

THE TRIANGULATION ALGORITHMIC: A TRANSFORMATIVE FUNCTION FOR DESIGNING AND DEPLOYING EFFECTIVE EDUCATIONAL TECHNOLOGY ASSESSMENT INSTRUMENTS

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ABSTRACT

This paper discusses the implementation of the Tri-Squared Test as an advanced statistical measure used to verify and validate the research outcomes of Educational Technology software. A mathematical and epistemological rationale is provided for the transformative process of qualitative data into quantitative outcomes through the Tri-Squared Test as an efficient technique for the rapid testing of software applications designed for the educational setting. The design of fixed-parameter algorithms for software engineering problems can prove to be beneficial to the Educational Technologist who has to efficiently and rapidly evaluate a software tool for deployment. This novel methodology provides elegance and has a wide range of applicability to practically address important and relevant problems in the "trial and error" process of application evaluation. This research is the continuation of a dynamic mixed methods approach that is a transformative process which changes qualitative data into quantitative outcomes through the Tri-Squared statistical measure first introduced in i-manager's Journal of Mathematics.

Keywords: Algorithmic Model, Algorithmics, Cartesian Coordinates, Geometric Vectors, Mathematical Models, Meta-Cognitive Analysis, Research Engineering, Triangulation, Trichotomy, Tri-Squared, Tri-Squared Test, and Vectors.

INTRODUCTION

One of the most challenging areas of research in education involves the construction of specific instruments that are designed to measure qualitative outcomes and data. Although there are a great many measurement tools that analyze the cognitive and psychomotor domains, there remains a vacuum in the number of instruments especially designed to accurately measure the adequacy of specific software designed for the educational setting. This void is further expanded when the specific event under investigation is unique, specialized, has specific characteristics, serious legal constrictions, and issues regarding time. This often requires the research investigator to design an instrument that ideally measures the variables under investigation.

The Tri-Squared Test is an efficient and elegant technique that was introduced in 2012. It is a four step process for determining the appropriate Research Effect Size, Sample Size, and Alpha Level (Osler, 2012). The Tri-Squared research procedure consists of a four step approach designed to provide the researcher with a clear and

precise set of data to conduct research, analyze data, and determine the level of significance required to either validate or reject the initial research hypothesis.

The process of designing instruments for the purposes of assessment and evaluation is called "Psychometrics". Psychometrics is broadly defined as the science of psychological assessment (Rust&Golombok, 1989). The Tri-Squared Test pioneered by the author, factors into the research design a unique event-based "Inventive Investigative Instrument". This is the core of the Trichotomous-Squared Test. The entire procedure is grounded in the qualitative outcomes that are inputted as Trichotomous Categorical Variables based on the Inventive Investigative Instrument. The specific assessment of the variables is completely dependent upon the outcomes determined by the researcher's instrument. The creation, production, and deployment of the trichotomous Inventive Investigative Instrument requires that the research investigator adopt the role of a "Trichotomous Psychometrician" (Osler, 2013a). A "Trichotomous Psychometrician" is an Educational Scientist that uses

trichotomous-based psychometrics to develop a qualitative Inventive Investigative Instrument specifically designed capture qualitative responses during a specific event. A description of the entire Tri-Squared research process follows and is described in detail to provide the reader of the precise steps undertaken in the process of developing, designing, and ultimately implementing an Inventive Investigative Instrument (Osler, 2013b).

Defining the Field of Educational Science

The field of "Education Science" is also represented by the term "Eduscience" which is a portmanteau of the two terms "Education" and "Science". Similar to the field of "Bioscience", Eduscience is the study of education wherein applicable sciences (such as ergonomics, statistics, technology, etc.) are applied to enhance and improve learning. The primary purpose of the field of Eduscience is the study and application of solutions to improve and enhance the learning environment and learning in general. Eduscience is solution-driven and is actively concerned with the transfer and dissemination of knowledge. Education Science is a broad field and its professionals are directly involved in the field. Those who are actively involved in Eduscience can be referred to as "Education or Educational Scientists". Educational Scientists or "Eduscientists" are multifaceted professionals who have a variety of areas of expertise. They can assume multiple roles in the educational environment and can serve in a variety of offices and in a multitude of capacities. The primary positions that Eduscientists assume are in the following areas: Administration (as Leaders, Organizational Heads, and Organizational Management Professionals), Instruction (as Teachers, Professors, and Facilitators), Practice (as Practitioners in a variety Specified Areas and Arenas), and Technology (as Educational Technologists, Instructional Technologists, and Information Technologists). In these positions Eduscientists effectively use, analyze, study, and deploy novel instructional learning theories, methodologies, strategies, solutions, tools, and techniques in both traditional or virtual (pedagogical and andragogical) settings to bring about learning (Osler, 2012).

