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Multiple Solutions Approach (MSA): Conceptions and Practices of Primary School Teachers in Ghana

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Abstract

The study explored the curriculum guidelines and primary school teachers' conceptions and practices of the Multiple Solutions Approach (MSA) in teaching mathematics using basic qualitative research design. Informal conversation interviews (ICIs), observations, video and document analyses were used to collect data. Participants included a purposive sample of five practicing teachers from three districts of the Central Region of Ghana. Descriptive analytical procedures were applied to the quantitative data while the qualitative data were analyzed thematically. Findings indicated that the curriculum materials lacked concrete direction for MSA and less than 22% of sub-topics were presented in different ways: a provision that is insufficient to guide and stimulate teachers to teach in multiple ways. Teachers also lacked a clear understanding of the MSA. Although the curriculum policy encouraged MSA, teachers' practices were inconsistent with the curriculum guidelines. The implementation of MSA was limited by inadequate curriculum provisions and teacher knowledge.

Key words: Multiple solution approach (MSA); Mathematics teaching; Problem solving; Ghanaian teachers

Introduction

In Ghana, the Education for All policy and the curriculum premise that "all students can learn mathematics and that all need to learn mathematics" (MOE, 2010, p. ii), gives rise to high enrolment of children with different backgrounds, learning abilities, and learning styles in the mathematics classroom today. The enrolment of children with such diverse characteristics in classroom settings poses challenges to mathematics teachers. The mathematics teacher is challenged to provide equal learning opportunities and attention for children with different experiences, cognitive abilities, and learning styles.

Indeed, the educational reforms (MOE, 2004; 1995) in Ghana were motivated by the need to provide for diversity and to make education more relevant. The mathematics curriculum content was revised to equip students with mathematical knowledge and skills to fit into different programmes of interest. One theoretical lens that guided the revision of the current mathematics curriculum to provide for diversity in learning is constructivism. The adoption of constructivism as theory in the mathematics curriculum process is based on the recognition that there are different ways of conceptualizing reality (Clements & Battista, 2009; Kamii & Lewis, 2009). The constructivist principles provide space for cognitive diversity and allow the possibility of solving problems in different ways. Mathematics teachers, who are the implementers of the mathematics curriculum, are to facilitate students' construction of mathematical knowledge from different perspectives. They are also to create a classroom culture that enables students to actively explore different problem solution strategies to develop the skills delineated by the mathematics curriculum. Constructivism as a theoretical lens for Ghana's curriculum design and implementation process is a provision for multiplicity − multiple presentations, solutions, learning abilities and styles.

One way of meeting the mathematical learning needs in a diverse classroom setting is using multiple modes of representations that integrate children's prior experiences and interest in solving mathematical problems (Chen & Weiland, 2007) or engaging learners in multiple solution tasks [MSTs] (Levav-Waynberg & Leikin, 2010). MSTs explicitly provide for solving problems in multiple ways (Levav-Waynberg & Leikin, 2010) and the process of guiding learners to solve mathematical problems in different ways to arrive at the same solution is described by Bingolbali (2011) as Multiple Solutions Approach (MSA). Ghana's mathematics curriculum encourages multiple solutions to mathematical problems through the use of open-ended tasks, projects, investigations and problem solving. Understanding that a problem can be solved in more than one way is critical in making sense of mathematics (Cramer, Monson, Whitney, Leavitt, & Wyberg, 2010).

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Teachers' Knowledge-Base for MSA

Although Ghana's mathematics curriculum reform initiative incorporated MSA as a way of providing space to meet the needs of all learners, "interpreting reform ideas, managing the challenges of change, using new curriculum materials, enacting new practices and teaching new content all depends on teachers' knowledge of mathematics" (Ball, Lubienski, & Mewborn, 2001, p. 437). Teaching is a complex activity and according to Shulman (1986), teachers need to have the subject matter content knowledge (SMCK), pedagogical content knowledge (PCK) and curriculum knowledge (CK) in order to teach. The teacher must be able to explain for students to understand the accepted truths in the domain, why a particular proposition is deemed acceptable, why it is important knowing it and how it relates to other propositions both within the discipline and without, both in theory and in practice (Shulman, 1986). Building on Shulman's ideas, other researchers Ball and Bass (2000) and Ball, Lubienski and Mewborn (2001) developed the idea of mathematical knowledge for teaching (MKT). The MKT or PCK specifies the pedagogically useful mathematical knowledge (Ball & Bass, 2000) without which the mathematics teacher lacks the relevant tools for practice (Ball, Lubienski, & Mewborn, 2001; Ekawati, Lin, & Yang, 2015). PCK intertwines content with pedagogy (Ball & Bass, 2000) and equips the teacher with the relevant skills and knowledge required to unpack mathematical ideas into elemental components in ways that are visible to the learner (Ball & Bass, 2000; Morris, Hiebert, & Spitzer, 2009). It forms the foundation for interpreting different mathematics solution pathways and explaining to learners why a given solution approach works or is worth knowing.MKT is essential and crucial in teaching problem solving in ways that are sensitive to individual differences.

