A Case Study on Pre-service Secondary School Mathematics Teachers' Cognitive-metacognitive Behaviours in Mathematical Modelling Process

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Abstract   The aim of the present study is to investigate pre-service secondary mathematics teachers' cognitive-metacognitive behaviours during the mathematical problem-solving process considering class level. The study, in which the case study methodology was employed, was carried out with eight pre-service mathematics teachers, enrolled at a university in Erzincan, Turkey. We collected data via think aloud protocols in which metacognitive supports are provided and not provided and with unstructured metacognitive protocols related to the process. The metacognitive support provided to the pre-service teachers and this case caused an increase in the numbers of cognitive-metacognitive behaviours in class level in general. Any pattern was not encountered for the sequences of cognitive-metacognitive behaviours in the process of mathematical modelling during the stages in which metacognitive support was given and was not given but some little patterns were detected. It was noticed that there was no relationship with respect to the metacognitive support and without metacognitive support between and frequencies of cognitive-metacognitive behaviour mathematical modelling progress. The 1st and 2nd grade-pre-service teachers claimed that the metacognitive support contributed to them such factors as understanding and detailing, the 3rd and 4th graders expressed that it enabled them to follow, reach alternative ways of solution. The participants, during the stage in which the metacognitive support was provided, demonstrated more success during the process of mathematical modelling compared with the first stage.

Keywords   Metacognition, Cognitive-metacognitive Behaviours, Mathematical Modelling, Mathematical Modelling Process, Pre-service Mathematics Teachers

1. Introduction

Metacognition, which is one of the most talked about and mentioned concepts [54], its being noticed and using it dates back to 557 B.C. [31]. This concept entered in the literature in 1976 by Flavell [33] (p.232) as the definition of 'metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them'. The researchers related to this concept in time as the following examples have carried out many definitions;

- It was defined as: individuals' having knowledge about their cognitive functions and arranging these cognitive functions and this definition, at the same time, is the answer of questions 'what do individuals know about their own cognitive functions and how do they arrange their cognitive activities throughout any task performance by [25]'.
- Individuals' knowledge about their own cognitive processes to achieve a certain goal and their observing-arranging these cognitive processes [54]
- A multi-dimensional skill set including thinking about thinking [46]

As these definitions relevant to metacognition are investigated, it can be noticed that knowledge related to the cognition and arrangements to the cognition are common in all definitions. One of the two main parts of metacognition, which seems to be different but highly interrelated, is cognitive knowledge, and the knowledge of cognition consisting of beliefs [36]. The knowledge of cognition, which is also called as metacognitive knowledge, is knowledge that individuals have about their weaknesses as a problem solver or learner, or their own cognitive skills of knowing where, how and why they are to apply a strategy [25]. The second main part of metacognition is regulation of cognition depending on checking and arranging cognitive events [36]. The regulation of cognition is related to the metacognitive skills, which consist of observing and arranging activities to which individuals apply during problem solving and learning process [25]. While the metacognitive knowledge are mostly declarative knowledge depending on the interaction between three features of

individual, task and strategy, metacognitive skills are mostly procedural knowledge concerning the arrangements of problem solving and learning activities [52].

Discovering the instructional effects of these two significant components of metacognition caused metacognition to be studied in other disciplines. As one of the disciplines is mathematics, and because of metacognition contribute to the acquisition of things to be learned, making it meaningful and applying it, that it has a significant role in learning mathematics by the students were emphasized by [9,36,42]. Moreover, deficiency of metacognitive skills is considered to be one of the main reasons of students' failure in mathematics [21,24].

It is thought that adding such educational activities enable students to make their own choices, make their own decisions, follow and check these decisions, interpret the results of their decisions in teaching-learning process may contribute their metacognitive skills development. Because, these behaviours consisting of arranging, observing, planning and evaluating are related to metacognition associated with individuals' thoughts are related to their own cognitive processes rather than cognition [54]. Problem solving process, as it consists of those sorts of educational activities, is a former activity among the other ones that can be used for the development of metacognition. Mathematical modeling, in which unstructured problems are used in its process [65], can also be used with the same purpose. In addition to the problem solving activities, as mathematical modelling activities enable students not only solve a problem but also configure the problem and check, help students more than problem solving [28,63]. That is, while even a simple problem solving experience has four steps of modelling process, mathematical modelling problems consist of multi-modelling processes, in short[74].

The stages named as modelling periods should be considered as steps; carrying a problem from real life to the mathematical world and converting it to a mathematical problem, developing an appropriate model for the problem, performing mathematical operations to solve the model, and interpreting the solution to the real life [41, 56, 58] and as a periodical or repetitive process [73]. So, this process of solving problems gathered from real life is named as mathematical modelling [34]. Therefore, this dynamic structure of mathematical modelling process enabling the students to explain their mathematical thinking, testing and correcting through revising [33] gives them more opportunities to demonstrate their metacognitive behaviours.

These interrelations among problem solving, modelling and metacognition stated as topics of many studies. These researches, firstly, started with the metacognitive researches which are driving forces in problem solving by Lesh, Silver and Schoenfold in 1982 [27]. Metacognition, since the mentioned date, has become one of the most the most researched two topics (the other is problem solving) in mathematics [8] and still remains popular. As the studies on metacognition carried out in mathematics are investigated, it is noticed that metacognition is the largely focused topic among those studies.

That is, during a task of problem solving, solvers, before applying investigating a problem, understanding it, evaluating the sufficiency of given data, organizing these data, making a plan for solving and applying the plan, they come across with tasks such as predicting its usefulness then, applying this plan and evaluating the validity of its results [8,10,57,59]. But, it is extremely difficult to explain these activities during the problem solving with cognition consisting of only knowledge organization and thinking [31]. Individuals, in their experiences of problem solving-mathematical modelling, can be better problem solvers by facing them with metacognition training and increasing or powering their metacognition [4,16,19,57,58,59,61,62]. Veenman[53] stated that, in acquisition and teaching of a metacognition effectively, metacognition training being integrated to the relevant topic rather than a general perspective is one of their basic principles. It was pointed out that the direct and obvious metacognitive training rather than those given indirectly, are more beneficial to maintain metacognitive thinking and skills [12,13,67]. Because, one of the most effective ways to activate metacognition is to give directions to the students and provide their compliances to the learning process encourage their prudential thoughts [81]. The metacognition support is given under the control of teachers, systematically and organized, it will have a significant role on learning performance.(22, 26,48,54,82]. In addition, Lester, Garafalo & Kroll [26] pointed out three roles of teachers seem to be different during the metacognition instruction but, at the same time, extremely relevant, a) as a metacognitive monitor, (b) as a facilitator of students' metacognitive awareness, and (c) as a model of a metacognitive-adopted problem solver. In some of the studies, giving participants different sorts of metacognition supports, variables such as problem solving process and its success were investigated. The samples of this situation, placed in the literature, [7,11, 20,21,26, 30,39, 43,51,53, 66, 68,77,29] in addition, most of those studies were carried out with 3th-12th graders in secondary education. In terms of the teachers and pre-service teachers, studies exist in literature as follows.

In some of the carried studies, teachers were observed in classroom and it was claimed that they had significant role in students' gaining metacognition skills [1,75,78]. Yet, the facts that teachers have little knowledge about metacognition, their answers to the questions about the metacognition were not more than It is independent learning, and they could not answer the question of how can metacognition be applied, are among the results of the studies [17,55]. In another research, it is pointed out that teachers enthusiastic in teaching metacognition in their lessons, but they have not got the tools and possibilities to provide this.[54]. In addition, there are studies pointing out that to provide the metacognitive activations of teachers with developing a model consisting of the social and cooperative situations and with the help of this model provided the metacognitive activations of them [82]. Teachers' knowledge and

awareness about their own learning processes are extremely crucial for both in terms of being a good model for their students and teaching these behaviours to their students [18, 76]. Therefore, it is thought that investigating and learning metacognitive behaviours that the pre-service teachers have in problem solving process just before the service and the change of those behaviours supported with metacognition will be significant.

Takahashi & Murata[81], in their experimental studies, compared an education activating metacognition with a problem-solving-based education in terms of the time, which the pre-service teachers, without any education, wasted on problem solving. As a result of the study, they reached a result that metacognition teaching has a function of activating metacognition in terms of understanding and managing problem solving process. In a study by Yimer and Ellerton [16], metacognitive behaviours that the pre-service teachers demonstrated during the problem solving process, without any education support, were analysed with the help of a five-stage-model consisting of engagement, transformation- formulation, implementation, evaluation and internalization. The results of the analysis indicated that the metacognitive behaviours of pre-service teachers in this five-stage model were not linear or single-directional. A study was carried out by Demircioğlu, Argün and Bulut [32] aiming to evaluate the metacognitive behaviours of pre-service secondary school mathematics teachers in problem solving process. From the results of the study, it was reached that there was no relation between the academic success of pre-service teachers and the frequency of demonstrated metacognitive behaviour, yet, it was realised that the type of the problem affected this frequency. Wilburne [40] investigated the effects of metacognitive strategy education on problem solving successes of secondary school pre-service mathematics teachers and expressed that the support provided increase in the pre-service secondary school mathematics teachers' levels of problem solving success.

It was noticed that although the metacognitive competencies in the process of mathematical modelling [44, 45, 64, 74] were significant in mathematical modelling, studies on this topic were mainly focused on cognitive dimension [69, 70]. Therefore, researches enabling the development of metacognitive behaviours in mathematical modelling process will contribute to the process. Accordingly, in the study, it is aimed to investigate the differences of cognitive-metacognitive behaviours demonstrated by pre-service teachers in the occasions with and without outer support. In accordance with this aim, following research questions were asked;

1. Do the numbers of cognitive –metacognitive behaviours demonstrated by pre-service teachers within the stages of mathematical modelling vary in terms of the class variable in the occasions with and without metacognitive support?

2. Does the sequence of demonstrated cognitive-metacognitive behaviours by pre-service teachers in mathematical modelling processes vary according to the occasions with and without metacognitive support in terms of the class variable?

3. How do the pre-service teachers evaluate the mathematical modelling processes in terms of with and without metacognitive support?

2. Materials and Methods

In this study, a case study from, considering the qualitative research models, was adapted in order to handle the problem deeply [38,71,79]. In this study, the participants' cognitive-metacognitive activities in mathematical modelling were tried to handle deeply based on this method.

2.1. Participants

This study was carried out with eight pre-service teachers who were students at the department of secondary school mathematics teacher training at a university, which is located in a medium sized province in the east of Turkey. In the study group, there were two students from the first, second, third and fourth grades.

The process of constituting the study group was completed in two stages. In the first stage, the research university was determined via the convenience sampling that enables constituting the research group among the ones who can easily be reached and economical in terms of time – money-effort etc. [50]. In the second stage, typical situation sampling method allowing some previously determined significance criteria, and preferred for enabling all the situations to be studied and revised [50] was determined. In determining the participants in this study, the criteria of constituting the participants can be listed as follows;

1. That the general average success of participants should be neither high nor low, that is, at medium level

2. That their self-expression skills should be at good level

3. That their readiness level on mathematical model and modelling should be same.

The first of those was considered as a criterion for a pre-service teacher who has an average success rather than one with high success or one with low success level in terms of demonstrating metacognitive behaviours during problem solving process and 8 participants from all of the grades were determined with the help of class advisor.

The participants' expressions their procedures during the study are crucial for the validity of the study. The second criterion is significant for this reason and to provide this criterion, the views of the lecturers in the department were taken into consideration. Along with these opinions, the numbers of participants were reduced from 8 to 5.

The third criterion was considered in terms of creating

same conditions before the participants' starting problem solving and in order to investigate this situation, the following process was taken into consideration:

A form consisting of two mathematical modelling problems was given to the pre-service teachers to read (They were told they could read as much as they wanted). This process continued about 5 minutes. After the feedback from the pre-service teachers about finishing reading, they were asked if they had encountered with this sort of problems before as first question. Then as a second question, the pre-service teachers were asked a question as; 'If you had to solve this problem, what sort of procedure would you follow?' After asking this question, the pre-service teachers were asked to make operations and they were asked to tell only their ideas of solution. As third and last question, the pre-service teachers were asked; 'Have you heard the terms of mathematical model, mathematical modelling, mathematical modelling problem before, if you have, can you explain?' At the end of this process, the participants which were previously 5 in each grade were reduced to 2 and totally 8 participants were determined for the study. Then, the participants were told and informed about the aim and topic of the study, they were ensured to participate this study voluntarily.

