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TEACHING RLC PARALLEL CIRCUITS IN HIGH-SCHOOL PHYSICS CLASS

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Abstract: This paper will try to give an alternative treatment of the subject "parallel RLC circuits" and "resonance in parallel RLC circuits" from the Physics curricula for the XIth grade from Romanian high-schools, with an emphasis on practical type circuits and their possible applications, and intends to be an aid for both Physics teachers and students eager to learn and understand more.

Key words: alternating current, RLC circuits, resonance, teaching

1. Introduction

Both Electricity and Magnetism has been known as basic subjects in Physics education, at all levels, because of several reasons: (a) both are the main source of knowledge about the structure, properties and applications of matter, (b) because of their practical applications have a great relevance in our everyday lives, under every aspect of it (social, cultural, personal, technological, etc.). Therefore, in several studies [1-7] regarding learning difficulties of Physics, subjects like steady state or dynamic electric circuits were used to measure the level of the problem (helding misconceptions, misunderstanding concepts, erroneous reasoning, conceptual difficulties, etc.). As expected, there are several studies of alternative methods to help students to overcome those difficulties. A fair survey of the literature [8-15] suggest two type of methodes: (a) a traditional one, with emphasis on experimental activities (laboratory) and (b) a modern one, with emphasis on computational resources using modeling and simulations.

Alternating current (AC) and related phenomena, physical quantities and applications are a very important part of the Romanian high-school Physics curricula for both Xth and XIth grades [16-17]. In the Xth grade it takes the students to an exciting journey from definition and generation, through circuit elements behavior and AC energy/power notions, to applications like transformers, electric motors or home appliances. Students from the XIth grade are taken to the "next level". They learn about the RLC circuits, electromagnetic oscillations and resonance, and some practical applications of oscillating circuits.

All these AC related knowledge could be a considerable challenge for the students due to two reasons: (a) the physical quantities have a very different dynamic behavior and properties, as compared to what they know until than (time and frequency dependence, periodicity, reverse in direction, phase, lead or lag relationship between voltage and current), (b) the math's that applies is quite difficult (trigonometry, operation with time dependent quantities, Fresnel type phasor diagram, differential equations, complex numbers, etc.).

One of the major problem with the Physics textbooks, designed and written according to the above cited curricula, is that they are using ideal, theoretical models and concepts that are, sometimes very far or not related at all with reality or the practical aspects of the subject. This is the special case of the parallel RLC circuit in the XIth grade curricula and accredited Physics textbooks [18-21].

This paper will try to give an alternative treatment of the subject "*parallel RLC circuits*" and "*resonance in parallel RLC circuits*" with an emphasis on practical type circuits and their possible applications.

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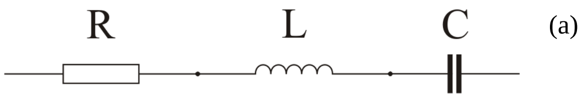
2. Basic aspects of the RLC circuits in XIth grade Physics textbook

According to the mentioned Physics textbooks [18-20] an RLC circuit is an oscillating electric circuit consisting of a resistor (R), an inductor (L) and a capacitor (C) connected in series or in parallel (see Figure 1 a and b).



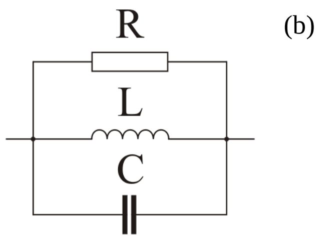


Figure 1. *Ideal RLC series (a) and parallel (b) circuits*

The name of the circuit is derived from the initials of the constituent passive components of the circuit, connected is that particular order. Obviously, if they would be connected in an other sequence, the circuit name will not be different, as it is expected!

Oscillating means that such a circuit is able to produce a periodic, oscillating signal by the periodical transfer of the stored energy between the two reservoirs (L and C), the resistance (R) being responsible for the dumping, the loss of some energy during the back and forth transformation via Joule heat dissipation, leading to the exponential decay of these oscillations.

The most important facts about the two circuits, stated in the textbook, are synthesized comparatively in Table 1.

Table 1. Summary of properties of RLC circuits based on XIth grade Physics textbook [19]

Type / **Property**	**SERIES**	**PARALLEL**
Impedance	$Z_s = \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2}$	$Z_p = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega \cdot L} - \omega \cdot C\right)^2}}$
Phase angle	$\tan \phi_s = \frac{\omega \cdot L - \frac{1}{\omega \cdot C}}{R}$	$\tan \phi_p = R \cdot \left(\frac{1}{\omega \cdot L} - \omega \cdot C\right)$
Resonance type	... of voltages and impedance is minimized	... of currents and impedance is maximized
Resonance frequency	$f_s = \frac{1}{2\pi \cdot \sqrt{L \cdot C}}$	$f_p = \frac{1}{2\pi \cdot \sqrt{L \cdot C}}$
Quality factor	$Q_s = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$	not given

3. Some critical aspects of the RLC circuits in XIth grade Physics textbook

When analyzing the information given about the parallel RLC circuit one can see that, it is significantly less than that given for the series circuit (which is also incomplete and deficient sometimes).

