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PRIMARY STUDENT TEACHERS' PERSPECTIVES OF THE TEACHING OF FRACTIONS

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Abstract: The aim of this study was to investigate primary student teachers' perspectives of the teaching of fractions, i.e. their PCK of fractions. The research design used for the study was a descriptive survey method. As data collection instrument, we conducted a questionnaire composing of 14 open and closed-ended questions. The questionnaire was administered to 126 third grade primary student teachers at the department of primary teacher education in Sinop University. There were 91 females and 35 males. In the analysis of open-ended questions, the participants' responses were qualitatively analysed to characterize patterns and categorize answers. Questionnaires were analysed by using open coding. The results of the study indicated that, when saying fractions, first thing which comes in the student teachers' mind is the meaning "part-whole" of fractions. By supporting on the relevant literature, it can be asserted that, only this meaning is not sufficient to provide better understanding of fractions among students. In parallel, activities that most of the student teachers prefer for the introduction to fractions are closely related to this meaning such as dividing a cake, a breath, etc. into equals parts or shading a region.

Key words: Mathematics teaching, primary education, fractions, teacher training.

1. Introduction

Mathematics is generally known as a discipline having a hierarchical structure. Mathematics subjects have a more ordered form than other lessons. Therefore, mathematics creates itself without any external contribution (Altun, 2005). Fractions are also one of the important concepts that allow us to understand this hierarchical structure of mathematics. Student' lack of understanding fraction concept raises many problems in the following topics such as fraction computation, decimal and percent concepts, and other concept areas which require the use of fractions, e.g., algebra (Brown & Quinn, 2007; National Mathematics Advisory Panel, 2008). Based on the results of NAEP tests, some researchers underline that fractions can also pose grave problems for students even into the middles grades (Sowder & Wearne, 2006; Wearne & Kouba, 2000). From some aspects, we think that it is normal to consider these challenges. As known, until their first encounter with fractions, students have represented whole quantities by using whole numbers. According to constructivism, new knowledge and meaning are constructed on the previous ones. When meeting situations with fractions, students naturally try to solve problems by using what they know about whole numbers. Based on the relevant research, Van de Walle, Karp and Bay-Williams (2010) identify the reasons of students' difficulties in fractions as follows:

Fractions include many meaning such as part-whole, measurement, division etc. Understanding fractions needs to understand all of these different meanings.

The written of fractions is an unusual for students.

The conceptual understanding of fractions is ignored in instructions.

Whole-number knowledge is overgeneralized by students.

Unfortunately, fractions are generally reduced to only a meaning part-whole by both textbook writers and teachers. As already mentioned, there are many meaning of fractions. Focusing on only one of them is not sufficient to understand fractions completely. Thus, the researchers such as Clarke, Roche and Mitchell (2008) and Siebert and Gaskin (2006) argue that students would better understand fractions with other meanings. We believe it would be very interesting to consider what and how many of these

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meanings of fractions are mentioned by the student teachers participating in this study. These meaning are summarized as follows (Alacacı, 2009; Wan de Walle, Karp & Bay-Williams, 2010):

Part-whole: It is the most known and used meaning. A whole is divided into equal parts and we select or take some of them. Sure, this whole can be a group of people or a length.

Division: This meaning is rather encountered in the situations of sharing. There is equally sharing some quantities between some people or things such as sharing 20 pens with 5 students.

Measurement: Sometimes, fractions are used to identify a length or a measurement piece to determine the length of an object. This meaning refers that fraction represent measurement of quantities such as length, area, weight or volume which are unable to be represented with whole numbers.

Operator: This meaning refers to enlarge or reduce a certain quantity. For instance, an image can be enlarged or reduced at a rate of 3/4 with a photocopier. After that, we can ask students what to do in order to restore it, i.e., at what rate it is necessary to enlarge or reduce the image. It is not difficult to consider the relationship between operator meaning and multiplication of fractions. In other terms, examples based on this meaning can help to understand the multiplication of fractions (Charalambous & Pitta-Pantazi, 2005).

Ratio: The ratio is another meaning of fractions. For instance, 2/5 can be considered to be the probability of an event being two in five. Ratios can be expressed in two different ways. The one is part-part such as the ratio of boys to girls in class and the other one is part-whole such as the ratio of boys to all class.