2.2. Data Collection Instruments and Data Collection

The verbal protocol technique was applied as a data collection tool in the study. A mathematical modelling problem was given to the participants to solve during the protocol and they were asked to solve it. It is predicted that, these sorts of problems, with the nature of their structures, giving the necessary mathematical data for the target and solution in a closed form [23] enable using metacognitive knowledge and skills during the problem solving process frequently.

Moreover, as it is indicated in Figure 1, considering the viewpoint of model and modelling as a parallel and interactive structure with the views on cognition and metacognition is considered as single-way and sequent –the same as the traditional definitions of metacognition [72]. That is, according to this point of view, throughout the process of given problem situation, the amount of energy, which the individual spent on cognitive activities, is the most important factor which effects the size of relevant metacognitive process. Therefore, some information is given related to the metacognitive activities which are demonstrated during the solving processes of modelling problems as in any problem solving process, can be monitored along the cognitive- metacognitive frame but this frame cannot be represented in a hierarchy.
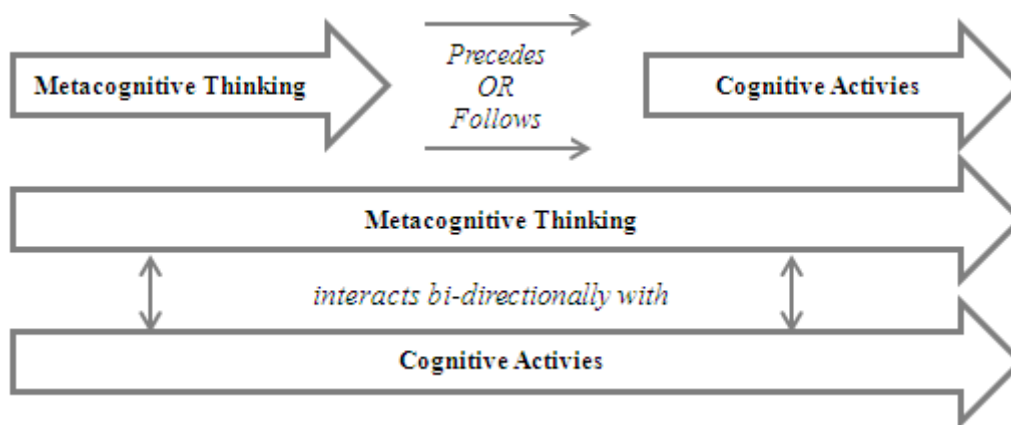

Figure 1. Relationships between metacognitive and cognitive

The mathematical modelling problem to be used in the research was decided as follows: Firstly, necessary literature scanning was carried out with three researchers, who are proficient in the field under discussion, many articles and theses on mathematical modelling were investigated and a question pool was constituted. The modelling problems existing in this pool, the grade they used for, the context they were used, the frequency of using, their difficulty levels were also considered and, therefore, as modelling problem a baseboard question was chosen. Moreover, the baseboard problem consists of five sub-problems, each problem aims to include keeping different things. With this aspect, it has a structure, which gives detailed information about the process compared with a single problem. After the pilot study, final decision was made about the chosen modelling problems were suitable. The baseboard problem is as follows;

Baseboard Problem: Ahmet wants to cover his room with basement. Firstly, in order to make covering on the walls and floors, he will have tacked baseboards around the room. The width of the room is 21m and its length is 28m. The baseboards are 10 and 16m in length.

1) How many of each baseboard does he need to buy to cover around the room?
2) If Ahmet wants to make as few joints as possible, how many of each baseboard does he need to buy?
3) If Ahmet wants to waste as few baseboards as possible, how many of each baseboard does he need to buy?
4) If 16m board is 1.25$ and 10m board is 1.10$, how many of each board does he need to buy with little money?
5) 16m boards are on sale and as its new cost is 0.85$, 10 m boards' costs are same, that is 1.10$. If Ahmet wants to spend the least money, how many of each board does he need to buy?

The information related to data collection tools applied in the research is as follows in order;

1. *Think-aloud Protocols*: Think aloud protocols are on-line measuring techniques which ask individuals tell-describe their thoughts verbally while they are working on a task (problem) [3, 48]. Think-aloud protocols provide the richest data which can be obtained about metacognitive processes throughout learning, as they are related to gathered metacognitive behaviours [3] during a learning task [52]. When the role of metacognition in predicting learning performance as a key variable was considered [1, 9], moreover, and the fact that the metacognition explain 37% of learning variance during a learning task was considered [54] it was decided that the most appropriate tool for the aim of this study was think-aloud protocols.

This protocol was carried out in two stages as with and without metacognitive support. The second stage, in which the protocol was carried immediately after the first stage, in order not to encounter with troubles endangering the validity of the study by affecting the participants' process (the participants' talking with somebody about the solving procedure of the problem, making investigation, producing new solutions, the weakening control of interviewer and participant on the problem solving process etc.). An environment enabling the participants no corruption in their attention in terms of noise and comfort during the process was created. Some documents, which they can use in each stage, were prepared for the participants in order to work comfortably on the problems. Each participant was coded as K.1.1, K.1.2 etc. in each grade and was prepared before the protocol. After all of the preparations, in order to determine the possible problems in practice, a pilot study was carried out with a pre-service teacher at the third grade.

During the first stage of the protocols, no metacognitive support was given to the participants by the researchers. Thus, it was aimed to investigate the cognitive-metacognitive behaviours demonstrated by the participants during the solving processes of mathematical modelling problems without any support. The participants were informed about the goals of the research at the beginning, that the most significant point for the research was the description or telling of what they think during the process, each idea they thought up, all the activities they drew, every symbol they made, equation, figure, graph… etc. that should be described were reminded to the participants. Furthermore, it was reminded to the participants that the period of the solving process is not an important factor, so the participants were provided not to feel under pressure thinking of 'I have to be the fastest solver'. The end of the first stage was decided with the expressions by the participants as 'enough, finished'. The think-aloud protocols with the participants changed as40-100 minutes. The researchers did not interrupt with any verbal, factual or symbolical interventions, which would affect the problem solving process of the participants during the period. Yet, during the situations in which the participants continued the solving process but preferred keeping silent rather than describing the solving process verbally, the researchers warned them expressing: ' Could you tell us what you do-think here'.

At the second stage of the protocols, the participants were supported with metacognition by the researchers. Thus, it was aimed to investigate the metacognitive activities of participants in the solving process of mathematical modelling problems when the participants got metacognitive metacognition support. The metacognition support here, was used in terms of asking questions directing the participants in situations in which they can demonstrate metacognitive behaviours. With this aim, following questions were asked to the pre-service teachers in each sub-problems of the baseboard problem?

Can you read the question……………….? Can you describe what you understand from this question? That is, what do you want to mean?

What do you have to do in this occasion? Have you got a

*plan for this? What's you plan? Which strategy do you think
to follow?*

Does it work? Was it useful? Did you reach the solution?

*Do you think this result is correct? Have you convinced
with this result? Is it the only solution for you? I wonder if
reach the solution with another procedure?*

These questions were formed depending on the three
metacognitive questions of Schoenfeld[11] and the
principles of Veenman[51]. These questions mean the
awareness of individuals about what they know and when
and how they apply this knowledge [83]. So, the questions
asked in the study were related to the behaviours of
metacognition knowledge and arrangement, which are two
significant components of metacognition. The questions
under discussion, even if they were asked sometimes in
different forms according to the problem solving processes
of the participants, it was taken care of not losing their
content meanings.

While the second stage of think-aloud was being carried
out, the documents consisting of the data related to the
solution of the problem and used by the participants in the
first stage were placed in which the participants can easily
see. The think-aloud protocols with the participants
continued for 25-60 minutes.

2.   *The Protocol After Thinking Aloud Protocols*: The third
stage of the protocols has unstructured characteristics.
The protocol is related to the participants' comparing
the solving processes of mathematical modelling
problem in the first and second stages. Thus, the
participants were asked to evaluate the first and second
stages of the think-aloud protocols in terms of problem
solving process. The period of the protocol differed
between 3-7 minutes.

The data in the study were collected thanks to the
protocols mentioned above. The study was carried out with
two different kinds of protocols. All of the protocols were
recorded with a video camera after the permissions of the
participants were taken. The collected data through the
think-aloud protocols with/ without metacognitive supports
and the protocols after the think-aloud protocols were sent to
the computer system and prepared for the analysis process

2.3. Data Analysis

In the analysis of the collected data, various analysis
methods were applied. During the study, to provide the
protocols with the participants to be long-term, applying to
experts views who are skilled in each process of the research
(expert analysis), showing the documents of the protocols
to the participants and taking the participants' confirmation
(participant confirmation), placing the protocols between the
participant-researcher related to the codes and themes
emerged during the study (detailed description), the criteria

of sampling selection, that the research reports being
investigated by different researchers (verification and
validation analysis) were considered. The analysis process
related to each problem can be detailed as follows.

For the *analysis of the think-aloud protocols*, firstly, the
documents of protocols were presented to the participants'
investigation. For the transcripts of think-aloud protocols,
descriptive analysis was applied. According to this analysis,
the collected data were summarised according to the
previously determined themes and interpreted and the aim is
to present the gathered data to the readers in an arranged and
interpreted form [14]

For the conceptual framework required in the process of
descriptive analysis, firstly the frameworks in the literature
were investigated. The researchers have developed many
models and frameworks during the period that enable the
individuals to describe their cognitive and metacognitive
activities during the process of mathematical problem
solving;

- read, analyse, explore, plan, implement, verify [9],
- orientation, organization, execution, verification
[36],
- understanding, analyse, planning, exploring,
implementing, verifying [4,5],
- engagement, resources, executive behaviours, beliefs
[80],
- report, discussion, monitoring, self-evaluation [60],
- orientation, planning, executing, checking [49],
- engagement, transformation, formulation,
implementation, evaluation, internalization [15],

In this study, the framework developed by Garofalo &
Lester [36],was applied. The basic reason for choosing this
framework is that the stages, which exist within the
framework, are substantially coincided with the
mathematical modelling cycles. This framework, according
to the aim of the study is used as follows: The four heading
placed in cognitive –metacognitive framework as orientation,
organization, execution and verification, were named as the
themes of 1, 2, 3 and 4. The behaviours under these themes
were associated with the letters orderly as A, B, C, D and
each behaviour was categorised as 1A, 1B…etc. In order to
accommodate these labels to the transcripts, the researcher
and three experts from the field of mathematical instruction
and the draft codes read all of the transcripts and indicators,
which represent the meaning of relevant label, were
determined. These draft codes and indicators were used for
testing for the analysis of some transcripts and their final
forms were prepared to use in the analysis of other transcripts.
In order to analyse the cognitive-metacognitive activities,
which the participants demonstrated during the solving
process of baseboard problem within the scope of the study,
the final form of the framework is indicated in Table 1.

Table 1. The cognitive-metacognitive behaviour framework applied in the analysis

Theme and Categories	Codes		Indicator	Sample Situation
Strategic behaviour to assess and understand a problem
Organization	1A	Comprehension strategies	Reading again, underlining the sentences, taking short notes (given, not requested), self-description of the problem	Firstly, we talk about a place which is 21m width 28m length.
	1B	Analysis of information and conditions	Self-talks, writings, trying to understand the concepts used in the problem	Here says as less addition as possible. What do the baseboards mean? I do not understand.
	1C	Assessment of familiarity with task	Comparing the similarities	Here, then this is the changing aspect of the problem.
	1D	Initial and subsequent representation	Drawing shapes such as table, graph, pictures	Let's draw the room. Write the dimensions.
	1E	Assessment of level of difficulty and chances of success	The gestures, mimics and comments about the easiness and difficulty of the problem.	This is actually complex. I didn't understand the problem.
Planning of behaviour and choice of actions
Organization	2A	Identification of goals and sub goals	Self-describing the requests as an aim, determining a strategy	My aim is to calculate it with minimum cost,
	2B	Global planning	Providing a way map for the solution	I should use more pieces of 16m boards
	2C	Local planning (to implement global plans)	Further customizing and minimizing the drawn map of way	I should use 7 pieces of 16m boards and 5 pieces of 10m boards
Regulation of behaviour to conform to plans
Execution	3A	Performance of local actions	Making mathematical calculations	7x0.85=5.95
	3B	Monitoring of progress of local and global plans	Turning back to previous operations and revising the procedure	I have used 5 pieces of 10 m boards for its width. I have used 7 pieces, it costs 1 lira, I have used 8 pieces, which are 2 liras.
	3C	Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)	Making a decision statement relevant to the results of the calculations	Totally, to sum up, then, that is, I should use 7 of them.
Evaluation of decisions made and of outcomes of executed plans
Verification	4A	Evaluation of execution	Expressions, checking and evaluations related to understanding the problem and comparing the results of the chosen strategy	I realized that it costs much, I disregard this, that is definitely the correct answer
	4B	Evaluation of execution	The check and evaluations of the reached solution, crosscheck and alternative ways of solutions	We have made an operational mistake, thus we were able to check it. Another way of solving is…

Table 1 was considered as a base for the analysis of each participant transcripts included in the study. For instance, if a transcript was labelled with 2C that means it belongs to the theme no. 2 category no. C. The meaning of 2C label is constituted with the combining the meanings of theme 2 and the category C. For instance, the theme number 2, in sum, means; making a plan for the solution of the problem, the category number C means; making special plans, the labelling 2C means making special plans, which can be applied in the solution of the problem. In the transcript analysis of the think-aloud protocols, in order to provide an integrity, thus, making the analysis more reliable the points that have been considered throughout the analysis are summarised and illustrated below.

