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Local conjecturing process in the solving of pattern generalization problem

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This aim of this study is to describe the process of local conjecturing in generalizing patterns based on Action, Process, Object, Schema (APOS) theory. The subjects were 16 grade 8 students from a junior high school. Data collection used Pattern Generalization Problem (PGP) and interviews. In the first stage, students completed PGP; in the second stage, work-based interviews were conducted by the researchers to understand the process of conjecturing. These interviews were video tapped. The results show that the local conjecturing process can be grouped into two, namely local conjecturing based on proximity and local conjecturing by contrast. The local conjecturing process occurs at the stage of action in which subjects build a conjecture by observing and counting the number of squares separately. At the stage of process, the object and scheme were perfectly performed.

Key words: local conjecturing, problem solving, and pattern generalization.

INTRODUCTION

Pattern generalization is an important aspect in the activities of school mathematics (Dindyal, 2007; Vogel, 2003; Zazkis and Liljedahl, 2002). In line with this idea, Küchemann (2010) states that generalizations should be the core of the activities of mathematics at school. Generalization is certain kind of conjecture, which is obtained from particular to general reasoning (Yerushalmy, 1993). Further, a generalization based on inductive reasoning is called a conjecture (Ramussen and Miceli, 2008).

A conjecture is a logical statement, but whose truth has not been confirmed (Cañadas and Castro, 2005; Ontario Ministry of Education, 2005; Mason et al., 2010; Reid, 2002). The process of producing conjecture is called the conjecturing process. Conjecturing process is the mental activity used in building a conjecture based on one's knowledge (Sutarto et al., 2015). Mental activity in building conjecture is a process that occurs in the mind that can be seen through the behaviour of students in problem solving.

Associated with the conjecturing in the school mathematics is a process of building new knowledge for the students. This is in accordance with the statement by Lee and Sriraman (2010) that conjecturing in mathematical problem solving in learning process is to construct new knowledge for the students according to the existing knowledge that students already have.

A significant contribution to the conjecturing process

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has been given by several researchers (Cañadas, 2002; Cañadas et al., 2007; Polya, 1967; Reid, 2002). Polya (1967) suggests a four-step process of inductive reasoning in solving problems, namely (1) observation of particular cases, (2) conjecture formulation based on previous particular cases, (3) generalization, and (4) conjecture verification with new particular cases. Reid (2002), in the process of inductive reasoning in the context of empirical induction of a finite number of discrete case, uses the following stages of (1) observation of a pattern, (2) the conjecturing (with doubt) that this pattern applies generally, (3) the testing of the conjecture, (4) the generalization of the conjecture, and (5) using generalizations to prove. Cañadas (2002) suggests seven steps to describe the process of inductive reasoning, namely (1) observing cases, (2) organizing cases, (3) searching for and predicting patterns, (4) formulating a conjecture, (5) validating the conjecture, (6) generalizing the conjecture, and (7) justifying the generalization. Furthermore, Cañadas et al. (2007) use seven stages that describe the process of inductive reasoning of Cañadas (2002) as one type of conjecturing process of empirical induction of a finite number of discrete cases. These studies have not revealed the conjecturing process done by students in pattern generalization problems.

Associated with the process of conjecturing, Cañadas et al. (2007) argue that one of the familiar conjecturing processes in mathematical problem solving is conjecturing of empirical induction of a finite number of discrete cases. This type of conjecturing is frequently found in problems involving pattern of numbers. In solving problems involving numbers to a consistent pattern, the seventh stage of conjecturing process does not always happen as there are many factors that influence such type of task or characteristics of the students involved (Cañadas and Castro, 2005).

Sutarto et al. (2015) explained that subject classification based on observing the case in conjecturing process: 1) observing and counting the number of the square at the down-left side up-right, the primary, the left and right without discerning the Black and White boxes (3 subjects), 2) observing and counting the number of Black and White boxes separately (5 subject) , 3) Observing and counting the number of the box without discerning the Black and the white boxes (8 subject).

Based on that explanation, conjecturing processes of students in the solving of pattern generalization problems are grouped into two: local conjecturing and global conjecturing. The local conjecturing process is the activity in building a conjecture by observing the problems separately, while the global process of conjecturing is a activity in building a conjecture by observing the problem as a whole. The local conjecturing process is rarely performed by students when solving pattern generalization problems. Therefore, in this study we will describe the process of local conjecturing in pattern generalization problems.