The Origins of the term "Trichotomy"

The term is pronounced ['trahy-kot-uh-mee'], spelled

"trichotomy", and is a noun with the plural written form "trichotomies". A "Trichotomy" in terms of philosophy can be referred to as a threefold method of classification. Philosopher Immanuel Kant adapted the Thomistic acts of intellect in his trichotomy of higher cognition-(i) Understanding, (ii) Judgment, (iii) Reason-which he correlated with his adaptation in the soul's capacities-(i) Cognitive faculties, (ii) Feeling of pleasure or displeasure, and (iii) Faculty of desire (Kant, 2007). In terms of mathematics, Apostol in his book on calculus defined "The Law of Trichotomy" as: Every real number is negative, 0, or positive. The law is sometimes stated as "For arbitrary real numbers a and b , exactly one of the relations $a < b$, $a = b$, and $a > b$ holds" (Apostol, 1967).

It is important to note that in mathematics, the law (or axiom) of trichotomy is most commonly the statement that for any (real) numbers x and y , exactly one of the following relations holds. Until the end of the 19th century the law of trichotomy was tacitly assumed true without having been thoroughly examined (Singh, 1997). A proof was sought by Logicians and the law was indeed proved to be true. If applied to cardinal numbers, the law of trichotomy is equivalent to the axiom of choice. More generally, a binary relation R on X is trichotomous if for all x and y in X exactly one of xRy , yRx or $x = y$ holds. If such a relation is also transitive it is a strict total order; this is a special case of a strict weak order. For example, in the case of three elements the relation R given by aRb , aRc , bRc is a strict total order, while the relation R given by the cyclic aRb , bRc , cRa is a non-transitive trichotomous relation. In the definition of an ordered integral domain or ordered field, the law of trichotomy is usually taken as more foundational than the law of total order, with $y = 0$, where 0 is the zero of the integral domain or field. In set theory, trichotomy is most commonly defined as a property that a binary relation $<$ has when all its members $\langle x, y \rangle$ satisfy exactly one of the relations listed above. Strict inequality is an example of a trichotomous relation in this sense. Trichotomous relations in this sense are irreflexive and anti symmetric (Sensagent: Retrieved, May 9, 2012 from the following website: [http://dictionary.sensagent.com/trichotomy+\(mathematics\)/en-en/](http://dictionary.sensagent.com/trichotomy+(mathematics)/en-en/)). It is from these logical and mathematical definitions that the author derives the definition of

“Research Trichotomy” and applies it to the qualitative and quantitative analysis of the affective domain of learning.

The Psychometrics of Trichotometric Analysis

One of the most challenging areas of research in education involves the construction of specific instruments that are designed to measure qualitative outcomes and data. Although there are a great many measurement tools that analyze the cognitive and psychomotor domains, there remains a vacuum in the number of instruments especially designed to accurately measure the affective domain (the learning domain that contains attitudes, opinions, emotions, perception, and perspectives). This void is further expanded when the specific event under investigation is unique, specialized, has specific characteristics, serious legal constrictions, and issues regarding time. This often requires the research investigator to design an instrument that ideally measures the variables under investigation.

The process of designing instruments for the purposes of assessment and evaluation is called “Psychometrics”. Psychometrics is broadly defined as the science of psychological assessment (Rust&Golombok, 1989). The Tri-Squared Test pioneered by the author, factors into the research design a unique event-based “Inventive Investigative Instrument”. This is the core of the Trichotomous-Squared Test. The entire procedure is grounded in the qualitative outcomes that are inputted as Trichotomous Categorical Variables based on the Inventive Investigative Instrument. The specific assessment of the variables is completely dependent upon the outcomes determined by the researcher’s instrument. The creation, production, and deployment of the Inventive Investigative Instrument requires that the research investigator adopts the role of a “Trichotomous Psychometrician” or “Trichotometrician”. A “Trichotomous Psychometrician” is an Educational Scientist that uses trichotomous-based psychometrics to develop a qualitative Inventive Investigative Instrument specifically designed capture qualitative responses during a specific event. A description of the entire Tri-Squared research process follows and is described in detail so that the reader is informed of precisely how an Inventive Investigative Instrument is

developed, designed, and ultimately implemented.