- Sentences and clauses were coded considering the solving process of the problems that they related to. During the protocol with metacognitive support, while K.2.2 coded participant was solving the 3rd problem,

he expressed that, 'for 28 m, let's use 16, 2C is 4 from one side and 4 from the other side, 3A totally 8 pieces left over, said 3C' with his clause; 'totally 8 left over' he put restriction in the third question saying: 'provided the least of wasting', thus, considering the expressions, it was coded as 3C.

- Each clause constituting the sentences was analysed one by one. K.1.1 coded participants' expression, in protocol with metacognitive support, while solving the 2nd problem as 'If I use 1 piece of 16m, 2C, there remains 8m 3A, no there are 12m 4B, if I used 1 more pieces of 16m 2C, we will use 12m of them 3A, then, here is 1 joint 3A, 1 more joint opposite, totally I have used 4 pieces of 16m, 3C' can be given as an example to this situation.

- As the room described in the study is rectangular, while the participants expressing the required numbers of baseboards, they mostly specified separate results

for its width and length then stated the numbers of the baseboards that they will use for the entire room. Each sentence in which the participants expressed the numbers of the baseboards used for the width and length of the room, as the each sentences expressing the numbers of the baseboards used for the entire room, were coded as 3C. In the protocol with metacognitive support, during the K.3.1 coded participant was solving the 5th problem, the expressions as, 'That is, I used 3 pieces of 16m is enough for its width, 3C' can be given as an example to this situation. Yet, 1A coded participant, in solving the 2nd problem with metacognitive support used the expression as: 'We have made totally 6 joints, 2 in length and 4 in width, said,3C' , as the results of width and length used for the general result, they were coded as not 3 of 3C but 1 of 3C.

- Because of the rectangular shape of the room described in the problem used in the study, throughout the protocols both with and without metacognitive support, after the participants had finished the operations related to the width and length of the room, they frequently used the expressions as: 'this is same for the opposite wall', 'the above will also be same to this'. These expressions and all of the sentences, which are same in meaning with these expressions, were evaluated within the scope of 2C.

- The meaning of the sentences was taken into consideration. For instance, while a transcript as 'It may be more appropriate to use all the boards from 16.', was coded as 2C, as the aims were suitable for the planning of procedure, in general, the transcript 'If I use all of them from 16m, 2m left over, this is disadvantageous for me' was coded orderly as 2C, 3A, 4A.

- If the answers do not include a definite determination to the questions asked by the researchers in metacognitively supported protocols that means; the coding will not applied. In a metacognitive supported protocol, the answer of K.3.2 coded participant 'I have no idea now' to the question, during the process solving the 3rd problem, 'Is there a way of solution in which there is no boards left?' can be given as examples for this situation.

- In metacognitive supported protocols, participants, after expressing the result of the problem, were asked some questions by the researchers as 'I wonder if another way of solution can give you this result' or 'Do you want to operate through a different way'. Although the answers related to this question by the researcher seems to be appropriate to the code of 3C as it was an alternative way of solution, it was coded as 4B. While the K.2.2 coded participant, in a protocol with metacognitive support, was solving the 2nd problem, the expression of '6 pieces of 16m and 1piece of 10 m also give the solution 4B, 8 pieces of 16m gives the solution, too. 4B' can be given as an example

to this situation. In this transcript, as the participant suggested two different way of solving, 4B code was used twice.

The data analysis process was carried out with two experts studying simultaneously and independently. For the think-aloud protocols in whit metacognitive support was given and was not given, the numbers of consensus and disagreement were determined and the reliability of the research was calculated using the formula of Miles and Huberman [47] Reliability = Consensus / (consensus + disagreement).

At the end of the coding, for the think-aloud protocols without metacognitive support, the reliability was found as 90%. In 90 % of the coding direct consensus was provided, 7% of them an agreement was provided in one code of the two researchers, 1,3 % of them a coding was decided although none of the two researchers had been coded, 1,7 % of them, although both of the researchers had coded, it was decided not to make coding, 0,1 % of them a code which was completely different from the codes of the two researchers was used.

For the think-aloud protocols with metacognitive support, the reliability was found as 85%. In 85% of the coding a direct consensus was provided, 12% of them an agreement was provided in one code of the two researchers, 0,4% of them a coding was decided although none of the two researchers had been coded, 2,3% of them, although both of the researchers had coded, it was decided not to make coding, 0,4% of them a code which was completely different from the codes of the two researchers was used. In qualitative researches, that the consensus being over 70%, is regarded adequate in terms of the reliability of the research[47]. The findings whose reliabilities were provided are shown in tables and graphs using frequencies and percentages.

For the *analysis of the mathematical modelling stages* the steps in which Voskoglou[56] described the mathematical modelling process were determined as a basic framework. The basic reason for choosing this framework is that Voskoglou[56] describes the mathematical modelling process steps in an analytical way mostly, that gives the possibility to turn-back within the process and the content of this framework provides the opportunity to couple with used cognitive-metacognitive framework. Voskoglou[56] stated that mathematical modelling process consists of five main stages starting with S1and finishing with S5.
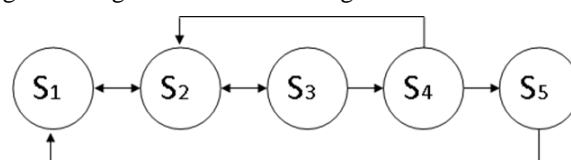

Figure 2. Voskoglous' mathematical modelling stages

These stages are described as follows: S1: Understanding the problem: Realizing the content and limits of the real problem situation and expressing them. S2: Forming it to mathematics: It means building the model prepared for

mathematical procedures, in other words, formulizing the real situation. S3: Solving the Model: Doing appropriate mathematical operations. S4: Checking the Model: Before the interpretation of the model, comparing the adequacy of the mathematical model with the mentioned problem. S5: Interpretation: In order to respond the problem, adapting the model to the real situation and interpreting the final mathematical results.

Considering the contents of this framework, it was determined that the codes as 1A, 1B, 1C, 1D and 1E used in cognitive-metacognitive framework was used to represent the stage of S1 in mathematical modelling process; the codes as 2A, 2B and 2C to represent the stage of S2, the codes as 3A, 3B and 3C to represent the stage of S3 and 4A, 4B codes to represent S4 and S5 stages. Therefore, an expert who dealing with cognitive activities in mathematical modelling process was applied for to take opinion.

For the analysis of the protocols after the think-aloud protocols, the views of the participants were described. To do this, firstly, the transcripts of the protocols with the participants were constituted and applied the confirmations of the participants.

3. Findings

In this section, findings related to the sub-problems of the study are presented in order.

3.1. Findings Related to the Research Question of 'Do the numbers of cognitive –metacognitive behaviours demonstrated by pre-service teachers within the stages of mathematical modelling vary in terms of the class variable in an occasion with and without metacognitive support?'

In order to find answer to this sub-problem, the findings gathered during the occasions of with and without metacognitive support are presented in order considering the class level as follows.

3.1.1. The Numbers of Cognitive-Metacognitive Behaviours of K.1.1 and K.1.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.1.1 and K.1.2 coded participants during the stages with and without metacognitive support is indicated in Table 2.

As the Table 2 is investigated, it is noticed thatK.1.1 and K.1.2coded participants, during the stage of S1 modelling period, have used 1A and 1B coded behaviours most. It is noticed that, there is generally a reducing or equality the use of 1A coded behaviour at the stage of the protocols with metacognitive support, yet an increase and equality was observed in 1B. 1E coded behaviour at the stage of S1, in the protocols without metacognitive support, and this participant has never demonstrated 1C-coded behaviour, in the protocols with metacognitive support. During the stage of S2, it was observed that the participants mostly demonstrated 2C coded behaviour and secondly demonstrated 2B coded behaviour. This behaviour have demonstrated increase generally in the protocols with metacognitive support (except for the 4th question of 1B). Mostly demonstrated behaviour by the participants during the stage of S3 was 3A coded behaviour and demonstrated increase in the protocols with metacognitive support. Mostly demonstrated behaviour during the stage of S4 was 4A coded behaviour and again its demonstration increased in general during the protocols with metacognitive support.

Table 2. The numbers of cognitive-metacognitive behaviours of K.1.1 and K.1.2 coded participants

Modelling circles	Coding	K.1.1					K.1.2				
		Sub-problems					Sub-problems				
		1	2	3	4	5	1	2	3	4	5
S1	1A	4- 2	2-1	2-1	2-2	1-1	4-3	1-2	2-1	2-2	1-1
	1B	3- 6	3-6	1-1	1-1	0-1	2-8	1-2	1-3	1-1	0-3
	1C	0- 1	0-3	1-0	1-1	1-0	0-0	0-0	0-0	0-0	0-0
	1D	0- 1	0-2	0-2	0-0	0-0	2-1	1-2	1-0	1-0	1-1
	1E	0- 0	0-0	0-0	0-0	0-0	0-0	0-0	1-0	0-0	0-1
S2	2A	0- 1	0-0	0-0	0-1	0-0	0-1	2-2	1-1	0-1	0-0
	2B	1- 2	0-3	0-1	0-1	1-2	0-1	2-2	0-2	1-0	3-2
	2C	1- 5	2-10	1-5	2-4	2-4	3-4	6-5	3-5	3-1	3-5
S3	3A	1- 2	2-10	1-5	3-6	2-5	1-1	3-6	3-6	6-1	5-7
	3B	0- 0	0-1	2-0	0-1	1-1	1-5	0-2	0-1	0-2	0-0
	3C	0- 2	2- 6	1-2	0-3	2-0	4-5	3-3	1-1	1-0	3-1
S4	4A	1- 5	0-3	1-6	2-5	2-3	0-0	1-2	2-2	1-4	2-2
	4B	0- 2	0-5	0-3	1-3	0-2	0-9	0-5	0-1	0-0	0-1
Total		11-29	11-47	10- 26	12-28	12-19	17-38	20-33	15-23	16-12	18-24

For each sub-problem, the first column indicates the numbers of behaviours without the metacognitive support and the second column with the metacognitive support.

K.1.1 and K.1.2 coded participants demonstrated totally 56-86 cognitive-metacognitive behaviours in the process of mathematical modelling in the stage without metacognitive support, and they demonstrated 149-130 cognitive-metacognitive behaviours in the stage with metacognitive support.

3.1.2. The Numbers of Cognitive-Metacognitive Behaviours of K.2.1 and K.2.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.2.1 and K.2.2 coded participants during the stages with and without metacognitive support are indicated in Table 3.

As the Table 3 is investigated, it is noticed that K.2.1 and K.2.2 coded participants during the stage of S1 modelling period, have used 1A and 1B coded behaviours most, It is noticed that, during the stage of S2, they demonstrated 2C coded behaviours most. 2C coded behaviour, in the protocols with metacognitive support increased in general (except for the 1st question of 2B). During the stage of S1, while 1C coded behaviour was never demonstrated in the protocols without metacognitive support, 1C coded behaviour is the least demonstrated behaviour during this stage. Mostly demonstrated behaviour by the participants during the stage of S3 is 3A coded behaviour and increased in the protocols with metacognitive support. The most demonstrated behaviours during the stage of S4 are 4A and 4B coded behaviours, however, it also increased with metacognitive support.

