

*Full Length Research Paper*

# University students' metacognitive failures in mathematical proving investigated based on the framework of assimilation and accommodation

Nizlel Huda<sup>1\*</sup>, Subanji<sup>2</sup>, Toto Nusantara<sup>2</sup>, Susiswo<sup>2</sup>, Akbar Sutawidjaja<sup>2</sup> and Swasono Rahardjo<sup>2</sup>

<sup>1</sup>PMIPA Mathematics Department, Jambi University, Jambi, Indonesia.

<sup>2</sup>Mathematics Department, State University of Malang, Indonesia.

Received 25 February, 2016; Accepted 25 April, 2016

This study aimed to determine students' metacognitive failure in Mathematics Education Program of FKIP in Jambi University investigated based on assimilation and accommodation Mathematical framework. There were 35 students, five students did not answer the question, three students completed the questions correctly and 27 students tried to solve problems but unfortunately made the same mistake. Out of 27 students involved in the study, two students were taken as the research subjects. The research was a qualitative research; while the research instruments is the test items on proving the mathematical equations. The research data was the result of the students' works and transcripts of interviews about the activities of metacognition of the problem solving. The benefits of the research could be used as a material consideration and metacognitive information regarding the failure of students in mathematical proofs. The results were obtained from the data from two research subjects, namely student one (S1) and student two (S2). The S1 used assimilation process as much as 7 times and the accommodation process as much as 4 times with failure metacognitive, such as metacognitive blindness, mirage metacognitive, and metacognitive vandalism. The S2 used a process of assimilation as much as 12 times and accommodation process as much as 6 times with the metacognitive failure only the metacognitive vandalism.

**Key words:** Metacognition failure, the framework of assimilation and accommodation, mathematical proofs.

## INTRODUCTION

Proving was an activity that could not be separated from mathematics. This was because the structure of the mathematical form was the theorems which must be substantiated or proved related to its truth. Mathematical

proving was taught to learners when they were dealing with a problem that was not commonly encountered in problems solving task (Zaslavsky et al., 2012).

According to Hernadi (2013), the proofs in mathematics

\*Corresponding author. E-mail: nizlel@yahoo.com.

include: What the evidence was (what was the proof), the reason we prove (why do we prove), what do we need to prove (what do we proof) and the mechanism to prove (how do we prove). Further Hernadi explained that there were at least six motivational importance to prove, namely: To prove a fact with certainty (to establish a fact with certainty), to gain an understanding, to communicate the ideas with others, to challenge, to be creative to create something beautiful, to construct a mathematical theory that was much broader.

Proving and reasoning had an important role in the learning of mathematics (Verghese, 2009). The statement was coherence with NCTM (2000) that stated that the proving and reasoning become one of the standard processes in the school of mathematics. Process standard contains several indicators, among others, so that the students could develop ideas and explore a phenomenon. There was a relationship between the proving and problem solving. According to Savic (2015) there was an overlap between the proving and problem solving, while Downs et al. (2013) suggested that the problem solving aspects were within the proving. The test item for proving could also be seen as an item for problem solving. This was in accordance with the opinion of Weber (2001) who argued that the proving could be seen as a process of problem solving.

Problem-solving activities were closely related to the metacognitive activity called metacognition. The metacognitive function which was to encourage or trigger the problem solver to switch to the various problem-solving stages used in understanding a problem, planning the completion strategy, making decisions about what to do, and make the decision about it.

Metacognition was thinking to think (Schoenfeld, 1992; Toit and Kotz, 2009; Flavel, 1979; Brown, 1978; Garofalo and Lester, 1985). Metacognition components consist of metacognitive knowledge and metacognitive regulation (Brown et al., 1978; Veenam et al., 2006; Scott and Leviy, 2013).

There were three functions of metacognition, namely (1) metacognitive awareness relating to individual awareness of where they were in the learning process, (2) metacognitive regulation which occurs when individuals modify their thinking (3) metacognitive evaluation refers to the individual could make a decision on the effectiveness of thinking and the strategy chosen (Wilson and Clarke, 2004; Magiera and Zawojewsky, 2011).

