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Advanced Mathematical Thinking and Students' Mathematical Learning: Reflection from Students' Problem-Solving in Mathematics Classroom

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Abstract

Mathematical teaching in Thai tertiary education still employs traditional methods of explanation and the use of rules, formulae, and theories in order for students to memorize and apply to their mathematical learning. This results in students' inability to concretely learn, fully comprehend and understand mathematical concepts and practice. In order to overcome this learning deficit, it is necessary that the concept of "reflection" be implemented in the teaching of this subject. It is believed that the adoption of this teaching concept will allow students to learn mathematics by themselves. This article is aimed at presenting mathematical problem-solving of undergraduate students on Calculus I. Concrete problems were assigned to students to participate, to improve students' way of mathematical thinking, and to encourage the students' mathematical learning and advanced mathematical thinking. The study was a qualitative research project conducted with first-year undergraduate students of Rajamangala University of Technology Phra Nakhon who had enrolled for Calculus I. Data were collected from interviews and field notes, along with video recordings. Findings showed that students succeeded in solving mathematical problems from simple to complex levels and using the subject fundamentals to connect to several methods of higher levels of thinking. Students also created effective means of problem-solving and applied these concepts to solve new problems.

Keywords: advanced mathematical thinking, mathematical learning, problem-solving

1. Introduction

Nowadays, mathematics education in Thai classrooms employs traditional methods of explanation, focusing on theories and formulae. Students are asked to complete significant amounts of exercises so as to improve their test-taking fluency. Mathematical content, however, is highly abstract (Rodjai, 2009). Tall (1991) proposed that a mathematics curriculum requires high abstract thinking, and as a result students must encounter complexity in mathematics learning. Teachers generally do not pay enough attention to each students' cognitive process—a compulsory element for students to acquire in order to develop their knowledge (Ministry of Education, 2010; National Education Committee, 2000).

The Ministry of Education (2008) has compelled teachers to rely on student-centered learning, but in practice past education reforms in the mathematics curriculum have not been a success (Tipkong, 2002; Worawan Na Ayutthaya, 2009) and have failed to develop Thai students' advanced thinking and problem solving skills (Worawan Na Ayutthaya, 2009). The Ministry of Education (2010) has indicated that proper mathematics education can only be achieved when teachers realize that students' thinking must be encouraged and successfully turn their classroom into meaningful self-learning classrooms in order that students can attain sufficient knowledge application of their subjects.

Byness (2001) proposed that advanced mathematical teaching is one that enables students to create their own mathematics concepts in which skills and principles of mathematics are applied. Tall (1991) stated that advanced mathematical thinking is related to students' existing concepts and learning from procedural association. In order to attain this level of advanced mathematical thinking, Tall stated that a concept is specific knowledge by which individuals can explain or practice to acquire theory, formulae and mathematical apprehension (Evitts, 2004). Advanced mathematical thinking not only requires knowledge creation but also knowledge enhancement (Tall, 1991, 1995).

Mathematics learning is associated with advanced mathematical thinking in that it allows students to create concepts and mathematical connections by themselves (Ito-Hino, 1995). Mathematics education should therefore encourage students to effectively improve their learning abilities based on student-centered learning (Ministry of Education, 2008).

Mathematics problems play an important role in helping students to participate in problem-solving activities and stimulate their learning abilities. The problems should be challenging enough for students to solve in order to enhance their knowledge and comprehension. This type of activity develops students' curiosity in solving mathematics problems and successfully enhances their cognitive processes (Henninsen & Stein, 1997). Mathematics contents that can challenge students to think and learn depend on how teachers implement these principles (Stigler & Hiebert, 1999). Therefore, mathematics education that encourages students to think for themselves is significant to their learning, allowing them to be curious to learn. The acquisition of mathematics concepts is essential for teachers to develop in their students.

Consequently, the researchers are interested in studying advanced mathematical thinking of students as reflected in-class mathematics problem solving and the creation of mathematics concepts.