Algorithmics

Algorithmics is the mathematics of program construction. A Software Engineer conducts program development through systemic, cyclic, and sequential trial and error. They carefully observe a formal and tedious methodology to carefully construct an application. This same process can be used by an Educational Technologist to evaluate educational software. The elements of this process can be observed holistically in the following model displayed in Figure 1.

This same model can be used to evaluate educational software. A research design is then needed to develop an instrument to test the software based upon the aforementioned model. Thus, the Triangulation Algorithmic Model is provided and defined to aid in understanding the process of measurement instrument construction and research design.

The Triangulation Algorithmic Model for the Tri-Squared Test

The Algorithmic Model of Triangulation is of the form (Figure 2).

Where,

Vertex a = $\angle a$ = “authoring” = The Initial Tri-Squared Instrument Design;

Vertex b = $\angle b$ = “building” = The Tri-Squared Qualitative Instrument Responses; and

Vertex c = $\angle c$ = “conveying” = The Final Tri-Squared Test Outcomes in a Quantitative Report.

Thus, the Triangulation Model is symbolized by a Right Triangle written as: “ ∇ ”. This symbol called “Trine” (meaning a group of three) is written mathematically as “ $\nabla = abc$ ” and is simplified into the mathematic geometric expression: **∇abc** (meaning “Triangulation Model abc” or more simply

The Software Engineering Model		
Variable a=“Author” (Inquiry)	Variable b=“Build” (Deployment)	Variable c=“Convey” (Outcome)
Theoretical	Testing	Practical
Algorithmics [Modeling]	▶	Buildable [Construction]
Efficient [Effective]	▶	Correctable [Error-Free]
Sustainable [Lasting]	▶	Stable [Reliable]
Logical [Practical]	▶	Recycle [Re-Usable]

Figure 1. The Holistic Model of the Entire Software Development Process

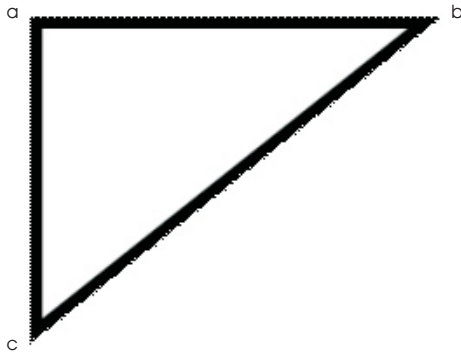


Figure 2. Algorithmic Model of Triangulation

“Trine abc”) (Figure 3).

The angles have the following angular measurements in degrees: $\angle a = 36.86$, $\angle b = 90$, and $\angle c = 53.14$, that all add up to the standard 180° of a traditional triangle ($36.86^\circ + 90^\circ + 53.14^\circ = 180^\circ$). The connective points (i.e., the lines between points a, b, and c respectively are geometric “vectors” (lines with both size [magnitude] and direction) making the model a systemic or cyclic process. This is illustrated in terms of Cartesian Coordinates as shown in Figure 4.

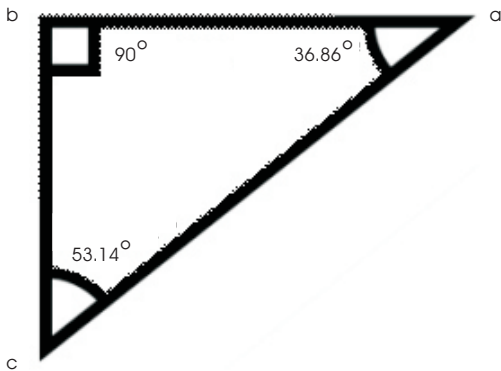


Figure 3. Triangulation Model or “Trine abc”