Based on the opinion of Magiera and Zawojewski (2011), the indicators of metacognition activity could be structured as follows: (1) An indicator of metacognitive awareness include: (a). Consciousness in the process of thinking about what was already known; (b) Awareness in the thought process of trying to remember had to solve the problem like that before; (c) Awareness in the thought process to remember what had been done by past could help the problem solving at the time; (d) Awareness in the

Prove: $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
--

Figure 1. Proving of the test item.

thought process to find out what had been done, and (e) Awareness in the thought process to determine the type of problem; (2) Indicators of metacognitive regulation include: (a) Setting the thought process to make the settlement plan; (b) Setting the thought process to create a different way to solve the problem; (c) Setting process of thinking about what to do next, and (d) Adjustment of the process was thought to change the way work; (3) An indicator of metacognitive evaluation include: (a) Evaluate the process of thinking about how to do it; (b) Evaluate whether the thought process had done what was planned; (c) Evaluating whether the thought process of thinking was correct; (d) Evaluate the process of thinking to be able to do.

Metacognitive process was intended to make people keep thinking on the right track. Some researchers had suggested that metacognitive process could improve the results of solving the problem (Artz and Amour-Thomas, 1992; Goos and Galbarath, 1996).

The problems encountered among the under-graduate students of Mathematics Education department were that they could not produce the valid proof of a statement. The researcher of the recent study gives a test item of proving to the 5th semester of under-graduate students of Jambi University in Mathematics Education department as in Figure 1.

One of the students' answers on the questions above could be seen in Figure 2. In the figure, the particular student failed to do the metacognitive thinking process that resulted in the wrong answer in proving the equation.

The metacognition process failure could lead to failure of metacognitive. According to Goss (2002) and Stillman (2011), there were three types of failure metacognitive, namely: (1) Blindness metacognitive was a failure of metacognitive when someone made a mistake in the process of problem solving and was not aware of the "red flag". Its indicators: (a) Use strategies wrong; (b) Ignore the incorrect calculations. (2) Mirage metacognitive was a failure of metacognitive when someone does not make mistakes but realize it as a "red flag". Its indicators: (a) Any work causing the deadlock; (b) Change the problem so it was not in accordance with the structure of the concept. (3) Vandalism metacognitive a failure metacognitive marked discrepancies to the concept and context of the issue when responding to "red flag". Its indicators: (a) Change the problem so that the degree of difficulty to be lost; (b) An error to use the strategy; (c) Changing the calculation but contain errors; (d) Reject the correct answer. Metacognitive failures could occur

Jwb:  $\frac{n(n+1)(2n+1)}{6}$

$n = 1^2 + 2^2 + 3^2 + \dots + n^2$

maKa:  $\frac{14(14+1)(2(14)+1)}{6}$

$= \frac{240}{6}$

$= 40$

Mahasiswa mengalami kesalahan dalam proses berpikirnya

## Translated version

Answer:  $\frac{n(n+1)(2n+1)}{6}$

$n = 1^2 + 2^2 + 3^2 + \dots + n^2$

than:  $\frac{14(14+1)(2(14)+1)}{6}$

$= \frac{240}{6}$

$= 40$

Students experience an error in his thinking process

Figure 2. The student's answer.

accompanied by the activity of metacognition (Stillman, 2011).

In doing the problem solving activities using metacognitive processes, it could occur "red flag". The metacognitive "Red flag" indicated a need for someone to stop or re-examine the problem solving process (Goos, 2002; Stillman, 2011). "Red flag" may appear on the stage of the problem solving process and could happen in metacognitive activities (Stillman, 2011). According to Goss (2002) and Stillman (2011), there were three "red flags" that happened and could identify the metacognitive failures, as it was mentioned as follows: (1) There was no progress in the process of finding a solution (lack progress); (2) Detection of an error (error detection) in the troubleshooting process; (3) Their ambiguous on the final answer (anomalous result).

