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Integration of Digital Technology and Innovative Strategies for Learning and Teaching Large Classes: A Calculus Case Study

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Abstract

Successful science and engineering programs require proficiency and dynamics in mathematics classes to enhance the learning of complex subject matter with a sufficient amount of practical problem solving. Improving student performance and retention in mathematics classes requires inventive approaches. At the University of Central Florida (UCF) the Department of Mathematics developed an innovative teaching method that incorporated computers and MyLabsPlus software with application sessions in large calculus classes. Introduction of new technology, in-class problem solving and application (or discussion) sessions are important factors in the enhancement of students' deep understanding of mathematics. We will detail various components of the course (daily online homework sets, online skills tests, application sessions and projects with Teaching Assistants, in-class tests, and comprehensive final exam) and discuss how we obtained optimal results enhancing the traditional teaching techniques. The instructional delivery involves group work combined with the use of computer technology to analyze the relationship between the physical problems and the mathematical models. We hope that the details of our experiences and the lessons we learned along the way will be helpful to others who are struggling with the same issues. Furthermore, this technique can be used to teach large classes not only in Science, Technology, Engineering and Mathematics (STEM) disciplines but also in social sciences.

Key words: STEM, calculus, technology, redesign, large class, case study, teaching and learning

Introduction

Calculus has always been a central part of the American baccalaureate degree program in science and engineering. As disciplinary specialization has grown throughout the twentieth century, calculus has become the course that assures all students, regardless of specialization or intended career, become acquainted with history, science, engineering and mathematics. It is also a major vehicle for cultivating capacities such as communication, critical thinking, quantitative reasoning, and integration of knowledge (Ratcliff, Johnson, La Nasa, and Gaff, 2001).

A strong foundation in the knowledge and application of calculus provides a distinct advantage in the learning of engineering concepts. Learning and retaining complex subject matter in science and engineering greatly depends on students' established mathematical proficiency. Effectively teaching calculus establishes a strong base of knowledge from which all future learning of these concepts is supported. The University of Central Florida has embarked upon an innovative approach for the effective teaching of mathematics. Several dynamic techniques have been initiated to deliver a result-centered learning environment.

The development of new digital technologies must have a positive impact in the learning process of Mathematics, but the speed that is characteristic of this development limits the time needed to understand the importance of these resources and their inclusion in the courses. On the other hand, a traditional curriculum, the standard in many classrooms, actively resists questioning and creates difficulties in the establishment of defined criteria that can guide us into making allies out of technologies currently available.

When relating technology to the Calculus learning process, the result undoubtedly steers toward graphing software, be it for computers or calculators. Nevertheless, the integration of these well-known technological resources should not be taken as a guarantee of a better learning process. In our judgment, innovation can only come when the software is implemented correctly and thoughtfully in the curriculum, and when it brings a significant, notable improvement in the teaching and learning process. In this work, we provide some issues to consider when pondering the impact that digital technologies can have when introduced in a visual learning

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process for Calculus. For this, we have two important factors to consider: the characteristics of the technological environment and the design of didactic activities centered on harnessing these characteristics.

The aim of this work is to describe a didactic experience that uses a kind of software created specifically for the improvement of the mathematics learning process. We want to share the design of a sequence of activities carried out in the Calculus course with the help of said software. It is part of a broad work of educational research that we have been practicing in our institution in order to give elements of discussion about the opportunity to transform curriculum by means of taking advantage of the new technologies that offer great expectation about the way Mathematics could be learned. The textbook collection edited by Pearson, provides an approach to Calculus where the desired interaction promotes the development of visualization, modeling, and flexibility between the different mathematical representations. In order to do this, the integration of different specialized mathematical technologies, like graphing software and spreadsheets, has become a key element. The didactic design that textbooks promote, provides a new structure of mathematical knowledge that presents an integration of the concepts of derivative and integral from the very beginning of the Calculus learning process. It uses technology to favor the interaction a student can have with a main problem: The prediction of values of a magnitude that is changing.