The process of local conjecturing in problem solving of pattern generalization is analysed by using the APOS theory, because it is a theory that can be used as an analytical tool to describe the development of a scheme by a person on a mathematical topic as totality of knowledge related (consciously or unconsciously) to the topic (Dubinsky, 2001).

This theory is based on the hypothesis that a person's knowledge of mathematics will be a tendency to cope with a situation of mathematical problem by building actions, processes, and objects and arrange them in a scheme to understand the situation and solve the problem (Dubinsky and McDonald, 2001). Asiala et al. (1996) considered that APOS theory begins with manipulating previously constructed mental or physical objects to form actions; action is then interiorized to form processes which are then encapsulated to form object. Object can be encapsulated back to the processes from which they were formed; Finally, action processes and objects can be organized in schemas. This theory is called the APOS (actions, processes, objects, and scheme).

Indicators of local conjecturing process

Researchers describe the process of local conjecturing using the theory of Cañadas et al. (2007) on the seven stages of the process of conjecturing of empirical induction of a finite number of discrete cases. The explanation on the seven stages of the conjecturing process is as follows: (1) Observing cases is the initial activity carried out on particular cases of the problem posed; (2) Organizing case is an activity that involves the use of strategy facilitating the work in certain cases.

The most common strategy used in organizing cases is registering or sorting the data; (3) Searching for and predicting patterns is one's activity when observing repetitive and regular situations, one naturally imagines that the pattern may apply to the next cases of unknown; (4) Formulating a conjecture is making a statement about all possible cases, based on empirical facts, but with an element of doubt or in other words conjecture is a statement that has not been validated; (5) Validating the conjecture is the activity performed to justify the conjecture generated based on specific cases but not in general; (6) Generalizing the conjecture is an activity on changing confidence related to the generated conjecture, that the conjecture is valid in general; (7) Justifying generalization is the activity performed to justify generalizations. Justifying the generalization involves giving reasons that explain conjecture with the intention of convincing others that the resulting conjecture is correct.

Based on the explanation of the seven stages and indicator of conjecturing processes adapted from Sutarto

Table 1. Indicators of local conjecturing process

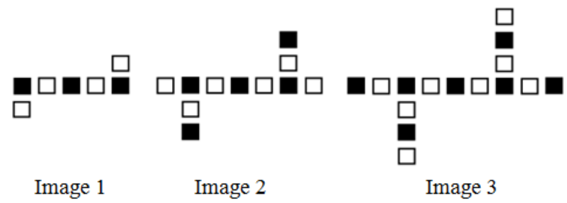
Stages of conjecturing process	Indicator
Observing the case	Initial activities against specific cases of problems presented as: - Observing and counting the number of square section on lower left, upper right, main, left and right without distinguishing a black square and a white square - Observing and counting the number of horizontal and vertical square-section without distinguishing a black square and a white square - Observing and counting the number of black squares and white squares separately
Organizing the case	Activities that involve the use of strategies that facilitate work in certain cases such as: - Writing down the number sequence - Making a list or table to associate number 1 with the picture number 1, number 2 with picture number 2, number 3 with picture number 3, and so on - Writing down symbol that indicates a similar pattern as the bottom line, circle, or other
Searching for and predicting the pattern	Activities of observing certain objects either organized or disorganized and thinking about the next object that is not yet known such as: - Calculating the difference between a square to the first, second, and third image and thinking about the next object - Calculating the difference between a black square and a white square to the first, second, and third image and thinking about the next object
Formulating the conjecture	Making a statement about all possible cases, based on empirical facts, but have not been validated as: - Stating pattern or formula of image n applicable - Declaring the n formula of black square 2n + 1, white square 2n + 2 and the n formula for black and white squares (2n + 1) + (2n + 2) - Stating the general formula or the number of squares of n image = 4n + 3
Validating the conjecture	Activities carried out to establish the truth of the conjecture produced by certain new cases but not in general, such as: - Validating the particular case to establish the truth of the conjecture generated, for example, for the first, second, third image, ... - To sketch the next object that represents the next reachable pattern to establish the truth of the conjecture generated, for example, for the fourth, fifth, sixth image, ...
Generalizing the conjecture	Changes related to the confidence of the generated conjecture, that the conjecture is valid in general such as: - Stating pattern or formula of then image - Believing the formula of n black square as 2n + 1, white square as 2n + 2 and the formula for n black and white squares (2n + 1) + (2n + 2) - Believing the general formula or the number of squares for n = 4n + 3 applied in general.
Justifying the generalization	Giving the reasons that explain the generalization with the intention of convincing others that the resulting generalization is correct, such as: - Justifying generalizations based on specific cases.

et al. (2015), we obtained indicators for local conjecturing process in problem solving of pattern generalization as presented in Table 1.