As there are many factors that can influence it, mathematics teaching is a complex process. Sure, one of the most important factors is the teacher. The belief that the well-teaching of mathematics strongly requires an adequate mathematic background is very common in society (Baştürk & Dönmez, 2011). Sure, the importance of the subject matter knowledge is not deniable, but only having this kind of knowledge is not sufficient to be an effective teacher. In the literature, we consider that teacher knowledge is defined in different ways and its many components have been discussed (Fennema & Franke, 1992; Grossman, 1990; Hill, Ball, & Schilling, 2008; Shulman, 1986). With his studies on teacher knowledge, Shulman (1986, 1987) is one of the first researchers in this subject.

Content knowledge and pedagogical knowledge were considered to be independent from each other for years. Shulman (1986) combined them and called this new component of teacher knowledge as pedagogical content knowledge (PCK). According to Shulman, PCK is as follows: "The most useful forms of content representation, the most powerful analogies, illustrations, examples, and demonstrations-in a word, the ways of representing and formulating the subject that makes it comprehensible for others" (p. 9). This knowledge also includes "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9).

In mathematics education, there are some information resources for the teaching of a concept. The teacher is one of the most important of them. Therefore, the identification of teachers' perspectives of teaching and learning mathematical concepts may contribute to develop their teaching and learning (Baştürk & Zeybek, 2007). Teacher knowledge on subject matter, students' learning difficulties, teaching strategies, representations, curriculum etc., provide us to realize insight into the nature of mathematics teaching. As student teachers reflect the profile of teachers in future, examining such knowledge becomes more important. Thus, this study aimed to investigate primary student teachers' perspectives of the teaching of fractions, i.e. their PCK of fractions. All the education to be given to student teachers during their training should provide them with required knowledge and skills for, and positive attitudes towards the profession they will do (Baştürk, 2009a; Johnson & Howell, 2005; Senemoğlu & Özçelik, 1989). Identifying student teachers' PCK of fractions is quite important in the context of helping us understand at what points the present teacher training programs should be reviewed and improved. As known, one of the most important missions of teacher training programs is to determine how to train an effective teacher, how to equip student teachers with the characteristics of such a teacher and to evaluate the current program. Therefore, it can be asserted that the results of this study will contribute to improve the teaching of fractions in teacher education.

2. Method

The research design used for the study was a descriptive survey method. Thus, student teachers' perspectives of the teaching of fractions were identified without influencing them in any way. This kind of researches has a pioneering role to quantitative research designs. As quantitative experiments often take a lot of time and are expensive, descriptive research design is very useful to decide what is worth studying (Shuttleworth, 2008).

2.1. The participants

The study was conducted with 126 third grade primary student teachers at the department of primary teacher education in Sinop University that is a public university in the north of Turkey. There were 91 females and 35 males. The courses related to mathematics and mathematics teaching taken by primary student teachers until the experience were basic mathematics I (2 credits), basic mathematics II (2 credits), mathematics teaching I (3 credits) and mathematics teaching II (3 credits). The content of these courses are presented in Table 1. As can be seen from the table, the research group were able to understand and answer to a questionnaire designed to determine student teachers' subject matter knowledge and PCK of fractions.

Table 1. *Content of the courses related to mathematics and math teaching*

Courses	Content of the courses
Basic mathematics I	Definition of mathematics, relation with other science. Sets and operation at sets. Foundation of number system, natural number. Different based number, integer. Divisibility rules, Greatest Common Factors (GCF) and Least Common Multiple (LCM) concepts and applications. Ratio and proportion, compound ratio concepts and application. Real number, exponentiation and rational exponents. Cartesian product, correlation, function, operation concepts and their graphics. Equality and ordering correlation. Data collection and summary of data, their graph, measure of central distribution.
Basic mathematics II	Algebraic expression, equation and identity concepts, operation at algebraic expressions, equation and inequality concepts, system of equation and inequality. Two variable functions, graph of continuously and discontinuous functions. Foundation of geometry. Plenary Shape and their area and circumference. Solids and their area and volume. Congruent, similarity, right triangles, correlations of right triangles, geometric place and basic drawing. Fundamental concepts of trigonometry. Analytical examination of line and circle.
Mathematics teaching I	Historical development of teaching mathematics, teaching methods in mathematics education, learning-teaching process, primary mathematics education program, adaptations of general teaching methods to mathematics education, special teaching methods and strategies in mathematics education, measure and evaluate in mathematics education, problem solving, concept of number, natural numbers, basic operations with natural numbers.
Mathematics teaching II	Fractions, students' misconceptions about learning fractions, different meanings of fractions, fraction models, equality, compare, set up in order, operations with fractions, decimal fractions, operations with decimal fractions, suitable activity examples for targets of programme, geometry, development of geometric thinking of children, teaching 2 and 3-dimensional geometry concepts, suitable activity examples for targets of geometry in programme, measure, development of thinking about measure of children, dimension, field, volume, time measures, weigh, money, suitable activity examples for targets of measure in programme, data management, tables and graphics, suitable activity examples for targets of data in programme, measure and evaluation in mathematics education, multiple measure and evaluation techniques.