There is a current trend among institutions of higher education to attempt to do more with less without sacrificing the quality of education. As a result, it is not uncommon for freshmen and sophomore classes to be taught in a large class format. Educators and students are worried about the quality of teaching and high DFW rates which affect time to graduation and retention rates. Hence, at UCF, we came up with an innovative teaching program for our large calculus classes with an emphasis placed on understanding and retaining the calculus skills. Furthermore, an emphasis is placed on applying these skills when addressing real world problems we come across within our personal and professional lives. This teaching method can be modified appropriately and applied to science, engineering and humanity classes.

Redesigned Course Structure

In the redesigned Calculus course students spent three hours per week in a large lecture class of 180 students, complemented by two hours per week of required small group class meetings facilitated by their GTA mentors and peer tutors. There were at least four hours per week of optional help sessions.

Part of the planned redesign was a change in the mix of instructional staff teaching the course. Table 1 provides the mix of instructional personnel teaching the sections pre-project as well as the redesign project mix. Pre-project sections were consistently about 49 students per section. For the redesign project, there were up to 49 students in the traditional sections and up to180 students in the redesigned sections.

Table 1. Mix of instructional personnel teaching the sections

	Adjunct	GTA	Visitor	Instructor or Lecturer	Tenure or Tenure Track
Pre-Project: Percentage of the 65 sections taught by each type of instructor (fall 2008-spring 2010)	57%	12%	2%	8%	22%
Project Percentage: Percentage of the 1320 students taught by each type of instructor (fall 2010 and spring 2011)	22%	18%	3%	10%	47%

As shown in the table, the vast majority of Calculus I sections were previously taught by non-permanent faculty which was problematic as it often resulted in course drift and variability in quality across sections. With this project, we were able to reduce the percentage of non-permanent faculty teaching the course which resulted in a more consistent, better quality instruction. As the Calculus I course affects every course in which Calculus I is a prerequisite, including the remaining courses in the calculus sequence as well as many physics and engineering courses, the overall positive effect will continue beyond the semester the student is enrolled in Calculus I. The comparison of type of instructor is provided for the first year of the redesign with the percentage of adjuncts, GTAs, and Visitors decreasing in subsequent terms. Currently, with the exception of sections taught for special programs such as our honors program, all Calculus I sections are taught by either tenure or tenure track faculty or permanent instructors in the redesigned format with an increase in the size of the large lecture of up to 450 students compared to the project upper limit of 180 students.

Newly Introduced Course Requirements

Once the decision to redesign our calculus course was made, a change in the course requirements was necessary. The pedagogy of engaging students during class, requiring them to be active learners with their classmates, and the provision of immediate feedback were top priorities of the redesign. The following four components comprised the course requirements:

Online Homework

Graded online homework, which utilized the MyLabsPlus software packaged with the textbook were introduced into the redesigned course. As these assignments were completed online, students were expected to have access to a computer or they could use a computer in one of the computer labs on the main campus. Typically, there was an online homework assignment for each section in the textbook. These assignments included about seven to ten questions selected from the respective section in the textbook. When multiple attempts were allowed on a question, the last submitted answer was used for grading purposes. Online quizzes were assigned as well.

Online Skills Tests

During the semester, each student completed four online skill tests which were proctored in our Mathematical Assistance and Learning Lab. Students scheduled testing appointments for 75 minutes however the actual test time was limited to 50 minutes. Test settings were possible within the online system to provide eligible students electing to take their tests with extended time accommodations. The skills tests were designed to include skill and drill type questions as opposed to conceptual based questions. The software used for the online homework was ideal for testing the students on these types of questions which were graded without the provision of partial credit. Immediate feedback was provided for the student as they were able to view their grade as soon as they submitted their completed skills test (see the appendix for a sample skills test).