K.2.1 and K.2.2 coded participants demonstrated totally 8-83 cognitive-metacognitive behaviours in the process of mathematical modelling in the stage without metacognitive support, and they demonstrated 130-115 cognitive-metacognitive behaviours in the stage with metacognitive support.

Table 3. The numbers of cognitive-metacognitive behaviours of K.2.1 and K.2.2 coded participants

Modelling circles and coding		K.2.1					K.2.2				
		Sub-problems					Sub-problems				
		1	2	3	4	5	1	2	3	4	5
S1	1A	2-1	1-4	1-2	0-2	0-2	6-1	1-1	1-1	2-1	3-2
	1B	0-7	0-3	0-1	0-1	0-1	4-5	1-2	0-2	1-2	2-0
	1C	0-0	0-0	0-0	0-0	0-0	0-1	0-0	4-0	0-0	3-0
	1D	2-1	0-0	0-1	0-1	0-1	3-0	1-0	1-1	1-0	1-0
	1E	1-1	0-0	0-0	0-0	0-0	2-0	0-0	0-0	0-0	0-0
S2	2A	1-3	0-0	0-0	0-0	0-1	0-2	1-1	0-0	0-0	2-0
	2B	0-0	0-4	0-0	0-1	0-2	5-7	0-0	0-0	2-3	1-2
	2C	0-11	0-4	0-2	0-6	0-6	5-2	2-4	3-7	2-3	1-1
S3	3A	0-12	0-4	0-2	0-6	0-4	2-11	1-2	2-7	3-4	0-1
	3B	0-1	0-0	0-1	0-1	0-2	0-0	0-0	0-2	0-1	0-0
	3C	0-0	0-3	0-1	0-1	0-0	5-15	2-4	1-4	1-2	1-1
S4	4A	0-4	0-1	0-2	0-4	0-4	0-4	0-2	1-1	0-4	0-0
	4B	0-4	0-1	0-3	0-1	0-0	2-3	0-2	1-4	0-1	0-1
Total		6-44	1-24	1-15	0-24	0-23	34-40	9-18	14-29	12-20	14-8

For each sub-problem, the first column indicates the numbers of behaviours without the metacognitive support and the second column with the metacognitive support.

3.1.3. The Numbers of Cognitive-Metacognitive Behaviours of K.3.1 and K.1.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.3.1 and K.3.2 coded participants during the stages with and without metacognitive support are indicated in Table 4.

Table 4. The numbers of cognitive-metacognitive behaviours of K.3.1 and K.3.2 coded participants

Modelling circles and coding		K.3.1					K.3.2				
		Sub-problems					Sub-problems				
		1	2	3	4	5	1	2	3	4	5
S1	1A	2-1	2-2	1-0	1-4	2-1	4-3	1-1	2-1	1-1	0-1
	1B	4-4	3-1	1-2	0-0	0-1	7-4	1-2	1-1	1-2	0-0
	1C	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-1
	1D	1-2	1-0	1-1	1-0	1-1	4-1	0-1	0-1	0-0	0-0
	1E	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-1	4-0	0-0
S2	2A	4-0	0-1	0-0	0-2	0-0	0-0	0-1	1-0	0-0	0-1
	2B	1-3	2-1	4-10	0-7	1-4	7-1	5-7	0-0	3-1	0-1
	2C	5-3	6-6	5-8	3-2	4-0	12-2	2-2	0-0	1-0	0-0
S3	3A	5-0	7-3	8-12	2-2	1-1	16-8	5-3	0-0	2-3	0-0
	3B	0-0	5-1	0-1	0-2	2-2	0-0	2-0	6-1	1-0	0-0
	3C	3-1	7-8	5-8	2-4	2-3	9-1	4-4	3-0	3-1	0-1
S4	4A	0-2	3-5	1-3	1-4	1-2	1-1	2-3	2-1	1-2	0-1
	4B	0-4	3-5	0-8	0-1	0-1	5-1	2-1	0-0	1-0	0-1
Total		25-20	39-32	26-47	10-28	14-16	48-22	24-25	15-6	18-11	0-7

For each sub-problem, the first column indicates the numbers of behaviours without the metacognitive support and the second column with the metacognitive support

As the Table 4 is investigated, it is noticed that K.3.1 and K.3.2 coded participants during the stage of S1 modelling period, have demonstrated 1A and 1B coded behaviours most, It is noticed that, during the stage of S2, they demonstrated 2B coded behaviours most. During the stage of S1, the participant support never demonstrated 1C and 1E coded behaviours in the protocols without metacognitive. During the stage of S3, the most demonstrated behaviours by the participants are 3A and 3C coded behaviours. The most demonstrated behaviour by the participants during the stage of S4 are 4A and 4B coded behaviours and again increased in the protocols with metacognitive support.

K.3.1 and K.3.2 coded participants demonstrated totally 114-105 cognitive-metacognitive behaviours in order, in the process of mathematical modelling during the stage without metacognitive support, and they demonstrated 143-71 cognitive-metacognitive behaviours during the stage with metacognitive support.

3.1.4. The Numbers of Cognitive-Metacognitive Behaviours of K.4.1 and K.4.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.4.1 and K.4.2 coded participants during the stages with and without metacognitive support are indicated in Table 5.

Table 5. The numbers of cognitive-metacognitive behaviours of K.4.1 and K.4.2 coded participants

Modelling circles and coding		K.4.1					K.4.2				
		Sub-problems					Sub-problems				
		1	2	3	4	5	1	2	3	4	5
S1	1A	5-2	2-3	3-1	1-1	0-2	8-1	4-1	2-1	3-2	2-1
	1B	2-0	2-1	1-1	1-0	0-0	1-2	1-2	0-1	1-1	0-1
	1C	0-1	1-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0
	1D	1-0	1-1	1-0	0-0	0-0	1-1	1-1	1-0	0-0	0-0
	1E	1-1	1-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0	0-0
S2	2A	0-0	0-0	2-0	2-0	0-0	0-1	0-0	1-1	1-2	0-0
	2B	2-1	0-4	0-0	5-4	3-2	4-1	0-6	0-0	0-1	1-0
	2C	0-3	1-1	1-0	2-0	0-0	1-1	2-5	2-0	0-0	0-0
S3	3A	10-2	3-1	3-0	6-3	1-0	3-8	3-6	2-1	2-1	0-1
	3B	0-1	0-0	1-2	3-1	0-1	0-0	0-0	0-1	0-1	0-1
	3C	2-0	2-5	1-0	1-1	2-0	1-0	5-5	4-1	1-2	1-1
S4	4A	0-2	1-5	1-1	5-2	0-1	0-1	1-4	0-1	2-7	0-3
	4B	3-2	0-8	3-0	1-3	1-1	0-3	0-7	0-4	0-2	0-2
Total		20-21	13-29	17-5	29-15	7-7	19-19	17-37	12-11	10-19	4-10

For each sub-problem, the first column indicates the numbers of behaviours without the metacognitive support and the second column with the metacognitive support.

As the Table 5 is investigated, it is noticed that K.4.1 and K.4.2 coded participants during the stage of S1 modelling period, demonstrated 1A and 1B coded behaviours most, It is noticed that, during the stage of S2, they demonstrated 2B coded behaviours most. During the stage of S1, the participants never demonstrated 1C and 1E coded behaviours in the protocols with metacognitive support. During the stage of S3, the most demonstrated behaviours by the participants are 3A and followed by 3C coded behaviours. The most demonstrated behaviour by the participants during the stage of S4 are 4A and 4B coded behaviours and again increased in the protocols with metacognitive support.

K.4.1 and K.4.2 coded participants demonstrated totally 86-62 cognitive-metacognitive behaviours in order in the process of mathematical modelling during the stage without metacognitive support, and they demonstrated 77-96 cognitive-metacognitive behaviours in the stage with metacognitive support.

As considering the findings within the scope of the first sub-problem, those can be expressed; K.1.1, K.1.2, K.2.1, K.2.2, K.3.1, K.3.2, K.4.1 and K.4.2 coded participants, placed in the study group, demonstrated totally 56, 86, 7, 84, 114, 122, 91, 62 behaviours in order, throughout the mathematical modelling process, during the stage without metacognitive support; during the stage with metacognitive support they demonstrated totally 149, 130, 130, 116, 152, 70, 71, 96 metacognitive behaviours, in order. When the frequencies in which metacognitive support were not given to the participants subtracted from the total frequency with metacognitive support, the difference came out in 1st and 2nd grades as 93, 44, 123, 32, and in 3rd and 4th grades as 38, -52, -20, 34.

3.2. Findings Related to the Research Question 'Does the sequence of demonstrated cognitive-metacognitive behaviours by pre-service teachers in mathematical modelling processes vary according to the occasions with and without metacognitive support in terms of the class variable?'

In order to find answer to this sub-problem, the findings gathered during the occasions of with and without metacognitive support are presented in order considering the class level as follows.

3.2.1. Sequences of Cognitive-Metacognitive Behaviours of K.1.1 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.1.1 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 6.

According to the data in Table 6, in both of the occasions in which metacognitive supports exist or not, the participant started the questions with 1A. In the protocols without metacognitive support, 1A behaviour was demonstrated 6 times; in the protocols with metacognitive support 1A behaviour was demonstrated twice. 1B behaviour was generally demonstrated after 1A and 1B in the protocols with and without metacognitive support. 1C behaviour was demonstrated after 1A in the protocols without metacognitive support, in the protocols with metacognitive support was demonstrated both after 1A and 4A.

While 2A behaviour did not exist in protocols without metacognitive support, in the protocols with metacognitive support, 2A behaviour was demonstrated once and 2B and 2C behaviours followed this behaviour in order. The participant demonstrated 2C 5 times in the protocols without metacognitive support and 3A followed this behaviour 21 times. For demonstration of 3B, a general pattern was not come out. This behaviour was demonstrated after various behaviours but generally 2B and 2C followed this behaviour. The 3C behaviour was generally demonstrated after 3A, 3C and 4A. The behaviour sequence as 2B→ 2C→ 3A, during the protocols with metacognitive support, was demonstrated once during 3th, 4th and 5th questions.

Table 6. Sequences of cognitive-metacognitive behaviours of K 1.1 coded participants

Metacognitive Support (No)	First Problem	1A→ 1A→ 1A→ 1B→ 1B→ 2B→ 1A→ 1B→ 2C→3A→ 4A
	Second Problem	1A→ 1B→ 1B→ 1A→ 2C→ 3A → 3A→ 1B→ 2C→ 3C→ 3C
	Third Problem	1A→ 3B→ 3B→ 1B→ 1A→ 1C→ 2C→ 3A→ 4A → 3C
	Fourth Problem	1A→ 1B→ 1A→ 1C→ 2C→ 3A → 3A→ 4A → 2C→ 3A → 4B→ 4A
	Fifth Problem	1A→ 1C→ 3B→ 2B→ 2C→ 4A→ 3A→ 2C→ 4A→ 3C→ 3A→ 3C
Metacognitive Support (No)	First Problem	1A→ 1A→ 1C→ 1D→ 1B→ 1B→ 1B→ 1B→ 2A→ 2B→ 2B→ 1B→ 2C→ 2C→ 3A→ 4A → 2C→ 3A→ 4A → 4A→ 4A→ 3C→ 4A→ 1B→ 2C→ 3C→ 2C→ 4B→ 4B
	Second Problem	1A→ 1D→ 2B→ 1B→ 2C→ 3A → 2C→ 3A → 3C→ 1B→ 4B→ 2C→ 3A → 2C→ 2C→ 3A → 4B→ 2C→ 3A→ 3A→ 3C→ 3C→ 3C→ 1B→ 3B→ 2C→ 1B→ 3C→ 4B→ 2B→ 4A→ 2C→ 3A → 3A→ 3A→ 4B→ 1B→ 1D→ 2B→ 4A→ 1B→ 2C→ 3A → 2C→ 3C→ 4A→ 4B
	Third Problem	1A→ 1B→ 1D→ 2C→ 3A → 1D→ 4A→ 4B→ 4A→ 2B→ 2C→ 3A → 2C→ 3A→ 2C→ 3A → 3A→ 2C→ 3C→ 4A→ 4B→ 3C→ 4A→ 4A→ 4B→ 4A
	Fourth Problem	1A→ 1B→ 1A→ 2A→ 2B→ 2C→ 3A → 4B→ 4B→ 2C→ 3A→ 4A → 3B→ 2C→ 3A→ 3C→ 3A→ 4A → 3A→ 4A → 2C→ 3A→ 4A → 3C→ 3C→ 4A→ 1C→ 4B
	Fifth Problem	1A→ 2B→ 3B→ 2B→ 2C→ 3A→ 2C→ 3A → 2C→ 3A → 3A→ 4B→ 2C→ 3A→ 4A → 1B→ 4A→ 4A→ 4B

The participant finished all the questions with 4A or 4B during the protocols with metacognitive support. During the protocols without metacognitive support, the participants demonstrated only one behaviour within the scope of 4B and this behaviour was followed by 4A. The demonstration of the 4A and 4B behaviours during the protocols with metacognitive support was more than the demonstration of the behaviours without metacognitive support. 4A behaviour was demonstrated after 3A most during the protocols without (3 times) and with metacognitive support (7 times).

