The Effects of Mathematical Modelling on Students' Achievement-Meta-Analysis of Research

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Abstract

Using meta-analytic techniques this study examined the effects of applying mathematical modelling to support student math knowledge acquisition at the high school and college levels. The research encompassed experimental studies published in peer-reviewed journals between January 1, 2000, and February 27, 2013. Such formulated orientation called for extracting individual effect sizes of student achievement from the accumulated research conducting a moderator analysis. A systematic review of literature resulted in locating13 primary research articles involving 1,670 participants. The overall mean effect size; ES = 0.69 (SE = 0.05, 95% CI: 0.59–0.79) of a medium magnitude and positive direction supported the claim that mathematical modelling helps students understand and apply math concepts. A subsequent moderator analysis revealed differences of the effect sizes due to different modelling designs, aim of the modelling process, grade levels, and content domains. The research findings along with the discussion can be of interest to mathematics curriculum designers and practitioners who use modelling in their teaching practice.

Keywords: Mathematical modelling; Meta-analysis; Student achievement.

Introduction

Mathematical modelling (MM) is defined in literature in various ways; Pollak (2007) a precursor of introducing MM to school practice described modelling as a process of formulating a problem from outside of mathematics, understanding the problem, visualizing, and solving it. Lesh and Harel (2003) defined MM as an activity of finding quantifiable patterns of a phenomenon and its generalization. A more comprehensive description of MM was proposed by Confrey and Maloney (2007) who stated that MM is:

> The process of encountering an indeterminate situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to bear to transform the situation. The modelling produces an outcome - a model - which is a description or a representation of the situation, drawn from the mathematical disciplines, in relation to the person's experience, which itself has changed through the modelling process (p.60).

MM as an educational development (Pollak, 1968) was initiated in engineering and sciences settings, and then spread to other fields. Its purpose was to elevate the gap between reasoning in a mathematics class and reasoning about a situation in the real world (Blum, Galbraith, Henn, & Niss, 2007). Situated in contexts, MM provides methods for analyzing data, formulating theories—often expressed in symbolic mathematical forms—and testing those theories as well as it helps with contextualizing problem solving processes. The process of MM can be exercised using various learning settings; from deductively arranged authentic problem modelling activities (e.g., English & Sriraman, 2010) to inductively organized inquiries leading the learners to formulating general patterns (e.g., Sokolowski & Rackly, 2011). Due to being context driven, knowledge acquisition by the processes of modelling plays a vital role in developing students' skills not only in mathematics classes but also in other disciplines, especially in sciences (Lesh & Harel, 2003; Wells, Hastens, & Swackhamer, 1995). As Confrey (2007) claimed "the strongest arguments for modelling are based on the view that it will be advantageous for the development of student thinking" (p.125) which is being accomplished by shifting the learning focus from finding unique solutions to enhancing skills of developing general solution processes through transforming and interpreting information, constructing models, and validating the models (Lim, Tso, & Lin, 2009). Through these processes, students learn math to "develop competency in applying mathematics and building mathematical models for areas and purposes that are extra — mathematical" (Niss, Blum, & Galbraith, 2007, p.5). Concerning the underlying learning theory, modelling is "based upon a constructive paradigm; hence, the assumption that learning is a self-regulated activity which cannot be controlled from the outside but which can be encouraged at best" (Hussmann, 2007, p. 344). This orientation requires the teacher to guide the students through MM processes not provide direct solutions.

Theoretical Background

Lingefjärd (2007) stated that "mathematical modelling *is not* a body of mathematical knowledge but rather a collection of general principles which experience has proved to be helpful in the process of applying mathematical know-how to analyze problems" (p. 476). As an activity helping students apply the concepts of mathematics *outside* of mathematics classroom, MM is characterized by a unique structure called often modelling cycles (e.g., see Blum, 1996) and components.

Organization of modelling Activities and Mathematical Models

MM structure consists of several stages. Blum and Leiss (2007) proposed to following: understanding the problem (constructing), simplifying (structuring), mathematizing, working mathematically, interpreting, validating and exposing. Transitioning through these stages involves observations, measurements, interactions - described together as data, coding systems, methods of sampling, and data collection (Confrey & Maloney, 2007). The process of MM can be supported by various means with a real experimentation to be the most common and recommended (Thomas & Young, 2011). Since conducting real experiments is difficult in mathematics classrooms—that are traditionally not designed for that purpose—there is a need for trying other means, for example computerized experiments. Podolefsky, Perkins, and Adams (2010) proved that virtual experiments can substitute for real experiments in science, thus their adoption for enhancing MM has become more tangible in contemporary math classrooms. While interactive software may serve as a means of providing new mathematical insight, Alsina (2007) warned that it cannot replace *learning by making*, which implies that while using interactive software students need to be given opportunities to manipulate on the system variables and then discover the underlying principles by themselves. While progressing through stages of MM, the learners can achieve multifaceted cognitive goals and consequently increase their competencies in applying math in other disciplines. MM activities not only provide opportunities for constructing models but they also expand students' views of mathematics by integrating it with other disciplines, especially sciences and engage students in the process of mathematization of real phenomena (Bleich, Ledford, Hawley, Polly, & Orrill, 2006).

The MM processes usually conclude with a formulation of mathematical representations called models—that are themselves key artifacts of the modelling processes (Confrey and Maloney, 2007). Elicited models are to be simplified, but accurate representations of some aspect of the real world (Winsberg, 2003). Models, can take various forms, ranging from three- to one-dimensional physical objects, statistical expressions—mainly in forms of *general linear models*— to algebraic and differential equations, all of which symbolize system variables and model their behavior. "The generic purpose of constructing and making use of a model is to understand problems seen in a broad sense, encompassing not only practical problems but also problems of a more intellectual nature that aim at designing parts of the real world" (Niss et al., 2007, p. 8).

Modelling at high school and college levels

MM can be exercised at any school level, yet the search undertaking for the purpose of this meta-analysis revealed that the majority of the research concentrates on high school and college levels. MM on these levels focuses the learners on "learning mathematics so as to *develop competency in applying mathematics and building mathematical models* for areas and purposes that are basically extra-mathematical" (Nish et al., 2007, p.5). Developing such competencies requires putting explicitly MM activities on the agenda of teaching and learning of mathematics. Research (Nish et al., 2007) shows that there is no automatic transfer of learned mathematics concepts to being able to apply them in real-life situations. MM activities possessing exploratory character are to help students make the transfer more adaptable to their experiences. The content for exercising modelling depends on the schooling level. While at the secondary level, students are introduced to modelling dynamic phenomena, at the university courses, students are expected to be able to use calculus to model given situations and produce analytical results from analyzing their models (Alsina, 2007). At the university level, modelling activities often constitute a separate course aimed at training pre-service teachers (Lingefjärd, 2007). In addition to acquiring competencies, MM activities at university level "open an excellent opportunity for revising the traditional assessment of course work and written examinations and go into the fruitful collection of good assessment practices" (Alsina, 2007, p. 472). At both levels; high school and college, the activity of modelling will require students to coordinate results of applied inquiry and construct and justify formulated models (Confrey & Maloney, 2007).