Application Sessions and Projects

During the semester, each student was scheduled to attend one application session each week. While in their application sessions, students worked in small groups to explore and discuss course content and gain experience working as a team while solving Calculus based application problems. Graduate teaching assistants answered student questions and completed examples of applied problems. The GTA mentors monitored student progress throughout the semester and intervened with an action plan to help them succeed in the course when it was deemed appropriate. Also, some applied problems arising in science and engineering were assigned as project problems for credit (see the appendix for a sample project).

Tests and Final Exam

There were three (paper and pencil) tests throughout the semester and a comprehensive final exam. These tests were graded using a rubric (see the appendix for a sample test and final exam).

Grading Policy

The following five components were used in grading:

(i) Online Homework: 10%
(ii) Participation in Application sessions: 10%
(iii) Skill tests (4 tests in total): 20 %
(iv) In class tests (3 tests in total): 36%
(v) Final exam: 24 %

The participation component was calculated based on the successful completion of assigned projects, as well as attendance and participation in the large lecture classes, and GTA led applications sessions.

A typical grading scale on the 10% marks was used for the course. At UCF, there are certain courses which utilize a No Credit (NC) grade. No course credit is given for an "NC" grade, nor will it satisfy any requirements or subsequent courses' prerequisites. However the student's UCF grade point average is not penalized for the "NC". As calculus is one of these courses, the grade of D was not given in the course. A student that did not successfully complete the course earned either an NC or F.

Use of Technology

Technology was used in the large lectures, application sessions, formative and summative assessments, and outside of class assignments. Communication has been shown to have an important role in learning and understanding mathematics (Knuth & Peressini, 2001). This was a motivating factor in the decision to redesign our calculus course to provide immediate feedback to students whenever possible. Given the large class size, technology was an essential tool used by faculty and GTAs to provide immediate feedback to students.

Faculty used PowerPoint slides, a document camera, interactive figures, and a classroom response system (CRS) commonly referred to as clickers in the large lectures. PowerPoint slides provided basic content for the course but given concerns of attention span, we felt that built in breaks during the lecture were necessary. A change in presentation format from PowerPoint slides to the document camera provided the most subtle type of break. The use of interactive figures during the large lecture class provided another type of break in addition to helping the student with a visual learning process. Asking a clicker question resulted in the student transitioning from a passive learning environment to an active environment.

The inclusion of clicker questions was the preferred type of break for our students. This was not a surprise to us as there are multiple studies which have shown that clickers in the classroom improve student awareness of their misunderstandings, increase their sense of involvement in the class, and make it more enjoyable (Nicol & Boyle, Using Classroom Communication Systems to support interaction and discussion in large class settings, 2003; Elliot, 2003; Caldwell, 2007). Researching ways of improving conceptual learning in the sciences, the Galileo Project Group at Harvard found that breaking up lecture inputs with concept questions and in-class discussions was beneficial (Crouch & Mazur, 2001). The Physics Education Research Group (PERG) at the University of Massachusetts found similar findings in support of the use of clickers to improve conceptual learning (Dufresne, Gerace, Leonard, Mestre, & Wenk, 1996). The benefits of using clickers in the classroom include instant feedback on the students' understanding or misconception for the instructor, instant feedback to the student regarding how they compare with the rest of the class, and students who are actively participating in the course. Multiple studies have shown that students believe the use of clickers helps them understand class expectations, makes them more aware of their misunderstandings, and makes them more involved in classes that use clickers when compared to traditional classes (Nicol & Boyle, 2003, Elliot, 2003, Bode, Drane, Kolikant, & Schuller, 2009, Dufresne, Gerace, Leonard, Mestre, & Wenk, 1996) One study reported that students were twice as likely to work on a problem presented during class if answers were submitted by a clicker or some other group response system than by show of hands and if credit was given for answering correctly, the likelihood of working on the problem increased even more (Cutts, Kennedy, Mitchell, & Draper, 2004).