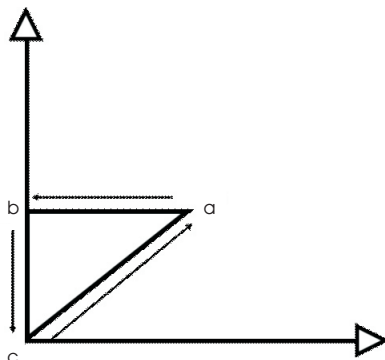


Figure 4. Cartesian Coordinates

In terms of Vectors, the Triangulation Model Right Triangle ∇abc is equal to three vectors that illustrate the movement in direction and magnitude from one completed task into another. The entire process is both cyclical and sequential with a “Trine Vector Equation” written as:

$$\nabla = [-x] \rightarrow [+y] \rightarrow [+z]$$

Defined as Trine = “Concentration of Vector x into Concentration of Vector y into Concentration of Vector z”, which is simplified into a more standardized Trine Vector Equation form written as:

$$\nabla = \vec{x} \rightarrow \vec{y} \rightarrow \vec{z}$$

Where vectors x, y, and z respectively are indicated on the “Algorithmic Triangulation Data Model” as shown in Figure 5.

Indicating that the standardized form of the vectors x, y, and z are equivalent to the following geometric vectors that are the sequential Cartesian Coordinates relative to the size and magnitude of the research engineering phases that sequentially connect the respective angles $\angle a$, $\angle b$, and $\angle c$. This is written as follows:

$$\begin{aligned} \vec{x} &= \overline{ab} \\ \vec{y} &= \overline{bc} \\ \vec{z} &= \overline{ac} \end{aligned}$$

The Complete Tri-Squared Algorithmic Triangulation Model

The three numeric Vector Operational Phases of the Triangulation Model are defined in the following manner:

The entire Triangulation Model as a Research Engineering methodology begins with a breakdown of the Trine $[\nabla]$ Operational Research Engineering Parameters and Geometric Vectors in the following manner:

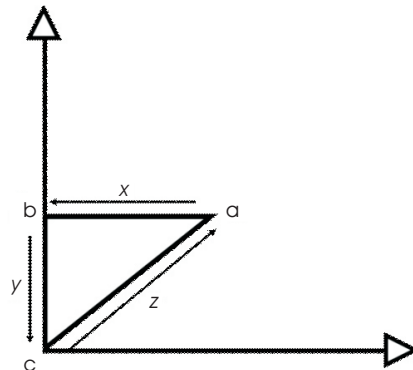


Figure 5. Algorithmic Triangulation Data Model

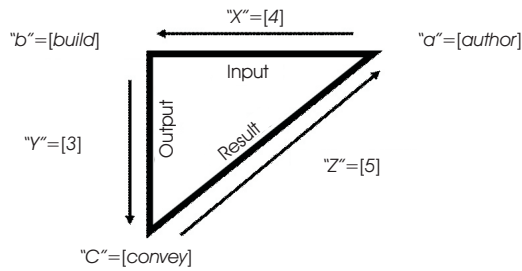


Figure 6. Complete Tri-Squared Algorithmic Triangulation Model

The Tri-Squared Triangulation Model Research Engineering Process (highlighting the Operational Parameters and Phases of the Tri-Squared Test)

Geometric Vertex $a = \angle a =$ "authoring" = The Initial Tri-Squared Instrument Design = Operational Parameter "a" = "author" = absolute value of $a = \text{modulus } a = |a| = \text{"Trine } a" = \sqrt{a}$ = The creation of the Tri-Squared Inventive Investigative Instrument. This process can be seen in the following model: (Figure 6).

This in turn, leads into $[-]$ vector $x =$ Geometric Vector $x = \leftarrow x = \overline{x} = \overline{ab} =$ The Initial Tri-Squared Instrument Construction = Operational Phases "x" = absolute value of vector $x = \text{"norm } x" = ||x|| = \text{"Trine } x" =$ The creation of the Tri-Squared Inventive Investigative Instrument = The Pythagorean Triple of the Triangulation Model = "4" = The 4 Phases of Tri-Squared Inventive Investigative Instrument Construction which is composed of the following 4 Operational Phases: (Figure 7).