1.1 Research Question

To what extent is there advanced mathematical thinking used to solve mathematics problems among students in Thailand?

1.2 Research Objectives

This research aims to study advanced mathematical thinking and students' mathematical learning in mathematics classroom.

1.3 Theoretical Frameworks

The study of Thai students' advanced mathematical thinking in solving in-class mathematics problems employed an adapted Inprasitha's theoretical framework (Inprasitha, 2010), for students in Rajamangala University of Technology Phra Nakhon (RMUTP).

Four stages of the instruction were developed:

Step 1 A teacher introduced mathematics problems to the students. Students were required to study the questions by themselves, with suitable teacher's guidance.

Step 2 Problem solving was based on Tall's (1991, 1995) theoretical framework to develop advanced mathematical thinking of the knowledge previously learned to create new knowledge and extend further knowledge. In this step, students can learn by themselves, and teachers can gather students' solutions to the mathematics problems.

Step 3 Teachers and students discussed mathematics problems together and compared students' solutions to specific problems. Students were asked to demonstrate their solutions to solve mathematics problems, and teachers focused on drawing a link with mathematical concepts.

Step 4 Students drew conclusions in order to link their solutions with mathematical concepts. Teachers drew summary links to enable students to consolidate their knowledge of mathematical formulae. Afterwards, the students were allowed to review what they had learned in class by asking questions to their classmates or teachers.

According to Tall's (1991) framework, advanced mathematical thinking requires basic thinking in order to develop the formation of concepts. In other words, the accumulation of previous knowledge to create new concepts is crucial. Tall (1995) states that advanced mathematical thinking not only formulates new concepts but also expands and enhances historical ones.

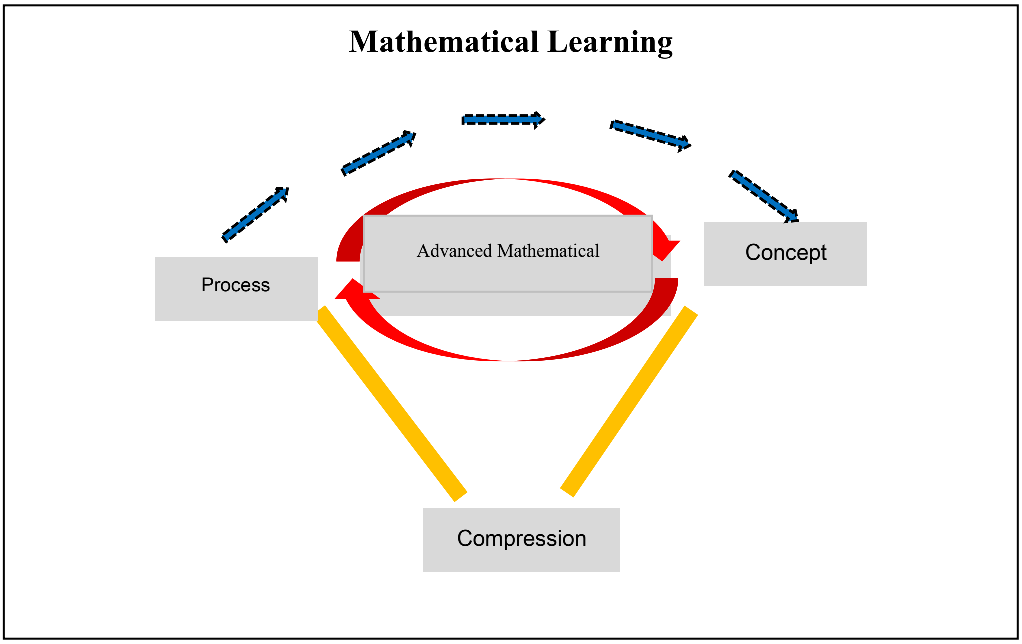


Figure 1. Advanced mathematical thinking and mathematical learning, developed from Tall's (1991) advanced mathematical thinking framework

According to Figure 1, advanced mathematical thinking takes place during concept-creating process in combination with compression to associate thinking procedures and concepts in order to create and develop mathematical concepts. This is the learning from the link between the procedures and the concepts—the relation of knowledge which each student is able to explain or carry out so as to acquire principles, formulae, and mathematical senses.