Pattern generalization

Many mathematicians claim that mathematics is referred

Look at the pattern of squares formed by Image 1, Image 2, and Image 3 below!



Find the general formula to decide the total number of squares n from the pattern formed!
Explain how you come to the general formula!

Figure 1. The Pattern Generalization Problem (PGP)

to as the science of patterns (Resnik, 2005; Tikekar, 2009). Learning about pattern is important and needs to be taught early. NCTM (2000) recommends that students participate in patterning activity from a young age, in the hope that they will be able to: (1) make generalizations about geometric and numerical patterns, (2) provide justification to their conjecture, (3) state the rules of the patterns and functions through verbal forms, tables, and graphs. Based on some of these opinions, it can be concluded that patterns are an important matter to be taught from an early age to train children to reason.

Pattern generalization is the activity to make a general rule pattern based on specific examples. Specific examples can be graphic, numeric, verbal and algebraic pattern (Janvier, 1987). For the aim of this study, the information provided through specific cases stated in the linear-shaped graphic pattern, because the graphic pattern allows students to observe in different ways. Wertheimer (1923) states Gestlat law of proximity, similarity and closure. The law of proximity states that when individuals perceive an assortment of objects they perceive objects that are close to each other as forming a group. The law of similarity states that elements within an assortment of objects are perceptually grouped together if they are similar to each other. The law of closure states that individuals perceive objects such as shapes, letters, pictures, etc., as being whole when they are not complete.

In particular, patterns are seen by some researchers as path to transition to algebra because they are a fundamental step to build the generalization that is the essence of mathematics (Zazkis and Lijedahl, 2002). In generalizing patterns, it is not enough to declare a general rule and order patterns verbally but must also state the general rule of pattern with symbol.

The aim of the study

The aim of this study is to describe local conjecturing process in the solving of pattern generalization problem based on APOS theory.

METHODOLOGY

Subjects

Researchers asked 42 students at junior high school (SMPN 3 Malang) to complete pattern generalization problem. After experiencing saturation data in the subject, 16 students produced a formula or general rule symbolically.

Instrument

There are two types of instruments used, main and auxiliary instruments. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Pattern Generalization Problem (PGP) and interviews. The problem given aims to obtain a description of the process of conjecturing of the students, while the interview used was unstructured interview. The PGP is presented in Figure 1.

Procedure

In the first stage, students completed PGP. In the second stage, the researchers conducted work-based interviews to understand the process of conjecturing and then the researchers recorded them by using a handy cam.

Data analysis

This study is a qualitative research with descriptive exploratory approach. At the data analysis stage, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data reduction, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field (3) encoding the data from PGP answer sheet and interviews refer based on indicators of local conjecturing process are presented in Table 1, (4) describing the local conjecturing process in the solving of pattern generalization problem based on APOS theory, and (5) conclusion.

RESULTS AND DISCUSSION

Based on the results of the analysis of the PGP answer sheets and interviews, we obtained data on the local conjecturing process undertaken by students in solving

Table 2. The results of the conjecturing process presented by students when solving pattern generalization problem.

Conjecturing process		
Local		Global
Proximity	Contrast	
3	3	10
18.75 %	18.75 %	62.5 %

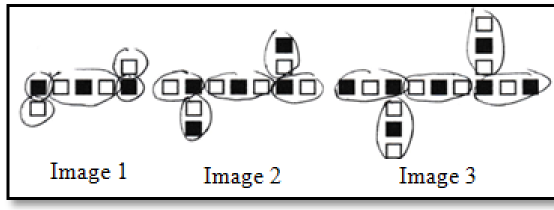


Figure 2. The work of S_3.

pattern generalization problem. The local conjecturing process in pattern generalization problem solving can be grouped into two general categories: (1) local conjecturing based on proximity and (2) local conjecturing by contrast. Of the 16 students who did conjecturing process and produced a formula or symbolic general rule, 10 students did global conjecturing process, and 6 students did local conjecturing process. Six students were grouped into two, 3 students (subject S_1, subject S_2, dan subject S_3) did the proximity conjecturing, and 3 students (subject S_4, subject S_5, dan subject S_6) by contrast. The results of the process of conjecturing are presented in Table 2.

