledge of content, pedagogy, and technology. The field experience may provide them real time experience of classroom complexities dealing with students of different interest, nature, and motivation. Sometimes, their expectations do not meet in the classes, and they may go through situations of frustrations and dissatisfaction from their teaching. Their self-reflection on teaching learning process in the complexities may help them understand the non-linearity of theory and practice of using technological tools. They may understand that building a positive and a caring relationship is the first thing before even use of any sophisticated tools for teaching mathematics.

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