Compression is a thinking operation used to explain the formation of mathematical concepts. This mental process can be considered to be the means of a disciplined problem-solving process and from which concepts are developed.

Procepts are the procedural concepts employed as the concept-making mechanism in advanced mathematical thinking to solve problems which eventually lead to concept formation. Therefore, a procept, along with compression, is a tool to create concepts and a continuous procedure in the continuing development of advanced mathematical thinking.

2. Research Methodology

This article is part of the research conducted to exemplify students' mathematical thinking in mathematics classrooms.

2.1 Research Design

This qualitative research emphasized natural settings of the researchers, as instruments, to focus on procedures and meaning explanations (Bogdan & Biklen, 1992). It was designed to be case-study research of undergraduate students of Rajamangala University of Technology Phra Nakhon who enrolled Calculus I (unit of analysis). Data collection included semi-structured interviews, field notes, and video recordings.

2.2 Sample Group

The Target group of this study was first-year students who enrolled in Calculus 1 at RMUTP. All of them were purposively selected.

2.3 Instrumentation

Instruments used for data collection and interpretation including:

One mathematics problem on Calculus I,

1) A 10-item interview on students' advanced mathematical thinking, and

2) A field note used to observe students' process of developing advanced mathematical thinking.

2.4 Research Procedures

The study employed qualitative research methods by illustrating the interpretation and analysis of interviews, field notes, and video recordings. Data were collected at the end of each period.

Research methodology is addressed below.

1) Create 5 problems (Problem Design).

2) Teach the simulated problems to the students (Teaching Observation).

During this stage, students' way of solving problems is observed by taking field notes, recording videos, and interviewing the students so as to make interpretations.

3) Bring the data in stage 2 to analyze by employing triangulation—field note, video analysis, and interview data.

4) Make an interpretation.

3. Results

3.1 Exempla Analysis

The activity, finding maximum and minimum scores, was divided into 2 activities.

Activity 1. Box Building

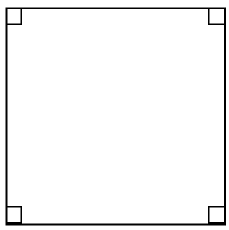
Directions

1) Build various sizes of rectangular boxes without a cover from the provided paper.

2) From 1, select the box of the most volume and explain why the selected box has the most volume.

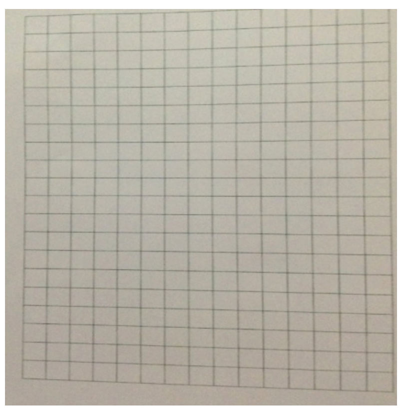
3) Give a presentation in front of the class.