- $\alpha_0 =$ The Instrument Name (Asset Security Optional);
- $\alpha_1 =$ Section One of the Research Instrument. Constructed from the first series of instrument items (a. through c.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the Initial Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as

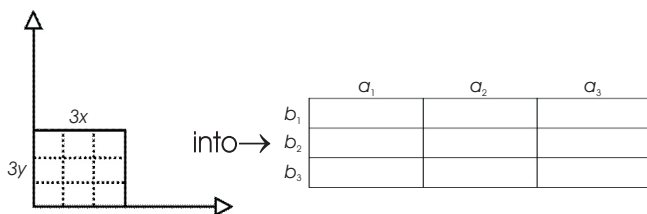


Figure 7. Tri-Squared Triangulation Model Research Engineering Process

the Resulting Outcome Output Variables = $b_1, b_2,$ and b_3 respectively).

- $\alpha_2 =$ Section Two of the Research Instrument. Constructed from the second series of instrument items (d. through f.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the secondary Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as the Resulting Outcome Output Variables = $b_1, b_2,$ and b_3 respectively); and
- $\alpha_3 =$ Section Three of the Research Instrument. Constructed from the third series of instrument items (g. through i.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the tertiary Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as the Resulting Outcome Output Variables = $b_1, b_2,$ and b_3 respectively);

The following table provides the metrics for the construction of the Inventive Investigative Instrument following the parameters indicated in phases 1. through 4. of the first vector [= 4] of the Triangulation Model, (Table 1).

This Tabular Triangulation Model can be fully exemplified in the aforementioned example provided by Table 1.

$\alpha_0 =$	Inventive Investigative Instrument —Name— [Optional Asset Security]		
$\alpha_1 =$	Section 1. Research Question One. The First Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below Responses: [Select only one from the list.]▶ b_1 b_2 b_3		
	a.	Item One	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	b.	Item Two	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	c.	Item Three	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
$\alpha_2 =$	Section 2. Research Question Two. The Second Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below Responses: [Select only one from the list.]▶ b_1 b_2 b_3		
	d.	Item Four	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	e.	Item Five	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	f.	Item Six	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
$\alpha_3 =$	Section 3. Research Question Three. The Third and Final Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below. Responses: [Select only one from the list.]▶ b_1 b_2 b_3		
	g.	Item Seven	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	h.	Item Eight	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
	i.	Item Nine	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>

Table 1. Inventive Investigative Instrument for Research Questions 1, 2 & 3

The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument ©

A. Has the 9th Grade Academy, Center, or Center Model been:

	Yes	No	Missing
1. Successful?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2. Made a Difference?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3. Aided in Retention?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

How as the Academy/Center been successful, made a difference, or aided in retention, if at all?

	Yes	No	Missing
4. Positive Impact?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5. Active Participation?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6. Decline in Dropout Rate?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

How as the Academy/Center been positive, aided in participation, or decreased the dropout rate, if at all?

How did the 9th Grade Academy, Center, or Center Model have an impact on the following:

	Yes	No	Missing
7. Positively Effect Standardized Testing?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8. Increase Graduation Rate?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9. Increase Attendance?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

How as the Academy/Center positively affected testing, graduation rates, and attendance, if at all?

How long has the model/program (freshman/Ninth Grade Academy been operation in your school?

How long has the interviewee (yourself) been (working) there and what is the level of his or her involvement in the program such as Assistant Principal, Principal, teacher or other staff member?

What is the role of those interviewed during the interviews, and their knowledge of the program, whether their knowledge was medium knowledge low level of knowledge etc.

This is defined in the first Operational Geometric Vector (see Figure 3)

Table 2. The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument (Courtesy Osler-Waden, 2012)

Thus, Table 2 is provided again (in an enlarged form) to illustrate a sample Inventive Investigative Instrument:

This is defined in the first operational geometric vector.

Where, the Operational Geometric Vector phases are defined respectively as:

α_0 =The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument ©;

α_1 =Has the 9th Grade Academy, Center, or Center Model been;

α_2 =Did the 9th Grade Academy, Center, or Center Model Result in the following;

α_3 =How did the 9th Grade Academy, Center, or Center Model have an impact on the following;

b_1 = Yes;

b_2 = No; and

b_3 = Missing.