3.2.2. Sequences of Cognitive-Metacognitive Behaviours of K.1.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.1.2 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 7.

According to the data in Table 7, in both of the occasions in which metacognitive support exists or not, the participant started the questions with 1A. The 1A behaviour was demonstrated 4 times after cognitive–metacognitive behaviour during the protocols with and without metacognitive support. 1B behaviour was generally demonstrated after 1A, 1B and 1D behaviours during the protocols without and with metacognitive support. This

participant in none of the two stages never demonstrated 1C and 1E behaviours.

During the protocols without metacognitive support of the participant, 3A behaviour followed 2C 12 times and 12 times during the protocols with metacognitive support. While 3B behaviour was demonstrated only once during the protocols without metacognitive support, it was demonstrated several times after various behaviours during the protocols with metacognitive support. And 3C behaviour followed 3A several times. The behaviour sequences as 2B→ 2C→ 3A were demonstrated twice during the second question during the protocols without metacognitive support, and once in 4th and 5th questions. This behaviour sequences were demonstrated once in second question during the protocols with metacognitive support.

During the stage without metacognitive support, the participant finished the problem solving processes with 3C 5 times, in the occasions with metacognitive support finished the processes twice with 4A, twice with 4B and once with 1B. There is no behaviour which can be evaluated within the scope of 4B code was demonstrated by the participant during the protocols without metacognitive support. The 4A behaviour was demonstrated after 3A most (5 times) during the protocols without metacognitive support, and it was demonstrated after 4A most (4 times) during the protocols with metacognitive support.

Table 7. Sequences of cognitive-metacognitive behaviours of K 1.2 coded participants

Metacognitive Support (No)	First Problem	1A→ 1D→ 1A→ 1D→ 1B→ 1A→ 1A→ 1B→ 2C→ 3A → 2C→ 3C→ 3C→ 3C→ 3B→ 2C→ 3C
	Second Problem	1A→ 1B→ 2A→ 1D→ 2B→ 2C→ 3A → 2C→ 2C→ 2B→ 2C→ 3A → 4A → 2C→ 2C→ 2A→ 3C→ 3A→3C → 3C
	Third Problem	1A→ 1D→ 1B→ 1E→ 2A→ 4A→ 2C→ 3A → 2C→ 3A→ 4A → 2C→ 3A →, 1A→ 3C
	Fourth Problem	1A→ 1D→ 1B→ 3A→ 2B→ 2C→ 3A → 2C→ 3A → 2C→ 3A→ 4A → 3A→ 3A→ 1A→ 3C
	Fifth Problem	1A→ 1D→ 2B→ 2B→ 2C→ 3A → 2C→ 3A→ 4A → 3C→ 3A→ 2B→ 3A→ 2C→ 3A→ 4A → 3C→ 3C
Metacognitive Support (No)	First Problem	1A→ 1B→ 1A→ 1B→ 2B→ 3B→ 4B→ 4B→ 4B→ 3B→ 4B→ 2C→ 3A→ 3C → 1D→ 1B→ 2C→ 1B→ 2C→ 2A→ 3B→ 4B→ 2C→ 4B→ 1B→ 3C→ 4B→ 1B→ 1A→ 1B→ 4B→ 3B→ 3C→ 3B→ 3C→ 3C→ 4B→ 1B
	Second Problem	1A→ 1A→ 1B→ 2A→ 2B→ 1D→ 2C→ 3A → 3A→ 2C→ 2A→ 2C→ 3A→ 3C → 4B→ 3B→ 3B→ 3A→ 3A→ 4B→ 1D→ 4B→ 4B→ 2C→ 3C→ 3C→ 2B→ 2C→ 3A → 1B→ 4A→ 4A → 4B
	Third Problem	1A→ 1B→ 1B→ 2A→ 1B→ 2B→ 3A→ 2B→ 3A→ 2C→ 3B→ 2C→ 3A → 2C→ 3A → 2C→ 3A→ 3C → 4A→ 2C→ 3A→ 4A → 4B
	Fourth Problem	1A→ 1A→ 3B→ 2A→ 1B→ 2C→ 3B→ 3A→ 4A→ 4A → 4A→ 4A →
	Fifth Problem	1A→ 1B→ 1B→ 2B→ 4A→ 2B→ 1B→ 1D→ 2C→ 3A → 3A→ 2C→ 3A → 2C→ 3A → 2C→ 3A → 2C→ 1E→ 3C→ 3A→ 3A→ 4B→ 4A

Table 8. Sequences of cognitive-metacognitive behaviours of K 2.1 coded participants

Metacognitive Support (No)	First Problem	1A→1D→2A→1E→1A
	Second Problem	1A
	Third Problem	1A
	Fourth Problem	
	Fifth Problem	
Metacognitive Support (No)	First Problem	1A→1D→1B→2A→1B→2A→1B→3A→2C→2A→2C→3A→2C→1B→1B→2C→3A→3B→3A→4A→4B→2C→3A→2C→3A→4A→4A→1B→4B→4B→2C→3A→3A→1B→2C→3A→2C→3A→2C→3A→2C→3A→4A→4B
	Second Problem	1B→1A→1B→1B→2B→2C→3A→2C→2B→1A→2B→2C→3A→3A→2B→2C→3A→1A→1A→3C→3C→3C→4B→4A
	Third Problem	1A→1A→1D→1B→3B→3C→4B→4B→2C→3A→2C→3A→4A→4B→4A
	Fourth Problem	1A→1D→1A→1B→2B→2C→3A→2C→3A→3B→2C→3A→2C→4A→3A→2C→3A→4A→2C→3A→4A→4A→3C→4B
	Fifth Problem	1A→1D→1A→3B→2B→2A→2C→1B→2C→3A→3B→2B→2C→3A→4A→3A→2C→3A→4A→2C→2C→4A→4A

Table 9. Sequences of cognitive-metacognitive behaviours of K 2.2 coded participants

Metacognitive Support (No)	First Problem	1A→1B→1D→1B→2B→1D→1E→1A→2C→1B→1A→2B→2C→3A→3C→1B→1A→1A→1A→1B→2C→2B→1E→1D→2C→2B→3A→2C→3C→4B→2B→3C→3C→3C→4B
	Second Problem	1A→1B→2A→1D→3A→2C→2C→3C→3C
	Third Problem	1A→1C→1C→1C→1C→1D→2C→4B→2C→3A→2C→3A→4A→3C
	Fourth Problem	1A→1A→1D→1B→2B→3A→2C→2C→3A→2B→3C→3A
	Fifth Problem	1A→1C→1B→1C→1A→1A→1C→1D→2A→2B→2C→2A→1B→3C
Metacognitive Support (No)	First Problem	1A→1B→1B→1B→1B→1C→2B→2B→2A→3A→2B→3A→2C→3C→4A→4B→2B→3A→2C→3C→4B→2B→3C→4B→3C→3C→4A→2B→2A→2B→4A→3A→3A→3A→3A→3A→3A→3A→3A→4A
	Second Problem	1A→1B→2A→2C→4A→2C→3A→3C→2C→3A→2C→3C→3C→3C→1B→4B→4B→4A
	Third Problem	1A→1B→1D→2C→3A→3C→2C→3A→3C→3C→4B→3B→2C→1B→2C→3A→2C→3A→2C→3A→4B→2C→3A→3C→4B→3A→3B→4A→4B
	Fourth Problem	1A→1B→2B→2B→1B→2B→3A→2C→3A→4A→2C→3A→3B→2C→3A→4A→3C→4B→4A→3C→4A
	Fifth Problem	1A→1A→2B→2B→2C→3A→4B→3C

3.2.3. Sequences of Cognitive-Metacognitive Behaviours of K.2.1 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.2.1 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 8.

According to the data in Table 8, the participant finished the process thinking that he cannot solve the problems without metacognitive support. The participant, during the occasion with metacognitive support, demonstrated behaviours within the scope of all of the codes except for 1C and 1E. Problem solving process started with 1A. During the protocols without metacognitive support, the 1A behaviour was demonstrated once after cognitive-metacognitive behaviour, and during the protocols with metacognitive support 6 times. The participant demonstrated 1B coded behaviour in all of the solving processes intensively except for the 4th and 5th questions during the protocols with metacognitive support.

The participant demonstrated 2C behaviour 22 times before 3A behaviour, however, in second, fourth and fifth questions, he demonstrated 2C after 2B, and 3A after 2C. The behaviour 3B was usually demonstrated after 3A. Furthermore, the participant demonstrated 3C behaviour

quite a few times, and during some solving processes, he did not demonstrate it at all. The behaviour sequences as 2B→ 2C→ 3A were demonstrated by the participant during the protocols with metacognitive support, 3 times in second question and once in 4th and 5th questions.

The participant finished the problem solving processes with 4A three times and with 4B twice. The 4A and 4B coded behaviours were demonstrated not only at the end of the problem solving process but also in the middle of the process and mostly after each other. During the protocols with metacognitive support, the behaviour 4A was demonstrated after 3A most (8 times).

3.2.4. Sequences of Cognitive-Metacognitive Behaviours of K.2.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.2.2 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 9.

According to the data in Table 9, the participant in each situation, started all of the problem solving process with 1A. 1A behaviour was demonstrated 6 times after the cognitive-metacognitive behaviour during the protocols without metacognitive support, and once during the

protocols with metacognitive support. The participant demonstrated 1C in all of the solving processes intensively in the occasion without metacognitive support but he demonstrated the behaviour only once during the stage with metacognitive support.

During the protocols without metacognitive support, 3A followed 2C 4 times, and during the protocols with metacognitive support 12 times. Moreover, 3A followed 2B 3 times during the protocols with metacognitive support. While 3B behaviour did not take place during the protocols without metacognitive support, it was demonstrated twice after 3A and once after 4B. 3C behaviour was demonstrated after 3A and 3C behaviour most. The behaviour sequence as 2B→ 2C→ 3A was demonstrated once in the 1st question during the protocols without metacognitive support, and once in the 5th question with metacognitive support.

While the participant finished the problem solving processes 3 times with 3C and once with 3A and once with 4B during the stage without metacognitive support, he finished the processes 3 times with 4A, once with 4B and once with 3C during the stage with metacognitive support. During the protocols without metacognitive support, 4A was demonstrated once and before it 3A was demonstrated. During the protocols with metacognitive support, 4A behaviour was demonstrated most (3 times) after 3A behaviour.

3.2.5. Sequences of Cognitive-Metacognitive Behaviours of K.3.1 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.3.1 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 10.

According to the data in Table 10, the participant, except for the third question in the stage with metacognitive support, started the problem solving process with 1A. During the protocols without metacognitive support, 1A behaviour was demonstrated 3 times after a cognitive –metacognitive behaviour, but during the protocols with metacognitive support, this demonstration was 6 times.

During the protocols without metacognitive support of the participant, 3A behaviour followed 2C 12 times, and 6 times during the protocols with metacognitive support. Moreover, during the protocols without metacognitive support of the participant, 3A behaviour followed 2B once, during the protocols with metacognitive support 8 times. 3C behaviour was demonstrated after 3A and 3C both during the protocols with and without metacognitive support. The behaviour sequences as 2B→ 2C→ 3A were demonstrated 3 times in the 3rd question during the protocols without metacognitive support and twice in the 3rd question during the protocols with metacognitive support.