2.2. Data collection and procedures

In order to understand the primary student teachers' perspectives of teaching fractions, a questionnaire composing of 14 open and closed-ended questions was administered. Based on the objectives of fractions of Turkish elementary mathematics curriculum, the studies on student teachers' PCK and students' misconceptions about fractions (Baştürk, 2009b; Eroğlu, 2012; Chang, 1997; Haser & Ubuz, 2003; Johnson, 1998; Mack, 1990; Pesen, 2007; Soylu & Soylu, 2005), the questionnaire items were developed. The questionnaire aimed to determine the student teachers' content knowledge and PCK (e.g., teaching methods, learning difficulties, students' misconceptions about fractions, their reasons and overcoming strategies for these misconceptions). In this paper, we only presented the results of the analysis of three open-ended questions directly related to the research problem.

2.3. Data Analysis

The participants' responses to three questions were qualitatively analysed to characterize patterns and categorize answers. The analysis was based on using open coding. It is the part of analysis that pertains specifically to the naming and categorizing of phenomena through the close examination of data (Yıldırım & Şimşek, 1999). In order to test the inter-judge reliability, the researcher and two experts from the department of primary teacher education examined data and response categories. Disagreements between the experts were solved by discussing and at this way it was tried to arrive a great common agreement on categories and coding (Lincoln & Guba, 1985).

3. Results

In this section, the results of the analysis of the questions will be presented based on the frequency and percentage tables and quotations from typical responses.

3.1. First things coming to student teachers' mind when saying fractions

In the first question, the student teachers were asked to indicate what is coming to their mind when saying fractions. With this question, we aimed to consider their "concept images" of fractions in the terminology of Tall & Vinner (1981). The results are indicated in Table 2:

Table 2. *Fractions According to Student Teachers*

Categories	*f*	*%*
Category 1 (C1): Part-whole	58	46
C2. Statements containing elements of fractions such as numerator, denominator, fraction bar etc.	54	42,9
C3. Division	37	29,4
C4. Type of fraction (improper, proper, equivalent, etc.)	13	10,3
C5. Sharing or equal distribution	12	9,5
C6. Examples comprised of partitioning a whole into fractional parts	1	0,8
C7. Numbers able to be written decimals	1	0,8
C8. Non-integer	1	0,8
Other responses	2	1,6
Non response	0	0

When saying fractions, first things the most of the student teachers think are related to the part-whole meaning of fractions (58%). 54% of them indicate elements of fractions such as numerator, denominator, and fraction bar etc., while 37% give responses evoking the division meaning. On the other hand, there are 13% and 12% who answer to the question by respectively writing type of fractions (improper, proper, equivalent, etc.) and sharing or equal distribution. As a result, it can be asserted that two meanings of fractions are more common among the student teachers: the part-whole meaning and the division meaning. In addition, based on the categories C2 and C4, we can suppose that fractions' types and elements take an important place in the fractions learning and teaching of the student teachers. Some excerpts from student teachers' typical responses are given below:

> Parts which are selected from a whole (Student Teacher98).

> Fractions mean the expression of case of a part in whole. We use fractions to symbolize that this part corresponds to how many portion of the whole (ST33).

> 3/4, Numbers which have numerator and denominator (ST110).
>
> Fraction bar first comes to my mind, then slice of cake, graphs and figures (ST51).
>
> When saying fractions, the ratio of a number to another comes to my mind such as a/b (ST114).
>
> There are numerator and denominator. If the numerator is less than the denominator, the fraction is called proper. If the numerator is greater than the denominator, the fraction is called improper (ST11).

As a result, the student teachers' first images of fractions largely focus on the part-whole and division meanings of fractions. The other meanings like measurement, operator and ratio do not exist or they are very limited. On the other hand, the elements used in the writing of fractions such as numerator, denominator, fraction bar etc. firstly appear in many student teachers' mind when they hear the word of fraction.