There is no information about the half-power frequencies and bandwidths. A very superficial definition of quality factor Q is given and only for the series circuit. Both AC powers (active, reactive and apparent) and power factor are defined and given for just the series case.

The chapter ends with just two possible applications of AC circuits, i.e. transformers and electric motors, presented and treated very briefly.

All the components were considered to be ideal, but both inductor and capacitor usually have a loss resistance - therefore it is not even necessary to have an extra resistor, the dumping being assured by these loss resistances.

Such a real parallel RLC circuit is depicted in Figure 2, where R_L and R_C are the loss resistances of the inductor and the capacitor, respectively.

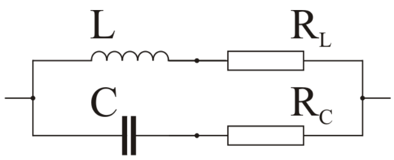


Figure 2. *Two branch RLC parallel circuit*

Studying such a parallel circuit is considerably more difficult than those depicted in Figure 1, even if in the case of the ideal circuits it is customary to say that the parallel circuit is "dual" of the series one [22] - the current and the voltage exchange roles, the parallel circuit has a current gain instead of the voltage gain found for the series one, the impedance will be maximized for the parallel oscillator at resonance rather than minimized, as it is for the series one.

We may suspect that it will be somewhat similar for the real parallel circuit, but this is not so obvious at the first sight and has to be proven later. All this uncertainty is mainly due to the fact that, in the case of Figure 2, each branch of the circuit will have its own phase angle and they cannot be combine simply, like in for ideal ones.

When studying AC circuits, the following three methods are available: a) Analytical method; b) Fresnel phasor (vector) method; c) Complex number method. Each one has its own advantages, but drawbacks too, mainly due to the not so easy maths.

The analysis of the two branch real parallel circuit can be found rarely in the literature, authors privilege the easier ideal models. Despite of this, some excellent treatment of the realistic RLC circuits are available: a very short and synthetic, but useful presentation [23] or a more detailed one, with several calculus examples [24].

4. Detailed analysis of the practical RLC parallel circuit

In order to give a real aid for the curious and enquiring high-school students, let's presume that the capacitor is ideal ($R_C \approx 0$). This is a presumption very close to reality, the dielectric found in capacitor are almost lossless when used under working conditions, and will lead us to a much approachable and treatable circuit called practical RLC parallel circuit (see Figure 3).

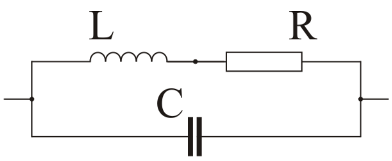


Figure 3. *Practical RLC parallel circuit*

Let us consider the practical parallel circuit presented in Figure 4, where the circuit is connected to a signal generator providing a time varying input signal described by the expression:

$$u(t) = U_0 \cdot \sin(\omega t)$$

where U_0 is the peak value of the voltage (unit: V) and ω represents the angular frequency (unit: rad/s) defined using the physical frequency f (number of cycles per second, unit: Hz) like being $\omega = 2\pi \cdot f$.

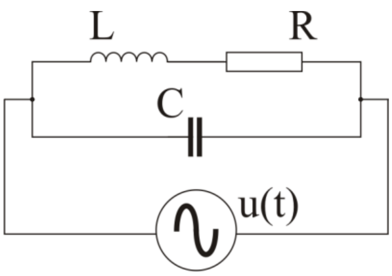


Figure 4. *Circuit diagram for the idealized LRC parallel circuit*

For such a configuration, the instantaneous voltage across the branches will be the same and the time-varying analytical expressions for the current in the main branch of the circuit and in the two parallel branches become, respectively:

$$i(t) = I_0 \cdot \sin(\omega t + \phi)$$

$$i_L(t) = I_{L0} \cdot \sin(\omega t - \phi_L)$$

$$i_C(t) = I_{C0} \cdot \sin(\omega t + \phi_C)$$

where the indexes "L" and "C" are depicting the branches containing the inductor and the capacitor, respectively, index "0" indicates the peak values of the currents and the Greek letter ϕ (*phi*) is standing for the phase angle appeared between currents and voltage, the "+" or "–" signs are denoting the leading or lagging relationship between the current and voltage. The value of the phase angle in the branch of the capacitor is $\phi_C = \pi / 2$ (or 90°) because it was considered to be ideal. The phase angle in the inductors branch will depend on both loss resistance and inductance, $\phi_L = tan^{-1}(\omega L/R)$.