The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument © [with Asset Security = ©]

α_1 = Has the 9th Grade Academy, Center, or Center Model been:

Responses: [Select only one from the list.] ► b_1 Yes b_2 No b_3 Missing

a. = 1 Successful?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 2 Made a Difference?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 3 Aided in Retention?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

α_2 = Did the 9th Grade Academy, Center, or Center Model Result in the following:

Responses: [Select only one from the list.] ► b_1 Yes b_2 No b_3 Missing

a. = 4 Positive Impact?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 5 Active Participation?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 6 Decline in Drop Out Rate?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

α_3 = How did the 9th Grade Academy, Center, or Center Model have an impact on the following:

a. = 7 Positively Effect Standardized Testing?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 8 Increase Graduation Rate?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
a. = 9 Increase Attendance?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Table 3. The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument (illustrating the instrument variables, items, and overall design). (Courtesy Osler-Waden, 2012).

This is immediately followed by (Table 3).

Geometric Vertex $b = \underline{\quad} b =$ "build" = The Tri-Squared Qualitative Instrument Responses = Operational Parameter "b" = "build" = absolute value of b = "modulus b " = $|b|$ = "Trine b" = ∇b = The effective deployment of the Tri-Squared Inventive Investigative Instrument to elicit responses and aggregate the response data. This process can be seen in the following model (Table 4).

A Standard 3×3 Tri-Squared Table of Comprehensive Inputted Qualitative Research Responses would resemble the following model represented by Table 5. Where, T_n = The Total Number of Responses (based upon a one to one ratio of each Trichotomous Testing Input Variables contrasted directly with each Trichotomous Results Output Variable) in each of the individual cells of the Standard 3×3 Tri-Squared Table.

	α_1	α_2	α_3
b_1	$\alpha_1 b_1$	$\alpha_2 b_1$	$\alpha_3 b_1$
b_2	$\alpha_1 b_2$	$\alpha_2 b_2$	$\alpha_3 b_2$
b_3	$\alpha_1 b_3$	$\alpha_2 b_3$	$\alpha_3 b_3$

Table 4. The Standard 3×3 Tri-Squared

	α_1	α_2	α_3
b_1	$T \alpha_1 b_1$	$T \alpha_2 b_1$	$T \alpha_3 b_1$
b_2	$T \alpha_1 b_2$	$T \alpha_2 b_2$	$T \alpha_3 b_2$
b_3	$T \alpha_1 b_3$	$T \alpha_2 b_3$	$T \alpha_3 b_3$

Table 5. Standard Responses the 3×3 Tri-Squared

Where Table 5 is defined as,

$T_{a_1b_1}$ = Total Number of Responses for Cell One in the Standard 3×3 Tri-Squared Table;

$T_{a_2b_1}$ = Total Number of Responses for Cell Two in the Standard 3×3 Tri-Squared Table;

$T_{a_3b_1}$ = Total Number of Responses for Cell Three in the Standard 3×3 Tri-Squared Table;

$T_{a_1b_2}$ = Total Number of Responses for Cell Four in the Standard 3×3 Tri-Squared Table;

$T_{a_2b_2}$ = Total Number of Responses for Cell Five in the Standard 3×3 Tri-Squared Table;

$T_{a_3b_2}$ = Total Number of Responses for Cell Six in the Standard 3×3 Tri-Squared Table;

$T_{a_1b_3}$ = Total Number of Responses for Cell Seven in the Standard 3×3 Tri-Squared Table;

$T_{a_2b_3}$ = Total Number of Responses for Cell Eight in the Standard 3×3 Tri-Squared Table; and

$T_{a_3b_3}$ = Total Number of Responses for Cell Nine in the Standard 3×3 Tri-Squared Table.

This leads into $[\rightarrow]$ vector y = Geometric Vector y = " $\downarrow y$ " = $d \text{ s } \overline{y} = \overline{bc}$ = The Initial Tri-Squared Instrument Construction = Operational Phases " y " = absolute value of vector y = "norm y " = $\|y\|$ = "Trine y " = ∇y = The creation of the Tri-Squared Inventive Investigative Instrument = The Pythagorean Triple of the Triangulation Model = "3" = The 3 Phases of Tri-Squared Inventive Investigative Instrument Deployment which is composed of the following 3 Operational Phases

- Inventive Investigative Instrument Deployment;
- Inventive Investigative Instrument Completion; and lastly
- Inventive Investigative Instrument Data Aggregation.