Prior Research Findings

In supporting a need for this study, we searched for meta-analyses and other types of research syntheses on MM using ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest Educational Journals, as well as Science Direct and Google Scholar. Although several meta-analytic research studies aiming at various aspects of conceptualization of mathematical ideas were located, a meta-analysis specifically targeting research on MM or a synthesis of quantitative research was not found. A lack of such undertaking further supported a need for conducting this study.

Dekkers and Donatti (1981) in their meta-analysis focused on computing the effect of using computer simulations as a medium for enhancing instructional strategies. The findings gathered from 93 empirical studies "did not support the contention that simulated activities cause an increase of students' cognitive development (ES = - 0.075) when compared with other teaching strategies" (Dekkers & Donatti, 1981, p. 425). In light of these findings and to provide suggestions for further research, they suggested that "attention should be given to reporting details of methodology employed" (p. 426). The lack of promising results was associated with inadequate teaching methods that simulations were supposed to support. While summarizing effects of technology on creation of new environments for intellectual work in mathematics, Fey (1989) uncovered that technology, at that stage of development, was not helping students with graph interpretation, as was expected and suggested developing projects that will address and investigate eliminations of these difficulties. He also noted a need for a change in teachers' perception regarding graph introduction—from teaching students "*how to* produce a graph to focusing more on explanations and elaboration on *what the graph is saying*" (p. 250). Another advantage of using computers in math education is their capability of creating micro-worlds that allows students to make changes in their environments (Balacheff & Kaput, 1996).

Quantification of learning effect sizes when the use of computer simulations were compared to traditional methods of learning was examined by Lee (1999), who meta-analyzed 19 empirical studies and concluded that they produced a moderate (ES = 0.54) learning effect size. Lee pointed out that "specific guidance in simulations helps students to perform better" (p. 81). In light of this finding, he advocated a need for placing more emphasis on the designing instructional support. A meta-analytic study conducted by Kulik (2003) who located 16 research studies published between 1990 and 2003 on the effectiveness of computerized exploratory environments in secondary schools revealed a moderate effect size of 0.32. This study did not provide further details on how the media of learning were embedded in the lesson cycles or discussed the design of instructional support. A substantial meta-analysis including studies published after 1990 was conducted by Li and Ma (2010) who extracted a total of 85 independent effect sizes from 46 primary studies representing all grades from elementary to senior secondary school. These researchers computed the effect sizes of the impact of computer technology on mathematics education in K-12 classrooms. The overall effect size of ES = 0.28 supported the claim that using technology in mathematics classes improves students' achievements. A corresponding subgroup moderator analysis revealed that effect of using simulations (ES = 1.32) outpaced the effect of tutorials (ES = 0.68). They also investigated moderator effects such as the type of learning environment and found out that "using technology in school settings where teachers practiced constructivist approach to teaching produced the larger effects on students' mathematics achievement than using technology in school setting where teachers practices traditional teaching methods" (Li & Ma, 2010, p. 234). This finding supports Confrey's and Maloney's (2007) thesis that "knowledge should be subjected to criteria of *functional fitness* that is akin to the constructivist concept of viability" (p. 58). In a similar vein, Hussmann (2007) argued that technology can support to situate in constructivist paradigm two important mathematical objectives; "function construction contributing to building ideas, and function iteration that initiates a change of concept" (p.348). Other researchers focused on investigating more specific constructs. Legé (2007) found out that having students formulate mathematical models and then having them validate the models generates higher learning effects as opposed to having students use prearranged models (formulas). He further claimed that the difference is accounted for the degree of ownership in model enacting: students who were involved in formulating the models varied given key assumptions and linked the keys together using selection criteria, whereas students in the control group passively constructed their models based on a single consideration. Lingefjärd (2005) also concluded that after being immersed in MM activities, students handled word problems better than those taught by conventional methods. Research conducted by McBride and Silverman (1991) revealed that MM used during integrated lessons increased students' achievement in all involved subjects, not only math.

Despite MM competencies being wider accepted as a part of mathematical literacy, modelling still faces unresolved issues that prevent the process of its conceptual framework design from solidifying. One such issue involves the stage of model validation. Zbiek and Conner (2006) suggested that students be given multiple opportunities to verify derived models. They also pointed out that MM supported only by pen-and-pencil might be lacking a reality aspect. Bleich and colleagues (2006) expressed concerns about inadequate teacher methodological preparation in inducing graphical representations of motion problems. A similar conclusion was reached by Sokolowski and Gonzalez y Gonzalez (2012) whose research revealed that teachers face obstacles in finding methodology that would help them guide the students through transitioning from observation to mathematization.

In sum, the major meta-analyses along with other research reported positive learning effects when MM was applied to enhance math learning objectives. Yet, the information associated with the type of instructional support that appears to be of high significance along with the extent to which contents from other disciplines should be induced into modelling activities is limited. This synthesis has also revealed that there are also unanswered questions regarding instrumental implementation of this learning method. By undertaking this study, we attempted to fill in the gap. The purpose of this meta-analysis was to synthesize peered- reviewed quantitative research findings on MM at the high school and college (tertiary) levels and search for ways of advancing the techniques of developing students' modelling competencies. Although "*research in mathematics education* has shown that the success of the modelling approach in mathematics at tertiary level does exist" (Alsina, 2007, p. 473), a study that would quantify the effect sizes of larger pool of research has not been yet undertaken.

Research Methods

We undertook a meta-analysis developed by Glass (1976) as a research method because meta-analysis helps to (a) to integrate the findings of individual research to formulate more general inferences about the effects of heuristic techniques applied during MM activities (b) to address some of the limitations of the previous research by allowing for construct formulations and evaluation and (c) evaluate effectiveness of MM activates using larger research pool since such a method has not been found in the prior literature. Zawojewski (2010) identified two types of research objectives on MM: (a) development and evaluation of the models formulated by learners, and (b) instructional tools and learning media applied during the modelling activities. This research intended to examine the findings of the former; effectiveness of instructional tools and learning media. Furthermore, through undertaking subgroup moderator analysis and identifying conditions that generate the highest learning effects, we hoped to also formulate suggestions for improving students' performance on MM tasks.