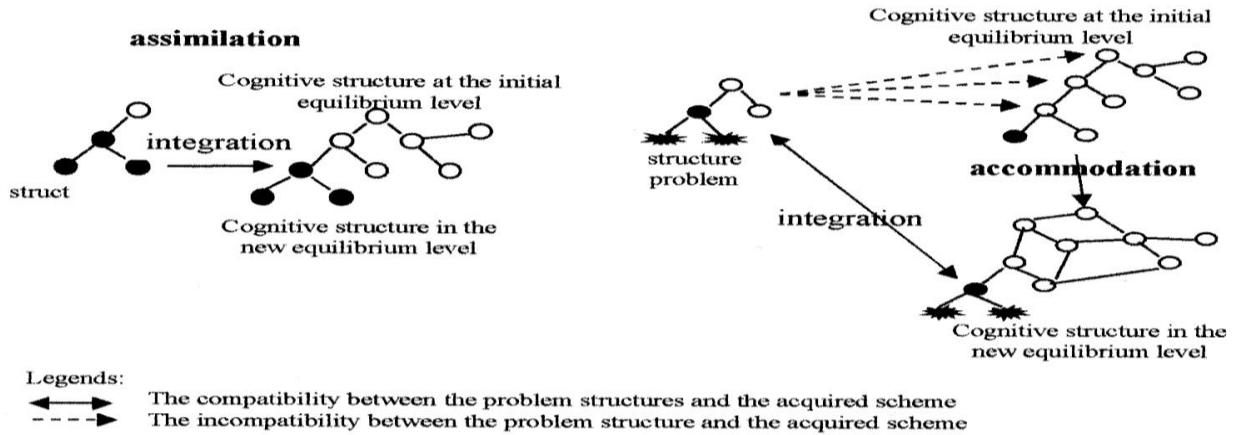
Metacognitive failures in problem solving could be assessed based on the framework of assimilation and accommodation. Piaget explained that in the case when a person interacted with the environment, there would be a cognitive process, namely assimilation and accommodation (Rajiden and Ahman, 2015). In the case, when a person interacted with the environment, there

would be a process of adaptation. At the time of adapting, someone experienced two cognitive processes, namely assimilation and accommodation, as shown in Figure 3.

According to Piaget, the assimilation process occurred when a child brought new knowledge into their existing schemes and the accommodation process occurred when children replaced their scheme to match new information or knowledge (Ultanir, 2012). The balance between assimilation and accommodation was called state of equilibrium and disequilibrium occurred when the child was in a new environmental phenomenon that did not fit into the child mental schemes (Blake and Pope, 2008). In research, the researcher investigated the failure of metacognitive based on the theory of Goos (2002) and Stillman (2011) and the subsequent failure was analyzed based on the framework of assimilation and accommodation.

## RESEARCH METHODS

The research was a qualitative research. Researchers conducted the study toward the under-graduate students of Mathematics Education Department in Jambi University. Out of the thirty-five



**Figure 3.** The Process of assimilation and Accommodation (Adopted from Rajiden et al, 2015).

students involved in the recent study, five students did not answer the question, three students completed the questions correctly and twenty-seven students unfortunately were found to make the same mistake to solve the test item. Therefore, it had been decided to select two college students from those twenty-seven students as the research subjects. The answers of the students were analyzed to investigate the students' metacognitive failures based on the framework assimilation and accommodation that include; (1) metacognitive blindness, (2) mirage metacognitive and (3) metacognitive vandalism. The data was collected by providing the proving test items to the students, and then they were interviewed about the activities of metacognition in solving those test items which were characterized by the "red flags" that occur while the students worked on the test items.

**FINDINGS**

The findings in the research on metacognitive failures investigated based on the framework of assimilation and accommodation was gained through investigating the process of students' thinking in mathematical proving, which includes; (1) metacognitive blindness, (2) metacognitive mirage and (3) metacognitive vandalism by "red flags" that occurs during the process of the test items solving.

The notations used to describe the findings of the recent study are given in Table 1.

**The thinking process of student one (S1) investigated based on assimilation and accommodation framework**

The thinking process of the student one (S1) investigated based on assimilation and accommodation framework in solving the test items could be seen in Diagram 1.

Based on the Diagram 1, the process of student thinking was dominated by the process of assimilation. At the beginning of the process of metacognition the student one (S1) realized in the thinking process that the test item was familiar, and then to try to recall what she already

knew. In the case there was a process of assimilation in the process of thinking of metacognition of student one. The particular student realized that she might have ever worked on a matter like this. The student also set up her thinking processes to recall what he had done before when solving the problems like what he had in the present. The student could recall what she had been done but she forgot how to solve the problems. It was known from research interview with the student as follows:

Researcher: "Were you trying to remember if you ever worked on a matter like this?"

Student one (S1): "I remember I once worked on a matter like this in the Introduction to Basic Math course. I know what was known and what was asked in this matter, but I forgot how to solve this problem (Awareness)".