Examples of Building Square Grid Paper Boxes:

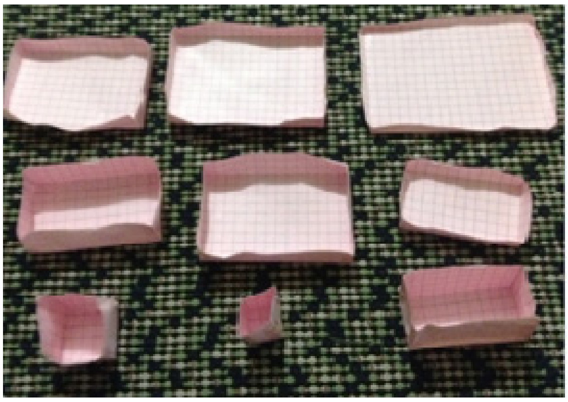
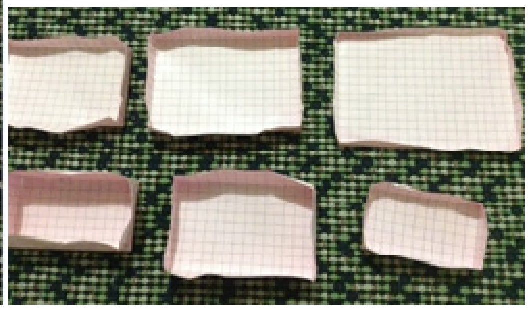


Tools

Tools comprised square grid paper of 15 x 20 square meters, adhesive tape and scissors.

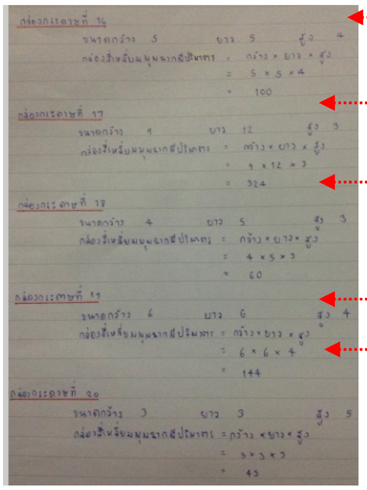


Students in each group used the grid paper to build boxes of various sizes, as shown below.

After building the rectangular boxes without a cover, students calculated the volume of the boxes from several methods as follows.

Method 1. Use formulae to calculate the volume



No. 16

W×L×H = 5×5×4 = 100

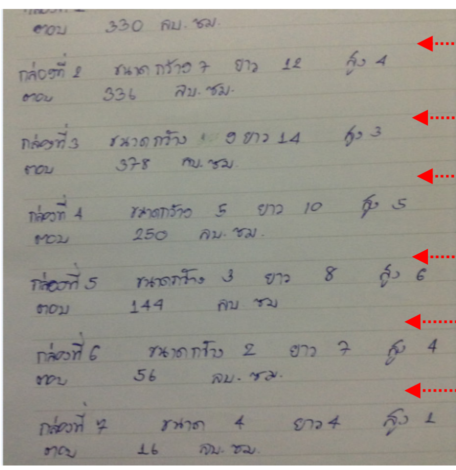
No.17

W×L×H = 9×12×3 = 324

No.18

W×L×H = 4×5×3 = 60

Method 2. Calculate with no calculating procedures



Box 2.

Wight 7 length 12 high 4 Ans. 336

Box 3.

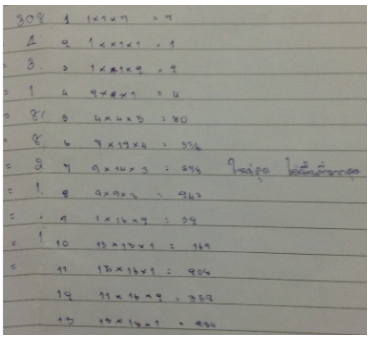
Wight 9 length 14 high 3 Ans. 378

Box 4.