After experiencing saturation in the process of data collection, there were 6 students that did the local conjecturing. Of the 6 students, we chose one subject that did local conjecturing process based on proximity, that is, the subject S_3, and one subject who did local conjecturing process by contrast, that is, the subject S_6.

Local conjecturing based on proximity

At the stage of action, subject S_3 realized that the images (first, second, and third image) formed a pattern. S_3 observed cases by observing and counting the number of square section at the lower left, upper right, main, left and right without distinguishing black square and white square of the first, second, and third image. Here are excerpts of the interview and the work of S3.

I : What did you think when reading this issue?
$S_3$1 : The pattern, off course.
I : What kind of patterns?
$S_3$2 : The pattern of addition.

It is always 4, namely from right, top, left and bottom, to the first, second, third, fourth, and fifth and onwards (pointing to the results of the work). Based on the number of squares on the first, second, and third images, subject S_3 organized the case by writing a symbol indicating a similar pattern by circling the first, second, and third images. This was confirmed by the transcript of the interview $S_3$3 and the work of S_3 in Figure 2.

S_3 3: To be able to see a pattern, I saw the addition at each end of the square that I circled.

At the process stage, S_3 internalized the action by finding and predicting patterns. The activities were carried out by calculating square difference of the first, second, and third image, that was 4 squares, as well as thinking about the fourth, fifth, sixth image and so on. This was confirmed by the transcript of the interview $S_3$2 and the work of S_3 (Figure 3).

At the object stage, S_3 encapsulated process to formulate a conjecture by connecting between the first, second, and third image to the difference and the number of the main part of the image given. For example, for the first image, it was written $(1 itten\ l$; the student wrote one for the first image, wrote four because there were four different images, wrote 3 for the number of main section was 3. For the second image, S_3 wrote $(2 ote\ tud$, and the third image, S_3 wrote $(3 rote\ he$. Furthermore, S_3 formulated the general formula for *n* image, that was $(x \times 4) + 3$. This was confirmed by the results of the work of S_3 (Figure 4) and interview transcript $S_3$4. S_3 validated conjecture by calculating conformity of the formula with the number of square of the fourth image, namely $(4 mely\ ed$. The formula obtained was correct as the number of square at the third image plus 4 equal to 19. This is evidenced by the following interview transcript of $S_3$5 and the work of S_3.

S_3 4: So for the first image I created a formula; for example, the first image was multiplied by the addition, and I got 4, then right, up, left, down was one, and I added all, and the result was 4. Once multiplied, then added with the main figure.
$S_3$5: I'm sure, I tried to count in the fourth image $(4 age\ sure$, the result is the same as for the third image, 3 plus 4 is 19.

At the scheme stage, S_3 believed the general formula $(x + 4) + 3$ produced was correct, based on the results of the validation. By believing that formula, S_3 was generalizing. Next on justify generalization stage, S_3 pinpointed the specific examples as done in validating the conjecture in order to convince others that the conjecture was generated correctly. This can be shown from the following interview excerpts.

I: Ok. How did you explain to others that the resulting

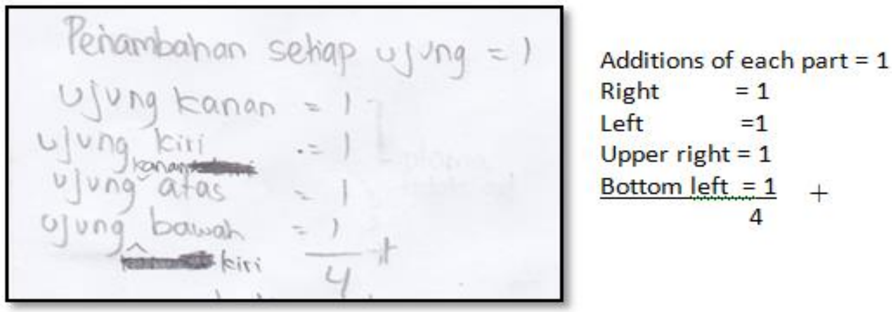


Figure 3. The work of S_3.