Using the Tri-Squared Triangulation Model for Application Testing: The Efficacy of Trichotometric Analysis through the Implementation of the Tri-Squared Test

The Tri-Squared research methodology: A four step process for determining the appropriate Research Effect Size, Sample Size, and Alpha Level (Osler, 2012). The Tri-Squared research procedure consists of a four step approach designed to provide the researcher with a clear and

precise set of data to conduct research, analyze data, and determine the level of significance required to either validate or reject the initial research hypothesis. The four Tri-Squared steps are as follows:

- Design of an Inventive Investigative Instrument that has Trichotomous Categorical Variables and Trichotomous Outcome Variables.
- Establish the Research Effect Size, Sample Size with associated Alpha Level.
- Establish Mathematical Hypotheses.
- Use the Tri-Squared Test as the Data Analysis Procedure following implementation.

The Tri-Squared Research Design

Step One: Design of an Inventive Investigative Instrument that has Trichotomous Categorical Variables and Trichotomous Outcome Variables.

Step Two: Establish the Research Effect Size, Sample Size with associated Alpha Level.

The Tri-Squared Effect Size Formula

$$Trif_{Eff}^2 = [T_{CR} - (T_C T_R : n_{Trif})] : C_S R_S (n_{Trif} - 1)$$

Step Three: Establish Mathematical Hypotheses.

Sample Mathematical Hypotheses for Holistically Testing an Application

$$H_0: Trif^2 = 0$$

$$H_1: Trif^2 \neq 0$$

Step Four: Use the Tri-Squared Test as the Data Analysis Procedure following implementation.

Sample Tables of qualitative and quantitative data follow to illustrate how the Tri-Squared Test displays the outcomes of affective domain inquiry (Osler, 2012).

Sample Table (A)

A Sample 3×3 Table of the Qualitative Input Outcomes of the Tri-Squared Test [TBD = To Be Determined.]

$n_{Trif} = 0$ $\alpha = \text{TBD}$	TRICHOTOMOUS TESTING INPUT VARIABLES			
	α_1	α_2	α_3	
TRICHOTOMOUS RESULTS OUTPUT VARIABLES	b_1	0	0	0 = Tr_1
	b_2	0	0	0 = Tr_2
	b_3	0	0	0 = Tr_3
	T_{C_1}	T_{C_2}	T_{C_3}	$T_{C_1} = T_{Trif}$
	$Trif^2 \text{ D.f.} = [C - 1][R - 1][3 - 1][3 - 1] = 4 = Trif_{Eff}^2$			

The Tri-Square Test Formula for the Transformation of Trichotomous Qualitative Outcomes into Trichotomous Quantitative Outcomes to Determine the Validity of the Research Hypothesis:

Tri^2 Critical Value Table = TBD (with d.f. = 4 at α = TBD). For d.f. = 4, the Critical Value for $p > TBD$ is TBD. The calculated Tri-Square value is TBD, thus, the null hypothesis (H_0) is rejected by virtue of the hypothesis test which yields the following: Tri-Squared Critical Value of TBD < or > TBD based upon the Calculated Tri-Squared Value.

Sample Table (B)

$n_{oi} = 0$ $\alpha = TBD$	TRICHOTOMOUS CONVERSION INPUT VARIABLES		
TRICHOTOMOUS CONVERSION OUTPUT VARIABLES	α_1	α_2	α_3
b_1	$\alpha_1 b_1 = \frac{T_{11} T_{c1}}{T_{T1}}$	$\alpha_2 b_1 = \frac{T_{12} T_{c1}}{T_{T1}}$	$\alpha_3 b_1 = \frac{T_{13} T_{c1}}{T_{T1}}$
b_2	$\alpha_1 b_2 = \frac{T_{21} T_{c1}}{T_{T1}}$	$\alpha_2 b_2 = \frac{T_{22} T_{c1}}{T_{T1}}$	$\alpha_3 b_2 = \frac{T_{23} T_{c1}}{T_{T1}}$
b_3	$\alpha_1 b_3 = \frac{T_{31} T_{c1}}{T_{T1}}$	$\alpha_2 b_3 = \frac{T_{32} T_{c1}}{T_{T1}}$	$\alpha_3 b_3 = \frac{T_{33} T_{c1}}{T_{T1}}$
$Tri^2 d.f. = [C - 1][R - 1] = [3 - 1][3 - 1] = 4 = Tri^2_{[R]}$			

A Sample 3 x 3 Table of the Quantitative Output Outcomes of the Tri-Squared Test is above.