Wight 5 length 10 high 5 Ans. 250

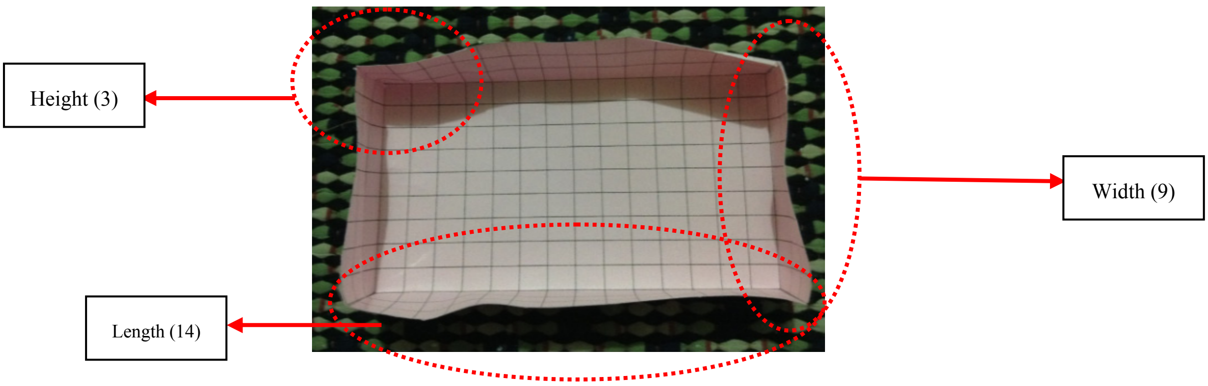
Box 5.

Method 3. Calculate by slowly increasing the cutting size of each box



1. $1 \times 1 \times 7 = 7$
2. $1 \times 1 \times 1 = 1$
3. $1 \times 1 \times 2 = 2$
4. $2 \times 2 \times 1 = 4$
5. $4 \times 4 \times 5 = 80$
6. $7 \times 12 \times 4 = 336$
7. $9 \times 14 \times 3 = 378$
8. $9 \times 9 \times 3 = 243$
9. $1 \times 16 \times 2 = 32$
10. $13 \times 13 \times 1 = 169$
11. $13 \times 16 \times 1 = 208$

Students found that the box of the most volume was the box with 9 x 14 x 3 (width x length x height) = 378 cubic centimeters. They labeled it as “the biggest, with the largest volume”.



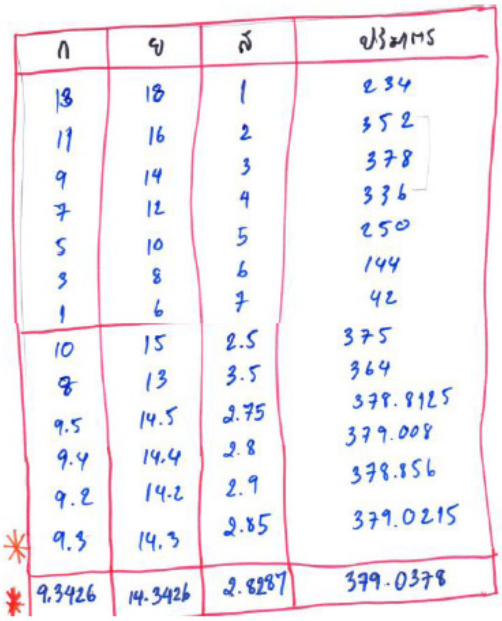
Activity 2. Calculating the box volume

Directions

1) Find as many alternative solutions, different from Activity 1, to calculate the box volumes.

2) Give a presentation in front of the class.

Method 1. Create a table



ก	ย	ส	ปริมาตร
13	18	1	234
11	16	2	352
9	14	3	378
7	12	4	336
5	10	5	250
3	8	6	144
1	6	7	42
10	15	2.5	375
8	13	3.5	364
9.5	14.5	2.75	378.8125
9.4	14.4	2.8	379.008
9.2	14.2	2.9	378.856
9.3	14.3	2.85	379.0215
9.3426	14.3426	2.8287	379.0378