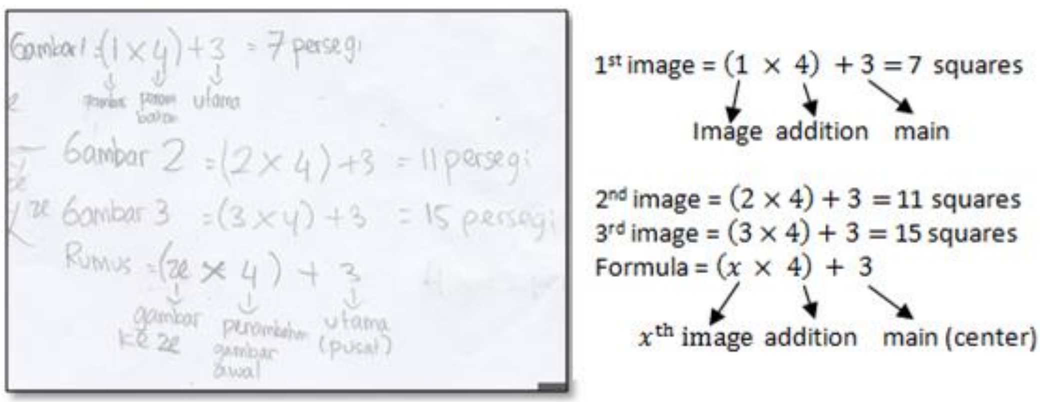


Figure 4. The work of S_3.

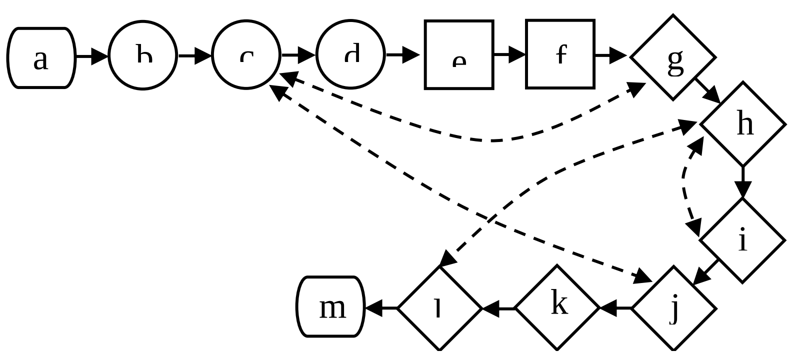


Figure 5. S_3 thinking structure.

formula is true (pointing to the work of S3).
$S_3$6: I'm going to show you an example.
I: Examples like what?
$S_3$7: Suppose the first image $(1 \mathit{Suppose}$ is true, the second image $(2\ \mathit{true}, t$ is true, the third image $(3s\ \mathit{true},$ is true, then $(4\mathit{en}\ \mathit{rue},$ is true and so on.

Based on these data, the structure of thinking of S3 is based on the stages of the conjecturing process and APOS theory. S3 thinking structure is presented in Figure 5 and code description of thinking structure Table 3.

Local conjecturing by contrast

At the action stage, subject S_6 had been aware that the

Table 3. code description of S3 thinking structure.

a :	The problem posed that is to find out general formula to decide the number of square on image *n*	k :	Believing that the *n* formula is$(x \times 4) + 3$
b :	Observing and counting the number of squares on the first, second, and third image separately	l :	Justifying the *n* formula
c :	Counting the number of squares on the bottom left, upper right, main, left, and right without differentiating the black and white squares on the first, second, and third image	m :	Finish
d:	Making a list to order the pattern	→	Order of activity
e:	Counting the difference of the first, second, and third image and thinking about the next object	◀- -▶	Validating, example from i to h, back to i; from j to c, back to j, and so forth
f:	Stating the difference of the numbers sequence, that is 4	○	Action
g:	Connecting the first, second, and third image by adding the main part	□	Process
h:	Example, the first image$(1 \times 4) + 3 = 7$ The second image$(2 \times 4) + 3 = 11$ The third image$(3 \times 4) + 3 = 15$	◇	Object
i:	The general formula of the *n* image is $(x \times 4) + 3$	⬡	Scheme
j:	Validating using the fourth image$(4 \times 4) + 3 = 19$	▢	Initial and end of activation

first, second, and third image formed a pattern. S_6 observed and counted the number of black and white squares of the first, second, and third image separately to find a common formula of the number of square-*n*. Here are excerpts from the interview with S_6.

