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Instrumental Appropriation of a Collaborative, Dynamic-Geometry Environment and Geometrical Understanding

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Abstract

To understand learners' appropriation of technological tools and geometrical understanding, we draw on the theory of instrumental genesis (Lonchamp, 2012; Rabardel & Beguin, 2005), which seeks to explain how learners accomplish tasks interacting with tools. To appropriate a tool, learners develop their own knowledge of how to use it, which turns the tool into an instrument that mediates an activity between learners and a task. The tool used in our study is the Virtual Math Teams with GeoGebra (VMTwG) environment. It contains a chat panel and multiuser version of GeoGebra. The learners are seven middle and high school mathematics teachers who participated in a professional development course in which they collaborated synchronously in VMTwG to solve geometrical tasks. We use conventional content analysis to analyze the work of a team consisting of two high school teachers. Our analysis shows that the teachers' appropriation and application of the dragging feature of VMTwG shaped their understanding of geometrical relations, particularly dependencies. This informs the broader question of how and what mathematical knowledge learners' construct using certain technologies.

Introduction

Understanding geometry is important in itself and for understanding other areas of mathematics. It contributes to logical and deductive reasoning about spatial objects and relationships. Geometry provides visual representations alongside the analytical representation of a mathematical concept (Goldenberg, 1988; Piez & Voxman, 1997). Pairing learning geometry with technological tools of Web 2.0 can allow learners to investigate collaboratively geometrical objects, properties, and relations and develop flexible understanding of geometry. The Common Core State Standards for Mathematics underscores that mathematics educators should seek to develop students' mathematical practices so that they "use appropriate tools strategically," including dynamic geometry environments (Common Core State Standards Initiative, 2010, p. 7). Although teaching with technology is recommended, meta-analytic studies show that teaching mathematics with technology cannot guarantee positive influence on learning (Kaput & Thompson, 1994; Wenglinsky, 1998). Consequently, careful investigations are required to understand the appropriation of technology and how it shapes mathematics learning. To contribute to this understanding, we describe the influence of learners' appropriation of online dynamic geometry tools on their geometric understanding. This paper responds to the question: How does learners' appropriation of an online, collaborative dynamic geometry environment shape their geometrical understanding?

Literature Review

Researchers investigated the use of technology in learning mathematics for different purposes. The first group of researchers focused on investigating the effect of introducing certain technologies in learning mathematics. For example, Hohenwarter, Hohenwarter, and Lavicza (2009) engaged 44 middle and high school mathematics teachers in four workshops to learn about different geometric and algebraic topics in a dynamic geometry environment (DGE), GeoGebra, to investigate changes in teachers' mathematical knowledge in general. Similarly, Sinclair and Yurita (2008) investigated teacher's changes in mathematical discourse after introducing the use of dynamic geometry software in classroom. These studies did not focus primarily on how the teachers interacted with DGE; their main goals were investigating the impact of introducing such technologies on learning and teaching mathematics.

Other studies focused on certain aspects of interacting with DGEs. Arzarello, Olivero, Paola, and Robutti (2002) studied the dragging action in DGE and the cognitive processes behind each different type of dragging. They identify two levels of cognitive processes linked to dragging: ascending (moving from drawings to theory) and descending processes (moving from theory to drawings). Ascending processes allow users to investigate the drawings freely to look for patterns and invariants. Descending processes are used with a theory in mind to validate or test properties. Dragging within those two cognitive levels can vary between wondering dragging, bound dragging, guided dragging, dummy locus dragging, line dragging, linked dragging, and dragging test (Arzarello et al., 2002). Baccaglini-Frank and Mariotti (2010) used the work of Arzarello et al. to develop a model that tries to explain cognitive processes behind different types of dragging. They used four different types of dragging: wondering dragging, maintaining dragging, dragging with trace activated, and dragging test. Wondering dragging is dragging that aims to look for regularities while maintaining dragging is dragging base points so that the dynamic figure maintain certain properties. Dragging with trace activated is dragging base points with trace activated on them. Drag test is dragging base points to test whether certain properties will meet certain conditions (Baccaglini-Frank & Mariotti, 2010).

The last group of studies looked at the instrumental transformation of technological tools into instruments that mediate users' activity. An example of these studies is Guin and Trouche's (1998) study. They investigated the instrumentation process that a group of students used to transform graphing calculators to mathematical instrument. They conclude that instrumentation is a complex and slow process and that interacting with technological tools in learning mathematics might not result in transforming the tool into an instrument. Similarly, Guin and Trouche (2002) and Trouche (2003) show how learners transform technological tools into mathematical instruments with more focus on the teacher's and the environment's roles that support this transformation.

Based on our review, there is a need for studies that investigate how users, through the instrumentation process, interact with each other in DGEs. The users' interactions with DGEs have an influence on their thinking and learning of geometry (Hegedus & Moreno-Armella, 2010; Rabardel & Beguin, 2005), which makes investigating how learners appropriate an online, collaborative dynamic geometry environment important. It can help mathematics educators understand how DGEs shape learners' geometrical understanding.

Theoretical Perspective

To understand learners' appropriation of technological tools, we draw on a Vygotskian perspective about goal-directed, instrument-mediated action and activity. Instrumental genesis (Lonchamp, 2012; Rabardel & Beguin, 2005) theorizes how learners interact with tools that mediate their activity on a task. To appropriate a tool, users (teachers, students, or learners in general) develop their own knowledge of how to use it, which turns the tool into an instrument that mediates activity between users and a task. The basic concept of the theory is that users engage in an activity in which actions are performed upon an object (matter, reality, object of work…) in order to achieve a goal using an artifact (technical or material component). Rabardel and Beguin (2005) emphasize that the instrument is not just the tool or the artifact, the material device or semiotic construct, it is "a mixed entity, born of both the user and the object: the instrument is a composite entity made up of an artifact component and a scheme component." (p. 442). An instrument is a two-fold entity, part artifactual and part psychological as utilization schemes. The user acquires a utilization scheme and applies it to the artifact.