weight	length	high	volume
13	18	1	234
11	16	2	352
9	14	3	378
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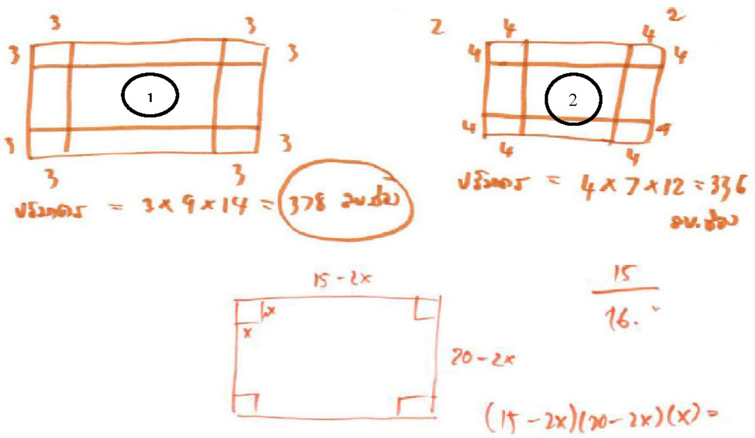
9.5	14.5	2.75	378.8125
9.4	14.4	2.8	379.008
9.2	14.2	2.9	378.856
9.3	14.3	2.85	379.0215
9.3426	14.3426	2.8287	379.0378

Method 2. Draw pictures and calculating them

Teacher activated the group who created a table, by asking them "Are you sure that the box you obtained is the biggest?"

Students applied decimals to calculate. Some groups also demonstrated by using an equation.