$S_6$1: Counting this, one by one, how many black squares, how many white squares (pointing at the image)
I: What do you mean?
$S_6$2: There are 3 black squares on the first image, 4 white squares. There are five black squares and 6 white squares. There are four black squares and 5 white squares on the fourth image. Each image has 2 different squares. And so on.

Based on the number of black and white squares of the first, second, and third image, S_6 organized case by ordering number sequences. This is demonstrated by the work of S_6 in Figure 6.

At the process stage, the subject S_6 internalized the action by finding and predicting patterns. The activities were carried out by calculating the difference between the second and the first image, between the third and the second image respectively for black and white squares. The difference of black and white square was 2. S_6 then thought the difference for the fourth image, the fifth image, and so on. The interview excerpt with $S_6$2 confirmed this.

At the object stage, S_6 performed encapsulation process to formulate a conjecture by looking at the relationship between the first image with the number of black squares plus white squares of the first image. S_6 also looked at the relationship between the second image with the number of black squares plus white squares of the second image, and so on. By looking at the relationship, S_6 formulated a conjecture to determine the number of black squares on image *n* = 2n + 1, to the white square on image *n* = 2n + 2. Furthermore, S_6 validated conjecture by looking at the suitability on the number of squares on the first, second, and third image, and counted the number of squares on the fourth and fifth image. Here is the interview excerpt with S_6 and the work of S_6 in Figure 7.

$S_6$3: Er .... No, no, I mean it. This must have something to do with this (pointing to the work). We are told to count pattern of the first image, and *n*. From the first image, meaning we must count the next pattern, the hundredth image, or maybe thousandth image for example so we could easily do that. We also have to count the relationship with this (pointing to the work). It makes me conclude this. $3 \times 2 = 6,\ 6 + 1 = 7.\ 3 \times 2 = 6,\ 6 + 2 = 8.$ Thus, the second image is $2 \times 2 = 4,\ 4 + 1 = 5,$ this continues. $2 \times 2 = 4,\ 4\ 2 = 6.$
I : Then how?

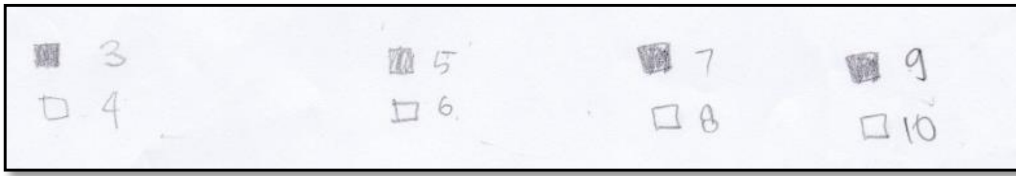


Figure 6. The work of S_6

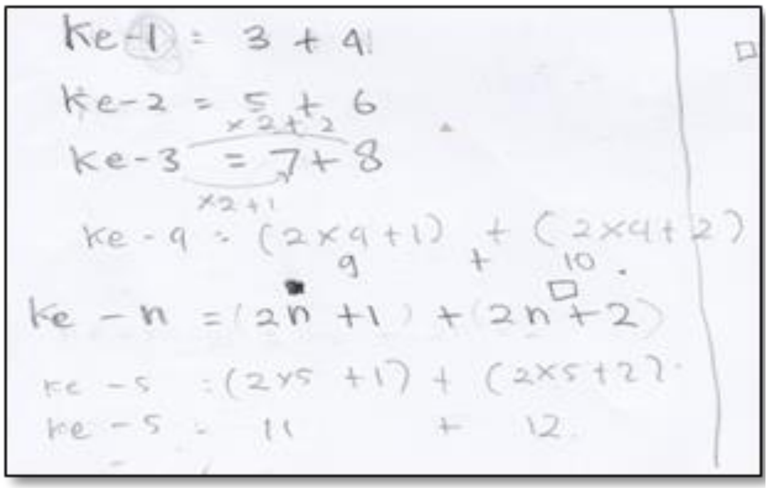


Figure 7. The work of S_6.

$S_6$4: Try to find the fourth image.
I : How can you be sure with this answer?
$S_6$5: Because I have counted the fourth image, the fifth image, and I matched with the first, second, and third image, and the result is correct.