Artifacts are subject to two kinds of utilization schemes. The first kind of utilization schemes is usage schemas, which are directly related to the artifact. It constitutes the basic knowledge of how to operate or use the artifact. For example, the knowledge about driving a car for an experienced driver such as changing gears or turning the steering wheel, or the knowledge of the components of a digital camera and how to use them. The second kind of utilization schemes is instrument-mediated action schemes, which are more related to the transformations that can be done to the object. These schemes are concerned with the activity that will lead the users, using the artifact, to reach a desired goal. For the example of driving a car, the instrument-meditated action scheme will be more focused on the other variables on the road that a driver needs to be aware of and react to their existence to be able to reach the final destination (Lonchamp, 2012; Rabardel & Beguin, 2005). The activity can be individual or collective depending on the number of users engaged in the activity.

Just interacting with an artifact is not an instrument-mediated activity. In instrument-mediated activity, instruments mediate users' activity or action to achieve a certain goal. While engaging in an activity, users monitor consciously the continuous transformation of an object towards their goal. This mediator role that instruments play governs the user-object relations, which might take epistemic or pragmatic forms. The epistemic mediation form focuses on the object and its properties. In this form, the instrument helps the user understands the object and its structure. On the other hand, in the pragmatic mediation form, the user is mainly concerned with the required actions while using the instrument to transform the object into the desired final result (Lonchamp, 2012; Rabardel & Beguin, 2005). The final result is the final transformation of the object, which might not match exactly the initial goal. The user may find certain form of the transformed object satisfying enough and, therefore, end the activity.

During an activity that is mediated by an instrument, it is understandable that the artifact affects the activity; however, users play a major role in shaping the activity. The users' interactions with an artifact shape the activity. Two different users can approach an artifact differently, develop different utilization schemes, and create two different activities and instruments.

The transformation of an artifact or tool into an instrument, or instrumental genesis, occurs through two important dialectical processes that account for potential changes in the instrument and in the learners, instrumentalization and instrumentation. The instrumentalization process is defined as "the process in which the learner enriches the artifact properties" (Rabardel & Beguin, 2005, p. 444). In this process, the user selects and modifies the properties of the artifact, for example, using a wrench as a hammer. The second process of instrumental genesis is instrumentation. This process is about the development of the learner side of the instrument. The development of the learner is basically the assimilation of an artifact to a scheme and the adaptation of utilization schemes. With the example of using a wrench as a hammer, the learner already had acquired the utilization scheme of a hammer and when a hammer was not available at the time of the action, the learner chose the wrench and associated it to the hammer utilization scheme. This is an example of "direct assimilation of artifact into a utilization scheme" (Rabardel & Beguin, 2005, p. 446), which changes the meaning of the artifact. During the act of assimilation, the learner employs previous utilization schemes to new artifact. In our example, acquiring the hammer utilization scheme led the learner to choose the wrench and not another tool because he is aware of the functions of the hammer and its structure that makes him look for a similar tool that can take the same scheme. In the situation where a new artifact cannot be assimilated to previously acquired utilization scheme, the learner adapts utilization schemes and makes the necessary modifications to it.

Through the two processes of instrumental genesis, instrumentation and instrumentalization, dialectically the tools influence the thinking of the learner and the learner influences the design of the tools. On the one hand, the structure and functionality of tools shape how the learner uses the tool, which result in shaping the learner's thinking. On the other hand, the learner's interactions with the tool also shape the tool and how it is used. In the case of dynamic geometry environments (DGEs), instrumentation occurs when users develop utilization schemes of how to use the environment. Utilization schemes include learning how to use a DGE's tools to construct and manipulate geometric objects. It also includes understanding the functions of those tools and their links to the theory of geometry and using the tools to explore, conjecture, and justify relations among geometric objects. The instrumentalization process concerns the design and the use of DGEs. Concerning the design, developers instrumentalize by deciding on what functions to include, how to organize the environment, how it reacts to user's actions, how to support users' activity, and so forth. Concerning the use of DGEs, instrumentalization occurs with users' decisions about how to use the environment. Users may use the environment and its functions as the developers intended. However, to achieve certain goals, users may use DGEs' tools differently from how developers intended. For example, constructing two parallel lines can be accomplished using "Parallel Line" tool, but users may use "Regular Polygon" tool to construct a square and then construct two lines on two opposite sides of the square. Of these two dialectically related processes, for this study, we are concerned with instrumentation in DGEs since we are interested in understanding how users develop their utilization schemes and learn to use the tools as intended. This understanding can provide insights into learners' geometrical knowledge and its development.

Further insights can be derived from how learners respond to the environment's actions. The feedback that the software gives to the user after manipulating dynamic objects affects the user's interaction with the software. The environment reacts to the users' actions through engineered infrastructure that responds according to the theory of geometry. These reactions can inform the users' actions and shape their thinking, which provide insights into how DGEs are used to mediate users' activity. During users' activity, dragging the base points or "hot-spots" of a dynamic figure can change the geometric properties of the figure and can provide insights into its construction process. Hot-spots are "points that can be used to construct mathematical figures, e.g. join two points with a segment, or construct a piecewise graph, and then used to dynamically change the construction." (Hegedus & Moreno-Armella, 2010, p. 26).