Research Questions

The formulation of the research questions was supported by (a) suggestions found in the prior literature, (b) development of contemporary views on the role of MM in school practice, and (c) the type of research methods employed. Intertwining these three venues, the following research questions were enacted:

1. What are the magnitude and direction of the learning effect size when the learning is situated in MM as compared to conventional methods of learning?
2. What are the moderators that affect students' achievement during modelling activities and what are their effect sizes?

Key Term Descriptions

The literature search was guided by the following operational terms: *mathematical modelling*, *model – eliciting activities, medium utilized for a model construction, mathematical model*, *experimental study*, and *effect size statistics*.

Mathematical modelling is a process of encountering a situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to transform the situation (Confrey and Maloney, 2007). Literature showed that the term mathematical modelling can describe two types of activity: translating the real— world system into mathematical terms for the purpose of solving a problem or for the purpose of analyzing a situation by applying various steps associated with accomplishing that goal (Gravemeijer, 1997). MM of both of these types of activities will be included in this study.

Model – eliciting activities (MEA) are defined as "problem solving activities that require students to express their current ways of thinking in forms that are tested and refined multiple times and that elicit a model" (Lesh & Yoon, 2007, p.162). In order to be termed MEA, an activity must satisfy six principles developed by Lesh and Kelly (2000): (1) the reality principle (2) the model construction principle (3) the self-assessment principle (4) the construct documentation principle (5) the model share-ability and reusability principle and (6) the simplicity principle.

Medium utilized for a model construction is defined as a form of information presented to the learners. The following are the possible media types: data tables, written text problems, computerized interactive simulations, or real experiments.

Model was operationally defined as a mathematical construct designed and formulated to study a particular real-world system or phenomenon (Confrey, 2007). Mathematical models can include, but are not limited to graphical, symbolic, and physical representations.

Experimental study is a type of research that seeks to determine whether an intervention had the intended casual effect on the participants. The following are the key components of an experimental study: (a) pre-posttest design (b) a treatment group and a control group and (c) random assignment of study participants (Shadish, Cook & Campbell, 2002).

Effect size statistics (ES) is a measure of strength of an outcome after treatment, in a form of MM was applied. ES was used to quantify student achievement in each of the located studies. The magnitude of the effect size was calculated using Hedge's (1992) formula.

$g = \frac{\bar{x}_1 - \bar{x}_2}{s*}$, where

$\bar{x}_1$ represents the posttest mean score of the treatment group (situated in modelling environment)

$\bar{x}_2$ represents the posttest mean score of the control group (taught by traditional methods)

$s^* = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}}$ represents coupled standard deviation; where

n_1, n_2 represent the sample sizes of the control and treatment groups respectively and

SD_1, SD_2 represent standard deviations of treatment and control groups mean scores.

Effect size will be expressed numerically in a decimal form.

Data Collection Criteria

The research included only peer-reviewed studies published in journals because such studies represented methodologically high quality research (Lipsey & Wilson, 2000). Although bias against null findings cannot be completely removed even in peer-reviewed journals (Cooper, 2009) high quality research provides means of computing moderator effects that was also intended in this study. The initial search criteria was restricted to the following constrains: (a) time span which included papers published between January 1, 2000, and February 27, 2013; and (b) experimental research that provided means for calculating effect size statistics (c) level and subject of teaching; high school and college math courses and (d) MM as process to transform a situation into a model and analyze or solve it. The section that follows defines, in more details, descriptive and inferential parameters that were extracted from each study.

Descriptive and Inferential Parameters

Descriptive parameters encompassed the following: the grade level of the group under investigation, *the locale* where the studies were conducted, the sample size representing the number of subjects in experimental and control groups, the date of the study publication, the duration of the study, and the total time interval that the subjects were under treatment. The total treatment time was introduced due to a high diversity of treatment frequency; thus, for instance, if the study lasted 2 months and the treatment was applied twice a week for 3 hours each session, the reporting is depicted as 2 months/48 h. Inferential parameters included posttest mean scores of experimental and control groups along with their corresponding standard deviations. If these were not provided, F-ratios or t-statistics were recorded. Although most of the studies reported more than one effect size describing also other constructs than students' achievement (see e.g., Schoen & Hirsch, 2003; Wang, Vaughn, & Liu, 2011), the current study focused on reporting effects of student achievement only. As experimental groups were under treatment of mathematical modelling, control groups were taught by traditional methods.

Descriptions of groups and their classes

A total of 14 classes were formulated and grouped according to their descriptive purposes in Table 1. The classes were used for coding purposes.

Table 1. Summary of Groups and Their Classes

Group	Classes
Study general characteristics	Research authors School level (high school or college) Subject area (calculus, statistics, algebra or geometry) Locale of the research (country where the study was conducted) Year of publication (year when research was published) Type of publication (peer-reviewed)
Study methodological characteristics	Instrumentation (computer-supported activity or pen and paper) Reliability of measure (researcher-developed instrument (local) or standardized tests) Type of research (quantitative) Group assignment (randomized or quasi-experimental) Sample size (number of participants in control and experimental groups)
Study design characteristics	Program used, research specifications (verbal descriptions) Duration of treatment (in semesters, weeks, or days) Frequency of treatment assignment (in hours per day or other metrics provided) Medium for model construction (computer or context provided on paper) Learning setting (student centered)

In the process of collecting the research literature, ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest educational journals, as well as Science Direct, Google Scholar, and other resources available through the university library, were used. The initial search terms were defined reflecting formulation of classes focusing mainly on study methodological characteristics and design. The following terms were utilized to locate the relevant literature: *mathematical modelling*, *model eliciting activities, simulations*, *computers in mathematics, mathematics education, student achievement*, *high school, college*. These search criteria returned 241 articles. After a review, it was revealed that eight studies satisfied the inclusion criteria. Most of the rejected studies focused on examining formulated models in the professional fields of engineering or medicine. In order to increase the pool, a further search was undertaken with broader conceptual definitions. This search included auxiliary terms that were found in descriptions of mathematical modelling activities, such as *investigations in mathematics, techniques of problem solving*, *exploratory learning in mathematics, and computerized animations and learning*. These modifications returned 82 research papers. After an additional scrutiny, 5 studies were added to the pool. The validity of the coding and the extracted data was supported by a double research rating that constituted of two teams; the primary authors and another professional who reviewed extracted studies for their adherence to selection criteria. The double rating was applied at the initial and at the concluding stages of the study. Any discrepancies were resolved.

Research Analysis

A total of 13 primary studies were used in this meta-analysis with a total of 1,670 participants. We realized that to have the most accurate data along with most accurate inferences, the modelling activities would have to be coded according to the MEA principles as defined by Lesh and Kelly (2000). However, such extractions were not feasible, due to MEA principles not being converted into quantitative constructs in these studies. Table 2 summarizes the studies' features.