Contemporary researchers (Ekawati, Lin, & Yang, 2015; Henriques & Ponte, 2014) have used PCK or MKT as a theoretical frame for teaching mathematics. Solving problems in different ways or MSA requires MKT that integrates mathematical content, pedagogical understanding and curricular knowledge. In this study, we assume MSA as a problem solving process teachers construct through experience in their classrooms. The meanings teachers give to their experiences in the MSA constitute their conceptions while the actions deployed to teach and solve problems in different ways and to address problem solving challenges reflect their mathematical knowledge of MSA.

Research on Multiple Solutions Approach (MSA)

Classroom studies (Ayub, Ghazali, & Othman, 2013; Chen & Weiland, 2007; Özgün-Koca, 2008) indicate that providing children with multiple solutions opportunities enables them to select solution options that make sense to them. Ayub, Ghanzali, and Othman's (2013) study of 5 and 6 year old preschool children's representation of number from multiple perspectives indicates that children use multiple strategies− subtilizing, counting in ones, and guessing− to respond to the number of dots in a problem. Children resorted to counting in ones when the number of dots was more than they can cope with using the subtilizing strategy. When sixth graders were given multiple opportunities to solve fraction problems in Cramer, Monson, Whitney, &Wyberg's (2010) study, the children relied on pictures to construct meaningful interpretation of fraction division on which future symbolic work can be built. Their solution strategies suggest knowing that a piece of fraction can be named in multiple ways is fundamental to the part-whole model.

Leikin and Levav-Waynberg (2007) observed that using MSA contributes to the development of students' creative and critical thinking skills, and the understanding that mathematics is embodied. Multiple solutions approach in mathematics problem solving develops and sustains students' interest and provides them with access to a range of solution strategies for the present and future. Silver, Ghousseini, Gosen, Charalambous and Strawhun (2005) argue that MSA can facilitate the connection of a problem to different elements of knowledge and thereby strengthening networks of related ideas. Although few research findings tend to indicate that mere access to multiple solutions does not necessarily lead to improved understanding (Ainsworth, 2006; Seufert, 2003), the advantages outweigh the disadvantages.

Researchers and professionals who believe in multiple perspectives in learning (Bingolbali, 2011; Leikin, Levav-Waynberg, Gurevich&Mednikov2006; Özgün-Koca, 2008) advocate MSA as an option to developing mathematical understanding. Solving mathematical problems in multiple ways develops students' creativity, critical thinking skills, interest, and ability to search for alternatives (Bingolbali, 2011; Leikin, 2007; Levav-Waynberg & Leikin, 2010). Multiple solutions approach enables pupils to develop better understanding of the basic mathematical ideas required for higher mathematics. Silver, Ghousseini, Gosen, Charalambous and Strawhun (2005) summarized the value of MSA asserting that you can learn more from solving a problem in many different ways than you can from solving many different problems, each in only one way. Approaches that give learners the opportunity to arrive at the solution of a mathematical problem in different ways invigorate learners and improve their performance.

Extensive literature (Bingolbali, 2011; Cai & Lester, 2005; Mewborn, 2003) suggest that what the student learns in the classroom is influenced by what teachers do. It is the teacher who probes, chooses and uses the approaches to a mathematical problem (Bingolbali, 2011). Their choice of representation influences students' representations (Cai & Lester Jr., 2005) and hence, their understanding of how problems are solved. Limitation in teacher's mathematical knowledge (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005) and the press to cover (Ball, Lubienski, & Mewborn, 2001) can block the use of multiple solutions in the classroom. Schools pre-occupied with test scores put pressure on teachers to emphasize basic skills, at the expense of providing multiple opportunities for students to construct multiple frames of reference.

Although these studies suggest students can learn more from solving problems in many different ways, Bingolbali's (2011) study of Turkish teachers found that teachers are not open to different solutions to mathematical problems and do not value different solutions. A review of literature (Nabie, Akayuure, & Sofo, 2013) on problem solving suggest that although real-life problems have multiple solution pathways, students can solve problems in different contexts if only teachers employ techniques that incorporate multiple solution strategies in their lessons. The teacher directs students' solutions pathways and prompts for alternatives. Teacher education in Ghana exposes teachers to educational theories and provides them with examples of structurally similar problem solutions in varying contexts but learning from such exposures may be superficial.