The relationship between the user and the DGE is a result of co-action between the two (Hegedus & Moreno-Armella, 2010). The notion of co-action has two sides: (a) the user's action can guide DGE and (b) DGE's reaction can guide the user. A dynamic geometry software allows users to act on it and, in turn, reacts to their actions. As users drag (click, hold, and slide) a hotspot of a geometric figure, the DGE redraws and updates information on the screen, preserving all constructed mathematical relations among objects of the figure. In redrawing, the DGE creates a family of not only visually but also mathematically similar figures. Users may then attend to the reaction of the DGE and experience and understand underlying mathematical relations such as dependencies. DGEs "remember" underlying mathematical relations among various objects of a construction. For instance, if a point P is the midpoint of a segment AB, then as the length or position of segment AB changes, P's relationship to AB remains invariant, namely that P is equidistant from the line segment's endpoints A and B.

Dynamic geometry environments are tools that learners can appropriate through an instrumentation process. Learners will need to acquire utilization schemes – usage schemes and instrument-mediated activity schemes – to appropriate the tool. In DGE, the usage schemes include knowledge about the software use and its functionalities. The second level of utilization schemes for a DGE includes knowledge of geometry and dependencies. When learners appropriate a DGE as an instrument, they will be able to use it to demonstrate geometrical concepts and solve geometrical problems. This appropriation may result in knowledge of how to use dynamic geometry software as well as knowledge of geometry. The geometrical knowledge can be a special type of knowledge shaped by DGE. Within DGEs, Straesser (2002) sees that geometry is "lived in differently, broader scope, has a new, more flexible structure, [and] offers easy access to certain heuristic strategies." (p. 331). Balacheff and Kaput (1996) claim that characteristics of DGEs result in creating new mathematics, a geometry that is different from Euclidian geometry in the plane.

Methods

Data come from a project that integrates a cyberlearning environment with digital tools for collaborative geometrical explorations grounded in a pedagogical approach that engages learners in developing significant mathematical discourse. The project investigates learners' actions as they occur through an iterative coevolution of the technology and curricular resources in the context of engaging, reflective collaborative learning experiences of significant mathematical discourse by in-service teachers and their students. The data for this paper come for an online professional development course for middle and high school teachers that occurred over 15 weeks in the fall 2014. In small teams, seven New Jersey middle and high school mathematics teachers engaged in interactive, discursive learning of dynamic geometry through collaborating to solve tasks in a computer-supported, collaborative-learning environment: Virtual Math Teams with GeoGebra (VMTwG). They also read and discussed articles about collaboration (Mercer & Sams, 2006; Rowe & Bicknell, 2004), mathematical practices (Common Core State Standards Initiative, 2010), accountable talk (Resnick, Michaels, & O'Connor, 2010), technological pedagogical content knowledge (Mishra & Koehler, 2006), implementing technology in mathematics classroom (McGraw & Grant, 2005), and validating dynamic geometry constructions (Stylianides & Stylianides, 2005) and analyzed logs of their VMTwG interactions to examine, reflect, and modify their collaborative and mathematical practices.

VMTwG, a product of a collaborative research project among investigators at Rutgers University and Drexel University, is an interactional, synchronous space. It contains support for chat rooms with collaborative tools for mathematical explorations, including a multi-user version of GeoGebra, where team members can define dynamic objects and drag the hotspots around on their screens. VMTwG records users' chat postings and GeoGebra actions. The research team designed dynamic-geometry tasks that encourage participants to discuss and collaboratively manipulate and construct dynamic-geometry objects, notice dependencies and other relations among the objects, make conjectures, and build justifications (Powell & Alqahtani, 2015).

For this paper, we analyze the work of Team 3, which consists of two high school mathematics teachers. Before this course, both teachers did not have any experience with dynamic geometry. The teachers met in VMTwG synchronously for two hours twice a week. We selected data of this team's interactions since they demonstrated conspicuously how team members built an understanding of the dragging affordances of the environment. To understand how teachers interact with VMTwG environment and how the environment shapes their geometrical knowledge, we used the discursive data generated from their work on three tasks, Task 8, which they worked on in the fourth week, Task 16, which they worked on in the fifth week, and Task 21, which they worked on in the seventh week. Task 8 (see Figure 1) asks the teachers to discuss the construction of equilateral triangle and then construct it and Task 16 (see Figure 2) asks the teachers to drag dynamically different triangles and discuss the dependencies involved in their construction. Task 21 (see Figure 4) asks teachers to construct a perpendicular line that passes through an arbitrary point. The discursive data includes the logs of teachers' chat communications and their GeoGebra interactions. Using conventional content analysis (Hsieh & Shannon, 2005), we analyzed their discursive data to understand the developmental process of instrument appropriation and the implications of that appropriation. In addition, we used the construct of co-action to understand when, why, and how do teachers interact with base points; what feedback do they perceive and what do they do with this feedback; and how does the feedback shape their subsequent actions.

Results

Our analysis focuses on understanding how the teachers' appropriation of VMTwG shapes their geometrical understanding. Specifically, our results show how through co-action teachers' interaction with VMTwG leads to shaping their understanding of affordances of dragging in GeoGebra while working on constructing an equilateral triangle (Task 8). The result from Task 16 shows how teachers used aspects of the environment to identify dependencies among geometric objects. Our results in a later VMTwG session also show how the VMTwG environment shapes the knowledge that the teachers develop while working to construct a perpendicular line that goes through an arbitrary point as well as their heuristic to solve the problem (Task 21).

