International Education Studies; Vol. 9, No. 2; 2016
ISSN 1913-9020 E-ISSN 1913-9039
Published by Canadian Center of Science and Education

Thinking Process of Pseudo Construction in Mathematics Concepts

Subanji[1] & Toto Nusantara[1]

[1] Department of Mathematics, Universitas Negeri Malang, Indonesia

Correspondence: Toto Nusantara, Department of Mathematics, Universitas Negeri Malang, Indonesia. E-mail: toto.nusantara.fmipa@um.ac.id

Received: July 2, 2015 Accepted: August 7, 2015 Online Published: January 25, 2016

doi:10.5539/ies.v9n2p17 URL: http://dx.doi.org/10.5539/ies.v9n2p17

Abstract

This article aims at studying pseudo construction of student thinking in mathematical concepts, integer number operation, algebraic forms, area concepts, and triangle concepts. 391 junior high school students from four districts of East Java Province Indonesia were taken as the subjects. Data were collected by means of distributing the main instrument and tracer instrument to the subjects respectively. Both instruments were deployed for the purpose of digging up the construction process. The construction was clustered on the basis of the pseudo contruction cases and followed with in-depth interviews to three subjects of each case. The findings show that pseudo construction was identified in four cases. The first case was associated with integer operations, operations of algebraic form, the concept of area, and the concept of triangle. They used analogy of "in debt" to construct concepts of negative number operation. In the second case, they used objects (book, pencil, and thing) to describe variables in the algebraic form operation. For the third case, students deciphered unit area (m^2) as multiplication *m* x *m*. In the fourth case, students did not pay attention to the requirements of the triangle. Although they gave a right answer to their work, their construction concept was completely false.

Keywords: cognitive maps, pseudo construction, negative number operation, area concept, triangle concept

1. Introduction

Indonesian junior high school students have a lower achievement in mathematics for International Studies TIMMS 2011. Mulis et al. (2012) reported that Indonesian 8th grade students' achievement in mathematics of international studies TIMSS2011 was at a low level with a score of 386 even lower than the score in 2007, namely 397. In 2011, Indonesian students ranked 38th out of 45 participating countries, while in 2007 ranked 36th out of 49 countries. The students' low achievement could be reflected in terms of content domain and cognitive domain, particularly in Numbers, Algebra, Geometry, and Data and Chance. This fact has been a serious concern in the context of education in Indonesia, and the government has tried to undertake two important policies: reforming the primary and secondary education curriculum, including curriculum 2007 and Curriculum 2013 and certifying teachers so that they become more professional. Both policies were undergone to improve students' quality of learning. However, a study conducted by Subanji and Nusantara (2013) indicates that middle school students in East Java Indonesia made mistakes when dealing with mathematics. These mistakes were closely related with three typical characteristics: pseudo thinking, mistakes in the use of analogies, and misconceptions.

Pseudo thinking is a thinking process that results in an answer to a problem or construction to a concept "that is not true". The construction concept does not represent the actual thinking. Subanji (2007) explains that the pseudo thinking can be classified into two forms: true pseudo and false pseudo. In the context of problem solving, true pseudo happens when a student answers a question correctly but the process of thinking is wrong. False Pseudo occurs when a student answers a question incorrectly, but he/she is able to reason correctly. In the context of the construction of concepts, true pseudo happens when the concept a student writes seems to be correct, but his/her understanding about the concept is wrong. False Pseudo happens when a student writes the concept wrongly, yet his/her understanding about the concept is correct. This study examines the students' thinking processes in constructing a true pseudo concept of integer operations and algebraic forms, hereinafter called pseudo construction

Pseudo thinking process has been studied by many researchers in different terms and contexts. For instance, Vinner (1997) used the term pseudo-analytic versus analytic in the context of routine mathematical problem solving. Lithner (2000) used the term Established Experience (EE) versus Plausible Reasoning (PR) in the context of non-routine problem solving. Pape (2004) used the term Direct Translation Approach (DTA) versus Meaning Based Approach (MBA) in the context of solving word problems. Leron and Hazzan (2009) applied the Dual Process Theory of Kahneman (process S1 versus S2 processes) in the context of solving algebra problems. Studies conducted suggested pseudo thinking, but not discuss in detail the process of formation of pseudo construction. Nevertheless, a study of pseudo thinking in students' problem solving is still limited. Subanji (2007, 2013), Subanji and Supratman (2015) explored the process of pseudo thinking more deeply by using the framework of Piaget's assimilation and accommodation. In this study the thinking process about the construction of the mathematical concept of "pseudo" is discussed in detail. It emphasizes pseudo thinking in the process of concept formation of integer operations, algebraic form, area concept, and triangle concept.