While the participant finished the problem solving process with 3C five times during the protocols without metacognitive support, 3 times with 4B, once with 4A and once with 1B during the protocols with metacognitive support. Moreover, during the protocols with metacognitive support, 4A and 4B coded behaviours were demonstrated in second, third and fourth questions intensively. During the protocols without metacognitive support, 4A behaviour was demonstrated after 2C most (5 times), during the protocols with metacognitive support it was demonstrated after 3C most (6 times).

Table 10. Sequences of cognitive-metacognitive behaviours of K 3.1 coded participants

Metacognitive Support (No)	First Problem	1A→ 1A→ 2A→ 3A→ 1B→ 2A→ 2A→ 3A→ 2A→ 1D→ 1B→ 1B→ 2C→ 3C→ 2B→ 1B→ 3A→ 2C→ 2C→ 3A → 2C→ 3C→ 3A→ 2C→ 3C
	Second Problem	1A→ 2B→ 1D→ 2C→ 3A → 2C→ 3A → 3A→ 1B→ 3B→ 2B→ 1B→ 4A→ 2C→ 3A → 2C→ 3A → 3A→ 3C→ 3C→ 4B→ 3C→ 3B→ 3B→ 3B→ 3C→ 3C→ 1A→ 1B→ 4B→ 3A→ 2C→ 4A→ 2C→ 4A→ 3B→ 4B→ 3C→ 3C
	Third Problem	1A→ 1D→ 1B→ 2B→ 2C→ 3A → 2B→ 3A → 3A→ 3A→ 2C→ 3A → 2C→ 3C→ 3A→ 3C→ 2B→ 2C→ 3A → 3C→ 4A → 2B→ 2C→ 3A → 3C→ 3C
	Fourth Problem	1A→ 1D→ 2C→ 2C→ 3A → 2C→ 3A → 4A→ 3C→ 3C
	Fifth Problem	1A→ 1D→ 2C→ 3A → 2C→ 3B→ 2C→ 1A→ 2C→ 3B→ 4A→ 3C→ 2B→ 3C
Metacognitive Support (No)	First Problem	1A→ 4B→ 1B→ 1D→ 2C→ 1B→ 2B→ 4A→ 4B→ 1B→ 4B→ 1D→ 2C→ 2C→ 2B→ 3C→ 4A → 2B→ 4B→ 1B
	Second Problem	1A→ 1A→ 1B→ 2A→ 2C→ 2C→ 3C→ 3A→ 3B→ 2C→ 3A → 2C→3C→ 2C→ 3A → 3C→ 3C→ 3C→ 4A → 4A→ 4B→ 4B→ 4B→ 3C→ 4A → 2B→ 2C→ 3C→ 4A → 3C→ 4B→ 4B
	Third Problem	1B→ 1B→ 2C→ 3A → 4B→ 1D→ 4B→ 2C→ 3A → 2C→ 3C→ 4A → 2B→ 2C→ 3A → 3C→ 2B→ 2C→ 3A → 3C→ 2B→ 2C→ 2B→ 3A→ 2B → 3A→ 2B → 3A→ 2B→ 3A→ 2B→ 3A→ 3C→ 3C→ 4B→ 1D→ 2B→ 3A → 3A→ 2C→ 3C→ 2B→ 3A → 2C→ 3C→ 3C→ 4A → 4B→ 3B→ 4B→ 4B→ 4B→ 4B→ 4A
	Fourth Problem	1A→ 2B→ 2A→ 2B→ 2B→ 3A → 2B→ 1A→ 3B→ 2C→ 2A→ 2B→ 3C→ 2B→ 2C→ 1A→ 2B→ 3C→ 1A→ 3A→ 4A→ 4A→ 3C→ 3C→ 3B→ 4A→ 4A→ 4B
	Fifth Problem	1A→ 2B→ 1D→ 3B→ 2B→ 4A→ 3C→ 3B→ 3C→ 3A→ 4A→ 2B→ 1B→ 2B→ 3C→ 4B

Table 11. Sequences of cognitive-metacognitive behaviours of K 3.2 coded participants

Metacognitive Support (No)	*First Problem*	1A→ 1B→ 1D→ 1D→ 1B→ 1B→ 1D→ 1B→ 1A→ 1B→ 1A→ 1B→ 3A→ 2C→ 3A → 2B→ 1A→ 2C→ 3A → 3A→ 3A→ 2B→ 3A → 1B→ 2C→ 3A → 4A→ 3A→ 3A→ 2C→ 3A → 2C→ 3A → 3C→ 3C→ 3C→ 2B→ 3C→ 4B→ 1D→ 2B→ 3C→ 3A→ 2C→ 2C→ 2C→ 3A → 3C→ 3C→ 2B→ 2C→ 3A → 4B→ 4B→ 3C→ 2C→ 3A → 2C→ 3A → 3C→ 4B→ 2B→ 2B→ 4B→ 2A→ 2C
	Second Problem	1A→ 1B→ 3B→ 2B→ 3A → 2B→ 4B→ 3C→ 2B→ 3A → 3B→ 2B→ 3A → 4B→ 2B→ 3C→ 3A→ 3C→ 4A→ 2C→ 3A → 4A→ 3C
	Third Problem	1A→ 1B→ 2A→ 3B→ 3B→ 3B→ 3B→ 3B→ 3B→ 3C→ 1A→ 4A→ 3C→ 3C→ 4A
	Fourth Problem	1A→ 1B→ 3B→ 3C→ 4B→ 3A→ 2B→ 3C→ 2B→ 3C→ 2B→ 2C→ 3A → 4A→ 1E→ 1E→ 1E→ 1E
	Fifth Problem	Participants have to terminate the process.
Metacognitive Support (No)	*First Problem*	1A→ 1A→ 1B→ 1D→ 1B→ 3A→ 1B→ 2C→ 2C→ 2B→ 3A → 3A→ 3A→ 3A→ 3A→ 3A→ 3C→ 1A→ 1B→ 4A→ 4B
	Second Problem	1A→ 1D→ 1B→ 2A→ 2B→ 3A → 3C→ 2B→ 3A → 3C→ 2B→ 3A → 2C→ 2B→ 2C→ 1B→ 2B→ 3C→ 4A→ 4B→ 3C→ 2B→ 2B→ 4A→ 4A
	Third Problem	1A→ 1E→ 1B→ 1D→ 3B→ 4A
	Fourth Problem	1A→ 1B→ 1B→ 2B→ 3A → 3A→ 3A→ 3C→ 4A→ 4A
	Fifth Problem	1A→ 1C→ 4B→ 2B→ 2A→ 4A→ 3C

Table 12. Sequences of cognitive-metacognitive behaviours of K 4.1 coded participants

Metacognitive Support (No)	*First Problem*	1A→ 1A→ 1B→ 1D→ 1A→ 1E→ 1A→ 1B→ 3A→ 1A→ 2B→ 3A → 3C→ 2B→ 3A → 3C→ 3A→ 3A→ 3A→ 3A→ 3A→ 3A→ 3A→ 4B→ 4B→ 4B
	Second Problem	1A→ 1A→ 1C→ 1D→ 1B→ 1E→ 3A→ 2C→ 3A → 3A→ 1B→ 4A→ 3C→ 3C
	Third Problem	1A→ 2A→ 1D→ 1B→ 3A→ 3B→ 2A→ 2C→ 4A→ 1A→ 1A→ 3A→ 3C→ 4B→ 4B→ 3A→ 4B
	Fourth Problem	1A→ 1B→ 2B→ 2C→ 3A→ 4A → 2B→ 3B→ 4B→ 3A→ 3B→ 3A→ 4A → 3B→ 2A→ 2B→ 2B→ 2C→ 3A→ 4A → 2B→ 3A→ 4A → 2A→ 3A→ 3C→ 4A
	Fifth Problem	2B→ 2B→ 3C→ 3A→ 4B→ 2B→ 3C
Metacognitive Support (No)	*First Problem*	1A→ 1A→ 1E→ 1C→ 3B→ 4B→ 2C→ 2C→ 2C→ 2B→ 3A → 3A→ 4A → 4A→ 4B
	Second Problem	1A→ 1B→ 2B→ 2B→ 3C→ 1A→ 3C→ 4A→ 4A→ 1A→ 3C→ 2C→ 4A→ 3C→ 2B→ 3A → 4B→ 4B→ 1D→ 2B→ 3C→ 4B→ 4B→ 4B→ 4A → 4B→ 4A → 4B→ 4B
	Third Problem	1A→ 1B→ 3B→ 3B→ 4A
	Fourth Problem	1A→ 4B→ 4B→ 2B→ 4A→ 2B→ 2B→ 3A → 2B→ 3A → 3B→ 3A→ 3C→ 4A→ 4B
	Fifth Problem	1A→ 2B→ 1A→ 2B→ 3B→ 4B→ 4A

3.2.6. Sequences of Cognitive-Metacognitive Behaviours of K.3.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.3.2 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 11.

According to the data in Table 11, the participant in each situation started solving all of the problems with 1A. During the protocols without metacognitive support, 1A behaviour was demonstrated 4 times after the cognitive-metacognitive behaviours and during the protocols with metacognitive support twice.

During the protocols without metacognitive support of the participant, 3A behaviour followed 2C 11 times, but during the protocols with metacognitive support, such a sequence was not encountered. Especially, during the protocols without metacognitive support, 3A behaviour followed 2B behaviour. While 3B behaviour was demonstrated after 3B during the protocols without metacognitive support, it was demonstrated only once during the protocols with metacognitive support. During the protocols without metacognitive support, 3C behaviour generally demonstrated after 3C, but during the protocols with metacognitive support, such a generalization was not encountered. The behaviour sequence as 2B→ 2C→ 3A was demonstrated once in 1st and 4th questions only during the protocols without metacognitive support.

Moreover, during the protocols without metacognitive support of the participant, 3A followed 2B four times, and 5 times during the protocols with metacognitive support. The participant, during the protocols without metacognitive support of the participant, finished the problem solving process with 2C, 3C, 4A and 1E in order, but during the protocols with metacognitive support, finished the process with 4A three times, 4B once and 3C once. During the protocols without metacognitive support, 4A behaviour was demonstrated after 3A most (3 times), during the protocols with metacognitive support it was demonstrated after 3C and 4A most (twice).

3.2.7. Sequences of Cognitive-Metacognitive Behaviours of K.4.1 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.4.1 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 12.

According to the data in Table 12, K.4.1 coded participant in each situation started solving all of the problems with 1A. During the protocols without metacognitive support, 1A was demonstrated 6 times after cognitive-metacognitive behaviour, during the protocols with metacognitive support 4 times. During the protocols without metacognitive support, while 1B behaviour generally was demonstrated after 1A and 1D, it was demonstrated after 1A during the protocols with metacognitive support.

During the protocols without metacognitive support of the participant 3A behaviour followed 2C 3 times, but during the protocols with metacognitive support, such a sequence was not demonstrated. While 3C behaviour was generally demonstrated after 3A during the protocols with metacognitive support, such situation was not demonstrated during the protocols with metacognitive support. The behaviour sequence as 2B→ 2C→ 3A was only demonstrated in the 4th question during the protocols with metacognitive support.

Moreover, during the protocols without metacognitive support of the participant, 3A behaviour followed 2B 3 times, but during the protocols with metacognitive support 4 times. During the protocols without metacognitive support, while the participant finished the problem solving process twice with 3C, twice with 4B, and once with 4A, but during the protocols with metacognitive support three times with 4B and twice with 4A. During the protocols without metacognitive support, 4A behaviour was demonstrated after 3A most (4 times), during the protocols with metacognitive support after 4B most (4 times).

3.2.8. Sequences of Cognitive-Metacognitive Behaviours of K.4.2 Coded Participants

The findings related to the numbers of cognitive-metacognitive behaviours demonstrated by the K.4.2 coded participants within the scope of baseboard problem during the stages with and without metacognitive support are indicated in Table 13.

According to the data given in Table 13, the participant in each situation, started solving all of the problems with 1A. During the protocols without metacognitive support, 1A behaviour was demonstrated after cognitive-metacognitive behaviour 10 times, during the protocols with metacognitive support, it was demonstrated once after cognitive-metacognitive behaviour.

During the protocols without metacognitive support of the participant 3A behaviour followed 2C 4 times, during the protocols with metacognitive support 4 times. During the protocols without metacognitive support, 3C behaviour was generally demonstrated after 3C during the protocols with metacognitive support, it was demonstrated generally after 4B. The behaviour sequence as 2B→ 2C→ 3A was demonstrate only in the 2nd question during the protocols with metacognitive support.