Table 2. General Characteristics of the Studies' Features

Authors	Date	Locale	RD	SS	School Level/ Subject	Research Duration/ Frequency	Learning Setting	Medium of Learning
Young, Ramsey, Georgiopoulos, Hagen, Geiger, Dagley-Falls, Islas, Lancey, Straney, Forde, & Bradbury	2011	USA	QE	265	College/ Calculus	1 semester 1h/week	SC	Comp
Wang, Vaughn, & Liu	2011	Taiwan	QE	123	College/ Statistics	1 semester NP	SC	Comp
Voskoglou & Buckley	2012	Greece	QE	90	College/ Calculus	1 semester NP	SC	Comp
Laakso, Myller, & Korhonen	2009	Finland	R	75	College/ Statistics	2 weeks 2h/week	SC	Comp
Milanovic, Takaci, & Milajic	2011	Serbia	QE	50	HS/ Calculus	1 week 4.5h	SC	Comp
Baki, Kosa, & Guven	2011	Turkey	R	96	College/ Geometry	1semester NP	SC	Comp
Bos	2009	USA	R	95	HS Algebra	8 days 55min/day	SC	Comp
Mousoulides, Christou, & Sriraman	2008	Cyprus	QE	90	HS Statistics and Geometry	3 months 3h	SC	Comp
Schoen & Hirsch	2003	USA	QE	341	HS Algebra	1 semester NP	SC	PP
Scheiter, Gerjets, & Schuh	2010	Germany	QE	32	HS Algebra	1 session 2h	SC	Comp
Eysink, de Jong, Berthold, Kolloffel, Opfermann, & Wouters	2009	The Nether-lands and Germany	QE	272	HS Probability	1 week	SC	Comp
Bahmaei	2012	Iran	R	60	College/ Calculus	1 semester 15 sessions	SC	PP
Baki & Guveli	2008	Turkey	QE/ MS	80	HS Algebra	1 semester NP	SC	Comp

Note. R = randomized, QE = quasi-experimental, RD = research design, SC = student centered, MS = mixed methods, Comp = computer, PP = pen and pencil, HS = high school, SS = sample size, NP = not provided.

The majority of the studies (9, or 70%) were designed as quasi-experimental, while 4 (30%) were randomized. The study durations ranged from 2 hours to 1semester. The average sample size for the study pool was 123 participants, with the highest of 272 participant conducted by Eysink and colleagues (2009) and the lowest sample of 32 students in a study by Milovanović and colleagues (2010). When categorized by school level, college and high school were uniformly represented, with six high school studies (or 46%) and seven college studies (or 54%). When categorized by learning setting, all of the studies were student centered, meaning that students worked on deriving models for the given problems using the teachers' expertise only when needed. Such organized MM activities "provided students with opportunities to discuss employed strategies with each other, explore alternative solution pathways, interpret and evaluate the reasonableness of arguments and solutions and explain both results and reasoning to others" (Antonius, Haines, Hojgaard, Niss, & Burkhardt, 2007, p. 295). Model formulation was supported by using computerized simulations in 11 (or 85%) of the studies; two studies (or 15%) used the traditional pen-and-pencil approach.

Inferential Analysis

The inferential analysis of this study pool was initially performed using SPSS 21 (Statistical Package for the Social Sciences) software. We used the program to verify the homogeneity of the study pool, as suggested by Cooper (2009) and calculated the effect size for each study using posttest means on experimental and control groups, as suggested by Lipsey and Wilson (2001). Such standardized individual effect sizes were then corrected for population bias, and weighted as suggested by Hedges (1992). After weighted effect sizes were computed, the overall mean effect size statistic along with homogeneity statistics for the entire pool was calculated. The calculated homogeneity statistics ($Q_T = 329.74$, with $d_f = 16$, $p < 0.01$) indicated statistically significant variation of the effect sizes; thus, a random-effect model was adopted for further data analysis. These computations allowed for answering the first research question:

What are the magnitude and direction of the learning effect size when the learning is situated in MM as compared to conventional methods of learning?

The mean effect size for the 13 primary studies (14 primary effect sizes) was reported to have a magnitude of 0.69 (SE = 0.05). A 95% confidence interval around the overall mean—C_{lower} = 0.59 and C_{upper} = 0.79—supported its significance (Hunter & Schmidt, 1990). The numerical value of the effect size of 0.69 is classified by Lipsey and Wilson (2000) as of a moderate size. Its positive direction indicated that the score of an average student in the experimental groups —who used MM to enhance problem-solving techniques— was 0.69 of standard deviation above the score of a student in the control groups, who was taught the same processes using traditional methods of instruction. One can claim that MM activities that involve rethinking the nature of givens and patterns and allow students' own ways of reasoning are more effective than traditional problem solving approaches that are characterized by Lesh and Yoon (2007) as getting from *pre-mathematized givens* to mathematical goals. Table 3 provides a summary of the individual effect sizes of the meta-analyzed studies along with their confidence intervals, standard errors, and general descriptions of the treatment and computer programs used as a medium for model eliciting.

Table 3. Effect Sizes of Applying Mathematical Modelling in High School and College

Study (First Author)	ES	SE	95% CI Lower	95% CI Upper	Reliability of Measure Used	Program Used, Research Findings, Research Specifications
Bos (2009)	0.70	0.21	0.18	1.01	Standardized Texas state assessment. Kuder-Richardon formula 20 for reliability: r_{pret} = .80 and r_{postt} = .90.	TI Interactive Instructional environment.
Young (2011)	0.61	0.13	0.10	1.09	(UCF) university faculty Math Department tests. Inter-rater reliability: r_{pret} = .82 and r_{postt} = .92.	Research modelling activities (Excel) supported by computer.