It will be useful to make a short detour here: in Reference [25] a very good suggestion is made for memorizing the current/voltage relationships in the case of capacitors and inductors - the mnemonic CIVIL is introduced.

Considering the positions of the letters in the word CIVIL we will have: C - I - V (in the case of the capacitor C, the current I will LEAD the voltage V) and V (repeated) - I - L (voltage V LEAD the current I, in the case of the inductor L), respectively.

In order to complete the analysis of such a circuit it will be necessary to calculate the currents in each branch (main and secondary), work out the phase relationship between the main current and the supply voltage, find the impedance of the circuit and the resonance frequency, and figure out the time (frequency) dependent behaviour of the circuit.

Beacuse of the necessity of solving differential equations, an exhaustive analytical treatment of the subject exceeds the Romanian high-school maths curricula. Anyway, in order to find out the currents, some analytical expressions has to be written down.

For the inductors branch, we will have the Kirchhoff's voltage law (KVL) for the instantaneous voltages:

$$u(t) = U_0 \cdot \sin(\omega t) = u_R(t) + u_L(t) = R \cdot i_L(t) + L \cdot \frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} = I_{L0} \cdot \omega \cdot \cos(\omega t - \phi_L) = I_{L0} \cdot \omega \cdot \sin(\omega t - \phi_L + \frac{\pi}{2})$$

$$U_0 \cdot \sin(\omega t) = R \cdot I_0 \cdot \sin(\omega t - \phi_L) + L \cdot I_{L0} \cdot \omega \cdot \sin(\omega t - \phi_L + \frac{\pi}{2})$$

For the capacitors branch, we have for the instantaneous value of the electric charge on the capacitor plates:

$$Q_C(t) = C \cdot u(t) = C \cdot U_0 \cdot \sin(\omega t)$$

The current through this branch will become:

$$i_C(t) = \frac{dQ_C(t)}{dt} = C \cdot \frac{du(t)}{dt} = C \cdot U_0 \cdot \omega \cdot \sin(\omega t + \frac{\pi}{2}) = I_{C0} \cdot \sin(\omega t + \frac{\pi}{2})$$

The current in the main branch of the circuit will be given by the Kirchhoff's current law (KCL):

$$i(t) = i_C(t) + i_L(t)$$

$$I_0 \cdot \sin(\omega t + \phi) = I_{L0} \cdot \sin(\omega t - \phi_L) + I_{C0} \cdot \sin(\omega t + \frac{\pi}{2})$$

These currents can be represented with the phasor diagram shown in Figure 5.

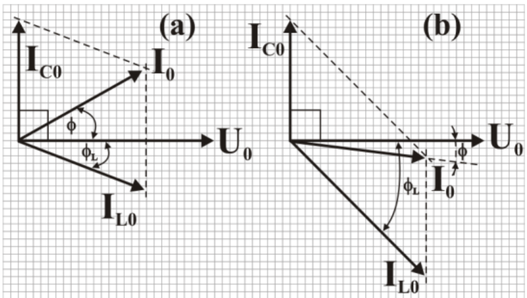


Figure 5. *Phasor diagram for the idealized LRC parallel circuit (a) with capacitive behavior (currents leads voltage), (b) with inductive behavior (currents lags voltage)*

Solving such diagrams using only geometry and trigonometry is not such an easy task.

Let's take a look on Figure 5a. The peak value of the current in the main branch will be:

$$I_0^2 = (I_{C0} - I_{L0} \cdot \sin\phi_L)^2 + (I_{L0} \cdot \cos\phi_L)^2$$

$$I_0^2 = I_{C0}^2 + I_{L0}^2 \cdot \sin^2\phi_L - 2 \cdot I_{C0} \cdot I_{L0} \cdot \sin\phi_L + I_{L0} \cdot \cos^2\phi_L$$

$$I_0^2 = I_{C0}^2 + I_{L0}^2 - 2 \cdot I_{C0} \cdot I_{L0} \cdot \sin\phi_L$$

This result lead us to the impedance and later, to the resonant frequency of the circuit.

In AC circuits the complex ratio of voltage to current is called impedance. This somehow extends the concept of resistance (known from direct current part of the curricula, Xth grade). Basicly, it is a generalized, extended resistance which has both magnitude and phase.

Thus for the impedance one can write:

$$\left(\frac{U_0}{Z_p}\right)^2 = U_0^2\cdot(C\cdot\omega)^2 + \frac{U_0^2}{R^2+(L\cdot\omega)^2} - 2\cdot U_0^2\cdot\frac{(C\cdot\omega)}{\sqrt{R^2+(L\cdot\omega)^2}}\cdot\sin\phi_L$$

with $\sin\phi_L = \dfrac{L\cdot\omega}{\sqrt{R^2+(L\cdot\omega)^2}}$.