Appropriating Dragging

Task 8 asks teachers to drag objects and then to discuss (in the chat window) what they notice about the given figure and then construct a similar one in GeoGebra (see Figure 1). Among other things, previous tasks engaged the teachers in noticing as they dragged hotspots variances and invariances of objects and relations of figures.

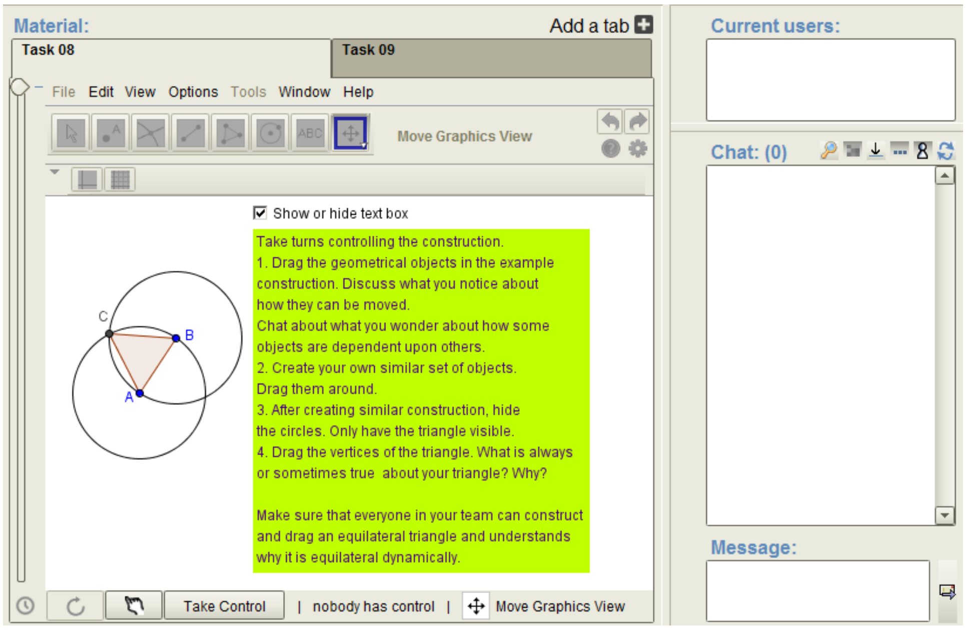

Figure 1. Task 8: Constructing equilateral triangle.

This task was intended to extend the teachers' experience with dragging and geometrical dependencies. Before this session, the teachers had already worked on some basic geometrical objects (such as lines, lines segments, circles, and circles whose radius is dynamically dependent on a line segment) and were asked to drag and notice relationships among the objects. Those tasks mainly aimed to familiarize the teachers with the functionality of the VMTwG environment and to the cognitive habit of noticing and wondering about the behavior of object and relations among objects. That is, the tasks engaged the teachers in becoming aware of co-active relations between their actions and reactions of the VMTwG environment. Below, their chat posting shows that they focused on relationships that were visually apparent. It also shows that the teachers, gouri and sophiak, felt the necessity to revisit their understanding of dragging after being instructed to create an equilateral triangle.

#	User	Chat Post
26	sophiak	It seems that point C is fixed but pts A&B are not. I am thinking somehow A&B were used to create the circles which is why the make the circles bigger or smaller.
27	sophiak	How about you try to explore now?
28	gouri	ok I'll continue on with #2 [the second instruction in Task 8] as well
29	sophiak	No, I would like to create the objects as well. I think it is valuable if we both explore
30	gouri	C does seem fixed/constrained
31	gouri	sure - how about i do it and then you do it as well after?
32	sophiak	Sounds good. Please type what you do.
33	gouri	So far I created 2 circles
34	gouri	and overlapped the D point as the raius point for E
35	gouri	one more try
36	gouri	ok - i deleted the other circle because i dont need it
37	gouri	I somehow thought i could create all 3 points, abc through two circles
38	sophiak	How did you create F?
39	gouri	I added a point
40	gouri	then the polygon tool for the triangle
41	sophiak	Did you want to explore your picture to see if it behaves the same way as the original?
42	gouri	ok
43	gouri	[after dragging for few minutes] I noticed that it's the points that make the circle dynamic
44	gouri	and not the circle (in black) itself

Our analysis of this excerpt reveals two aspects of this team's instrumentation process: collaboration and tool use, which parallels their mathematical understanding. From a collaboration point of view, the team was trying to establish collaborative norms by starting tasks by exploring the pre-constructed figures and then reproducing those figures. In lines 27 and 29, sophiak suggests explicitly that gouri explores before she constructs. This team's evolving collaborative norm seems to start with each member exploring and sharing noticings and then each member constructing the figures.

With regards to teachers' actions towards solving the task, the teachers started by stating their noticings of the construction. In line 26, sophiak mentions that point C is fixed and points A and B are not and states that points A and B are used to construct the two circles. She states that since dragging points A and B effects the circles then they are used in constructing the circles. It indicates how sophiak views the relationship between dependency and construction and how she is starting to identify the hotspots of the figure. Her comment at line 26 seems to indicate that she is connecting prior experience (A and B's independent role) with other tasks to what she notices about the size of the circles. The co-action—dragging points A and B with the change in the circumference of the circles—provides epistemic mediation since sophiak acquires knowledge about the relationship between the base points, A and B, and the circumference of the circles. The second team member, gouri, takes control, agrees that point C is "fixed/constrained", and tries to construct a similar figure. She successfully creates a similar figure to the task's figure. In lines 33 to 40, gouri describes to sophiak the process of her construction, and following sophiak's suggestion in line 41, drags and tests gouri's construction and the pre-constructed figure. She states after dragging in lines 43 and 45 that "the points that make the circle dynamic and not the circle (in black) itself". These comments suggest that gouri was concerned with what is being dragged in a dynamic geometry environment and what makes it dynamic.