Students completed weekly online homework assignments which were designed to provide them with practice problems. Using technology to complete homework assignments online provided immediate feedback to the students on their submitted answers. The online homework system was also used to facilitate online skills tests which assessed students on skill/drill learning objectives. These skills tests were scheduled via a test scheduling system with all appointments being scheduled outside of regular class time. Completing skills testing outside of regular class time provided additional time to test in class on conceptual learning objectives instead of a mix of conceptual and skill/drill questions. Application sessions lead by Graduate Teaching Assistants (GTAs) made use of technology as well. The GTAs monitored students' progress and sent students weekly feedback via email regarding their progress. The ability to provide this weekly feedback in a timely and efficient manner was made possible by utilizing the email by criteria option offered within the online homework system.

Assessment of Results

Success Rates

Comparison of two teaching formats: Includes all students enrolled after add/drop who earned a C or higher: During fall 2010 it was 60.4% (n=278) with redesigned teaching compared to 46.5% (n=551) for the fall 2010 sections taught with traditional teaching. Note that the special program sections such as our honors sections were not included in either of these groups.

Student Survey Responses

It is not unusual for students, particularly freshmen and sophomores, enrolled in large lecture classes to state that they "do not know anyone in the class". Our redesigned course structure provided the opportunity for students to be active learners with their classmates. Students were often working together during their application sessions to solve a problem as well as discussing the clicker questions that they were asked to answer during their large lecture classes. It is the belief of the authors that the increase in engagement with their classmates resulted in a better sense of belonging as evidenced by survey responses which showed 93% replied that they knew one or more classmates by name. When asked to expand on this 34% stated that they knew four to six classmates by name and 12% knew seven or more students by name. When the class size increases, the amount of personal interaction the teaching team has with the students often decreases. Engaging with the students and providing feedback resulted in survey responses indicating that 50% of the students had met with their application session leader or had been contacted by them and 81% responded that their application session leader had spoken with them personally about an exercise problem.

Conclusions

We found a more consistent student experience in terms of content and assessment in the redesigned course when compared to the courses taught in the traditional format. Also, when comparing the traditional teaching methodology with the redesigned course methodology during the same semester, with the same grade weights and assessments, we found a significantly higher success rate for the redesigned sections. During the semesters since the redesign project completion, the Calculus I course has been taught using several different course formats depending on the scheduling of the classes and teaching preferences of the faculty assigned to the course. Some of the changes have included a change in the textbook from a late to early transcendental approach, removal of the online skills tests from the curriculum, changing from a two hour application session to a one hour recitation session, and use of faculty from another department to teach one of the large lectures. While it would not be appropriate to compare the success rates of semesters in which the course methodologies were significantly different from the redesign course project, when the course format was consistent with the redesign project, the success rate was very similar with success rates typically ranging from 58.3% to 64.9%.

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Sample Test

MAC 2311 **Test 2** Print NAME:

NOTE: Work on these sheets for full credit:

1. (10 pts) Let $\sin y + 5x = y^2$.
   (a) Verify that $(\pi^2/5, \pi)$ is on the curve.
   (b) Determine an equation of the tangent line to the curve at the point $(\pi^2/5, \pi)$.

2. (12 pts) Suppose that the position of an object moving horizontally after t seconds is given by

$$S = f(t) = 2t^3 - 21t^2 + 60t, \quad \text{for } 0 \leq t \leq 6$$

   (a) Find the velocity function. When is the object stationary, moving to the left, and moving to the right?
   (b) Determine the velocity and acceleration of the object at $t = 1$.
   (c) Determine the acceleration of the object when its velocity is zero.

3. (16 pts) An observer stands 300 feet from the launch site of a hot air balloon. The balloon is launched vertically and maintains a constant upward velocity of 200 ft/sec. What is the rate of change of the angle of elevation of the balloon when it is 300 feet from the ground? Recall that the angle of elevation is the angle θ between the observer's line of sight to the balloon and the ground.