Statement of the Problem

Multiple solutions approach (MSA) to solving mathematical tasks is a tool for developing the intellectual capacity of children in doing things in different ways especially at primary school level where foundation skills are developed. Multiple Solutions Approach is a function of the teachers' conceptions and understanding of multiple solutions pathways. However, majority of research on problem solving in Ghana (Nabie & Nantomah, 2012; Nabie, Nantomah, & Akayuure, 2011) and teacher knowledge (Awanta, 2003; Nabie & Ngman-Wara, 2003) tend to focus on the teacher content knowledge and its impact on effective teaching and learning. Although studies consistently report poor mastery of basic mathematical concepts (MOE, 2011; Nabie, Anamuah-Mensah, & Ngwan-Wara, 2010) and that teachers play a critical role in choosing and presenting problems in different forms (Bingolbali, 2011), few or no study has examined Ghanaian teachers' conceptions and practices of multiple solutions approach to teaching mathematics that provides options for easy understanding.

Purpose of the Study

When learners are denied the opportunity to learn mathematics in multiple ways, they see mathematics as a one-way rule pattern and as disconnected pieces of information. Hence the study was designed to explore the provisions for, and teachers' conceptions and practices of multiple solutions approach (MSA) in teaching mathematics. Specifically, the study was designed to explore:

1. Curriculum guidelines on multiple solutions approach to teaching mathematics.
2. Primary school teachers' conceptions of multiple solutions approach for teaching mathematics; and
3. Teachers' use of multiple solutions approach in the mathematics classroom

Research Questions

The use of Multiple Solutions Approach (MSA) in teaching depends on teachers' conception of MSA and the curriculum provisions that guide the use of the approach. Hence, the research was designed to respond to the following questions:

1. What are the curriculum guidelines on Multiple Solutions Approach to teaching mathematics in primary schools?
2. What are primary school teachers' conceptions of Multiple Solutions approach in teaching mathematics?
3. How do primary school teachers practice Multiple Solutions Approach in teaching mathematics?

Significance of the Study

Problem solving is the heart of mathematics education in Ghana. Research literature in Ghana consistently indicates children's weakness and poor performance in solving problems (NEA, 2011) and teachers' challenges in integrating problem solving in mathematics lessons (Nabie, Akayuure, & Sofo, 2013). Educational stakeholders are therefore looking for possible ways to address this weakness. The current study on MSA can enlighten the mathematics education community on teachers' conceptions and practices of solving problems in ways that locate learning in the comfort zones of learners. The study would also serve as a guide to curriculum and textbook developers in designing curriculum materials to provide MSA in the training of prospective and practicing mathematics teachers. Also, supervisors of mathematics teachers can draw vital information from the study for routine comprehensive supervision sessions.

The study also adds to existing knowledge on what really happens in our mathematics classrooms to provide justification for in-service training programmes for teachers to update their skills on MSA to enhance students understanding. In particular, it would bring to light teachers' personal understandings of multiple solution approach and their practices perceived to be multiple pathways in solving problems.

Method

In this study, we sought to discover and to understand MSA as described in the Ghanaian curriculum and from teachers' perspective. Basic qualitative research design (Merriam, 1998) was used to explore curriculum provisions and teachers' conceptions and practices of MSA in the Ghanaian classroom. This design allowed for theoretical flexibility and multiplicity of data collection instruments.

Participants

Using a purposive sampling technique, three districts (two Local and one Municipal) out of the 17 districts in the Central Region of Ghana were identified for the study. Bearing in mind that a small sample of information-rich individuals adequate enough to answer the research question is what matters in qualitative research (Johnson & Christensen, 2000; Merriam, 1998), five practicing teachers [4 males, 1 female] were selected from the selected districts based on their experience and willingness to participate in the study. Two were from the Winneba municipality, two from the Gomoa East and one from the Awutu district. Participants were identified by T1, T2, T3, T4, and T5 for anonymity. A summary of the participants' background information is shown in Table 1

Table 1. Characteristics of research participants

Code	Gender	Professional Teacher	Academic qualification	Level Teaching	Teaching Experience	Class
T1	M	No	SSSCE/WASSCE	Primary	Less than 1yr	6
T2	M	Yes	Diploma	Primary	Almost a year	3
T3	M	No	Construction Technician Part III	Primary	4 years	3
T4	M	Yes	Bachelor's Degree	Primary	7 years	6
T5	F	Yes	Diploma	Primary	4 years	3

As shown in Table 1, three participants were professional teachers (two diplomas and one bachelor's degree holder) and two were non-professionals with at least a Senior High School Certificate (SSCE/WASSCE). Their teaching experiences ranged from less than a year to seven years. By their academic qualification, they all had the content knowledge for teaching primary mathematics. The two non-professional participants may be deficient in pedagogical content knowledge theoretically for teaching but had observational experiences of teaching when they were students and practical experience in the classroom.