3.2. Introduction to fractions in the first lesson

In one of the questions, we asked the student teachers to briefly describe their introduction to fractions in the first lesson by specifying what figure or figures they draw, what example or examples they give from daily life. Frequencies and percentages of the student teachers' responses are presented in Table 3.

Table 3. *Introduction to Fractions*

Categories	*f*	*%*
C1. Use of materials and examples from daily life (partitioning a breath, cake, apple, etc.)	118	93,7
C2. Using visual shapes especially geometric shapes	43	34,1
C3. Half, quarter, equivalent parts	29	23
C4. Division	23	18,3
C5. Using elements of fractions such as numerator, denominator, fraction bar etc.	5	4
C6. Unit fraction	1	0,8
Other responses	1	0,8
Non-response	1	0,8

Table 3 reveals that the majority of the student teachers (about 94%) introduce to fractions by using materials and examples from daily life such as partitioning a breath, cake, apple, etc. In terms of their sensitivity with respect to the use of concrete examples and materials, this result can be considered to be positive. Using visual shapes especially geometric shapes plays an important role in the introduction to fractions of 34% of the student teachers. Some student teachers (23%) underline the concepts half, quarter, equivalent parts in their first lesson, while some other (18%) introduce to fractions with cases based on their division meaning. All this is reflected in the following excerpts:

> I use an object or a shape which attracts children's attention. For example, a cake. I divide it into equal parts, and distribute them in the manner of one part for the one, one part for the other. Then, I ask students questions such as "How many parts do the cake consist of? How many parts are disturbed? How many parts are left? etc." (ST122).
>
> Firstly, I ask students what are coming to their mind when saying part and whole. Then, I give the definition of fractions. To provide that they can understand fractions, by giving examples I try to lead them to the definition. With the examples such as cake, walnuts, etc., I try to provide the stability of new concept (ST123).
>
> With concrete examples, I introduce to fractions. I draw the shape of a watermelon and divide it into equal parts. Then, I ask students to distribute them to two or three people (ST27).
>
> By drawing attention to myself, I start the lesson. In my teaching, I benefit from geometrical shapes such as circle, square etc. From daily life, I give cake and pizza sharing examples (ST14).

Consequently, to start fractions most of the student teachers choose the use of materials and examples from daily life such as partitioning a breath, cake, apple, etc. Some of them prefer using visual shapes especially geometric shapes. It is possible to conclude that the student teachers' introduction projects to

fractions are mostly based on the part-whole meaning. This is in accordance with the results of the analysis of the previous question.

3.3. Students' difficulties in fractions according to the student teachers

In one of the questions, we asked the student teachers to write students' most frequent difficulties in understanding fractions and how to overcome them. The results of the analysis of their responses are displayed in Table 4.

Table 4. *Students' Obstacles or Difficulties of Fractions*

Categories	*f*	*%*
C1. Ordering fractions in ascending or descending order	32	25,4
C2. Four operations on fractions (adding or subtracting fractions containing unlike quantities)	28	22,2
C3. Understanding proper or improper	25	19,8
C4. Understanding what mean numerator and denominator	20	15,9
C5. Reducing or extending fractions	7	5,6
C6. Recognizing that integer and decimal numbers are also fractions	7	5,6
C7. Equivalence of fractions	6	4,8
C8. Transforming fractions into shapes or vice versa	6	4,8
C9. Comparing fractions	5	4
C10. Problems with fractions	5	4
C11. Identifying fractions on number lines	3	2,4
C12. Non difficulty	2	1,6
Other responses	6	4,8
Non response	9	7,1

As can be seen from the table, the difficulties most frequently mentioned by the student teachers are as follows: Ordering of fractions in ascending or descending order (25%), four operations on fractions (22%), understanding proper or improper fractions (20%) and understanding what mean numerator and denominator in a fraction (16%). Despite their small percentages, reduction or extension of fractions (6%), recognizing integer and decimal number as a fraction (6%), equivalence of fractions (5%), transforming fractions into shapes or vice-versa (5%) and comparing fractions are also among difficulties cited by the participants. The following excerpts illustrate very well our analysis on the student teachers' responses:

> Students have difficulties in understanding that fractions such as 3/6 and 1/3 mean the same thing. If I were a teacher, in order to teach in a better way, I would present this case with shapes (ST12).

> I believe that they (students) have difficulties in ordering fractions in ascending or descending order. To prevent this, it is able to benefit from concrete objects (ST26).