This event shows that the teachers are distinguishing between dragging that affects other geometrical properties in addition to the location of an object–dragging the points that relates to the construction–and dragging that only affects the location of an object–dragging the circumference of a circle. The second teacher here is also showing her understanding of the hotspots of the figure. The DGE's reaction informed the teachers' dragging. The co-action between the teachers and the environment helped the teachers develop an understanding of the dragging functionality in DGE. This shows how teachers appropriate the environment through developing their understanding of dragging and dependencies. In this session, the teachers started to pay more attention to how constructions in DGE take place.

Identifying Dependencies

In the fifth week of the course, Team 3 worked on Task 16. The teachers were asked to drag dynamically different triangles and discuss the dependencies involved in the construction (see Figure 2).

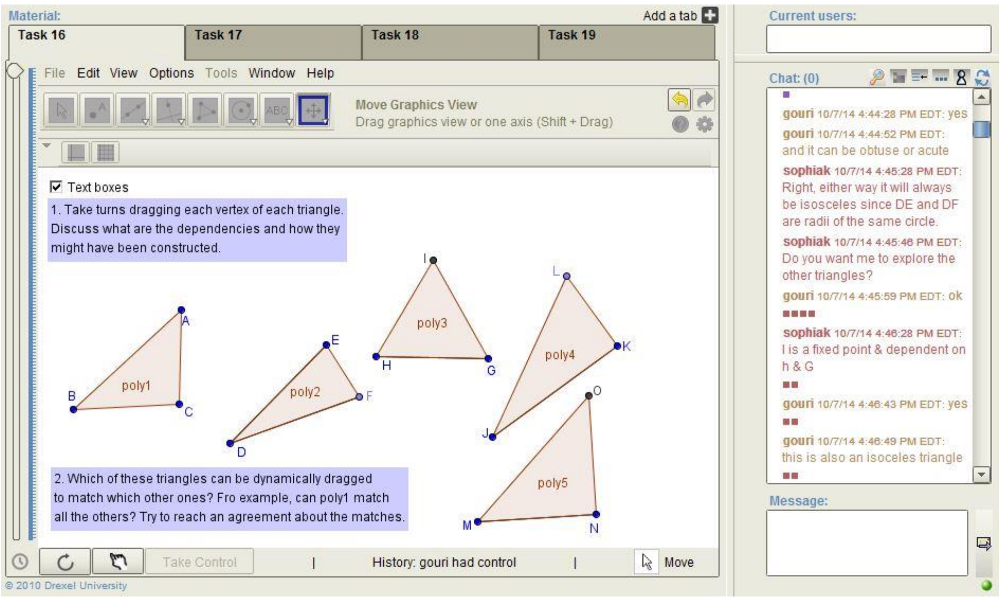

Figure 2. Task 16: Triangles with dependencies.

The teachers show fluent use of dragging dependency while discussing the triangles. They stated that the first triangle does not have any dependencies involved in its construction. While discussing the second triangle (poly2), they dragged the vertices D, E, and F vigorously then checked the Algebra View in GeoGebra to look for more relationships among triangle DEF objects. Algebra View is an analytical view that shows some properties of the objects in the graphic view. It also shows the hidden objects and their properties (see Figure 3). Team 3 was able to use Algebra View to explore the objects and their relationships.

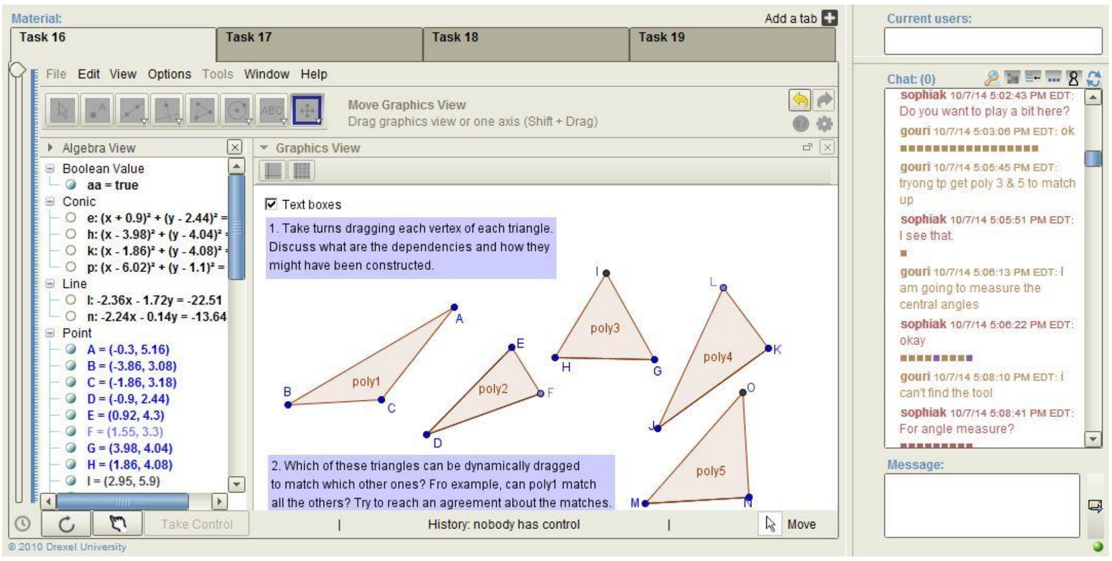

Figure 3. Algebra view for Task 16.