Data Collection Process

Informal conversational interviews (ICI) with field notes, observations, video tape, and document analyses were used to collect data. Informal conversational interview, a flexible data collection tool that enables multi-sensory channels (Cohen, Manion, & Morrison, 2007) and allows participants to voice their views unconstrained (Creswell, 2011) was used to collect participants' bio-data, conceptions and practices of MSA in teaching mathematics. Merriam (1998) described interviews as the best technique to use when conducting intense studies with few individuals. The fourth author conducted the one-to-one interviews with each participant in the school after classes. Each interview session on the average lasted 20 minutes. All interview sessions were audio-taped and notes taken during the process.

Observation which offers the opportunity to collect live data from participants' natural settings (Cohen, Manion & Morrison, 2007) was used for data on participants' MSA activity during teaching. Each participant's mathematics lesson was observed during normal instructional time of 60 minutes to collect data on how teachers solved mathematical problems using MSA. An observation checklist that outlined key MSA activities was used by the fourth author who ticked practices that reflected MSA as outlined on the checklist during observation. The lessons were also videotaped and later observed by the first three authors using the same observation checklist and compared notes to ensure consistency and that all relevant data was captured.

According to Fraenkel and Wallen (2003) the conscious and unconscious beliefs, attitudes, values and ideas of people are revealed in their communication documents (textbooks, syllabuses, articles etc.). The mathematics syllabus and the primary mathematics books that reflect the mathematical knowledge, beliefs, attitudes, and values for primary 3 and 6 were selected for analysis. These documents were individually analysed using a coding criteria developed by the researchers to collect data on activities, words, statements, and phrases in the curriculum materials that reflected MSA to teaching mathematics. The three coded results were compared for consensus.

To enhance the trustworthiness of the study, transcribed interviews and observation checklists were given to the participants for member checking to ensure that the data truly reflected their views and actions. When conducting the interviews, the interviewer listened extensively and recorded observations accurately. Thick descriptions were used to substantiate and illustrate participants' conceptions and practices of MSAs. Also the four researchers' individually analysed the documents using the same coding categories and compared their results to establish common grounds. The use of multiple analytical tools was to minimize instruments' biases and to provide space for an in-depth understanding of primary school teachers' conceptions and practices of the Multiple Solutions Approach.

Data Analysis

The study adopted qualitative research design that involved a mix of quantitative and qualitative data. Descriptive statistics was applied to the quantitative data from document analysis. Frequency counts were used to quantify the number of sub-topics in primary three and six mathematics textbooks and the corresponding sub-topics that were solved in multiple ways. The frequency counts were then converted into percentages and the results presented in tables. Observation data were tabulated in a matrix-table showing practices of multiple solutions approach. Qualitative data were read several times and recurring patterns or categories identified and coded. Categories that reflected the same code were re-categorised thematically.

Results

The study explored primary school teachers' conceptions and practices of multiple solutions approach in the mathematics classroom. Document analysis, interviews and observations and video tapes were used to collect data. Quantitative data were analyzed using descriptive statistics while the qualitative data were analysed qualitatively to reflect the purposes of the study.

Curriculum Guidelines for Multiple Solutions Approach

To ascertain what informed the use of Multiple Solutions Approach to teaching mathematics in the primary schools, the syllabus was analysed for curriculum guidelines. The syllabus emphasises mathematical knowledge, skills and values that will enable the child to function in society. It requires pupils to be competent in reading and interpreting numeral data, reason logically, solve problems involving calculations and mathematical reasoning, as well as communicate effectively using accurate mathematical data and interpretations (MOE, 2010). The syllabus recommended the *use of projects during and outside school hours.* In

Number work, the syllabus advocated the use of *both expanded and the short forms* of problem solution pathways to enable the leaner have a choice of strategy that he or she is most comfortable with.