Substituting and simplifying, we will find:

$$\left(\frac{U_0}{Z_p}\right)^2 = U_0^2\cdot(C\cdot\omega)^2 + \frac{U_0^2}{R^2+(L\cdot\omega)^2} - 2\cdot U_0^2\cdot\frac{C\cdot\omega}{\sqrt{R^2+(L\cdot\omega)^2}}\cdot\frac{L\cdot\omega}{\sqrt{R^2+(L\cdot\omega)^2}}$$

$$\left(\frac{U_0}{Z_p}\right)^2 = U_0^2\cdot(C\cdot\omega)^2 + \frac{U_0^2}{R^2+(L\cdot\omega)^2} - 2\cdot U_0^2\cdot\frac{C\cdot L\cdot\omega^2}{R^2+(L\cdot\omega)^2}$$

$$\left(\frac{1}{Z_p}\right)^2 = (C\cdot\omega)^2 + \frac{1}{R^2+(L\cdot\omega)^2} - 2\cdot\frac{C\cdot L\cdot\omega^2}{R^2+(L\cdot\omega)^2}$$

$$Z_p{}^2 = \frac{R^2+(L\cdot\omega)^2}{(R\cdot C\cdot\omega)^2+(\omega^2\cdot L\cdot C-1)^2}$$

Thus, the impedance of the practical parallel RLC circuit is frequency dependent and given by the relationship:

$$Z_p(\omega) = \sqrt{\frac{R^2+(L\cdot\omega)^2}{(R\cdot C\cdot\omega)^2+(\omega^2\cdot L\cdot C-1)^2}}$$

Resonance is a very important phenomenon in physics occuring in all sorts of systems. In the case of oscillating circuits, it occurs when the phase angle between the current in the main branch of the circuit and voltage across the circuit will be equal to zero.

From Figure 5a one can see that:

$$\tan\phi = \frac{I_{C0} - I_{L0}\cdot\sin\phi_L}{I_{L0}\cdot\cos\phi_L}$$

Substituting and simplifying, we will find:

$$\tan\phi = \frac{(C\cdot\omega)\cdot[R^2+(L\cdot\omega)^2]-(L\cdot\omega)}{R}$$

The resonance condition ($\tan\phi = 0$) will give for the resonant frequency of the practical RLC circuit the following expression:

$$f_p = \frac{1}{2\pi}\cdot\sqrt{\frac{1}{L\cdot C}-\left(\frac{R}{L}\right)^2}$$

One can see that, this value is slightly smaller, due to the loss resistance of the coil, than the resonance frequency

$$f_s = \frac{1}{2\pi \cdot \sqrt{L \cdot C}}$$

of the series RLC circuit obtained using the same components. Combining the two expressions for the frequencies, one can write:

$$f_p = f_s \cdot \sqrt{1 - \frac{R^2 \cdot C}{L}} < f_s$$

The value of the impedance of the practical parallel RLC circuit at the resonance frequency f_p, called dynamic resistance, and its expression will be:

$$Z_p(\omega)\Big|_{\omega = 2\pi \cdot f_p} = \frac{L}{R \cdot C}$$

As no one expects, at least after reading and learning from the XIth grade textbook, this impedance will be near to its maximum value, but will not be quite that maximum! The frequency at which the maximum impedance will occur is defined by an other, slightly higher frequency that the resonance one, as demonstrated in Figure 6. This frequency is determined by differentiating (calculus) the general equation for the parallel impedance with respect to frequency and then determining the frequency at which the resulting equation is equal to zero. The math's is quite extensive and cumbersome for the XIth grade, and will not be included here, but Physics teacher should deal with it easily. However, the expression for this frequency is the following:

$$f_m = f_s \cdot \sqrt{1 - \frac{1}{4} \cdot \frac{R^2 \cdot C}{L}}$$

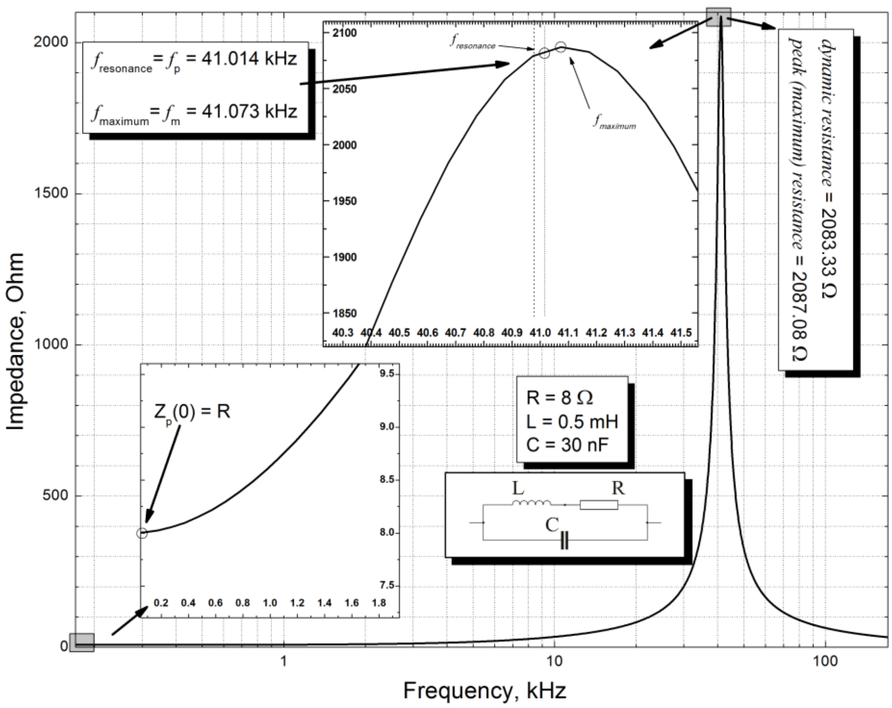


Figure 6. *Example of frequency dependence of the impedance of a practical RLC parallel circuit*

An other very important quantity, the quality factor (Q) of the resonant system is a measure of how "sharp" or narrow the frequency dependence of the impedance $Z_p = Z_p(\omega)$ is.

Accordind to one possible definition, the quality factor Q represents 2π multiplied by the ratio between the maximum energy stored and the total energy lost per cycle, at resonance.

For the maximum energy stored in the circuit we have:

$$E_{\max stored} = \frac{U_0^2}{2 \cdot C}$$

and the for the total energy lost due to the resistance in circuit, per cycle will find:

$$E_{total\ lost} = \frac{\frac{1}{2} \cdot I_{L0}^2 \cdot R}{f} = \frac{\frac{1}{2} \cdot \frac{U_0^2}{R^2 + (L \cdot \omega)^2} \cdot R}{f}$$

respectively.

Thus, for the quality factor Q, after substitution and simplifications, one can find:

$$Q = 2\pi \cdot \frac{E_{\max stored}}{E_{totallost}} = \frac{\frac{U_0^2}{2 \cdot C}}{\frac{\frac{1}{2} \cdot \frac{U_0^2}{R^2 + (L \cdot \omega)^2} \cdot R}{f}} = (C \cdot \omega) \cdot \frac{R^2 + (L \cdot \omega)^2}{R}$$

At resonance $\omega = \omega_p = 2\pi \cdot f_p = \sqrt{\frac{1}{L \cdot C} - \left(\frac{R}{L}\right)^2}$ the quality factor for the practical parallel RLC circuit will become:

$$Q_p = \sqrt{\frac{L}{R^2 \cdot C} - 1}$$

With the values given in Figure 6 one can see that the quality factor of such a circuit will be $Q_p = 16.11$.

The sharpness is a very important property because it depends upon how quickly the system loses the stored energy due to resistance in the circuit. When the curve is sharp or more narrowly peaked around the resonance frequency, one can say that the circuit has higher selectivity. This means that, the practical parallel RLC circuit can be used as a bandpass filter, a circuit which will let through signals with frequencies very close to the resonant frequency, but rejecting frequencies very different (lower or even higher) to f_p.

The influence of the value of the resistance R on the sharpness is depicted in Figure 7.