According to Subanji and Nusantara (2013), the mistake made by students in doing mathematics requires attention; the mistake would seriously affect the subsequent understanding of their mathematical concepts. In order to minimize the impact of the mistake in building the next concept, it is important to track the sources and causes of the mistake. The sources can possibly be found in the formation of student's thinking scheme called the construction process of student's concept. The construction can be seen in detail by using Piaget's framework, namely assimilation and accommodation, which is figured out in the form of cognitive map. Piaget (in Huitt & Hummel, 2003) stated that when someone gets a new stimulus, there are two processes used by the individual in its attempt to adapt: assimilation and accommodation. Both of these processes are used as the person increasingly adapts to the environment in a more complex manner. Assimilation is the process of using or transforming the environment so that it can be placed in preexisting cognitive structures. Accommodation is the process of changing cognitive structures in order to accept something from the environment. Both processes are used simultaneously and alternately throughout life. Piaget (in Subanji, 2007) explained that, in the learning process, someone constantly adapts and involves the process of assimilation and accommodation. The assimilation is a process of integrating a new stimulus into formed schemata. Assimilation occurs when there is a match between a given stimulus and schemata that already exists in the minds of students. Accommodation is the integration of a new stimulus to the schemes owned by changing the existing schemes. Accommodation occurs when the schema owned by the student is not in accordance with the stimulus. Someone needs to adjust the scheme by changing the old scheme or form a new scheme so that the scheme is formed according to the stimulus. Subanji (2007) illustrated Piaget's process i.e. assimilation and accommodation as shown in Figure 1.

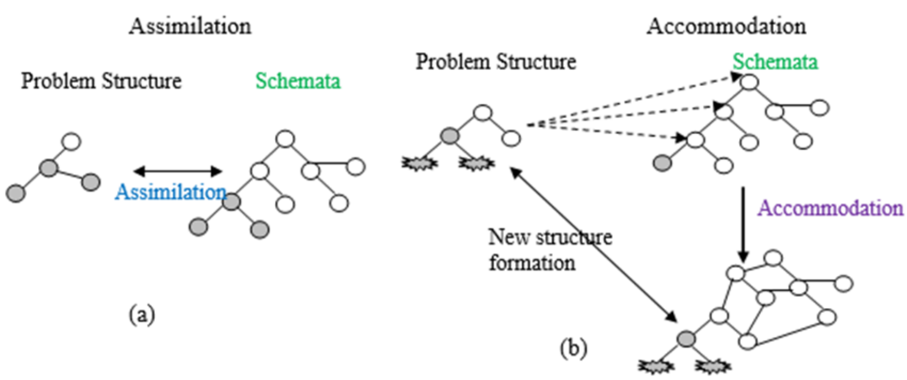


Figure 1. Assimilation and accommodation process

In Figure 1 (a), assimilation occurs when the problem structure is in accordance with the owned scheme, hence directly interprets in the correct way to form a new structure scheme. In Figure 1 (b), the scheme of structure thinking is not in accordance with the structure of the problem. In order to interpret in the correct way, there should be a conversion of the old scheme or a new scheme formation is made such that the structure of thinking can be aligned with the structure of the problem. Hence, the structure of the problem can be properly integrated into the new formed scheme.

The use of cognitive map to depict scheme formation has been addressed in several studies. For example, Jacobs (2003) revealed that the cognitive map indicates the direction of thinking. It can be used as guide for the next step of thinking. Pena et al. (2007) asserted that the cognitive map illustrates the causal relationship of the various phenomena and concepts, and can be modeled. Perdikaris (2012) described the cognitive style of students in solving geometry problems in term of Van Hielle Theory. Subanji (2007, 2011) used a cognitive map to assess pseudo thinking of students in solving mathematical problem. In this study, the schematic picture of students' thinking in constructing mathematical concepts using cognitive maps was depicted to capture the occurrence of mistakes in constructing the concepts.