Baki (2011)	0.81	0.26	0.09	1.11	PCVT test with KR-20 of r_{pret} = .82 and r_{pret} = .80 (Branoff, 1998).	Interactive geometry software.
Young (2011)	0.04	0.13	0.34	0.85	University (UCF) faculty Math Department tests. Reliability: r_{pre}= .82 and r_{post} = .92.	Research modelling activities (Excel) supported by computer.
Wang (2011)	0.45	0.26	0.08	1.11	Researcher-developed 20-item test, Conbach's α = .91.	Developed dynamic computer program that modeled real situations to test hypothesis.
Voskoglou (2009)	0.49	0.22	0.17	1.03	Researcher-developed test graded by two faculty members.	Contextualized differential equations using computer programs.
Laakso (2009)	0.61	0.24	0.12	1.07	Researcher-developed test.	Trakla2 to have learners developed probability principles.
Milanovic (2010)	0.67	0.29	0.02	1.18	Researcher-developed test, items the same on both pretest and posttest.	Developed simulated program to evaluate integrals. Used Macromedia Flash 10.
Mousoulides (2008)	0.31	0.22	0.17	1.03	Researcher-developed test.	Researcher-designed activities aimed at various math model formulations.
Schoen (2003)	0.53	0.11	0.38	0.81	Standardized calculus readiness test items.	Developed new math curriculum that focused on modelling.
Scheiter (2010)	0.57	0.36	-0.14	1.33	Researcher-developed test aligned with Reed (1999) categorization.	Computer programs to enhance modelling through animated situations.
Eysink (2009)	4.49	0.12	0.35	0.84	Researcher-developed 44-item test. Reliability was determined by Cranach's α = .64 and α = .82.	Different multimedia settings to investigate the effect on students' math inquiry skills.
Bahmaei (2012)	1.84	0.26	0.07	1.13	Researcher-developed test items.	Researcher-developed activities.
Baki (2008)	0.43	0.23	0.14	1.05	Researcher-developed test items with reliability of r_{postt} = .62.	Web-based mathematics teaching material (WBMTM).

Note. ES = effect size, SE = standard error.

All meta-analyzed studies reported a positive effect size when MM activities were used. The highest effect size of ES = 4.49 was reported by Eysink and colleagues (2009), who investigated the effect of multimedia on students' modelling skills, and the lowest of ES = 0.04 was reported by Wang and colleagues (2011), who investigated the effect of using computerized programs to model differential equations. In order to support reliability of the assessment instrument, several researchers (e.g., Wang et al., 2011) applied the Crnobach's α- coefficient or Kruder – Richardon's formula 20 (KR20). A reliability coefficient of the assessment instrument was applied in six (or 46%) of the studies. Table 3 contains also additional information provided by the primary researchers that distinguish the applied medium within the study pool. In the majority of the studies, the modelling activities were supported by researcher-developed contexts. In order to identify potential moderators, the studies were further aggregated (see Table 4).

Analysis of Moderator Effects

The process of computing subgroup effects allowed for uncovering moderators that optimized the magnitude of the effect size statistic and to answer the second research question:

What are the moderators that affect students' achievement during modelling activities and what are their effect sizes?

A set of four moderators was identified: *school level*, *type of medium used for MM*, *length of treatment*, and *mathematics content domain*. This categorization resulted in 10 subgroups whose effects were individually computed and summarized in Table 4.

Table 4. Summary of Subgroups and their Weighted Effect Sizes

Moderators and Their Groups	N	ES	SE	95 % CI	
				Upper	Lower
Grade Level					
High school	7	0.94	0.07	0.79	1.08
College	7	0.45	0.08	0.30	0.61
Medium Supporting MEA					
Computer simulations	12	0.72	0.06	0.60	0.85
Pen and paper activities	2	0.68	0.10	0.48	0.88
Treatment Duration					
Semester	8	0.46	0.06	0.34	0.59
Shorter than one semester	6	1.31	0.10	0.11	1.50
Content Domain					
Algebra	4	0.73	0.09	0.55	0.91
Calculus	5	0.38	0.09	0.19	0.56
Probability and Statistics	4	3.11	1.17	3.11	3.80
Geometry	1	0.81	0.26	0.09	1.11

Note, N = number of participants, ES = effect size, SE = standard error.

The mathematical calculations of the moderators followed Cooper (2009), who suggested giving more weight to effect sizes with larger sample populations. Calculation of corresponding confidence intervals and standard errors were also enacted. The following sections provide a more detailed discussion of the identified moderators and their effect on student achievement.

The effect of the school level on student achievement

This block was created to mediate the effect sizes of students' achievement between the high school and college levels. Although it was intended to differentiate not only among high school grade levels but also among college majors, due to the limited pool of studies, this idea was aborted and two general group levels—high school and college—were formulated. The effect size showed differences; high school students reported a large effect size of ES = 0.94 (SE = 0.07), versus college-level reporting a moderate effect size of ES = 0.45 (SE = 0.08). This evidences that that high school students benefit more by being involved in modelling activities than college-level students and the difference can be accounted for other mediators (silent in these studies), such as the difference in difficulty level of high school and college mathematics courses or better acquaintance of high school students with modern computerized modelling media. As modelling is a relatively new mathematics learning method, some college students might find it difficult to alter their habits of considering mathematics as a subject of *drill and practice* to a subject that provides a basis for hypothesizing, explorations and opportunities for genuine applications. We hypothesize that a prior experience with modelling at lower school level might also have an impact on students' achievement at the college level. The data accumulated in the pool did not provide the basis for supporting such claim though. However, if the information were available, an additional moderator could be formulated and further computations conducted. Developing modelling skills and techniques that require solving higher-order problems that involve analyzing and synthesizing knowledge of multiple subject areas requires certain time and effort. It seems though that the sooner developing such skills is initiated and brought forth, the sooner the learner will become acquainted and benefit from them.

The effect of medium used during MM activities on student achievement

Two learning media—computer simulations and written pen-and-pencil activities—were identified in the gathered pool. Computers were used in 12 (or 86 %) of the studies to support modelling activities, and written pen-and-pencil methods were used in two of these studies (or 14%). The learning effect size produced by simulations was higher (ES = 0.72, SE = 0.06) when compared to traditional pen-and-pencil activities (ES = 0.68, SE = 0.10), yet the difference was not that large. An advantage of computer simulations is their interactivity that allows to conveniently observing the system outputs due to manipulating on independent variable(s) (Scheiter et al., 2010) and also verifying derived model. The other advantage of using technology is engaging the learners in a new level of creative discovery that places them "in a situation where they naturally raise the question before being shown the result" (Pead, Ralph, & Muller, 2007, p. 315) which seemingly triggers revisions or consolidations of previously learned concepts. A word of caution must be placed here: the medium itself, as noted by Bos (2009), will not generate learning because concepts, principles, and ideas do not reside in physical materials or classroom activities but in what students actually do and experience. Kadijevich (2007) suggested to "view computers as tools that expand human mental function" (p. 352). As many students experience difficulties in transferring their knowledge from the mathematical world to real (e.g., see Crouch & Haines, 2004), technology according to Keune and Henning (2003), can help reduce such difficulties by enabling the students to concentrate and master the subtasks that cause the most difficulties in the transferring process. Careful inquiry planning coupled with availability of interactive media are the prerequisites for initiating students' engagement and knowledge transfer. Research also shows (e.g., see Young et al., 2011) that providing students with too detailed descriptions of procedures without letting them explore and discover relations on their own is not an effective inquiry and a balance between what students input should be and the degree of provided guidance needs to be established and controlled. A relatively low pool of located pen-and pencil research (2, 14%) suggests that such modelling medium is diminishing from the research and more sophisticated computerized environments are being used.