The following excerpt from Team 3's work on Task 16 shows how the team effectively used dragging and Algebra View to identify dependencies involved in the construction of triangle DEF.

#	User	Chat Post
11	gouri	I don't really see many dependencies between the triangles in this case
12	sophiak	I thought the same thing, poly 1 seems independent
13	gouri	[dragged point E] Poly2 is completely different

14	sophiak	What do you mean by different?
15	gouri	[dragged point E] When I drag point E, point F is dependent in that it follows E
16	sophiak	Does it move from either point F or D?
17	gouri	[dragged points D, E, and F and made the circle that was used in the construction visible] Points E and F are on a circle
18	gouri	they are radii and move accordingly
19	sophiak	How did you notice that?
20	gouri	[dragged points E and D] D is the center of the circle
21	gouri	I went to algebra view and unhid the conic e
22	sophiak	Oh, intersecting. DEF is a an isosceles triangle
23	sophiak	I meant to type interesting
24	gouri	yes
25	gouri	and it can be obtuse or acute
26	sophiak	Right, either way it will always be isosceles since DE and DF are radii of the same circle.

As mentioned above, the team states that the first triangle does not have dependencies in lines 11 and 12. In line 13 and 15, gouri drags point E, an independent point, and notices that point F is changing, which makes point F dependent on point E. Then sophiak asks about the other points in line 16. That question made gouri drag all the vertices of the triangle. In lines 17 and 18, gouri realizes that there is a hidden circle used in the construction and states that DE and DF "are radii and move accordingly" when changing the circle. In line 20, she states that point D is the center of the circle. The second teacher, sophiak asks "How did you notice that?" In line 21, gouri states that she used Algebra View to unhide the circle used in this construction. This allowed sophiak to know that the triangle is an isosceles triangle and justifies that in line 26 saying "it will always be isosceles since DE and DF are radii of the same circle." The team's work on this task shows that this team is using VMTwG with its different component efficiently to explore mathematical objects and their relations and to justify those relations.

Constructing Perpendicular Lines

Teachers' understanding of dragging different types of objects, hotspots and other objects, in DGE helped them appropriate the environment, which influenced the type of knowledge that teachers developed in later sessions in the course. To further illustrate this, we now discuss the teachers' work on Task 21 (see Figure 4). Between Task 8 (constructing equilateral triangle) and Task 21, the teachers used the compass tool to copy line segments and to construct different types of triangles. Task 21 invited them to construct a perpendicular line that passes through an arbitrary point on a given line (see Figure 4).

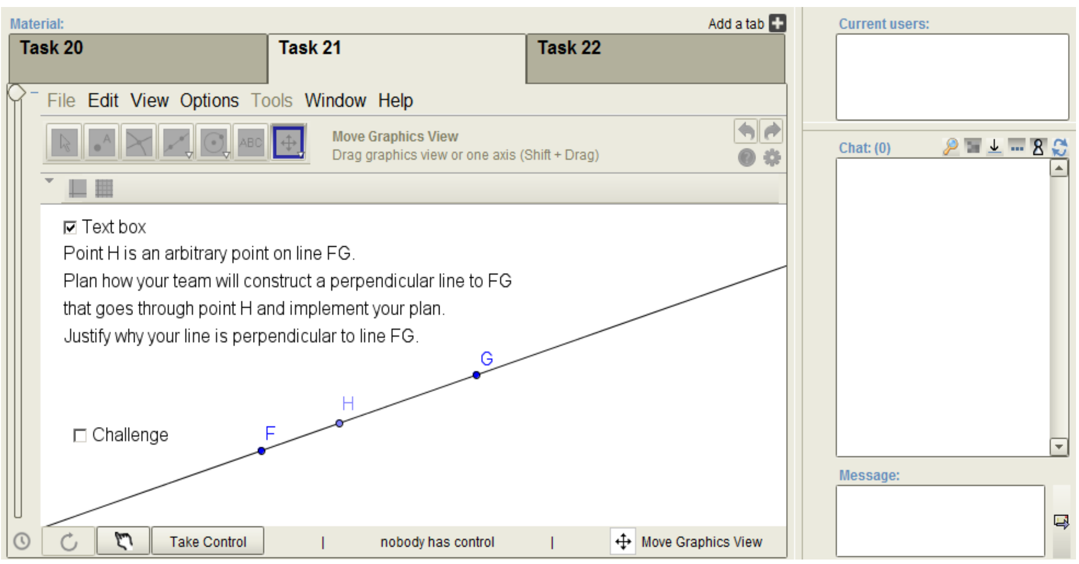

Figure 4. Task 21: Constructing perpendicular line that passes through an arbitrary point.

In the preceding task, Task 20, the teachers constructed a line perpendicular to a give line (see Figure 5). In this task, our intent was to enable the teachers to develop insight and skill to solve Task 21. However, after working on Task 20, this team of teachers were unable to solve Task 21 in their first attempt. As Mason and Johnston-Wilder (2006) note that "What is intended, what is activated (implemented), and what is attained or constructed by the learners are often rather different" (p. 27). In written feedback, we suggested that in their next synchronous session they revisit Task 20 and try to use its technique to solve Task 21.