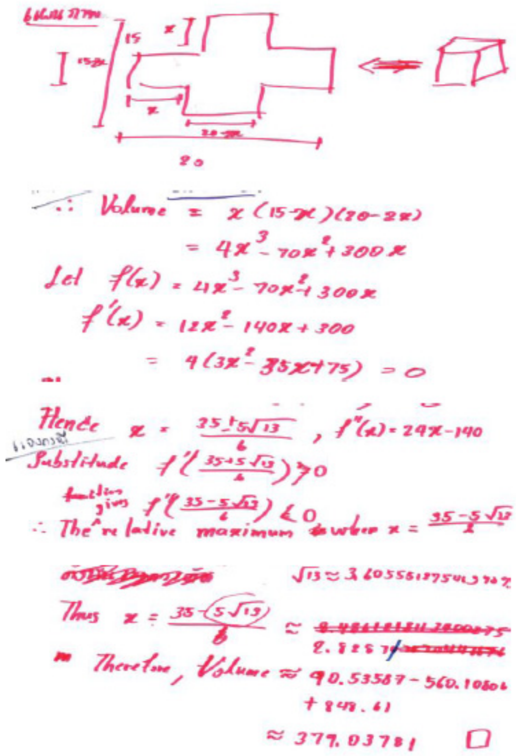
Method 3. Calculate by fixing variable

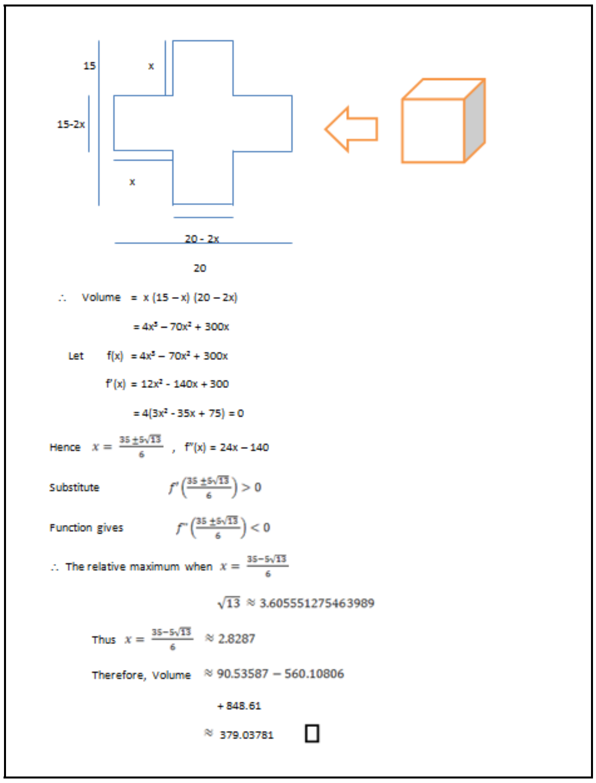


Volume 1 (1) $=378\ cm^3$

Volume 1 (2) $=336\ cm^3$

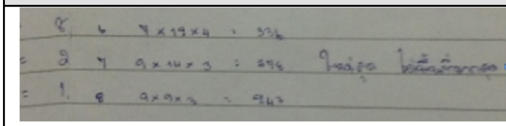
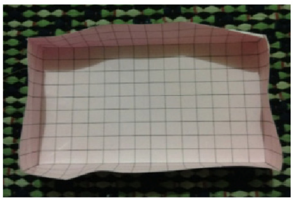
Method 4. Calculate by finding derivative

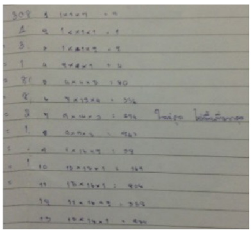
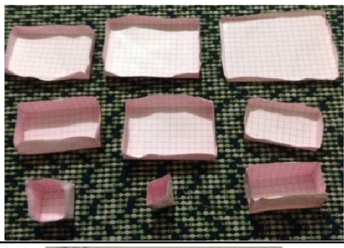


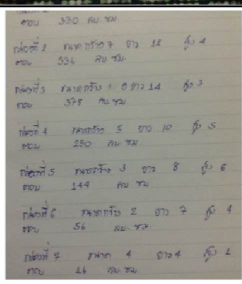
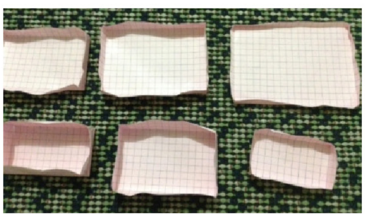


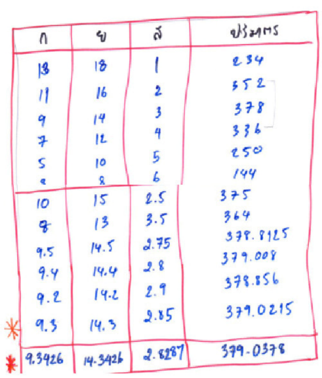
Students' methods of thinking were identified from the box building activity to find the maximum volume. After the activity, students felt more confident about using other various methods to find such volumes.

4. Data Analysis

<table>
<tr><th>Proof</th><th>Analysis</th><th>Interpretation</th></tr>
<tr><td>
</td><td>1. Students folded a piece of paper and write a calculation method.</td><td>- Students used a single method to solve the problem.

- Students tried to figure out the way to solve the problem by using what they had previously learned.</td></tr>
<tr><td>
</td><td>2. Students employed Various methods of calculation such as folding grid paper, writing down the calculation, and calculating.</td><td>- Students used a variety of methods to solve the problem.

- Students searched for the procedures to create their concepts.</td></tr>
<tr><td>
</td><td>3. Students considered how to fold a piece of paper, hoping to find the largest possible volume of the box. That volume was determined as $9\times14\times3=378$.</td><td>- Students realized that there are several ways to determine the solution to the problem and obtain accurate answers.