The effect of the treatment length on student achievement

Two different classes were formulated to answer this question: one semester and shorter than one semester. At the college level, some of the research was designed using a platform of a MM lasting one semester that examined the effects of modelling activities embedded during the course (e.g., Voskoglou & Buckley, 2012). The time span for other research was shorter ranging from a 2 h (e.g., Scheiter et al., 2010) to 3 months 3 h per week (e.g., Mousoulides et al., 2010). The effect size computation for this subgroup showed that shorter treatments produced a higher effect of students' achievement (ES = 1.31, SE = 0.10) than longer (ES = 0.46, SE = 0.06). The large effect size of ES = 1.31 that resulted primarily from computing the effect sizes of control groups taught by tightly choreographed traditional teaching methods and experimental using MM, supports the claim that providing students with opportunities for modelling problems where they "filter, interpret, relate, organize, or synthesize information" (Lesh & Yoon, 2007, p.169) brings positive effects in their learning even if such activities are shorter.

The effect mathematical content domain on student achievement

Four different domains were formulated for this subgroup: algebra, calculus, probability and statistics, and geometry. The frequency of studies in each level was highly dispersed, ranging from one study that examined modelling the concepts of geometry to five studies that focused on modelling calculus concepts. According to computations, probability and statistics, generated the highest effect size (ES = 3.11, SE = 1.17). The magnitude of this effect size was inevitably affected by an outlier of 4.49 (Eysink et al., 2009). If this study were removed, the effect size would have been ES = 0.21, with SE = 0.17. Lakoma (2007) identified several steps of students' natural reasoning that lead them to developing stochastic concepts: "exploring a situation involving randomness, formulating a problem, creating a local model of the phenomenon, analyzing the mathematical model in order to solve the problem and comparing solutions obtained using the model with results of observations of the random phenomenon" (p. 391). The high effect indicates that students learn the concepts of probability and statistics more effectively when the concepts are modeled.

The concepts of algebra and its predominated domain — function analysis— produced a moderately high effect size of ES = 0.73. Mathematical functions are difficult for students because they can be perceived structurally, as objects, and operationally, as processes (Sfard, 1991). Modelling challenges the students to connect objects and processes of various function representations, thus one can hypothesize that this is one of the reasons that modelling effectively supports function understanding. The lowest effect size of ES = 0.38, SE = 0.09) in this subgroup was attributed to activities involving calculus concepts. As calculus is driven by a high diversity of function representations, a closer look at this result and searching for ways of improving it seems indispensable. It is expected at the university level, that "students are able to use calculus to model situations and herby to be able to produce analytical results from analyzing models (Blomhøj & Jensen, 2007, p. 52). The task of model formulation, for instance differential equations (e.g., Milovanović et al., 2011) that are typically initiated by identifying a rate of change are poised to have bifocal difficulties; they require a deep understanding of principles of the underlying contexts and familiarity with various structures of differential equations.

As modelling is to promote better understanding of all involved subjects, not only its mathematical part, teachers need to be prepared to" help students clarify a real problem, generate and select variables, setup conditions appropriately and confidently […] and promote positive affective contexts about mathematics and the problem domain" (Kadijevich, 2007, p. 349). Knowledge of sciences as well as of other academia is required to be possessed by mathematics teachers to successfully lead students through the process of identifying embedded principles. Calculus, as a study of change and accumulation, provides a wide range of sophisticated apparatus for inducing mathematical modelling activities, but it seems that heuristic techniques applied during modelling calculus concepts can be revisited to better reflect students' needs and their experiences. While the focus of the current research is on applying MM to problem solving, attempts to use MM to model concept introduction such as, for example function differentiability, or the first fundamental theorem of calculus seem as suggestions for further research to improve the effects of MM on students achievement in calculus. It is hypothesized that introducing calculus concepts in contexts will simultaneously provide students with ample application examples thereby making the underlying theorems more related to students' prior experiences.

Summary and concluding remarks

Accumulating all of the inferences, it is concluded that modelling activities generate positive learning effects when compared to traditional teaching methods at high school and college levels in any content domain. As "a number of empirical and international comparative studies indicate that applications and modelling are less significant in everyday school life in many countries" (Kaiser & Maaß, 2007, p. 99), this study provides a robust support for a wider implementation of modelling to mathematical school practice as a support of students' mathematical learning through modelling activities as well as the development of students' modelling competencies.

A subsequent moderator analysis revealed that among the collected pool of studies, the setting that produces the most optimal learning effects is short activities conducted at a high school level supported by computer simulations as a medium for modelling. While real-world contexts provide legitimate sources for introduction of MM activities (Niss et al., 2007), more work needs to be done in bringing forth the heuristic techniques that MM has to offer such as "creating interplay between the real world and mathematics toward more realistic and less stereotyped problem situations and […] changing teachers' conceptions, beliefs and attitudes and change classroom culture by establishing new socio-mathematical norms" (Bonotto, 2007, p. 186).

This study uncovered also several concerns regarding domain specific instructional support that would guide the students through reasoning and firm their MM competencies (e.g., see Lim et al., 2009, Mousoulides et al., 2008). In school practice "the activity of modelling is a way to highlight a mathematical topic" (Makar & Confrey, 2007, p. 490) rather than focus on inferences that derived models can provide. Such organized activities might not benefit the learners in fully because they support "*realizing mathematics* –by pinpointing out applications of ideas and skills that are introduced […] not *mathematizing reality*" (Lesh & Yonn, 2007, p. 163). Since the goal of teaching mathematics today is to treat it as a language for communication and as a tool for predictions and explanations of reality (Freudenthal, 1983), there is a need to place more emphasis on the context and have the learner focus first on the underlined principles before attempting to translate it in a mathematical model. These concerns suggest pathways for further studies focusing on merging students reasoning skills learned in other subjects (e.g., sciences) in a unified comprehensive MM process.

If mathematics is to be taught as a language of communications, the learners need to be provided with directions on how to relate behaviors of variables of a given scenario with properties of specific functions. For example, students would need to realize that periodic occurrences will most likely be modeled by sinusoidal functions for which to know period of occurrence along with a maximum or minimum value is required (e.g., see Sokolowski & Rackly, 2011) or that two dimensional motion requires consideration of parametric equations. Students' familiarity with functions properties are important initial steps toward mastering modelling competencies but their skills to relate identified contexts principles with a corresponding function properties are anticipated to benefit the learner even more, especially at the high school and college levels.