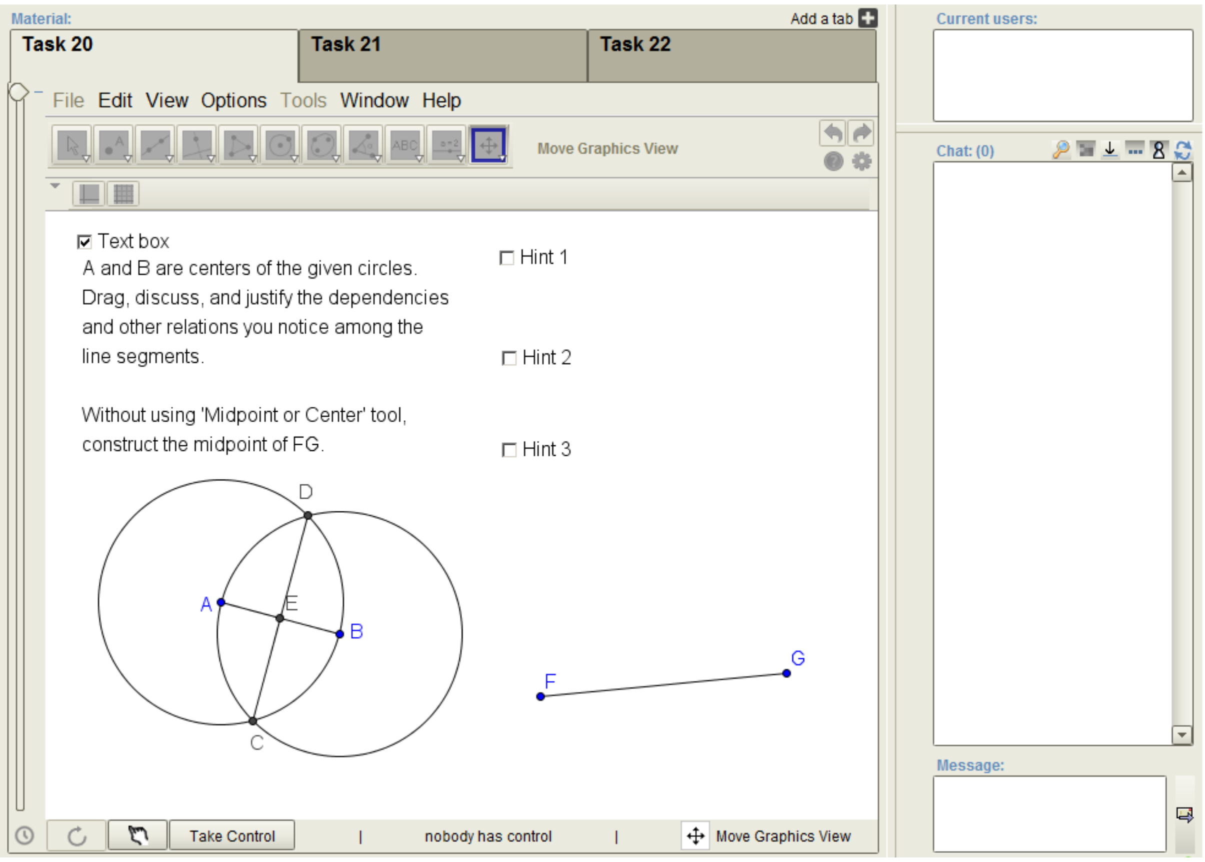

Figure 5. Task 20: Constructing perpendicular line.

The teachers met again to solve Task 21. To do so, they created a new GeoGebra workspace or tab within Task 21 and labeled it, "Task21-again" (see Figure 6) and successfully solved it. Interestingly, however, their solution did not rely on the solution of Task 20 in the way we anticipated. We intended that Task 20 would help the teachers see that one way to solve Task 21 is by constructing a circle that has the arbitrary point as center then mark the two intersection points of this circle with the given line to identify the radius for two other circles that will intersect in two points. Connecting those two intersecting points will create a perpendicular line that passes through the arbitrary point. Instead, the teachers develop another approach.

The teachers used some insights from Task 20 to construct perpendicular lines multiple times. They started by constructing a line AB and an arbitrary point C (see Figures 6). Then using the technique from Task 20, they constructed a line EF perpendicular line to AB (constructed circles with common radius AB, marked their intersections points E and F, then hid the circles) and dragged points A and B to test the construction. On that line, they marked point G and, employing the Task-20 technique, used it and point E to construct line IJ perpendicular to EF, which make IJ parallel to AB. After that, they construct circle EC and marked the intersection point of this circle with line IJ, point K. They dragged point C to test the behavior of the construction. Finally, they construct line KC, which is perpendicular to AB and passes through the arbitrary point C.