> Students may have difficulties in adding and subtracting fractions, especially when their denominators are not the same. I think that one can teach by visualizing (ST110).

> According to me, ordering fractions whose, the denominators are not the same may be difficult for students, because I also had difficulties in this subject. There can also have difficulties in reducing or extending fractions. To overcome difficulties, one can give more examples and solve more problems on this subject (ST32).

> They (students) have problems in converting improper fractions to proper and vice-versa (ST25).

> Students may not understand the relationship between whole and part. Examples underlining this relation more clearly, may help them to understand it better (ST41).

In conclusion, the student teachers' perspectives of students' difficulties and misconceptions of fractions are rather focused on the practice applications like ordering fractions, four operations in fractions etc. The number of the student teachers who mention the difficulties and misconceptions resulting from the conceptual understanding of fractions is very limited.

4. Conclusion and Discussion

This study aimed at investigating primary student teachers' perspectives of the teaching of fractions, i.e. their PCK of fractions through a questionnaire which was administered to 126 third grade primary student teachers. In the faculties of education, it is very important that student teachers should be well-equipped with subject knowledge and PCK. Thus, we think that the results of the present study will contribute to teacher training process.

The student teachers' content images of fractions are rather based on the part-whole and division meanings of fractions. The other meanings such as measurement, operator and ratio were very limited or did not exist. We assume that there can be two reasons: the one is that the part-whole meaning is an effective starting point for building meaning of fractions (Cramer & Whitney, 2010), and in line with the first, the other one is that the part-whole meaning is commonly used by textbook's writers and teachers (Van de Walle, Karp, & Bay-Williams, 2010). On the other hand, a significant proportion of the student teachers gave responses which refer elements of fractions such as numerator, denominator, fraction bar, etc. In order to explain it, we can propose a hypothesis that the nature of the student teachers' learning (and sure teaching) is (will be) based on external indices (i.e., "instrumental understanding") rather than on "relational understanding" (Skemp, 1977). Sure, with researches to be conducted in this subject, this hypothesis should be tested.

With regard to their introduction to fractions in the first lesson, in line with their first images of fractions, most of the student teachers choose the use of materials and examples from daily life such as partitioning a breath, a cake, an apple, etc. Starting from the same point, some of them prefer using visual shapes especially geometric shapes. As a result, it can be asserted that the student teachers' content images and introduction projects to fractions are very limited and rather based on the part-whole meaning of fractions. From the relevant literature, we can claim that only this meaning is not sufficient to provide better understanding of fractions among students (Clarke, Roche, & Mitchell, 2008; Siebert & Gaskin, 2006).

The studies about fractions have identified many misconceptions of students. The main reason for these misconceptions is that, in the teaching of fractions, one early passes to operations and numerical representations without understanding the important elements of fractions such as dividing whole into equal parts, identifying unit, unitizing and re-unitizing etc. (Bezuk & Bieck, 1993). Therefore, it is clear that if we want to enrich students' understanding of fractions and to help them overcoming their misconceptions, we should foremost start by overcoming the shortcomings of student teachers in this subject. Eroğlu (2012) highlights that teacher education programs have to make student teachers be familiar with students' common types of mistakes and sources of these mistakes. Knowledge of students' mistaken thinking processes would help them to prepare their lessons and teach mathematics effectively. Sure, the improvement of knowledge about students' thinking will allow student teachers to become more aware of and sensitive to students' needs and understanding. Therefore, it is important to develop student teachers' awareness and knowledge about students' mistakes and difficulties. When we look at the student teachers' opinions on students' difficulties and misconceptions of fractions, we consider that they are closely related to the results of the previous questions. The student teachers' nature of fraction learning are rather constructed on the instrumental understanding. Therefore, the difficulties and misconceptions mentioned by them are also instrumental rather than conceptual such as ordering fractions, four operations on fractions, understanding proper or improper, reducing or extending fractions etc.

As a result, the student teachers participating in this study mostly use the part-whole meaning of fractions to explain their first fractions images and to plan their introduction lesson to fractions. Their knowledge of students' mistakes (including misconceptions and difficulties) are limited to procedural mistakes. With the present study, it was tried to explore the primary student teachers' perspectives of the teaching of fractions. But this supported only what they wrote. However, in order to better understand the results of this study further research should find out how the student teachers will reflect their perspectives of teaching fractions on their teaching activities.

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