4. (16 pts) Let $f(x) = 2x^3 - 3x^2 + 12$.
   (a) Determine the intervals on which $f(x)$ is increasing or decreasing.
   (b) Use either the First Derivative Test or the Second Derivative Test to locate the local minima and local maxima.
   (c) Determine the intervals on which $f(x)$ is concave up or concave down.
   (d) Identify any inflection points.

5. (24 pts) Let $f(x) = \frac{x^2}{x^2-4}$. Given that $f'(x) = \frac{-8x}{(x^2-4)^2}$, $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}$. Graph the function $f(x)$. Use the following steps:
   (a) Identify the domain.
   (b) Exploit symmetry, if there is any.
   (c) Identify critical and possible inflection points, if any.
   (d) Find intervals on which the function is increasing/decreasing and concave up/down.

(e) Identify extreme values and inflection points.

(f) Locate vertical asymptotes and determine end behavior.

(g) Find the intercepts.

(h) Graph the function $f(x)$.

6. (14 pts) Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 3 ft by 4 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way, and verify that your solution is the absolute maximum.

7. (8 pts) Let $f(x) = \tan x$.

(a) Write an equation that represents the linear approximation at $a = 0$.

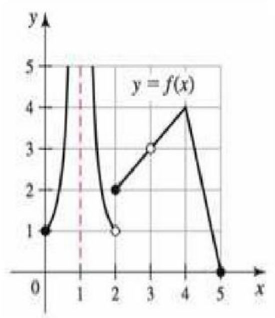
(b) Use the linear approximation to estimate $\tan(\pi/16)$.

Sample Final Exam

MAC 2311 **Final Exam** Print NAME:

NOTE: Work on these sheets for full credit:

1. Use the graph of f in the figure to do the following:



   (a) Find the values of x in the interval $(0, 5)$ at which f is not continuous.
   (b) Find the values of x in the interval $(0, 5)$ at which f is not differentiable.

2. A spring on a horizontal surface can be stretched and held 0.5m from its equilibrium position with a force of 50N. How much work is done in stretching the spring 1.5m from its equilibrium position? Recall Hookes Law: $F = kx$

3. Let $f(x) = \frac{x^2-9}{x(x-3)}$.
   (a) Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, and identify any horizontal asymptotes.
   (b) Find the vertical asymptotes. For each vertical asymptote $x = a$, evaluate $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

4. Find the area of the region enclosed by the curves $y = 12 - x^2$ and $y = x^2 - 6$.

5. Find the length of the curve $y = 3 + 2x^{3/2}$ from $x = 1$ to $x = 2$.

6. A spherical balloon is inflated and its volume increases at a rate of 15 in^3/min. What is the rate of change of its radius when the radius is 10 in.? Recall that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.

7. Find $\frac{dy}{dx}$ if $\tan(x+y) = 2y$.

8. Let $g(x) = x^4 - 2x^2 + 3$.

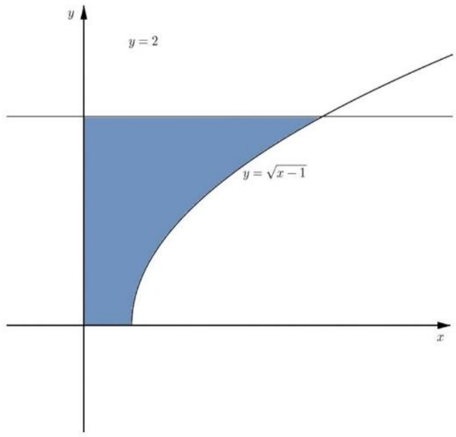
   (a) Find the intervals on which g is increasing or decreasing.
   (b) Find the local maximum and local minimum values of g.
   (c) Find the intervals of concavity and inflection points of g.

9. Find two positive numbers whose product is 100 and whose sum is a minimum. Be sure to verify that the sum is in fact a minimum.

10. Find the volume generated by rotating the region bounded by the curves $y = x^2$, $y = 0$, and $x = 1$ about the y-axis.