At the scheme stage, S_6 justifies generalization with the aim of convincing others that the conjecture produced is correct by way of explaining how to get the formula and calculate the number of squares as done in validating the conjecture. This is shown in the following interview excerpt.

I : How do you explain that the formula you produced is correct?
S_6 6: I explain how I come to and count the number of black and white squares (pointing at the work)
Based on these data, the structure of thinking of S_6 can be described based on the stages of the conjecturing process and APOS theory. S_6 thinking structure is presented Figure 8 and code description of S_6 thinking structure Table 4.

Local process conjecturing scheme based on APOS Theory

This section describes the scheme of local conjecturing process based on the APOS theory. In the action stage, S_3 and S_6 observed cases and organized a separate case. The activity then became the foundation in building conjecture. S_3 observed and counted the number of square from bottom left, top right, main, right and left separately. These activities were in accordance with the Gestlat law in the observation that is the Law of Proximity, in which a person tends to perceive elements adjacent to each other as a specific form (Wertheimer, 1923). S_3 observed cases by observing and counting the number of black and white squares separately. These activities were in accordance with the Gestlat law in the observation that is the low of similarity, whereby one tends to perceive the same stimulus as a whole (Wertheimer, 1923).

At the stage of process, subject internalized the action to find and predict patterns by looking at the difference between the number of squares of the first and second image, the second and third image, and thought that the next image had the same pattern. At the object stage, S_3 formulated conjecture by doing encapsulated process to formulate conjecture by connecting between the first, second, and third image to the difference and the number of the main part of the image given. From this activity, S_3 formulated a conjecture $(x \times 4) + 3$ to calculate the number of images to n square. S_6 did the encapsulation process to formulate a conjecture by looking at the

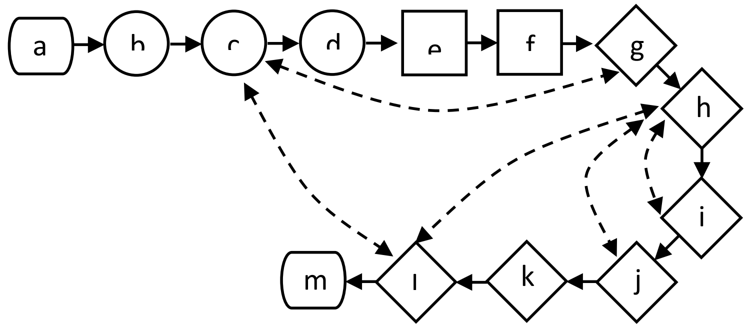


Figure 8. S_6 thinking structure.

Table 4. Code description of S6 thinking structure.

a :	The problem posed that is to find out general formula to decide the number of square on image *n*	k :	Believing that the general formula for black squares $n = 2n + 1$and white squares$n = 2n + 2$
b :	Observing the number of squares on the first, second, and third image separately	l :	Justifying the *n* formula
c :	Counting the number of black and white squares separately	m :	Finish
d :	Wiring down the number sequence of black squares 3, 5, 7, and white squares 4, 6, 8	⟶	Order of activity
e:	Counting the difference of the first, second, and third image	◄- - ►	Validating, example from g to c, back to g; from l to i, back to l, and so forth
f:	Stating the difference of the black squares 2 and the white squares 2, and thinking about the next object	○	Action
g:	Connecting the first, second, and third image by adding the number of black and white squares	□	Process
h:	Example: The first image= $1e\,fi$black squares The first image = $1e\,fi$white squares	◇	Object
i:	The general formula of black squares image $n = 2n + 1$and white squares image$n = 2n + 2$	⬡	Scheme
j :	Validating using the fifth image that is $= 2n + 1$+$2n + 2 = 11 + 12 = 23$	▢	Initial and end of activation

relationship of the number of black square plus white square of the first image. S_6 also looked at the relationship between the number black square plus white square of the second image and so on. The activity of S_6 to formulate a conjecture to determine the number of black squares on n image of $2n + 1$, and the number of white squares on n image of $2n + 2$. At the process and the object stage, this was done perfectly by the subject to produce a conjecture.

At the scheme stage, S_3 and S_6 generalized conjecture as to believe that the resulting conjecture was correct. In justifying generalizations, with the aim of convincing others that the resulting conjecture was true, S_3 and S_6 used specific examples obtained at the stage of action and object. In justifying generalizations, S_3 and S_6 performed it in their own way. This is in accordance with the statement of Caraher et al. (2008) that students do not just simply use the notation or symbols but also present and give a mathematical reason, make conclusions and generalizations on their own way. The scheme stage was also done perfectly.