Other statements such as *assist pupils to solve problems on direct proportion using unitary* and *ratio methods*, the use of *the distributive property to perform multiplication* and guiding *pupils to multiply in the vertical form* are found. Suggested lesson activities such as use of *structured base ten materials to illustrate the number of hundreds in a thousand, abacus and colour-coded counters to illustrate the same concept,* and *guiding pupils to use paper folding*, *fraction charts*, *Cuisenaire rods* among others to identify a part of a fraction were seen. These phrases reflect recommendations that suggest the use of MSA.

From the syllabus, although the multiple solutions concept is implicit, there are several words and phrases that suggest its use for teaching mathematics. As MSA is not explicitly suggested as a requirement in the teaching syllabus, the choice of solving problems in multiple ways tends to be at the discretion of the teacher.

MSA in Primary Mathematics Textbooks

Good textbooks provide alternative strategies, illustrations and activities for solving problems. To find out whether the mathematics textbooks provided for alternative solutions or solution presentations for problems, the primary class three and six mathematics textbooks were examined. The results of the analysis of sub-topics of topics with MSA in primary class three (P3) mathematics textbook is shown in Table 2

Table 2. Class 3 mathematics textbook topics that reflect MSA

Main Topics	Number of sub-topics	Number of sub-topics with MSA
Numbers and numerals	6 (9.84%)	0 (0.00%)
Addition and subtraction	5 (8.19%)	0 (0.00%)
Length and area	5 (8.19%)	0 (0.00%)
Fraction I	6 (9.84%)	2 (3.28%)
Collecting and handling data	2 (3.28%)	0 (0.00%)
Capacity and weight	4 (6.56%)	2 (3.28%)
Multiplication	7 (11.475%)	2 (3.28%)
Division	7 (11.475%)	2 (3.28%)
Shapes	3 (4.92%)	0 (0.00%)
Time and money	10 (16.39%)	5 (8.19%)
Fraction II	6 (9.84%)	0 (0.00%)
Total	61 (100.00%)	13 (21.31%)

Table 2 indicates that out of 61 sub-topics identified from the 11 main topics in the P3 mathematics textbook, only 13 (21.31%) sub-topics were presented in more than one way. As many as 48 (78.69%) sub-topics in the textbook were not presented with MSA. This means that most topics provided no direction to the teacher to engage in MSA. Time and Money had the highest number sub-topics (10) but only half the number representing 8.19% exhibited the use of MSA. Fraction I with six (6) sub-topics, Capacity and Weight with four (4) sub-topics, Multiplication and Division with seven (7) sub-topics all had two (2) sub-topics (3.28%) each applying the MSA. Numbers and Numerals (6 sub-topics), Addition and Subtraction (5 sub-topics), Length and Area (5 sub-topics), Collecting Data (2 sub-topics), Shapes (3 sub-topics) and Fraction II (6 sub-topics) each had no sub-topic (0%) approached in more than one way. However, the sub-topic 'thousands, hundreds, tens and ones' was illustrated with multiple representations. The data suggest there is insufficient provision in the P3 mathematics textbook to encourage teachers to engage in MSA in their instructional drive. Similarly, the results of the sub-topics with MSA in the primary class six (P6) mathematics textbook, are as presented in Table 3.

Table 3 indicates that very few sub-topics were treated in multiple ways. Out of the 82 sub-topics in the P6 textbook, only 15 representing 18.30% had problems solved in more than one way. A total of 67 sub-topics representing 81.70% of sub-topics in the P6 mathematics textbook were not presented in more than one way. This suggests that most topics do not provide direction to compel or motivate teachers to use MSA. Decimal Fractions and Percentages, Measurement of Length, Capacity and Mass, Shapes and Space, Measurement of Area and Volume, Money and the Number Plane all had one sub-topics (1.22%) each presented with MSA. Few sub-topics in: Set of Numbers, Numbers and Numerals from 0-9,999,999, Ratio and Proportion, Collecting and Handling Data, Multiplication and Division, investigating with Numbers, and Chance had two sub-topics (2.44%) presented with MSA. Only one topic, Operations on fractions, had three sub-topics (3.66%) presented with MSA. The provision was insufficient to motivate or compel a teacher to engage in multiple solution practices.