During the time of solving the test items, the student one (S1) experienced the disequilibrium stage about what to do first. The student then adjusted her thinking process for making a settlement plan that started from the equation  $a$  was more than  $b$ , as shown in Figure 4.

The student one (S1) realized if  $a$  was more than  $b$  then  $b$  must be added to something, say for example  $x$  was added to  $b$ , the value of  $b$  would remain the same if  $c$  was smaller or equal to  $0$ . By performing the setting and evaluation on the thinking process, the student was in the process of accommodation of the thinking process. This was proven from the finding of interview with research subject, the student, as follows:

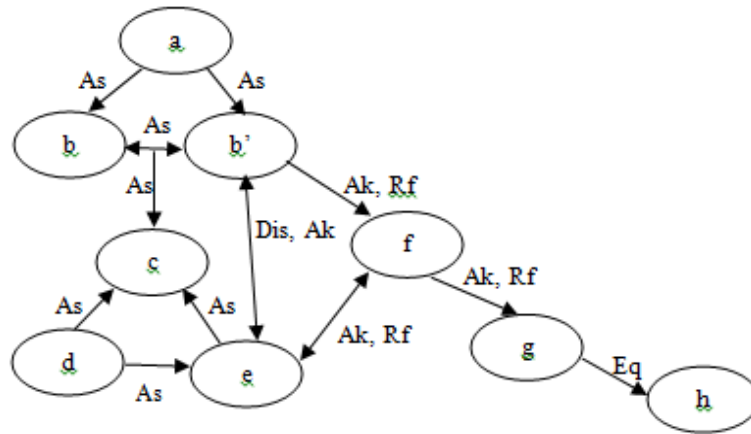
Researcher: "What do you think about  $a > b$ ?"

Student one (S1): "If  $a$  was greater than  $b$ , then  $b$  must be added to something, say for example  $x$ , but I got confused about  $c$  was less than or equal to  $0$  (regulator and evaluation)".

Furthermore, the student one (S1) evaluated her thinking processes if  $x$  was less than or equal to  $0$  then the

**Table 1.** The notations and the notations' meanings.

Notations	Meanings
A	Test item: Mathematical proving
B	The first given requirement: $a, b, c \in R$
B'	The second given requirement: $a > b$
C	The requirement that must be fulfilled: $ac \leq bc$ and $c \leq 0$
D	The average requirement: $ac \leq bc$
E	The necessary requirement: $c \leq 0$
F	The meaning of: $a$ was more than $b$
G	The multiplication of the negative number
H	Finish
I	The multiplication of the real number
J	The multiplication of one real number with zero
K	The multiplication of one positive real number to one negative real number.
L	When $a$ was more than $b$ , and then multiplied to $c$ was $0$ (zero)
M	When $a$ was more than $b$ , and then multiplied to $c$ the result was less than $0$ (zero).
N	The combination of $l$ and $m$
O	$c$ was equal to $0$ (zero)
O'	$c$ was less than $0$ (zero)
As	The process of assimilation happens
Dis	Disequilibrium
Ak	The process of accommodation.
Rf	Red flag
Eq	Equilibrium



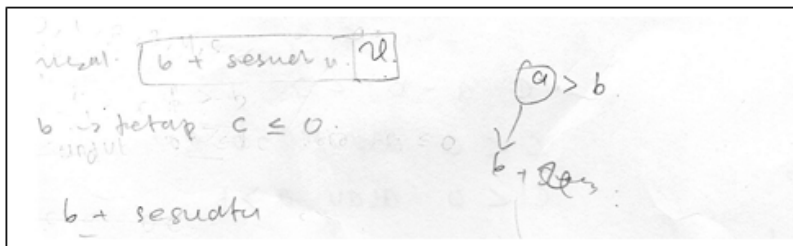
**Diagram 1.** The metacognition process of the under-graduate students investigated based on assimilation and accommodation framework.

equation  $c$  was less than  $0$  could be applied. If  $c$  was less than  $0$  then the result obtained was negative  $c$ . This could be seen in Figure 5. In this case there was a process of accommodation on the process of metacognition of the research subject, student one (S1).