Other suggestions for further research, that materialized might focus on (a) investigating the effect of MM on eliminating students' science and mathematics misconceptions and (b) the degree to which math modelling activities should be contextualized, whether MM should be limited to formulating mathematical representations or should it be perceived as a bridge linking mathematics with other academia and provide more opportunities for scientific investigations.

We are aware of certain factors limiting the study findings, one of which was the number of located primary studies and the aim that focused only on quantitative. While we focused on collecting available peer-reviewed studies published in journals, we realized that perhaps opening the research to other types of reports such as books, dissertations, technical reports, unpublished manuscript, conference proceedings and the like would increase the pool and consequently the significance of study findings. Extending the search criteria is suggested as an avenue for a further study along with conducting a parallel meta-synthesis of qualitative research. Another factor justifying the limited count of studies is the virtue of modelling that is not widely exercised in school practice yet despite its proven positive effects. However as this research showed, its popularity is gaining momentum; 46% of the collected studies (N = 6) were conducted within the past two years suggesting that a potential for a more comprehensive meta-analytic study exists.

References

** Note, the astrix indicates a study used in the meta-analysis.*

Al-Hammody, A. (2014). When a Facebook Group Makes a Difference: Facebook for Language Learning. ELTWorldOnline.com April 2014. Retrieved January 15, 2015, from http://blog.nus.edu.sg/eltwo/?p=4224 Volume 6.

Alisina, C. (2007). Less chalk, less words, less symbols ... more objects, more context, more actions. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 35-44). New York, NY: Springer.

Antonius, S., Haines, Ch., Hojgaard, J., Niss, M. & Burkhard H. (2007). Classroom activities and the teacher. In W. Blum, P. Galbraith, HW.Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 295-308). New York, NY: Springer.

*Bahmaei, F. (2012). Mathematical modelling in university, advantages and challenges. *Journal of Mathematical Modelling and Application*, *1*(7), 34-49.

*Baki, A., & Güveli, E. (2008). Evaluation of a web based mathematics teaching material on the subject of functions. *Computers & Education*, *51*(2), 854-863.

*Baki, A., Kosa, T., & Guven, B. (2011). A comparative study of the effects of using dynamic geometry software and physical manipulatives on the spatial visualization skills of pre-service mathematics teachers. *British Journal of Educational Technology*, *42*(2), 291-310.

Balacheff, N., & Kaput, J. J. (1996). Computer-based learning environments in mathematics. In *International handbook of mathematics education* (pp. 469-501). Netherlands: Springer.

Bleich, L., Ledford, S., Hawley, C., Polly, D., & Orrill, C. (2006). An analysis of the use of graphical representation in participants' solutions. *The Mathematics Educator, 16*(1), 22-34.

Blum, W. (1996). Anwendungsbezuge im mathematikumterricht—Trends und perspectiven. In G. Kadunz, H. Kautschitsch, G. Ossimitz, & E. Schneider (Ed.), *Trends und perspektiven* (pp. 15-38). Wien, Austria: Holder-Pichler-Tempsky.

Blum, W., Galbraith, P. L., Henn, H. W., & Niss, M. (2007). Modelling *and applications in mathematics education: The 14th ICMI Study*. New York, NY: Springer.

Blum, W., & Leiss, D. (2007). How do students and teachers deal with modelling problems. In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical Modelling* (pp. 222-231). Chichester, England: Horwood Publishing Limited.

Blomhøj, M., & Jensen, T. H. (2007). What's all the Fuss about Competencies?. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 45-56). New York, NY: Springer.

Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modelling. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 45-56). New York, NY: Springer.

*Bos, B. (2009). Virtual math objects with pedagogical, mathematical, and cognitive fidelity. *Computers in Human Behavior*, *25*(2), 521-528.

Chaachoua, H., & Saglam, A. (2006). Modelling by differential equations. *Teaching Mathematics and its Applications*, *25*(1), 15-22.

Coley, R. J., Cradler, J., & Engel, P. K. (2000). *Computers and the classroom: The status of technology in U.S. schools*. Retrieved January 15, 2015, from http://www.ets.org/research/policy research_reports/pic-compclssl.

Confrey, J (2007). Epistemology and Modelling-Overview. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education:The 14th ICMI Study* (pp. 125-128). New York, NY: Springer.

Confrey, J., & Maloney, A. (2007). A theory of mathematical modelling in technological settings. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 57-68). New York, NY: Springer.

Cooper, H. (2009). *Research synthesis and meta-analysis* (4th ed.). Thousand Oaks, CA: Sage.

*Cory, B. L., & Garofalo, J. (2011). Using dynamic sketches to enhance preservice secondary mathematics teachers' understanding of limits of sequences. *Journal for Research in Mathematics Education*, *42*(1), 65-97.

Crouch, R., & Haines, C. (2004). Mathematical modelling: transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology,* 55(2), 197-206.

Dekkers, J., & Donatti, S. (1981). The integration of research studies on the use of simulation as an instructional strategy. *Journal of Educational Research, 74*, 424-427.

English, L., & Sriraman, B. (2010). Problem Solving for the 21st Century. In *Theories of Mathematics Education: Seeking New Frontiers*. Monograph 1 in the series *Advances in Mathematics Education*. Berlin/Heidelberg: Springer Science. pp. 263-290.

*Eysink, T. H., de Jong, T., Berthold, K., Kolloffel, B., Opfermann, M., & Wouters, P. (2009). Learner performance in multimedia learning arrangements: An analysis across instructional approaches. *American Educational Research Journal*, *46*(4), 1107-1149.

Fey, J. T. (1989). Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics*, *20*(3), 237-272.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Reidel.

Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational Researcher*, *5*(10), 3-8.

Gravemeijer, K. (1997). Solving word problems: A case of modelling? *Learning and Instruction*, *7*(4), 389-397.

Hedges, L. V. (1992). Meta-analysis. *Journal of Educational and Behavioral Statistics*, *17*(4), 279-296.

Hunter, J. E., & Schmidt, F. L. (1990). *Methods of meta-analysis: Correcting error and bias in research findings*. Newbury Park, CA: Sage.

Hußmann, S. (2007). Building Concepts and Conceptions in Technology-based open learning environments. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 341-348). New York, NY: Springer.

Kadijevich, D. (2007). Towards a wider implementation of mathematical modelling at upper secondary and tertiary levels. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study.* (pp. 349-356). New York, NY: Springer.

Kaiser, G., & Maaß, K. (2007). Modelling in lower secondary mathematics classroom—problems and opportunities. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 99-108). New York, NY: Springer.

Keune, M., & Henning, H. (2003). Modelling and spreadsheet calculation. In Ye Q-X. et al. (Eds), *Mathematical Modelling in Education and Culture: ICTMA IO* (pp. 101-110). Chichester: Ellis Horwood.