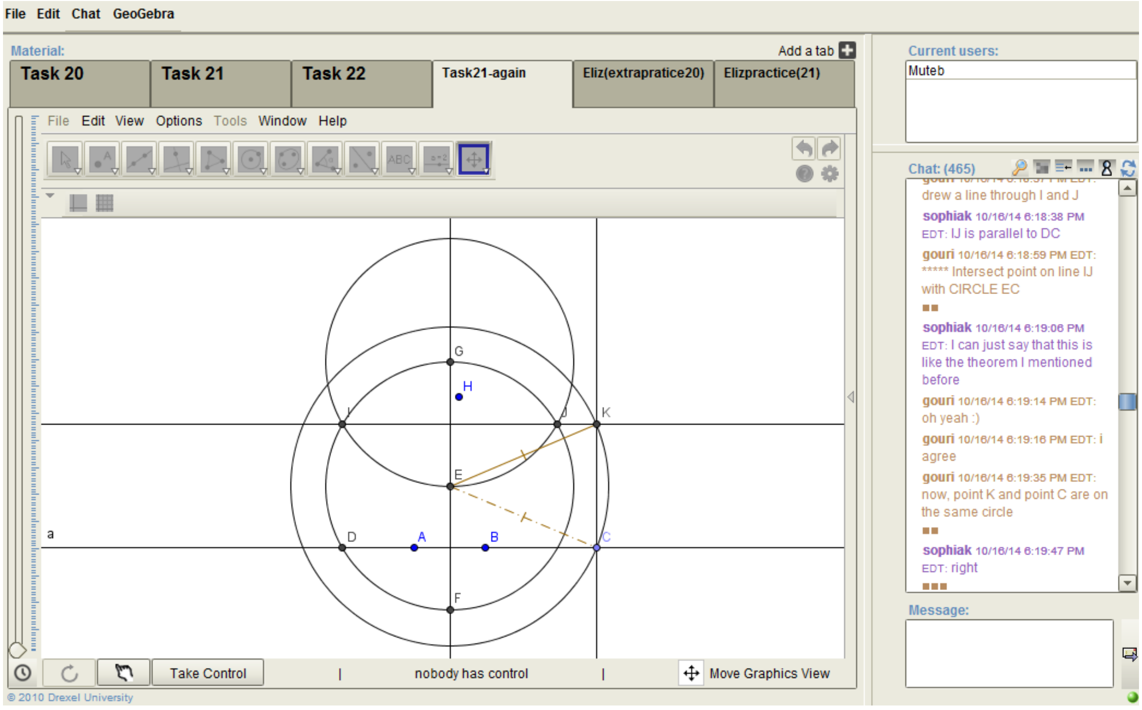

Figure 6. Team 3's solution to Task 21 In VMTwG.

Proving that KC is perpendicular to AB is beyond the scope of this paper; however, it can be done easily using triangle congruency. The teachers struggled for about an hour to solve this task. They passed the control of GeoGebra to each other and tried to make sense of each other's actions. They referred to Task 20 a few times and discussed how they could use it in solving Task 21. They collectively constructed their final solution. After each step of their construction, they dragged points A, B, and C to make sure that at each stage their construction maintained properties they intended. Their appropriation of dragging–what to drag, how to drag, and what to expect–was dominant in their problem solving of Task 21.

Discussion

A team of two high school teachers was introduced to collaborative, online, dynamic geometry environment, VMTwG, in a 15-weeklong professional course. During this course, the members of this team interacted in VMTwG to notice variances and invariances of objects and relations in pre-constructed figures or figures that they constructed and to solve open-ended geometry problems. Our analysis of their interactions allowed us to understand how they appropriated the environment and how this appropriation influenced their geometrical knowledge. At the beginning of the course, the teachers started by focusing on appropriating the dragging affordance of DGEs. They paid special attention to the characteristics of the objects that being dragged. Their interactions indicate that they see the significance of dragging the hotspots of a construction (Hegedus & Moreno-Armella, 2010). The co-action of the VMTwG environment that occurs while they drag different objects in Task 8 helped the teachers identify the hotspots and use them to test their construction and become aware of dependencies. The need for more than wondering dragging (Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010) in this task motivated the teachers to develop more purposeful dragging. They used maintain dragging to check if the triangle maintains its properties and later on, drag to test the validity of their construction (the drag test). Similarly, the teachers used combination of wondering dragging and maintain dragging to identify dependencies in Task 16. Their maintain dragging was more evident while trying to understand dependencies in the isosceles triangle.

Their process of appropriating VMTwG started with dragging and then looking at and understanding dependencies in dynamic constructions. Team 3's interaction with VMTwG suggests that while appropriating DGE, constructing dynamic figures in DGEs comes last, after dragging and dependencies. The Team's interaction also shows that the teachers attended to the analytical descriptions of objects in GeoGebra by using the Algebra View. They used this view in conjunction with explorations through dragging and visualization to satisfy their wondering and verify what they believed true.

While trying to solve Task 21, as mentioned above, the teachers referred to Task 20 multiple times and collectively constructed their solution for Task 21. They dragged the hotspots of their construction after each step. Their understanding of dragging hotspots helped them solve the task. While constructing a perpendicular line that passes through an arbitrary point, the teachers constructed a rectangle. Their procedure anticipates work they will be doing in a later task.

Understanding how instrumentation process occurs in DGEs informs implementing DGEs in learning and teaching mathematics. As Guin and Trouche (1998) found, instrumentation is a complex, slow process. The appropriation of dragging is an important aspect of transforming dynamic geometry tool into an instrument. The environment's reaction to dragging helps users identifying the independent, partially independent, and completely constrained points in any geometric figure, but users need to be attentive to dragging points and see what geometric relations and properties remain invariant. It is also important to anticipate DGE to shape users' knowledge by developing relational understanding of geometric notions.

Finally, further research is needed to understand what other aspects of DGEs are important in the instrumentation process. Investigating how the appropriation of different aspects or tools of DGEs might influence learners' knowledge is also needed. The instrumentalization process with DGEs is another important area of research since it can inform DGE design and implementation in teaching and learning mathematics. Understanding how learners use DGEs informs how researchers and educators design tasks for this environment and how to support learners' appropriation of this technological tool. Additionally, research is needed to investigate how teachers implement DGE collaboratively in their teaching to support students' strategic use of DGE tools and the discursive development of their geometric ideas.

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