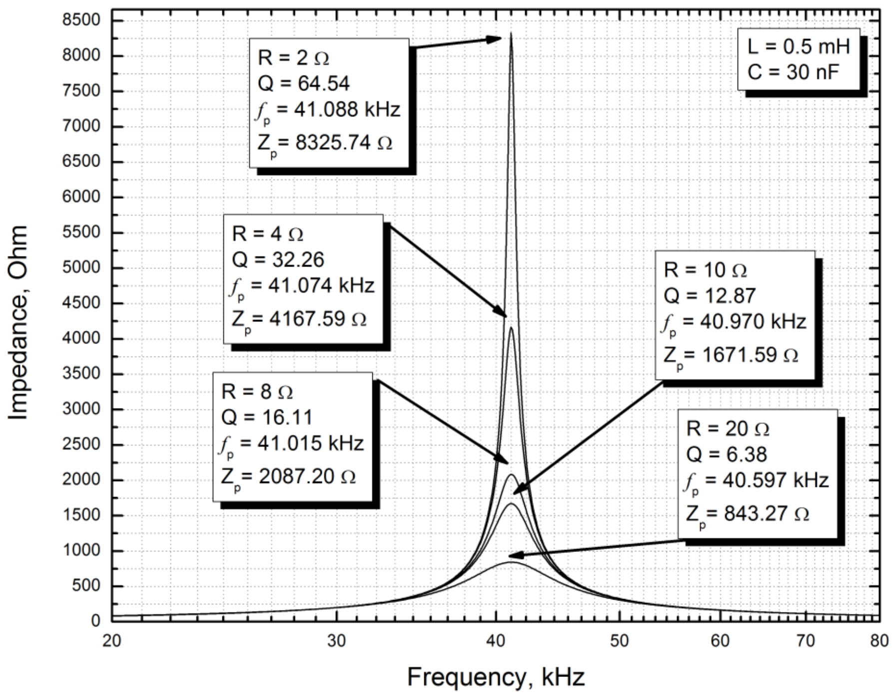


Figure 7. *Influence of loss resistance on the quality factor of a practical RLC parallel circuit*

The bandwidth (BW), the width of the resonant power curve at half maximum, depends upon Q:

$$BW = \frac{f_p}{Q_p} = \frac{f_p = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{L \cdot C} - \left(\frac{R}{L}\right)^2}}{\sqrt{\frac{L}{R^2 \cdot C} - 1}} = \frac{R}{2\pi L}$$

With the values given in Figure 6 we will get $BW = \frac{8}{\pi}\ \text{kHz} \approx 2.55\,\text{kHz}$.

This means that, around the resonant frequency (f_p = 41.015 kHz), there will be a passing window for such a frequency range (from 39.74 kHz to 42.29 kHz), the signals being attenuated less with less than 3dB. The signal with frequencies outside this range will be drastically attenuated and not allowed to pass.

5. The easy (or not so hard) way to deal with the practical RLC parallel circuit

If the treatment of the ideal RLC parallel circuit is still preferred, there is a compromise, a bridge towards the practical RLC parallel circuit: the series to parallel transformation. This technique will give the expressions of the elements of a parallel circuit as function of the elements of the series one. This subject can be treated easily with the complex number formalism, but such an approach is outside the high-school math curricula. Let us consider the two circuits depicted in Figure 8.

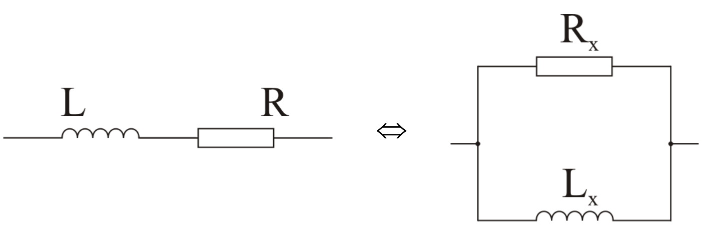


Figure 8. *Series to parallel equivalent circuits*

The two circuit will be equivalent, if both their impedances and quality factors are equal.

For the series circuit we have $Q = \frac{\omega \cdot L}{R}$ and $Z = \sqrt{R^2 + (\omega \cdot L)^2}$ and for the parallel one, the relationships will become $Q_x = \frac{R_x}{\omega \cdot L_x}$ and $Z_x = \sqrt{\frac{R_x{}^2 \cdot (\omega \cdot L_x)^2}{R_x{}^2 + (\omega \cdot L_x)^2}}$, respectively.

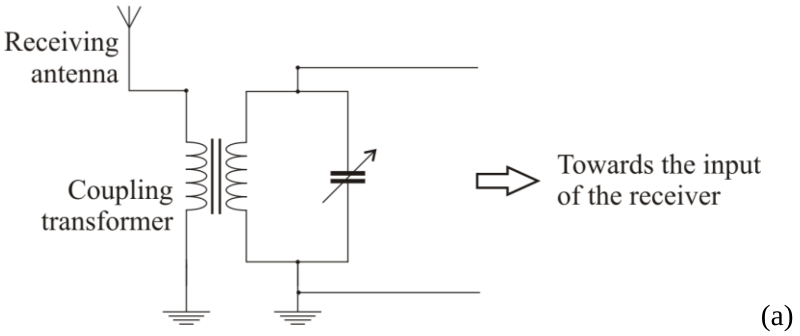
Using the equivalence conditions $Q = Q_x$ and $Z = Z_x$, respectively, for the elements of the parallel circuit we will find $R_x = R \cdot (1 + Q^2)$ and $L_x = L \cdot \frac{1 + Q^2}{Q^2}$ or in a more detailed form $R_x = \frac{R^2 + (\omega \cdot L)^2}{R}$ and $L_x = \frac{R^2 + (\omega \cdot L)^2}{L}$

Thus, the practical RLC circuit, formed by the parallel connection of a series RL branch with a capacitor C, could be easily transfromed this way in a parallel R_xL_xC circuit. One can see that both R_x and L_x are frequency dependent. This is the, not so obvious, reason why at resonance the impedance is not exactly maximum.