Sample Project

Project Week 14



1. Find the area of the regions between $y = \sin x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$.

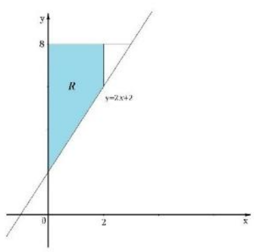
2. Find the area of the region bounded by $y = \sqrt{x-1}$, $y = 2$, $y = 0$, and $x = 0$.

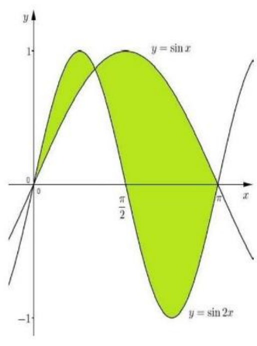
3. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x-1}$, $y = 2$, $y = 0$, and $x = 0$ about the y-axis.

4. Draw a picture of the region bounded by $y = x^2$ and $y = \sqrt{8x}$. Which solid gives the larger value, revolving the region about the y-axis or revolving the region about the x-axis.

5. Find the volume of the solid generated by revolving the region bounded by $y = |x|$, and $y = 12 - x^2$ about the x-axis.

6. Let R be the region bounded by $y = 8$, $y = 2x + 2$, $x = 0$, and $x = 2$. Find the volume of the solid generated by rotating R about the line $x = -2$.





Sample Skills Test

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1. Evaluate the following integral using the Fundamental Theorem of Calculus.

$$\int_{-9\pi/2}^{9\pi/2} (\cos x - 5)\mathrm{d}x$$

$$\int_{-9\pi/2}^{9\pi/2} (\cos x - 5)\mathrm{d}x = \boxed{}$$

(Type an exact answer, using π as needed.)

Answer: $2 - 45\pi$

ID: 5.3.36

---

2. Find the particular antiderivative of the following derivative that satisfies the given condition.

$$C'(x) = 6x^2 - 7x; \qquad C(0) = 1,000$$

$$C(x) = \boxed{}$$

Answer: $2x^3 - \frac{7}{2}x^2 + 1,000$

ID: 4.8.36

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3. Evaluate the following indefinite integral.

$$\int \left(12x^{11} - \frac{11}{x^{11}}\right) \mathrm{d}x$$

$$\int \left(12x^{11} - \frac{11}{x^{11}}\right) \mathrm{d}x = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $x^{12} + \frac{11}{10}x^{-10} + C$

ID: 4.8.20

---

4. Use analytical methods to evaluate the following limit.

$$\lim_{x \to 9} \frac{\sqrt[5]{2x+14}-2}{1/x-1/9}$$

$$\lim_{x \to 9} \frac{\sqrt[5]{2x+14}-2}{1/x-1/9} = \boxed{}$$

Answer: $-\frac{81}{40}$

ID: 4.7.39

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5. Evaluate the integral.

$$\int_1^2 \left(\frac{3}{x^2 - \frac{4}{x^3}} \right) \mathrm{d}x$$

$$\int_1^2 \left(\frac{3}{x^2 - \frac{4}{x^3}} \right) \mathrm{d}x = \boxed{}$$ (Simplify your answer.)

Answer: 0

ID: 5.3.74

---

6. Use the substitution $u = x^7 + 6x$ to find the following indefinite integral. Check your answer by differentiation.

$$\int (7x^6+6)\sqrt{x^7+6x}\mathrm{d}x$$

$$\int (7x^6+6)\sqrt{x^7+6x}\ \mathrm{d}x = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $\frac{2}{3}(x^7+6x)^{\frac{3}{2}} + C$

ID: 5.5.16

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7. Use a change of variables to find the indefinite integral. Check your work by differentiation.

$$\int \frac{x}{\sqrt{4-9x^2}}\mathrm{d}x$$

$$\int \frac{x}{\sqrt{4-9x^2}}\mathrm{d}x = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $-\frac{1}{9}\sqrt{4-9x^2}+C$

ID: 5.5.25

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8. Evaluate the following limit.

$$\lim_{x\to\pi}(x-\pi)\frac{1}{\tan x}$$

$$\lim_{x\to\pi}(x-\pi)\frac{1}{\tan x} = \boxed{}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

Answer: 1

ID: 4.7.27

---

9. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the following integral.

$$y = \int_0^{\sin x} \frac{\mathrm{d}t}{\sqrt{1-t^2}}, \quad |x| < \frac{\pi}{2}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \boxed{}$$ (Simplify your answer.)