- Students acquired the concept about the volume of the box.</td></tr>
<tr><td>
<table>
<tr><th>ก</th><th>ข</th><th>ส</th><th>ปริมาตร</th></tr>
<tr><td>13</td><td>18</td><td>1</td><td>234</td></tr>
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<tr><td>3</td><td>8</td><td>6</td><td>144</td></tr>
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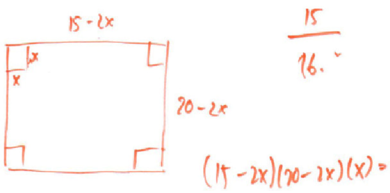
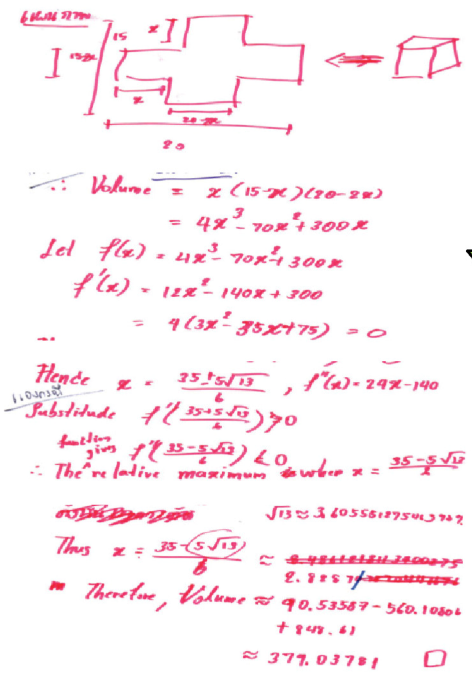
<tr><td>9.4</td><td>14.4</td><td>2.8</td><td>379.008</td></tr>
<tr><td>9.2</td><td>14.2</td><td>2.9</td><td>378.856</td></tr>
<tr><td>9.3</td><td>14.3</td><td>2.85</td><td>379.0215</td></tr>
<tr><td>9.3426</td><td>14.3426</td><td>2.8287</td><td>379.0378</td></tr>
</table></td><td>4. Students were aware of the ideas needed in order to obtain answers for the problem of the box, and that was by evaluating from the table and writing an equation to find a solution.</td><td>- Students realized the concepts that occurred.

- Students learned from process and procedures to form concepts.</td></tr>
</table>

	5. Students improved their methods in solving problems by evaluating from folding a piece of paper, drawing a table, and eventually creating an equation.	- Students applied what they had learned to expand their thinking. - Students employed the extended links from process and procedure to create concepts.
	6. Students continued their ideas previously obtained to evaluate the differentiation. Students applied what they had previously learned—maximum and minimum scores— to evaluate the most volume of the box by employing the differentiation.	- Students continued to develop more complex ideas. - Students used their previous knowledge to continue their learning. - Students obtained principles, formulae, and mathematical sense.

5. Discussions

Students developed their fundamental thinking into more complex levels of thinking. They began to think from a single method to formulate various methods in order to solve mathematical problems. They also succeeded in expanding their thinking ability by applying their previous methods of thinking from practice to formulate mathematical principles and formulae. Advanced mathematical thinking through these 6 stages is equivalent to Tall's (1991), stating that advanced mathematical thinking is formed through the formation of concepts—from employing several methods to solve the problems to developing the precept. In addition, the research findings are correlated with Evitts' (2004), which emphasized the relationship between procedures and concepts. Students are not familiar with mathematical open-ended problems. However, when they were asked, "Is there any other way to show which box is the biggest?" students responded by brainstorming to figure out other possible solutions. They attempted to find the answer, calculate, think systematically, draw pictures in the table provided, and eventually define variables. All of the students' responses demonstrate that students performed compression to attain concepts. Subsequently, students were encouraged by teachers to bring together all the necessary information to create equations which led to the solution of maximum and minimum scores.

6. Conclusion

Students succeeded in solving practical calculus problems. The problems were open-ended, and assigned to develop students' methods of thinking from the simple to the complex level. Students were able to develop experience in several methods of thinking in order to solve these problems. This experience helped develop the students' knowledge of the various methods used to solve mathematical problems. Students gained valuable experience in the practical use of mathematical principles and formulae. They also acquired some advanced mathematical thinking developed from their problem-solving methods to create concepts.

Acknowledgements

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