The student one (S1) evaluated if  $c$  was less than  $0$ , then  $b$  was multiplied by negative  $c$ , it was equal to  $c$  was

less than or equal to negative  $b$  multiplied by  $c$ . Here also aesthetically seen that the student had a process of accommodation in her thinking process. It could be seen from interviews with research subject, the student one, as follows:

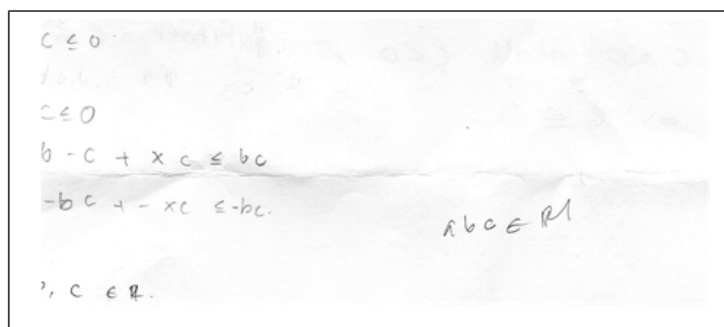
Researcher: "Were you sure that you had the correct



Translated version

For example,  $b + \text{something } (x)$ .  
 The value of  $b$  constant if  $c$  is less or equal to  $0$ .  
 $b$  is added by something.  
 $a$  is bigger than  $b$ , it means that  $b$  should be added by  $x$

Figure 4. The student one (S1)'s statement about  $a$  was more than  $b$ .



Translated version

$c$  is less or equal to  $0$   
 $c$  is less or equal to  $0$   
 $b$  multiplied by negative  $c$  added to  $x$  multiplied to  $c$  which is less or equal to  $b$  multiplied to  $c$   
 negative  $b$  multiplied by  $c$  added by negative  $x$  multiplied by  $c$  which is less or equal to negative  $b$  multiplied by  $c$ .  
 $b$  and  $c$  are the members of  $R$   
 $a, b, c$  are the members of  $R$

Figure 5. The student's answer about the equation  $c \leq 0$ .

answer?"

Student one (S1): "I was confused and not sure of the answer I gave. I was trying to recall if there were other ways I could do to solve the problem".

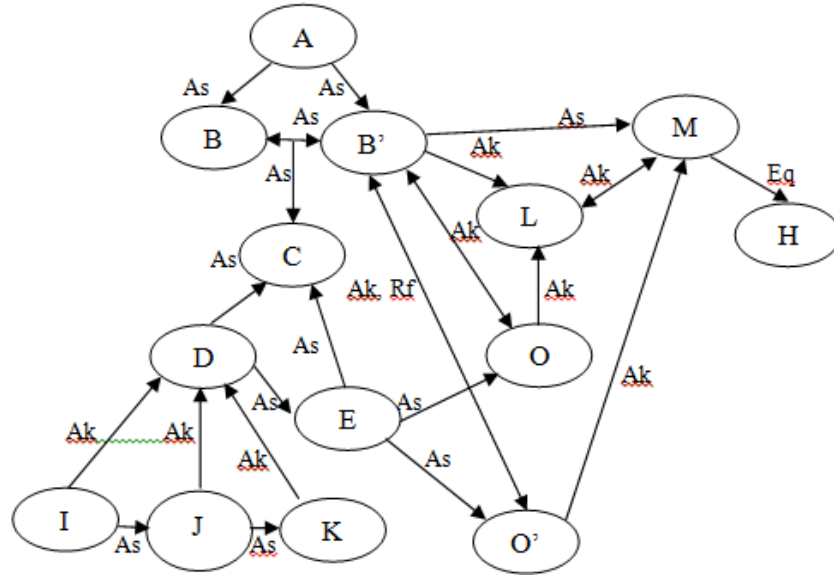
**Metacognition thinking process of student two (S2) investigated based on the framework assimilation and accommodation**

Metacognition thinking process of student two (S2) investigated based on the framework assimilation and

accommodation in solving the test items could be seen in Diagram 2.

In the beginning the process of metacognition of student two (S2) was dominated by the process of assimilation. Student two (S2) realized in the process of thinking that the test items were familiar and tried to re-evaluate what he already knew and whether he had ever worked on a matter like this and what he had done before when solving problems like this. It was known from interviews with the student two (S2) as follow:

Researcher: "Had you ever worked on a matter like this?"



**Diagram 2.** Metacognition thinking process investigated based on the framework assimilation and accommodation.

Misalkan  $a, b, c \in R$  dan  $a > b$ .