Answer: 1

ID: 5.3.85

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10. Use change of variables to evaluate the following integral.

$$\int \frac{\sec^2 x}{\tan^4 x} \mathrm{d}x$$

$$\int \frac{\sec^2 x}{\tan^4 x} \mathrm{d}x = \boxed{}$$

(Use C as the arbitrary constant.)

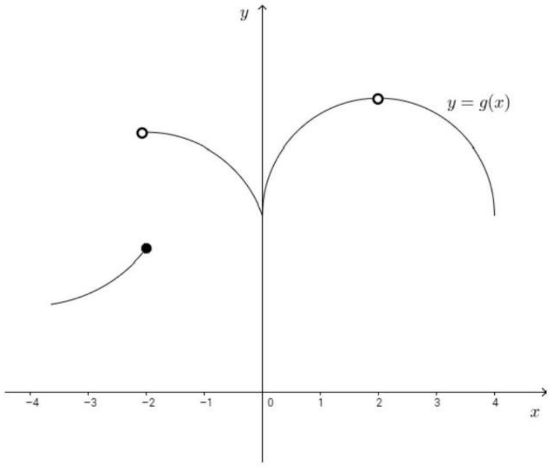
Answer: $-\frac{1}{3}\cot^3 x + C$

ID: 5.5.55

Sample Project

Project Week 3

1. Consider the graph of the function g.



   (a) Find the values of x at which the limit of g does not exist.
   (b) Find the values of x at which g is not continuous.
   (c) Find the values of x at which g is not differentiable.

2. **Projectile trajectory** The position of a small rocket that is launched vertically upward is given by $s(t) = -5t^2 + 40t + 100$ for $0 \leq t \leq 10$, where t is measured in seconds and s is measured in meters above the ground.

   (a) Find the rate of change in the position (instantaneous velocity) of the rocket, for $0 \leq t \leq 10$.
   (b) At what time is the instantaneous velocity zero?
   (c) At what time does the instantaneous velocity have the greatest magnitude, for $0 \leq t \leq 10$?
   (d) Graph the position and instantaneous velocity, for $0 \leq t \leq 10$.

3. **Gas mileage** Starting with a full tank of gas, the distance traveled by a particular car is $D(g) = 0.05g^2 + 35g$, where D is measured in miles and g is the amount of gas consumed in gallons.

   (a) Compute $\frac{dD}{dg}$. What units are associated with the derivative and what does it measure?
   (b) Find $\frac{dD}{dg}$ for $g = 0, 5$, and 10 gal (include units). What do your answers say about the gas mileage for this car?

(c) What is the range of this car if it has a 12-gal tank?

4. **Population of Las Vegas** Let $p(t)$ represent the population of the Las Vegas metropolitan area t years after 1950, as shown in the table and figure.

   (a) Compute the average rate of growth of Las Vegas from 1970 to 1980.

   (b) Explain why the average rate of growth calculated in part (a) is a good estimate of the instantaneous rate of growth of Las Vegas in 1975.

   (c) Compute the average rate of growth of Las Vegas from 1990 to 2000. Is the average rate of growth an overestimate of underestimate of the instantaneous rate of growth of Las Vegas in 2000? Approximate the growth rate in 2000.

Year	**1950**	**1960**	**1970**	**1980**	**1990**	**2000**
t	0	10	20	30	40	50
$p(t)$	59,900	139,126	304,744	528,000	852,737	1,563,282

Source: U.S. Bureau of Census