Table 3. Class 6 mathematics textbook topics that reflect MSA

Topics	Number of sub-topics	Sub-topics with MSA
Set of numbers	6 (7.32%)	2 (2.44%)
Operations on fractions	7 (8.54%)	3 (3.66%)
Numbers and numerals, 0- 9, 999, 999	8 (9.76)	1 (1.22%)
Addition and subtraction, 0- 9, 999, 999	2 (2.44%)	2 (2.44%)
Decimal fractions and percentages	12 (14.63%)	0 (0.00%)
Measurement of length, capacity and mass	10 (12.19%)	0 (0.00%)
Ratio and proportion	5 (6.09%)	1 (1.22%)
Shape and spaces	4 (4.88%)	0 (0.00%)
Collecting and handling data	4 (4.88%)	2 (2.44%)
Multiplication and division	5 (6.09%)	2 (2.44%)
Investigating with numbers	8 (9.76%)	1 (1.22%)
Measurement of area and volume	3 (3.66%)	0 (0.00%)
Money	3 (3.66%)	0 (0.00%)
Chance	2 (2.44%)	1 (1.22%)
The number plane	3 (3.66%)	0 (0.00%)
Total	82 (100.00%)	15 (18.30%)

Primary School Teachers' Conceptions of MSA

Effective teaching is a function of teachers' knowledge. To explore teachers' conception of MSA, participants were asked whether they have ever heard of multiple solutions approach. Three of the teachers (T1, T2 and T4) responded "*No*". Although the three participants responded no, all the participants were further asked what they thought MSA in teaching mathematics was all about.

In response, T1 said: *multiple solutions approach, I think, is using different strategies in teaching mathematics*. Similarly, T2 responded: *I think it is the act of using different methods in teaching mathematics*. On the contrary, T4 after a careful reflection said: *well looking at the term multiple solution approach, I understand it …* [as] *using multiplication in teaching or teaching using multiplication*. The other two participants T3 and T5 who had prior knowledge of MSA, explained respectively that *Multiple Solutions Approach is using different types of methods in teaching mathematics* (T5) and *it is simply to teach mathematics for pupils to understand* (T3).

The responses tend to suggest that teachers have varying and contrasting conceptions of MSA. Three respondents T1, T2, and T5 conceptualised MSA as using more than one way in teaching mathematics. One respondent, T4, wrongly inferred the meaning of MSA from the words as using multiplication in teaching. The responses suggest that some teachers had a clear view of MSA as using multiple pathways in teaching while others misconceived the concept as a basic operation − multiplication.

Knowing a concept is one thing and knowing its value or importance is yet another. To explore teachers' conceptions of the importance of MSA as required by the mathematics curriculum, participants were asked: "What are the advantages of teaching mathematics through MSA?" In response T1 said: *some children will not understand a particular strategy so using different approaches will help all the pupils in the class to benefit from what you are doing. …* [Also], *using multiple approaches can help others to bring out their problems for assistance*. Similarly, T2 explained that *using MSA helps a lot. Sometimes you will adapt one strategy and find out that children are not understanding or coping with what you are doing. If you use a different solution strategy it turns out the children understand what you teach better. Pupils understand things from different points of view. So you adapt different strategies or solutions to find where the child is capable.* Pausing for a moment, T2 continued: *for you the teacher, it will help you know the strengths and weaknesses of every child. You adapt a particular method and when you find out that your children are not scoring, you change it. You always adapt different strategies in your teaching.*

T3 and T5 both extolled what they conceived as the virtues of MSA to the teacher. T3 recounted that *it helps me to calculate mathematics easy whereas* T5 simply said *it makes my work easier because the pupils easily understand.* Both maintained that the *teacher gets the understanding* [in] *using different approaches.* T4's value of MSA reflected her initial misconceptions saying: *well one of the advantages is that when you employ this type of teaching it will help the children to understand multiplication better but if the method is not taught very well, it will make the children more confuse in learning multiplication*.

Teachers' responses on the importance of MSA reflected diversity. Two participants,' T1 and T2, valued MSA as a means to responding to individual differences in learning mathematics. It provides for children to work in their comfort zones for better understanding and makes the teacher's work easy. In this perspective, MSA benefits both the child and the teacher. Although two participants T3 and T5 valued MSA in terms of easing teacher work, T3's view of helping the teacher calculate mathematics is a misconception similar to T4 who thought of MSA as multiplication.

The responses suggest that some participants were well-informed on the value MSA in teaching and they saw the practice as a way of addressing learning difficulties and encouraging variety. Teachers with clear conception of MSA conceptualized the benefits of MSA as providing alternative paths for learners who may find it difficult to cope with a single approach. Using approaches children can cope with locate learning in their comfort zones and promote better understanding of mathematics.