- $a \in R$  dan  $c \in R \leq 0$   
 $b \in R$
- Maka setiap perkalian 2 bilangan riil selalu menghasilkan bilangan riil
- Dan setiap perkalian 1 bilangan riil dengan bilangan riil 0 selalu menghasilkan 0
- Untuk setiap perkalian 1 bilangan riil positif dengan bilangan riil negatif selalu menghasilkan bilangan riil negatif.

Translated version

For example  $a, b, c \in R$  and  $a > b$ . While  $a \in R, b \in R$  and  $c \in R \leq 0$ .  
 It can be concluded that every multiplications real numbers will always produce the real number.  
 And every one multiplication of real number to the zero real number will always produce zero.  
 For every multiplication positive one real number to negative will always produce negative real number.

**Figure 6.** Student two (S2) about the meaning of a, b, c as the members of R.

Student two (S2): “I once worked on a matter like this in the Introduction to Basic Math course. I try to recall what I had ever done to solve problems like this (awareness)”.

Furthermore, student two (S2) experienced a process of accommodation in his metacognition process. Student two (S2) then adjusted his thinking process of interpreting a was a member of R, b and c were the member of R as shown in Figure 5.

The student two (S2) evaluated if a was a member of the real numbers, b and c were the members of real

numbers, then the result of multiplying two real numbers as was obtained shown in Figure 6. The student two (S2) also evaluated whether he did it right. The information was obtained from interviews with the student two (S2) as follows:

Researcher: “What do you do after reading the test item?”

Student two (S2): “I know a, b and c were the members of the real numbers. I did check the results of multiplication of two real numbers based on the



dengan demikian, jika  $a > b$  dikalikan dengan  $c = 0$  maka  
 $a = b = 0$ . -- ①  
 jika  $a > b$  dikalikan dengan  $c < 0$  (bil. negatif) maka  
 berlaku  $ac < bc$ . karena jika bil. negatif semakin besar  
 maka nilainya semakin kecil. -- ②

Translated version

Therefore, if  $a$  is greater than  $b$ , and then multiplied by  $c$  which is equal to  $0$  means that  $a$  is equal to be equal to  $0$ .  
 If  $a$  is greater than  $b$ , and then multiplied by  $c$  which is less than  $0$  (negative number) means the value of  $a$  multiplied by  $c$  is less than the value of  $b$  multiplied by  $c$ . it is because if the bigger the negative number, the smaller the value.

**Figure 7.** Student two (S2)'s answer for the equation  $c$  was equal to  $0$  and  $c$  was less than  $0$ .

Dari ① dan ② dapat disimpulkan jika  $a, b, c \in \mathbb{R}$  dan  $a > b$ .  
 Jika  $ac \leq bc$  maka  $c \leq 0$ .

Translated version

Based on statement (1) and (2), it can be concluded that  $a, b, c$  are the member of  $\mathbb{R}$ , and  $a$  is greater than  $b$ . If the value of  $a$  multiplied by  $c$  is less or equal to the value of  $b$  multiplied by  $c$ , then  $c$  is less or equal to  $0$

**Figure 8.** Student two (S2)'s final statement.

requirements given (*regulation and evaluator*)”.

In the process of disequilibrium, the student two (S2) evaluated what would happen if  $a$  was more than  $b$  and  $c$  was equal to  $0$ . Furthermore, student two (S2) also evaluated what would happen if  $a$  was more than  $b$  and  $c$  was less than  $0$ . It could be seen from the results of the student two (S2)'s work in Figure 7.

As shown in Figure 8, student two (S2)'s answer for the equation  $c$  was equal to  $0$  and  $c$  was less than  $0$

The student two (S2) experienced disequilibrium process when student two (S2) organized and evaluated his thinking process in completing the equation  $a$  was more than  $0$  multiplied by  $c$  was equal to  $0$ . The finding could be seen from the result of interview between the researcher of this recent study and student two (S2) as follow:

Researcher: “Then what do you think?”

Student two (S2): “If  $a$  was more than  $b$ , then multiplied by  $c$  which was equal to  $0$  then the result obtained was  $a$  was equal to  $0$ ,  $b$  was equal to  $0$ ”.

Researcher: “Why was  $a$  equal  $0$  and  $b$  equal to  $0$ ?”

Student two (S2): “It was because  $c$  was equal to  $0$ ”.