Moreover, during the protocols with metacognitive support 3A followed 2B 3 times. During the protocols without metacognitive support, while participant finished the problem solving process with 3C four times and 4A once, during the protocols with metacognitive support, he finished the process 3 times with 4B and twice with 4A. During the protocols without metacognitive support, 4A behaviour was demonstrated most (twice) after 3C behaviour, during the protocols with metacognitive support after 4B most (7 times).

Table 13. Sequences of cognitive-metacognitive behaviours of K 4.2 coded participants

Metacognitive Support (No)	First Problem	1A→ 2B→ 1D→ 2B→ 1A→ 1A→ 3A→ 1A→ 1A→ 2B→ 2B→ 1A→ 1A→ 3A→ 3A→ 2C→ 1B→ 1A→ 3C
	Second Problem	1A→ 1D→ 1A→ 1A→ [2C→ 3A]→ [2C→ 3A]→ 3A→ 1A→ 3C→ 4A→ 3C→ 3C→ 1B→ 3C→ 3C
	Third Problem	1A→ 1D→ 2A→ [2C→ 3A]→ [2C→ 3A]→ 1A→ 3C→ 3C→ 3C→ 3C
	Fourth Problem	1A→ 1A→ 1A→ 2A→ 3A→ 4A→ 1B→ 3A→ 3C→ 4A
	Fifth Problem	1A→ 1A→ 2B→ 3C
Metacognitive Support (No)	First Problem	1A→ 1B→ 1D→ 2C→ 2A→ 1B→ [2B→ 3A]→ 3A→ 3A→ 3A→ 3A→ 3A→ 3A→ 3A→ 4B→ [4B→ 4A]→ 4B
	Second Problem	1A→ 1B→ 1D→ [2B→ [2C→ 3A]→ [2C→ 3A]→ [2B→ [2C→ 3A]→ 2C→ [2B→ 3A]→ 4A→ [2C→ 3A]→ 2B→ [2B→ 3A]→ 3C→ 3C→ 4B→ 3C→ 2B→ 4A→ 4B→ 4B→ 1B→ 4B→ 3C→ 4B→ 3C→ [4B→ 4A]→ [4B→ 4A]
	Third Problem	1A→ 1B→ 2A→ 3B→ 3A→ [4B→ 4A]→ 4B→ 3C→ 4B→ 4B
	Fourth Problem	1A→ 1B→ 2A→ 2A→ 1A→ 2B→ 4A→ 3B→ 3A→ 4B→ 3C→ 4A→ 3C→ 4A→ 4A→ [4B→ 4A]→ 4A→ 4A
	Fifth Problem	1A→ 1B→ 3B→ 4A→ 3A→ 3C→ [4B→ 4A]→ 4A→ 4B

3.3. Findings Related to the Research Question 'Does the sequence of demonstrated cognitive-metacognitive behaviours by pre-service teachers in mathematical modelling processes vary according to the occasions with and without metacognitive support in terms of the class variable?'

In order to give answer to this sub-problem, the answers to the question 'If you want to compare the 1st stage in which the metacognitive support is given with the second stage in which metacognitive support is not given, how do you evaluate' by the participants were presented in order considering the class level.

K.1.1coded participant expressed his thoughts, as 'I don't think that the problem is too difficult but it required attention, and I always have lack of attention. At first stage, as I did not think of the other side of the room, all of the procedure was incorrect. At the second stage, as I was enabled to see here, thus, the rest of the problem became correct easily. Generally, as a student, we quit directly after reaching the first solution, it is the same as in exams and as usual, there is no examine.' The participant, in the expressions, pointed out the attention that the second stage enabled him. The participant, immediately after reading and trying to understand the question at first stage, set the mathematical model. Calculating the perimeter of the room as 21+28=49 m, did not realise that he calculated it incorrectly, as he set his model as 10x+16y=49, he conducted all of the processes related to the problem according to this. The participant, at the second stage, noticed that he had made a mistake and stated that all the results should be doubled indeed. Yet, the participant did not use the mathematical model, which he settled both in the first and second stage during the solving process. Although he reached the correct solution in 3rd and 5th problems at the second stage, he reached the incomplete solution in the first and second problem and he finished the process with an incorrect solution in the fourth question. Therefore, the expressions of K.1.1 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.1.2 coded participant expressed his thoughts as 'Firstly, at the second stage, my ways of solution almost completely changed or alternative solutions came out. I do not think I completely understood the question at first stage. Yet, your interventions at the second stage, made me think more and I got better results.' The participant, in the expressions, pointed out that he could think more on the questions thanks to the second stage and as a result, changed the ways, found alternative ways of solution and reached better results. The participant, at first stage read the first problem during the solving process, drew a shape, made plans, calculated his plans and claimed the result of the problem as 10 of 10 m and 2 of 16 m boards should be used. The participant was not aware that the room's diameter is more than his use of such amount of baseboards. The participant did not develop any mathematical model in the second stage during the problem solving and used the expression, as 'There are many ways to solve this problem, because there is no restriction' several times. The participant tried to develop alternative ways for the solution of the problem but gave results, which exceeded the length of the room again. (For instance; 8 of 10m and 4 of 16). During the protocol without metacognitive support, the participant reached a correct solution for the second question but he could not reach a solution because of the incorrect calculations. Yet, the participant, in the second occasion, noticed the incorrect operations and recommended a different way of solution. In the third problem, while the participant found a result in which he increased 2 m at first stage, he was able to solve the problem without increasing. This participant found a solution (10 of 16m) even in the 5th question in which exceeded the length of the room fairly. The participant was not able to understand the problem much indeed. Therefore, the expressions of K.1.2 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.2.1 coded participant expressed his thoughts, as 'At first, I could not solve the problem. Then you did not give any information indeed but I could solve it then. Thanks to that you get us think about the questions continually, I started to solve the problem.' The participant expressed that he could think more on the questions thanks to the second stage. K.2.1 coded participant claiming that he found the questions enclosed at first stage he finished the problem solving process. At the second stage, the participant did not develop mathematical model in the first question. The participant gave some expressions, as 'As the baseboards are 10 and 16 m, taking their exact multiples I will calculate through trial and error'. After this process of trial and error, the participant expressed the only solutions reaching 98 exactly (3 of 16 m and 5 of 10m) and he increased at least 2 meters (5 of 16 m and 2 of 10 m). Therefore, the expressions of K.2.1 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.2.2 coded participant expressed his thoughts as 'I read the problem at first but I did not understand what it means. As you asked me question, I started to understand it and my procedure and solution changed automatically.' The participant pointed out that the second stage gave him the possibility to understand the problem. K.2.2 coded participant, at first stage, especially in the first problem, read it several times and tried to make it meaningful. As it is understood from the expressions of the participant: 'As it does not refer the least or most, I divide all of them with 1 and take the rest of them. Thus, I use 28 of them for the length and 21 for the width. That means I use 98' he really had difficulty in understanding the first question. At the second stage, the participant developed a mathematical modelling during the solving process of the first problem and reached the solution thanks to this model. Therefore, the expressions of K.2.2 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.3.1 coded participant expressed his thoughts as 'Turning the questions again and again I checked the results,

I was sure about some of them but this created a negative result for the second question in the problem. The first that I thought was more correct.' The participant pointed out that the second stage gave him the possibility to check the results but this became a negative way for a question for him. As K.3.1 coded participant, at the second stage, noticing the mistakes that he made at the first stage used some sentences as 'I have misunderstood here, I solved this incorrectly, and I check if this is correct or not, I think it is correct'. Therefore, the expressions of K.3.1 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.3.2 coded participant expressed his thoughts, as 'Of course there were differences between the first stage and the second stage. There were several occasion in which I said 'oh, I may make this here' and I did several things.' The participant pointed out that the second stage gave him the possibility to understand the problem. K.3.2 coded participant made a significant effort in order to solve the problems at first stage, claimed that there may be different ways of solving it during the process but could not limit the possibilities. During the second stage, the participant developed the mathematical model and thus made much faster correct solutions. When the participant developed the mathematical model at first stage, he wrote $10n+16m=98$ and he used an expression with smiling as 'that is to say, it depends on our point of view to the problem'. The participant, again at the first stage, claimed to use 9 of 10 m as an answer for the third problem but he did not notice that 90 m would not cover the room entirely. The participant, during the second stage, expressed that the model he developed for the problem as n=5 and m=3 that checked the model, that there is no waste and this procedure quite persuade him. The participant, during the first stage, after a few numbers of calculating the alternatives enabling the lowest cost for the fourth question, he expressed his thoughts as 'I, presumably, will continue with this problem till the morning, is this problem very difficult or I cannot solve it, I will solve through this way, but there are several ways, I take it there, I take this here.' Yet, at the second stage, this participant, calculating the costs of all the possibilities in his mathematical model, expressed the correct result with the lowest cost. Therefore, the expressions of K.3.2 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.4.1 coded participant expressed his thoughts as 'Question marks appeared in my mind. As we are usually in race with time, we cannot regard several things during the exams. Actually, at first stage, I knew that there were several ways of reaching the result but I thought that there was no need to try. I think that the second stage is useful that I could criticize myself.' In his expressions, the participant expressed that the second stage gave him opportunity for developing new solutions for the problems. K.4.1.coded participant, at first stage, during the solving process of the first problem, thinking that they can use both 10 m and 16 m baseboards, decided that they can cover the entire room with 10 of 10 m, or 7 of 16 m baseboards. Then, he finished the solving process of the first problem thinking of 'I might reach this result using two different kinds of baseboards, too but as the way I will have to follow completely depends on me, I may reach totally different results, I don't want to do this.' That is the K.4.1coded participant, actually, is aware that there are different ways to solve the first problem at first stage too, but he did not want to do these operations. Yet, as the participant, at the second stage, answered the question 'Are those the sole solutions?' with no, tried to solve the problem again, developed his mathematical model and then found out all the possible solutions of the problem. The participant, even in the second problem reached a solution with 8 joints most, developed different ways of solution in the second stage. The participant, during the first stage, claimed the results of the third problem without using mathematical model as using 5 of 10m or 3 of 16 m and then checked it as he stated in his expression 'The multiplication of those will give the diameter of the room and thus, we provide a crosscheck.' At the second stage, he answered the question 'what does this question mean' directed to him as 'the less boards waste, the better it is. In order to find it as little as possible, I found a solution giving no waste and I checked it.' Then the participant looking for the equations of his model developed in the first problem, he finished the process saying 'We have only once reached to 98. So, that is the correct solution, there is no more solution.' Therefore, the expressions of K.4.1 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

K.4.2 coded participant expressed his thoughts as '*In terms of the practicability, at the second stage, I noticed to think of different possibilities and different choices.*' The participant pointed out that the second stage gave him the opportunity to develop new solutions for the problems. At first stage, K.4.2 coded participant, during the solving process of the first problem, after demonstrating 1A and 2B codes, developed his mathematical model as $10x+16y=98$ and at first, thought about the equation giving it some values. Then, the participant following the operations of 98:16= 6 and 98:10=9, finished the process claiming that he can cover the room using 7 of 16 m or 10 of 10 m boards. In this stage, although the participant developed a mathematical model, he gave up calculating the numbers of 10 m and 16 m boards, which provide his model immediately, tried a different way of solution. Although this way and solution of the participant was correct, it was deficient as it was two ways of solution only. In the second stage, the participant found all the ways of solution, which provide his mathematical model. Moreover, he finished the solving process claiming *'Different ways may exist, this solution persuaded me.'* The participant stated that he could solve the problem with 8 joints at first stage and provide this solution using 6 of 16 m boards and 1 of 10 m board. At the second stage, the participant providing to protect the number of the joints as 8, developed two more ways of solution. The participant at first stage using 6 of 16 m and 2 of 10 m boards, he wasted 18 m,

at the second stage, finding the solution as 98 m through his mathematical model, stated that the room can be covered without any waste of boards. Therefore, the expressions of K.4.2 coded participant confirm the behaviours within the process of mathematical modelling in both of the stages.

4. Conclusions and Discussion

The carried study aims to investigate pre-service teachers' cognitive-metacognitive behaviours in the mathematical modelling problem-solving process with and without metacognitive support considering the class level. In this section, the results are going to be presented depending on the sub-problems.