[TBD = To Be Determined]

The Tri-Square Test Formula for the Transformation of Trichotomous Qualitative Outcomes into Trichotomous Quantitative Outcomes to determine the Validity of the Research Hypothesis:

$$Tri^2 = T_{Sum} \left[(Tri_x - Tri_y)^2 : Tri_y \right]$$

Tri^2 Calculated Tri-Squared = [0] + [0] + [0] + [0] + [0] + [0] + [0] + [0] + [0] = 0 (with d.f. = 4 at α = TBD). For d.f. = 4, the

Critical Value for $p > TBD$ is TBD. Thus, we can reject the null hypothesis (H_0) by virtue of the hypothesis test if: Tri-Squared Critical Value of TBD < or > TBD based on the Calculated Tri-Squared Value. Note: Comprehensive lists of Tri-Squared Tables are available that display and label Critical Values, Alpha Levels, and Sample Sizes based upon all calculated Tri-Squared Effect Sizes. The Tables for Hypothesis Testing and Probability Distribution follow (Tables 6 and 7 respectively). Together these two Tables summarize the entire outcome and relevance of the multi-test application development process that is analyzed through the Tri-Squared Statistical methodology.

Summary

The purpose of this paper is to provide support for the use of the Tri-Squared Test in application/software evaluation. The Tri-Squared Test Algorithmic Model and associated Statistical Methodology were provided to illustrate and display how method can be put to use practically and rapidly. The Total Transformative Trichotomy-Squared Research Design Methodology is a universally applicable in-depth investigative procedure that is an ideal way of examining the unique, diverse, and many times specialized procedures inherent in application testing and design. This process allows Educational Technologists to use quality control in the design and testing educational software testing process through an in-depth qualitative analysis. The qualitative outcomes can then be transformed into measurable quantitative outcomes to determine the outcome of testing process statistically. Both qualitative and quantitative methods are

Tri-Squared Distribution Table Displaying Primary Alpha Levels with Associated Critical Values for Hypothesis Tests

α Level	0.995	0.975	0.10	0.10	0.05	0.025	0.02	0.01	0.05	0.001	0.001
$Tri^2[x] = d.f. = 4$	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
$Tri^2 Eff = Effect Size$	Small4[4]	Small4[4]	Small4[4]	Small4[4]	Small4[4]	Medium4[16]	Medium4[16]	Medium4[16]	Large4[64]	Large4[64]	Large4[64] +
$Tri^2_{sm} = Sample Size [Intervals]$	1-16	17-33	34-40	41-57	58-74	75-139	140-204	205-269	270-526	527-783	784-1040+

Table 6. The Tri-Squared Hypothesis Test: Alpha Level, Effect Size, and Sample Size

Tri-Squared Probability Distribution Table

Number of Research Participants Placed in Intervals Based off of Tri-Squared Effect Size Magnitude: [Small, Medium, or Large] is Based off of the Tri-Squared Mean = [d.f.] = 4

Magnitude	Small Unit Intervals: Multiple of 1=4[4] = 4.4=16 Therefore, Interval has Increments of 16					Medium Unit Intervals: Multiple of 2=4[16] = 4[4.4]=64 Therefore, Interval has Increments of 64			Large Unit Intervals: Multiple of 3=4[4.4.4] = 4[64]=256		
Number of Participants	1-16	17-33	34-40	41-57	58-74	75-139	140-204	205-269	270-526	527-783	784-1040+
Probability P(x)	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001

Table 7. The Summative Comprehensive Tri-Squared Formula Probability Distribution

seamlessly incorporated into a unified data analysis methodology that delivers a reporting procedure that specifically aligns with the initial research objectives, hypotheses, and variables of the developer. This method of statistical analysis allows the engineer to use both qualitative and quantitative research methodologies by seamlessly combining the two opposing branches of research investigation for maximized statistical power. This provides Educational Technologists with a discrete yet comprehensively stringent data analysis tool that is purely objective, extremely reliable, and very valid. In this manner, the field of Educational Technology will continue to grow, expand, and empower present and future generations of learners.

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