Kulik, J. (2003). Effects of using instructional technology in elementary and secondary schools: What controlled evaluation studies say. *SRI International*. Project # P10446001. Retrieved January 15, 2015, from http://bestevidence.org.

*Laakso, M. J., Myller, N., & Korhonen, A. (2009). Comparing learning performance of students using algorithm visualizations collaboratively on different engagement levels. *Journal of Educational Technology & Society*, *12*(2), 267-282.

Lakoma, E. (2007). Learning Mathematical Modelling—From the Perspective of Probability and Statistics Education. In W. Blum, W. P. Galbraith, P., Henn, H.W. & Niss M. (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 387-394). New York, NY: Springer.

Lee, J. (1999). Effectiveness of computer-based instructional simulation: A meta- analysis. *International Journal of Instructional Media, 26*(1) 71-85.

Legé, J. (2007). “To Model, or to Let Them Model?” That is the Question! In W. Blum, W. P. Galbraith, P., Henn, H.W. & Niss M. (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 425-432). New York, NY: Springer.

Lesh, R., & Harel, G. (2003). Problem solving, modelling, and local conceptual development. Revealing Activities for Students and Teachers. *Mathematical Thinking and Learning: An International Journal, 5*(2/3), 157-190.

Lesh, R., & Kelly, E. (2000). Multi-tiered teaching experiments. In E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modelling. *Second Handbook of Research on Mathematics Teaching and Learning*, *2*, 763-804.

Lesh, R., & Yoon, C. (2007). What is distinctive in (our views about) models & modelling perspectives on mathematics problem solving, learning, and teaching?. In W. Blum, W. P. Galbraith, P., Henn, H.W. & Niss M. (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 161-170). New York, NY: Springer.

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students’ mathematics learning. *Educational Psychology Review*, *22*, 215-243.

*Lim, L. L., Tso, T. Y., & Lin, F. L. (2009). Assessing science students’ attitudes to mathematics: A case study on a modelling project with mathematical software. *International Journal of Mathematical Education in Science and Technology*, *40*(4), 441-453.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.

Lingefjärd, T. (2005). To assess students' attitudes, skills and competencies in mathematical modelling. *Teaching Mathematics Applications*, *24*(3), 123-133.

Lingefjärd, T. (2007). Modelling in teacher education. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 475-482). New York, NY: Springer.

Lipsey, M. W., & Wilson, D. B. (2000). *Practical meta-analysis.* Thousand Oaks, CA: Sage.

Makar, K., & Confrey, J. (2007). Moving the context of modelling to the forefront: Preservice teachers' investigations of equity in testing. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 485-490). New York, NY: Springer.

McBride, J. W., & Silverman, F. L. (1991). Integrating elementary/middle school science and mathematics. *School Science and Mathematics*, *91*(7), 285-292.

*Milovanović, M., Takači, Đ., & Milajić, A. (2011). Multimedia approach in teaching Mathematics: Example of lesson about the definite integral application for determining an area. *International Journal of Mathematical Education in Science and Technology*, *42*(2), 175-187.

*Mousoulides, N. G., Christou, C., & Sriraman, B. (2008). A modelling perspective on the teaching and learning of mathematical problem solving. *Mathematical Thinking and Learning*, *10*(3), 293-304.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, HW. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education:The 14th ICMI Study* (pp. 3-33). New York, NY: Springer.

Pead, D., Ralph, B., & Muller, E. (2007). Uses of technologies in learning mathematics through modelling. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 309-318). New York, NY: Springer.

Pollak, H. (1968). On some of the problems of teaching applications of mathematics. *Educational Studies in Mathematics, 1*(1/2), 24-30.

Podolefsky, N. S., Perkins, K. K., & Adams, W. K. (2010). Factors promoting engaged exploration with computer simulations. *Physics Review Special Topics—Physics Educational Research, 6*(2), 117-128.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Belmont, CA: Wadsworth Cengage Learning.

*Scheiter, K., Gerjets, P., & Schuh, J. (2010). The acquisition of problem-solving skills in mathematics: How animations can aid understanding of structural problem features and solution procedures. *Instructional Science*, *38*(5), 487-502.

*Schoen, H. L., & Hirsch, C. R. (2003). Responding to calls for change in high school mathematics: Implications for collegiate mathematics. *The American Mathematical Monthly*, *110*(2), 109-123.

*Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational Studies in Mathematics*, *79*(2), 215-237.

Schwarz, C. V., & White, Y. (2005). Metamodelling knowledge: Developing students' understanding of scientific modelling. *Cognition and Instruction, 23*(2), 165-205.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*, 1–36.

Sokolowski, A., & Gonzalez y Gonzalez, E. (2012). Teachers' perspective on utilizing graphical representations to enhance the process of mathematical modelling. In M. Koehler & P. Mishra (Eds.), *Research highlights in technology and teacher education: Society for Information Technology & Teacher Education* (pp. 157-164). Chesapeake, VA: SITE.

Sokolowski, A., and Rackley, R. (2011). Teaching harmonic motion in trigonometry: Inductive inquiry supported by physics simulations. *Australian Senior Mathematics Journal, 24*(2), 45–54.

Thomas, J. M., & Young, R. M. (2011). Dynamic guidance for task-based exploratory learning. In G. Biswas, S. Bull, J. Kay, A. Mitrovic (Eds.), *Artificial intelligence in education* (pp. 369-376). Berlin, Germany: Springer.

*Voskoglou, M. G., & Buckley, S. (2012). Problem solving and computational thinking in a learning environment. *Egyptian Computer Science Journal*, *36*(4), 28-45.

*Wang, P. Y., Vaughn, B. K., & Liu, M. (2011). The impact of animation interactivity on novices' learning of introductory statistics. *Computers & Education*, *56*(1), 300-311.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modelling method for high school physics instruction. *American Journal of Physics*, *63*(7), 606-619.

Winsberg, E. (2003). Simulated experiments: Methodology for a virtual world. *Philosophy of Science, 70*(1), 105-125.

*Young, C. Y., Georgiopoulos, M., Hagen, S. C., Geiger, C. L., Dagley-Falls, M. A., Islas, A. L., Bradbury, E. E. (2011). Improving student learning in calculus through applications. *International Journal of Mathematical Education in Science and Technology*, *42*(5), 591-604.

Zawojewski, J. (2010). Problem solving versus modelling. In R. Lesh, P. L. Galbraith, Ch. R. Haines, & A. Hurford (Eds.), *Modelling students' mathematical modelling competencies* (pp. 237-243). New York, NY: Springer.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modelling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics*, *63*(1), 89-112.