Primary School Teachers' Practice of MSA

To get an understanding of how MSA is practiced in the classroom, teachers were asked whether they ever taught mathematics using Multiple Solutions Approach. Four participants −T1 and T2 (who never heard of MSA), T5 and T3 responded in the affirmative. Only T4 responded in the negative. When those who practised the MSA were probed to give examples of topics they taught using the approach, both T1 and T5 mentioned *teaching of fraction*, T2 said *e...em addition and subtraction* while T3 mentioned *multiplication*. T4 after a little pause said: *ok…eh teaching multiplication?*

To ascertain whether participants truly taught mathematics using MSA, participants were asked to describe how they taught any topic/sub-topic using MSA. T1 and T2 described how they taught fractions and subtraction as expressed:
T1: *First of all, I took an orange and divided it into parts. I asked the children: How many parts have I divided the orange into? And they will mention it. Then I took some part out and I asked them: Tell me the number of parts that I have taken out. They mentioned the number. Then I brainstormed with them on what a fraction is about. They said a fraction is part of a whole number. I then wrote a fraction on the board. I asked them: The top number refers to what? And they said the numerator; and the down number refers to the denominator. I explained to them that denominator tells us the number of parts you divided the orange into and the numerator tells us the number of parts taken out of the divided orange.*

Describing his process, T2 said: *You see with the subtraction sometimes you can use, how I call it, comparing. You are going to compare two things. You are going to use comparing and you can also use matching. Assuming you have a group of items here and you have a group of items here* [pointing to two different places] *you match the ones which are not going to get partners. A-h…aa, then you are going to tell the children that if you have 5 here, you have 6 here after matching there is one left in the box without a partner. That one becomes the difference. So after taking 5 out of 6, 1 is left.* Explaining his process, T3 said: *I teach the children that may be* 2×2 *will give you 4,* 3×3 *will give you 9 and so on.* T5 who was alarmed by the request to describe how she used MSA to teach fractions responded: *m…mhewo! Ok you can use a sheet of paper in teaching. You can also draw on the blackboard for them to see.* However, T4 whose response on the use of MSA was negative explained: *As I said earlier on that multiple solution approach or concept is new to me. May be I might know it when you explain it further to me. May be I might be using it but I don't know that is it. So, as I said earlier on I have not used that. So I think if you explain it further to me may be I might know that I have been using it in my teaching but I didn't know*.

Participants' responses about the practice of MSA, in general, were at variance with one another. T1 who indicated that he had never heard of the multiple solutions concept described his traditional way of teaching fraction as his practice of the MSA. On the other hand, T2 who also indicated he had never heard of MSA described two approaches namely: comparing and matching as strategies he has been using to teach subtraction. In a way, T2 has been practicing MSA in teaching without knowing. Similarly, T5 described two strategies − paper folding activity and drawing on blackboard – which he has been using for teaching multiplication. From these descriptions, T1 and T3 erroneously perceived their unique traditional ways of teaching fraction and multiplication respectively as MSA. This suggests that what they had in mind about MSA is at variance with their practice. However, T2 and T5 actually employ the principles of the MSA. These responses suggest that some teachers lack a clear understanding of the concept and practice of MSA in teaching mathematics.

The Reality of MSA as Practiced

To ascertain whether or not participants' practice the MSA as claimed, each participant's lesson was observed using an observation checklist. The results of the lessons observed are presented in Table 4.

Table 4. Primary school teachers' classroom practices of MSA

MSA activity	T1	T2	T3	T4	T5
Lesson introduction					
Topic/sub-topic can be taught through MSA	√	√	√	√	√
Pre-informs learners of multiple approaches to arriving at a solution to problems	×	×	×	×	×
Lesson activities/development					
Leads learners through one approach to solving the mathematical problems	√	√	√	√	√
Encourages pupils to explore other approaches to solving mathematical problems	×	×	×	×	×
Lead learners through alternative approaches	√	√	×	×	√
Gives examples that can be solved using any of the approaches taught to arrive at the solution	√	√	×	×	×
Creates pupils' awareness on the need to know different approaches to solving a problem	×	×	×	×	×
Give learners the opportunity to solve problems using such approaches on the chalkboard	√	×	×	×	×
Motivates pupils to use the multiple solutions approach identified	×	×	×	×	×
Lesson evaluation					
Appreciates both solutions methods and award marks according to rubrics in exercise books.	–	–	–	–	–

Key √: Instructional practice performed; ×:instructional practice not performed; and –: no evidence of performance of instructional practice.

Table 4 shows that T1 taught addition of numbers, T2, subtraction, T3: column multiplication of two digits' numbers, T4, addition of six digit numbers and T5 taught a lesson on fractions (thirds and sixths). Three participants T1, T2 and T5 introduced learners to other approaches to arriving at the same solution. Two participants T1and T2 gave examples that could be solved using any of the methods taught. All the participants guided learners through a first approach but none pre-informs their learners that there are alternative approaches to arriving at the solution, encouraged their pupils to explore alternative approaches, created pupils' awareness on the need to know different approaches, nor motivate their pupils to use MSA.