At the end of problem solving, the student two (S2) organized was thinking process to combine two things that might happen when he multiplied  $a$  which was more than  $b$  to  $c$  which was equal to  $0$  and multiplied  $a$  which was more than  $b$  to  $c$  which was less than  $0$  as shown in Figure 8.

## DISCUSSION

In the beginning, the process of metacognition of student one (S1) was dominated by the assimilation process because the student one (S1) uses her knowledge when there was a new problem. This was in accordance with the opinion of Piaget (Ultanir, 2012) which said that a child brings new knowledge into their own scheme.

Furthermore, student one (S1) was in the accommodation process marked by when student one (S1) declared that  $a > b$  by adding  $b$  with something (eg



x). In facing this case, the student one (S1) replaced her schemes of thinking to match the new information or knowledge (Ultair, 2012) so the student was able to make the different ways of solving problems (Magiera and Zawojewsky, 2011). This resulted in a red flag occurred as a detection of an error (error detection) on the process of student one (S1) metacognition which was an indicator of metacognitive blindness. Therefore, it could be said that student one (S1) experienced the blindness metacognitive which it was some metacognitive failures when someone made mistakes in the problem solving process and did not realize the occurrence of the "red flag" (Goos, 2002). At the time of making the inequality of  $b + x \leq b$ , student one (S1) experienced a metacognitive failure in the thinking process in evaluating whether she figured it out correctly (Goos, 2002). It also caused the "red flag" changed the problem so it did not correspond to the concept of structure (Goos, 2002) which led to the mirage metacognitive which meant some metacognitive failures when one considered himself did nothing wrong but be aware as a "red flag".

In the next accommodation process, the student one (S1) adjusted her thinking process by taking  $c < 0$  and the result was the multiplication of  $-bc -xc \leq -bc$ . This phenomenon was marked with a "red flag" as a detection of an error (error detection) and there was no progress in the process of finding a solution (lack progress) (Goss, 2002). Thus, we could say that student one (S1) experienced vandalism metacognitive which meant some metacognitive failures that were marked by the noncompliance within the concept and context of the issue when responding to "red flag". The indicator was the errors in using the strategies (Goos, 2002).

Student two (S2)'s metacognition process was dominated by the assimilation process since the student (S2) used his knowledge when there was a new problem. This was in accordance with the opinion of Piaget (Ultanir, 2012) which says that a child brought the new knowledge into their own scheme.

Furthermore, when student two (S2) knew  $a \in R$ ,  $b \in R$  and  $c \in R$  then the student (S2) adjusted his thinking process to multiply two real numbers. There were three possible occurrence of the multiplications of two real numbers, namely (1) a positive real number was multiplied by a positive real number, (2) a positive real number was multiplied by a real number 0, and (3) the number real positive number was multiplied by real negative numbers. In this case, there was a process of accommodation since the student two (S2) replaced the scheme of thinking to match the new information or knowledge (Ultair, 2012) so that student two (S2) decided to make a different way of solving problems (Goos, 2002).

Student two (S2) adjusted his thinking process and evaluated when using  $a > b$  to  $c = 0$  which resulted the student (S2) concluded that  $a = b = 0$ . In this case there was a process of accommodation (Goss, 2002) which led

to the "red flag" that was characterized by the detection of an error (error detection) because the student two (S2) adjusted his thinking process to obtain results that were not in accordance with the requirements of  $a > b$ . Thus the metacognitive vandalism occurred which meant some metacognitive failures that marked by the noncompliance within the concept and context of the issue when responding to "red flag". The indicator was the student changed calculation but it contained some errors (Goos, 2002).

## Conclusion

The research subject, the student one (S1) used assimilation process as much as seven times and the accommodation process four times with metacognitive failures were the metacognitive blindness, mirage metacognitive and metacognitive vandalism. The research subject, the student two (S2) used a process of assimilation were as much as twelve times, and the accommodation process were as much as six times with the metacognitive failure was only vandalism metacognitive.

## Conflict of Interests

The authors have not declared any conflicts of interests.