The results within the scope of the first sub-problem of the research as 'Do the numbers of cognitive –metacognitive behaviours demonstrated by the pre-service teachers during the stages of mathematical modelling vary in terms of the class variable in a case with and without metacognitive support? 'can be described as follows;

With a general view, the demonstration sequence of the numbers of cognitive-metacognitive behaviours demonstrated during the mathematical modelling process from the most to the least within the scope of grades, during the protocols without metacognitive support is as 3-4-1-2 and during the protocols with metacognitive support as 1-2-3-4. Totally the behaviours were demonstrated by the participants among 3rd graders most and 4th graders least. Though the first and second classes demonstrated the most cognitive-metacognitive behaviours, there are also cases in which they were not able to be successful. This result is in parallel with the situation that shows the increase in the demonstration of metacognitive behaviours does not ensure the success [43].

The participants, during the grasp stage of the mathematical modelling process, in which *it is tried to understand the requirements and being aware of and expressing limitations of the real system*, they demonstrated mostly the behaviours of reading the problem and trying to understand the problem. During the stage of resembling to mathematics of mathematical modelling process in which *the real situation is prepared for the mathematical operations and, in a way, constructing the model and formulizing the real situation* they developed mathematical models mostly by making general and special plans. During the stage of solving the model, in which the mathematical model reach the result by appropriate mathematical operations and solve our problem, the applications of the model and the last mathematical results are interpreted to real situation of mathematical modelling process, the participants mostly made mathematical operations and demonstrated the result behaviours related to these operations. During the stage, in which the model is checked and interpreted mathematical modelling process *before solving the model, under the present condition, the behaviour of the real system redeveloped thanks to the model and achieved*, the

participants demonstrated mostly the behaviours of checking the results, making crosscheck and developing alternative solutions.

During the mathematical modelling process with and without metacognitive support, first graders mostly demonstrated the behaviours in order as *making mathematical operations and making special plans for the solution of the problem*. Related to this, it is possible to express that it is an indicator of providing metacognitive support does not change the behaviours, which the individuals prefer in mathematical modelling process much. Yet, providing metacognitive support contributed to the occurrence some cognitive-metacognitive behaviours and the frequency of some behaviour. For instance, while the behaviours of checking the procedure, crosschecking, developing alternative ways of solution was almost never demonstrated by several participants during the process with metacognitive support, during the protocols with metacognitive support was the most demonstrated behaviour. After the metacognitive support, the behaviour of *checking the consistencies of the reached results with the data in the problems* was the behaviour, which is the most frequent in every grade. Even in the study by Ellerton [60] during their problem solving experiences, it was proved that the students demonstrate higher metacognitive behaviours as they started to understand the role of reflection. Aydemir and Kubanç [30], pointed out that the students, who can use metacognitive skills, can develop alternative ways and reach the correct solution; on the other hand ones, who cannot use the metacognition skills reach the solution with ordinary methods.

The results within the scope of the second sub-problem of the research as 'Do the sequence of demonstrated cognitive-metacognitive behaviours by pre-service teachers in mathematical modelling processes vary according to an occasion with and without metacognitive support in terms of the class variable?' is described as follows.

All the participants placed in the study group, (except from one question of K.3.1 coded participant) started all of the mathematical modelling processes with reading the problem but they read the problem during the process from time to time, repeatedly. The behaviour of reading question was demonstrated after cognitive-metacognitive behaviour 38 times during the stage without metacognitive support and 27 times with metacognitive support. During the stage without metacognitive support, the numbers of re-reading metacognitive behaviours reduced. The behaviour of reading the question, which was demonstrated by the participants during the stage without metacognitive support, were observed mostly after the behaviour of reading the problem (18 times), trying to understand the problem (6 times) and making calculations (5 times), during the stage with metacognitive support were mostly after reading the problem (9 times), trying to understand the problem (4 times) and writing the result (3 times). That is, the participants re-read the problem, after reading the problem, after trying to understand the problem, after making the operations related

to the problem and after reaching the solution of the problem. Although the behaviour of re-reading the problem has the least frequency, it was demonstrated after drawing a figure related to the problem, after thinking of its difficulty, after making a plan related to the solution of the problem, and after checking the consistency of the gathered results with the problem. Yimer and Ellerton[16], also, stated that pre-service teachers might read the problem repeatedly for different purposes.

One of the patterns of the participants during the process of mathematical modelling process is demonstrating the behaviour of; *making mathematical operations* after *making special plans for the solution of the problem.* That is, the participants, after specialized with shortening their plans related to the solution of the problem, did the evaluations of these plans. While the sequence of these behaviours by K.1.1, K.1.2, K.2.1, K.2.2, K.3.1, K.3.2, K.4.1 and K.4.2 coded participants during the protocols without metacognitive support come out entirely, in order with the frequency of; 5, 12, 0, 3, 12, 11, 3, 4, during the protocols with metacognitive support orderly as the frequency of 21, 12, 22, 12, 6, 0, 0, 4. While the use of these behaviours shows increase or equality from without metacognitive support to with metacognitive supporting 1st and 2nd grades, it came out as reduce or equality in the 3rd and 4th grades.

One of the patterns encountered during the study was *making mathematical operations* after the behaviour of *making general plans about the solution of the problem*. That is the participants making some restrictions in their general plans related to the solution of the problem then made the calculations within the scope of these plans. While the sequence of the K.1.1, K.1.2, K.2.1, K.2.2, K.3.1, K.3.2, K.4.1 and K.4.2 coded participants' behaviours come out orderly, in the protocols without metacognitive support is totally with the frequencies as 0, 1, 0, 2, 1, 4, 3, 0, it occurred in the protocols with metacognitive support in order as 0, 2, 0, 3, 8, 5, 4, 3 frequencies. Demonstrating these behaviours one after another increases in all the classes from those without metacognitive support to those with metacognitive support, moreover, the highness in the frequencies in 3rd and 4th grades compared to 1st and 2nd grades is remarkable. Considering the reduce of the participants' demonstrating the behaviours *making mathematical operations* after the behaviour of *making special plans for the solution of the problem*, specifically in 3rd and 4th grades, the result mentioned about becomes considerable. Shortly, it can be expressed that the students in 3rd and 4th grades continued with calculations without specializing their plans related to the solution of the problem much.

Another point that is remarkable among the participants in the process of mathematical modelling is that the behaviour of *confirming the solutions with the data in the problem*, during the protocols without metacognitive support, coming after making calculations most (19 times),then, after *making special plans related to the solution of the problem* (5 times). The demonstration of this behaviour came after the behaviour of *making calculations* most (22 times) during the

protocols with metacognitive support, after the behaviour of *confirming the reached results with the data in the problem* most (21 times), and after r*esult check, crosscheck, developing alternative solutions* (17 times). That is, the participants, after making operations during the protocols without metacognitive support and making specialized plans related to the solution of the problem, confirmed those results with the data in the problem. During the protocols with metacognitive support, the participants again, after making operations and developing alternative solutions to the problem or after checking the results of the problem, confirmed those results with the data in the problem.

The remarkable point related to the demonstration of the behaviours *result check, crosscheck, developing alternative solutions* by the participants during the process of mathematical modelling is the demonstration of this behaviour mostly after itself. That is, the participants demonstrated the behaviours repeatedly after the behaviours of developing alternative solutions to the problem or checking the results, making crosscheck.

During the mathematical modelling process of the participants, apart from the binary patterns, a triple pattern was also encountered. The behaviour pattern as making a general plan related to the solution of the problem, specializing the plans and making calculations in every grades, was used 3 times, in average, sometimes without metacognitive support, sometimes with metacognitive support and sometimes in both of the two stages. In the occurrence of this behaviour pattern, any difference was not encountered according to the class level and metacognitive support.

Even in the study by Yimer and Ellerton[16], any general pattern was not encountered in the problem solving process of pre-service teachers demonstrating metacognitive behaviours, also it was noticed that the solving process of the same students in different problems were also different. Demircioğlu, Argün and Bulut [32] emphasized the same result. The researchers were not encountered with any pattern demonstrated by the pre-service teachers in terms of metacognitive in their academic success, but reached the result that different types of the problems affect the number of metacognitive behaviours.

The results within the scope of the third sub-problem of the research as 'How do the pre-service teachers evaluate the mathematical modelling processes in terms of with and without metacognitive support?' can be described as follows. Artzt, & Armour-Thomas[4], in their studies, claimed that the pre-service teachers returned to the steps reading the problem, analysing, discovering etc. again and again in the process of problem solving. Aydemir and Kubanç [30] pointed out that the students, who can use their cognitive skills, do not fulfill the stages of metacognition in order and they skip some steps during the problem solving process.

The participants expressed that the second stage enabled them the attention, thinking about the questions, checking and thinking of different ways of solutions. The expressions of the participants from the 1st and 2nd grades were mainly

on reading and understanding the problem but the participants from 3rd and 4th grades answers were mainly on following the operations, checking and confirming the results and developing different ways of solutions for the problems. Giving metacognitive support directed the participants to the behaviours as thinking on the problems more, understanding the problems, turning back to the questions again. Thus, it can be expressed that metacognitive support, enabling the participants to follow-check and find new ways of solutions, affected the mathematical modelling process positively. To sum up, considering the fact that participants are more successful in the process of mathematical modelling during the protocols with metacognitive support, it can be expressed that all of the cognitive-metacognitive behaviours as *confirming the results with the data in the problem, developing alternative ways of solutions* have significant role for the success in the process of mathematical modelling. This result is in parallel with most of the participants' claims that the metacognitive knowledge in modelling process contributed much to them in the study by Maab[44]. Similarly, it coincides with the expressions of most of the pre-service teachers as; *'By use of my metacognition, I remind myself to keep checking my solution with the problem to make sure I solve it correctly'* in the study by Yimer and Ellerton[16]. Consequently, it can be claimed that the participants are much more successful in the process of mathematical modelling during the stage in which metacognitive support was given to the participants. These results are in paralel with the results of the studies [4,19,20,29,30,40,48,66]   which show that the problem solving success increased with metacognitive support and teaching strategy. Takahashi & Murata[81] also reached the result showing the teaching of metacognition has a form, which activates metacognition for understanding and directing the problem solving process.

In general, it was noticed that giving metacognitive support during the process of mathematical modelling increased the numbers of cognitive-metacognitive behaviours. Within the scope of the study, it was recognised that the participants did not have a fixed template in the process of mathematical modelling both with and without metacognitive support. It was noticed that, in both stage, the participants demonstrated some cognitive-metacognitive behaviours one after another. It can be expressed that the metacognitive support contributed much advantages within the participants' point of view.

As the results were analysed within the perspective of mathematical model and modelling, it can be claimed that the metacognitive support helped the participants in developing mathematical model and thus, they were able to reach the correct and alternative solutions. Therefore, the participant could develop different ways of solutions thanks to their models or easily expressed that they were sure about the results. For instance, in the 3rd question, the participants could claim that is the result, the only result considering their model, because, only this solutions gives the zero. When the metacognitive support was not given, developing the

mathematical model was only developed by K.1.1 and K.4.2 coded participants but the participants did not apply the model as a tool in reaching the result. In the protocols with metacognitive support, five participants (K.1.1, K.2.2, K.3.2, K.4.1, and K.4.2) developed the mathematical model and the participants were able to use this model through their goals. When the mathematical modelling processes of the participant are investigated, the behaviours of developing mathematical modelling and reaching the results with the help of this model, it is noticed that these behaviours are achieved well in the grades of 3rd and 4th. Therefore, we can express that giving the metacognitive support in the process of mathematical modelling influenced the participants' developing mathematical model, and applying the model for their purposes positively, and this effect was higher in 3rd and 4th grades.

Another result which was reached in terms of mathematical model and modelling is that the participants always developed their mathematical models directly. These participants decided to develop a mathematical model because of the insolubility, the useless of the plans, disability to limit the possibilities, reached decisions as a result of the calculations which they encountered during the process. The behaviours of making mathematical operations on the model and reaching the result are the most encountered behaviours, which the participants demonstrated during the process with metacognitive support. The behaviours as testing the model, interpreting and evaluating are the behaviours, which are mostly demonstrated by the upper graders.

The study is limited with only one metacognitive support to the pre-service teachers in the process of mathematical modelling and so, it is not among the prior goals that the pre-service teachers to internalize these behaviours totally after the first metacognitive support. In following studies, it is thought that it will contribute to the literature much to investigate the effects of this situation on the mathematical modelling processes and the permanents of these effects with handling more detailed and systematically.

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