6. Applications of the parallel RLC circuit

Generally speaking, resonant RLC circuits are used in a variety of configurations in communication systems for coupling, filtering, tuning, generation of harmonic oscillations etc.

When listening to a radio, one should know that radio signals are broadcasted via electromagnetic waves through atmosphere towards the receving antennas, where small voltages are induced. From a quite wide range of frequencies only one or a narrow band must be extracted, transmitted to the receiver and amplified in order to be listened by us. Figure 9 shows some typical arrangements for antenna coupling (a), tuned amplification (b) and frequency selection (c), respectively.



Both coupling to the antenna and tuning to the desired frequency are realised via parallel resonant circuit formed by a real inductor and a variable capacitor.

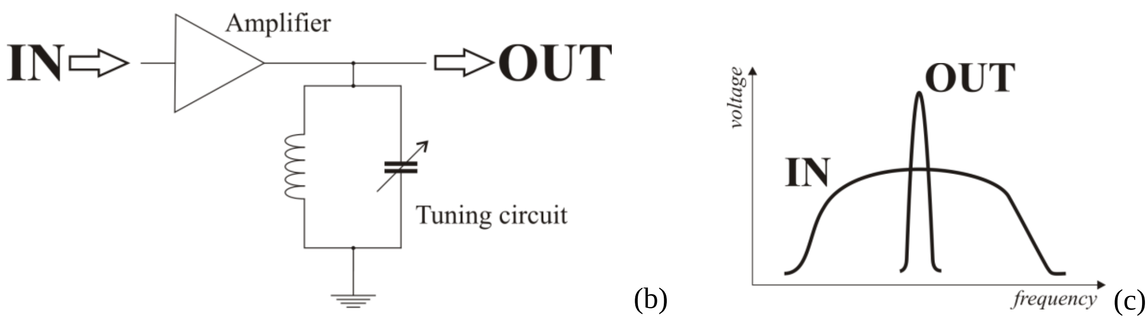


Figure 9. *Tuning and selection with parallel RLC circuits*

In TV receivers, the electronics must handle both video (pictures) and audio (sound) signal, in the same time. The sound is prevented from interfering with picture via so called wave trap or band-stop filter (Figure 10 a).

The sound is transmitted to an audio amplifier and speakers via band-pass filter tuned to the sound carrier frequency (Figure 10 b).and L_x are frequency dependent. This is the, not so obvious, reason why at resonance the impedance is not exactly maximum.

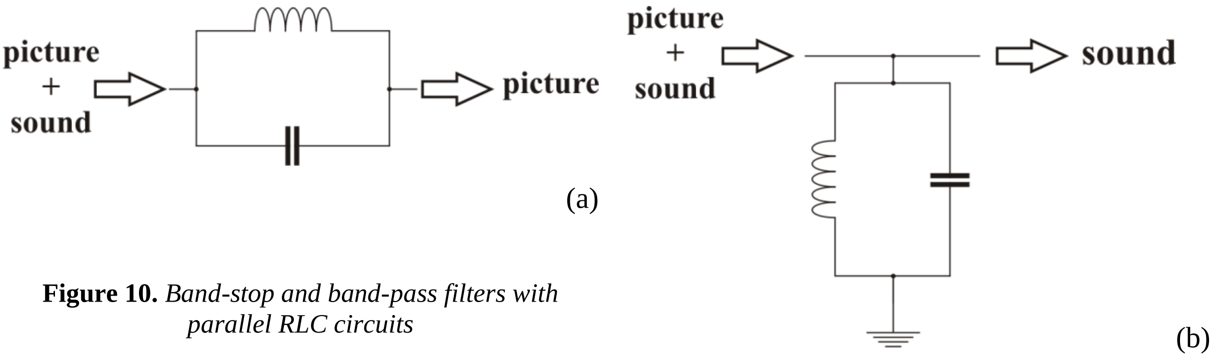


Figure 10. *Band-stop and band-pass filters with parallel RLC circuits*

Probably the most common application for the oscillating circuits is to generate a sine waveform at a constant frequency. The oscillator consisting of a capacitor connected in parallel with an inductor is called *LC oscillator*. Due to the loss resistance of the coil, in every cycle voltage will be lost and after a sufficiently long time, the oscillating will extinct. In order to avoid and overcome this, the switching technique or amplification with positive feed-back is used. An Armstrong type LC oscillator [26-27] is depicted in Figure 11.