Based on the description above, it can be described that the scheme of local conjecturing process by students in the solving of pattern generalization problem has been based on APOS theory. Local conjecturing process occurs at the stage of action, in which the subject built conjecture by observing and counting the number of squares separately and internalizing the action to the

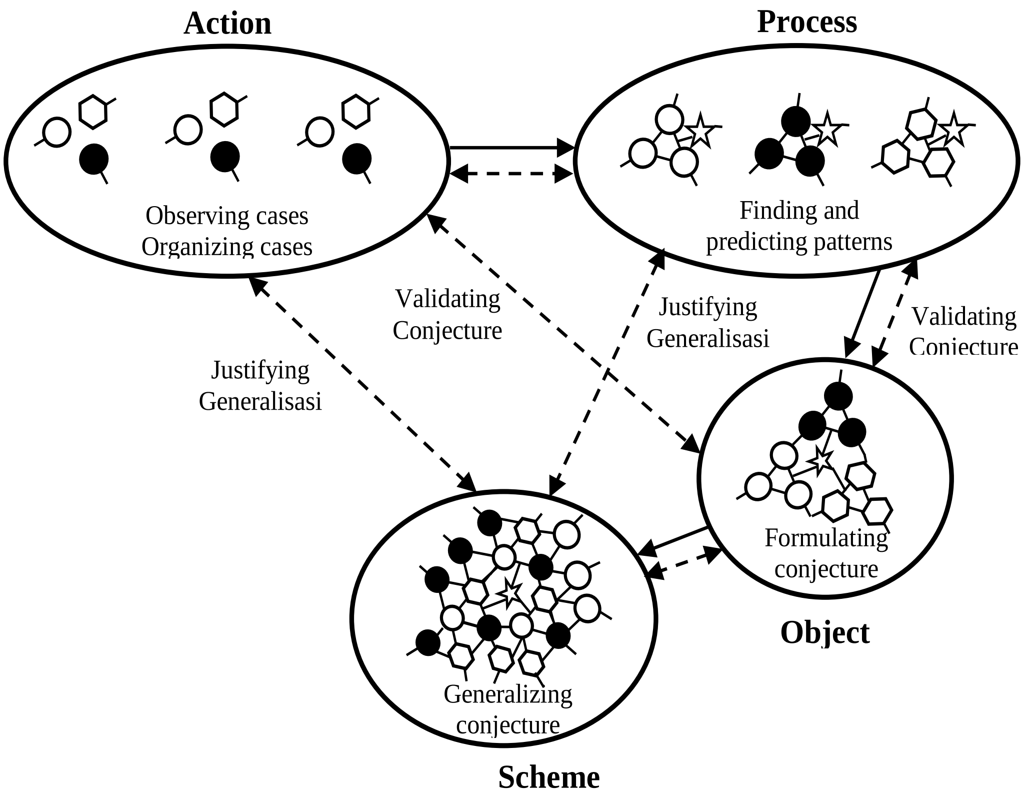


Figure 9. Scheme of local conjecturing process.

process stage, encapsulating the process to becoming an object. Then at the stage of scheme, all phases of APOS have been done perfectly. Here is the process of local conjecturing process scheme (Figure 9).

Conclusion

Local conjecturing process in the solving of pattern generalization problem is composed of local conjecturing process based on proximity and local conjecturing by contrast. Local conjecturing process based on proximity happens at the action stage, in which the subject builds a conjecture by observing and counting the number of square separately based on without proximity differentiating a black square and a white square; and at the process stage, object and scheme is done perfectly. Local conjecturing process by contrast occurs at the action stage, in which the subject builds a conjecture by observing and counting the number of squares separately between black squares and white squares, and at the process stage, object and scheme is done perfectly.

Certainly, the results of the current study mean much and have implications for the development of science. For the solving of graphic patterns problem, teachers need to consider aspects of observing the problem separately based on proximity and contrast in the process of building a conjecture.

Limitation of the study

The results of this study are limited to the data collected from the eighth grade students and this study has not described the process of global conjecturing in pattern generalization problem.

Conflict of Interests

The authors have not declared any conflicts of interest.

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