Discussion

The study explored the curriculum guidelines and primary school teachers' conceptions and practices of multiple solutions approach in teaching mathematics. The assumption was that teachers' use or non-use of MSA depends on their knowledge capacity of the curriculum content and its provisions. The curriculum resources are key components that guide the teacher's work. They provide information on what is valued and suggestions to guide curriculum implementation.

The results of the study indicate that the mathematics syllabus and textbooks have statements that reflect MSA. However, the statements lacked concrete direction and emphasis, and are insufficient to guide teachers to teach in multiple ways. Specifically, the textbooks lacked alternative directions on how teachers should approach most topics. Indeed, 21.31% of sub-topics in the class three mathematics textbook and 18.29% of sub-topics in the Primary six textbook identified were solved in more than one way. The provision of only few topics in the textbooks with more than one solution approach is an indication that MSA is not the emphasis of primary mathematics. The limited guidelines provided in the curriculum materials might have informed teachers' limited response to MSA in teaching mathematics. The provisions in the curriculum materials are inadequate to develop teachers Mathematical Knowledge for Teaching or Pedagogical Content Knowledge for multiple solutions approach.

The results also showed that MSA is conceived differently by different teachers and some teachers have no idea of the concept. Specifically, some teachers had a clear conception of MSA as solving problems in different or multiple ways consistent with (Leikin et al., (2007) view. A teacher viewed MSA as teaching for better understanding, a conception that reflects the ultimate goal of MSA. Another teacher misconceived MSA as a multiplication process. This suggests either that some teachers do not know what MSA is all about.

Despite the limited guidelines and lack of specific directions on MSA in the mathematics curriculum documents, teachers conceptualized the use of the approach as advantageous. MSA is seen as a means to respond to individual differences and an opportunity to provide for children to work in their comfort zones. MSA aids in diagnosing children's strengths, weaknesses and their understanding of mathematics from different perspectives. Learners also learn or develop alternatives strategies for solving problems. A teacher who conceived MSA as using multiplication in teaching mathematics also saw its benefits as using multiplication for better understanding. The conception of MSA in terms of multiplication is a misconception and lack of knowledge of the concept. This suggests that even though some teachers seem to be aware of some benefits of MSA, the practice of the approach is limited because they lacked the relevant knowledge and skills for interpreting and enacting its practice. The results affirm earlier studies (Nabie, Anamuah-Mensah, & Ngwan-Wara, 2010) that some basic school teachers lack the knowledge capacity to teach.

Evidence from classroom observations indicated that the conceptions teachers hold about MSA are hardly translated into practice. Teachers' ways of using multiple solutions approach during their lesson delivery neither reflected their conceptions of MSA nor followed any organized instructional pattern. Their MSA strategies or practices appeared accidentally as no conscious effort was made to aid learners in finding solutions to problems in different ways. Although some teachers' practice of teaching subtraction using “comparing” and “matching” reflected MSA, most topics identified to have possibilities for different approaches were treated in only one way. The results generally suggest that teachers are not open to different solutions to mathematics problems (Bingolbali, 2011) and do not consciously motivate children, in whatever form, to explore alternatives in solving problems. This can be attributed to limited knowledge and skills of teachers on what it takes to teach through MSA.

Conclusion

The main purpose of teaching and learning mathematics is to develop the individual to solve mathematically related problems in the society. Mathematical problems have more than one way to arriving at the solution. Enabling learners to find solutions to mathematical problems in more than one way has prospects. However, the study revealed that the curriculum documents neither provide adequate direction to empower teachers nor emphasise the use of MSA in problem solving. Consequently, teachers are limited by the curriculum provisions. Although some teachers in this study exhibited clear conceptions and knew the value of MSA in teaching, they hardly translated their conceptions and values into practice. Teachers do not make conscious effort to encourage MSA in their practice. We conclude that although the curriculum policy requires the use of MSA, practicing the policy is hindered by limited curriculum guidelines and teachers know-how. If MSA is to be valued and practiced in the Ghanaian classroom, conscious efforts must be made to explicitly incorporate it in the curriculum documents to emphasise its use in the classroom and in teacher education. If teachers are well informed about MSA and have the skills to practice it, students would develop the mathematical skills and competencies for solving problems in different ways.

Recommendations

The study only examined teachers' conceptions and practices of MSA without looking at their pedagogical knowledge base that underpinned their practice. Also, it did not look at the achievements of students of students taught using MSA and those who are not. Studies in these domains would be exciting.

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