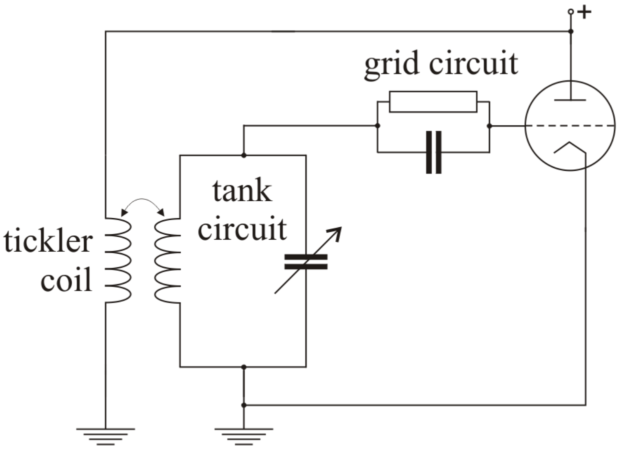


Figure 11. *Armstrong oscillator with triode*

This oscillator could be designed using both bipolar or field effect transistors, too.

The feedback from the parallel LC oscillator is ensured by a tickler coil wound next to the main coil of the tank circuit.

When apply power to the triode, voltage passes through the tickler coil to the plate, and a current begins to flow in the plate.

At this time, an electromagnetic field is developed across the tickler coil.

This gives feedback to the coil in the tank circuit (via electromagnetic induction).

The tickler coil acts as the primary, and the tank coil as the secondary of a transformer. The grid resistor drops the voltage, thus the grid becomes very negative with respect to the cathode. The grid capacitor keeps enough charge to keep the grid negative for at least one cycle of oscillation, it helps keep the grid negative when either side of the LC circuit is positive.

When the LC circuit's positive charge is at its maximum, the charge will balance with the grid capacitor, causing plate current to flow because there is no negative on the grid.

The grid controls the plate current in all vacuum tubes, thus if the grid oscillates this number of times, the plate will oscillate the same number of times, the tickler coil will give feedbacks at the same number of times. This is because of the tank circuit, which specifies the frequency.

The frequency can be adjusted when the coil and/or capacitor are adjusted.

7. Conclusion

An alternative treatment of the subject "*parallel RLC circuits*" and "*resonance in parallel RLC circuits*" from the Physics high-school textbooks was given, with an emphasis on practical type circuits and their possible applications.

As a general conclusion, Table 1, given in Section 2 of the paper, could be presented once again but completed with the information for the practical parallel RLC circuit, too.

This, almost complete comparison is given in Table 2.

Table 2. Comparison of properties of RLC oscillating circuits

Property \ Type	SERIES	IDEAL ← PARALLEL ↓	→ PRACTICAL ↓
Z	$\sqrt{R^2+\left(\omega\cdot L-\frac{1}{\omega\cdot C}\right)^2}$	$\sqrt{\frac{R^2\cdot(L\cdot\omega)^2}{(L\cdot\omega)^2+(\omega^2\cdot L\cdot C-1)^2}}$	$\sqrt{\frac{R^2+(L\cdot\omega)^2}{(R\cdot C\cdot\omega)^2+(\omega^2\cdot L\cdot C-1)^2}}$
ϕ	$\mathrm{atan}\left[R\cdot\left(\frac{1}{\omega\cdot L}-\omega\cdot C\right)\right]$	$\mathrm{atan}\left[R\cdot\left(\frac{1}{\omega\cdot L}-\omega\cdot C\right)\right]$	$\mathrm{atan}\left\{\frac{(C\cdot\omega)\cdot[R^2+(L\cdot\omega)^2]-(L\cdot\omega)}{R}\right\}$
f	$\frac{1}{2\pi\cdot\sqrt{L\cdot C}}$	$\frac{1}{2\pi\cdot\sqrt{L\cdot C}}$	$\frac{1}{2\pi}\cdot\sqrt{\frac{1}{L\cdot C}-\left(\frac{R}{L}\right)^2}$
Q	$\frac{1}{R}\cdot\sqrt{\frac{L}{C}}$	$R\cdot\sqrt{\frac{C}{L}}$	$\sqrt{\frac{L}{R^2\cdot C}-1}$
BW	$\frac{1}{2\pi}\cdot\frac{R}{L}$	$\frac{1}{2\pi}\cdot\frac{1}{RC}$	$\frac{1}{2\pi}\cdot\frac{R}{L}$
It magnifies	voltage by *Q*	current by *Q*	
Resonance type	of voltages	of currents	
At resonance	impedance is minimized	impedance is maximized	
Applications	filtering, tuning, coupling, generation of harmonic oscillations, generation of high alternating voltages		

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