## REFERENCES

- Artz AF, Armour-Thomas E (1992). Development of Cognitive-metacognitive framework for Protocol Analysis of Mathematical Problem Solving in Small Groups. *Cognition and Instruction*, 9(2):137-175.
- Blake B, Pope T (2008). Developmental Psychology: Incorporating Piaget's and Vygotsky's Theories in Classrooms. *J. Cross-Disciplinary Perspective Educ.* 1(1):59-67.
- Brown A (1987). Metacognition executive Control, Self-Regulation and Other More Mysterious, Mechanisms. In Fann Winert & Rainer Kluwe (Eds), *Metacognition, Motivation and Understanding*. London :LEA pp. 65-115.
- Downs JM, Downs M (2013). Problem Solving and Its Elements in Forming Proof. *The Mathematics Enthusiast*, ISSN 1551-3440. 10, 1(2):137-162.
- Flavel J (1979). Metacognition and Cognitive Monitoring. *American Psychologist*, 34:906-911.
- Garofalo J, Lester FK (1985). Metacognition, cognitive monitoring and mathematical performance. *J. Res. Math. Educ.* 16(3):163-176.
- Goos M (2002). Understanding, Metacognitive Failure. <https://espace.library.uq.edu.au/view/UQ:10303/JMB.Meta.failure.pdf>.
- Goos M, Galbraith P, Renshaw PA (2000). A money problem: A source of insight into problem solving action. *Intl. J. Math. Teaching Learning*, pp. 1-21.
- Hernadi J (2013). Metoda Pembuktian dalam Matematika. Makalah Seminar. [https://julanhernadi.files.wordpress.com/2013/11/method\\_of\\_proof.pdf](https://julanhernadi.files.wordpress.com/2013/11/method_of_proof.pdf).
- Magiera MT, Zawojewski JS (2011). Characterization of social-based and self-based context associated with students' awareness, evaluation, and regulation of their thinking small-group mathematical modelling. *J. Res. Mathe. Educ. New York: Macmillan.* 42(3):334-370.

- NCTM (2000). Principles and Standards for School Mathematics, NCTM.
- Rajiden S, Maedi SA (2015). The pseudo-covariational reasoning thought process in constructing graph function of reversible event dynamics based on assimilation and accommodation framework. *J. Korean Soc. Math Educ. Ser. D, Res. Math Educ.* 19(1):61-79.
- Savic M (2015). On similarities and differences between proving and problem solving. *J. Humanistic Math.* 5(2):60-89.
- Schoenfeld AH (1992). Learning to Think Mathematically : Problem Solving, Metacognition and Sense Making in Mathematics. In Douglas A. Grouws (ed) *Handbook of Research on Mathematics Teaching and Learning*. Micmillan Publishing Company, New York, pp. 355-358.
- Scott BM, Leviy MG (2013). Examining the Component of Fuzzy. *Educ. Res.* 2(2):1-12.
- Stillman G (2011). Applying Metacognitive Knowledge and Strategies in Applications and Modelling Task T Secondary Level. In Kaiser, G., Blum, W. Borromeo, F.R., & Stillman, G. *Trends in Teaching and Learning of Mathematical Modelling*, New York: Springer. pp. 165-180.
- Ultanir E (2012). An Epistemological Glance at the Constructivist Approach: Constructivist Learning in Dewey, Piaget, and Montessori. *International Journal of Instruction*, Vol. 1, No.2. e-ISSN: 1308-14070.
- Toit SD, Kotze G (2009). Metacognitive Strategies in the Teaching and Learning of Mathematics. *Pythagoras* 70:55-67.
- Veenam MVJ, Wolters VH, Afflerbach P (2006). Metacognition and Learning: Conceptual and Methodological Considerations. *Metacognition Learning* 1:3-14.
- Vergheze T (2009). IUMPST: Secondary-Level Student Teachers Conception of Mathematical Proof. *J. 1 (Content Knowledge)*.
- Weber K (2001). Student Difficulty in Constructing Proof: The need for Strategic Knowledge *Educ. Stud. Math.* 48(1):101-109.
- Wilson J, Clarke D (2004). Toward Modelling of Mathematical Metacognition. *Mathe. Educ. Res. J.* 16(2):25-48.
- Zaslavsky O, Nickerson SD, Stylianides AJ, Kidron I, Vinicky-Landman G 2012. The Need for Proff and Proving: Mathematical and Pedagogical Perspective. In G. Hanna and M de Villiers (eds), *Proof and Proving in Mathematics Education*. New ICMI Study Series. [http://dx.doi.org/10.1007/978-94-007-2129-6\\_9](http://dx.doi.org/10.1007/978-94-007-2129-6_9).