2. Method

The objective of study is to analyse the formation scheme of students' thinking by using cognitive maps. This study was conducted in early June 2014 at junior high schools. 391 students from four districts of East Java Province, i.e. Malang, Blitar, Tulungagung, and Jombang were taken as the subjects. In each district, two until four schools were selected carefully, depending on whether or not they were willing to participate in the research. The subjects in Malang and Blitar were selected from "good" schools. In contrast, the subjects in the regions of Tulungagung and Jombang were selected from the mainstream schools. The subjects were taken from IX grade junior high schools, after they had joined the National Examination. There were several reasons for it: (1) students had learned all of the materials required in the curriculum; (2) the research did not disturb the school activities; and (3) students were not burdened by the school exams; they could freely express their ideas.

Two types of instruments were used in this study: main instrument and tracer instrument. The main instrument was used to explore the student's thinking process in constructing concepts of integer operations, operation of algebraic forms, the concepts of area, and the concepts of triangle. The main instrument contained statements about the mathematical concepts with which the students could justify whether or not the concept is correct or wrong as shown in Table 1. They also gave reasons to strengthen their justification.

Table 1. Main instrument

No	Statement	Answer		Reason
		True	False	
1	$-4 - 3 = -7$			
2	$-4 - (-3) = -1$			
3	There is a triangle with sides length 6 cm, 7 cm, and 14 cm			
4	A rectangle with a size of 6 m x 5 m. The area of the rectangle is 30 m^2. A m^2 unit derived from m x m)			
5	$2x + 3x = 5x$			
6	2x + 3y = 5xy			

The tracer instrument was constructed to provide alternative variety of construction. The subjects were given an opportunity to provide reasons of their answers in order to reinforce, change, or alter their opinions. The tracer instrument was also used to justify the concept of construction process. The following tracer instrument was used in this study. This instrument was done by students after they had completed the first instrument, of course, after a given pause of two hours.

Table 2. Tracer instrument

No	Statement	Reason	Agree	Disagree
1	–4 –3 = –7	**True**, because it has a debt 4 and debt again 3 so that its debts to 7		
		False, because it has a debt 4 then pay by 3 so that its debts to 1		
		False, because negative 4 meets negative 3, it should be positive 7		
		True, because there are negative 4 and negative 3, then it becomes positive 7		
		True, because by using a number line, from negative 4 to move backward in three steps and becomes – 7		
		True, because by using the pattern, – 4 – 3 = –4 + (–3) = – 7		
2	–4 – (–3) = –1	**True**, because negative meets negative is positive; –4 +3 = –1		
		True, because negative times negative is positive; –4 + 3 = –1		
		True, because it has a debt 4 and is paid 3, the result is –1		
		False, because it has a debt 4 "minus". It means it has another debt 3, so that its debts becomes 7		
3	There is a triangle with sides length 6 cm, 7 cm, and 14 cm	**True**, because there are 3 (three) sides so it can be made a triangle		
		False, because when side of 6 cm is added by 7 cm, the result is 13 cm, shorter than 14 cm. So, it should be made longer.		
		False, because it does not meet Pythagorean Theorem's; $6^2 + 7^2$ is not equal to 14^2		
4	A rectangle with a size of 6 m x 5 m. The area of the rectangle is 30 m^2. A m^2 unit derived from m x m	**True**, because L = p x l = 6 m x 5 m = 6 x 5 mxm = 30 m^2		
		True, because number times number (6 x 5) and unit times unit (m x m)		
		False, because m^2in unit area (not mxm)		
5	2x + 3x = 5x	**True**, because, let x = book, then two books plus three books, the result is five books		
		True, because of distributive law's (2+3)x = 5x)		
		False, because 2x + 3x = $5x^2$		
6	2x + 3y = 5xy	**True**, because 2 books plus 3 pencils is 5 books pencils		
		False, because the variable is different so quantity isdifferent, let x = book and y = pencil, then book and pencil could be added		
		True, because 2 plus 3 is 5		
		False, because there is no property of summation (commutative, associative, and distributive) that guaranties		

The study was conducted in several steps. First, the subjects finished the main instrument that is related to statements of essential mathematical concepts. Here, the students' pseudo construction of mathematical concepts was obtained. Second, students did the tracer instrument the result of which was used to explore the construction process based on pseudo construction cases. Third, three students representing each pseudo construction case were interviewed in relation to integer operations, operation of algebraic forms, the concepts of area, and the concepts of triangle. The interviews were to triangulate the data and map the construction process of mathematical concepts. The construction process was then described through a cognitive map. The data analysis was carried out by grouping students' answers from the main and tracer instruments based on the types of pseudo constructions. The interviews were qualitatively analyzed and the construction process was explained by using a cognitive map. The students' pseudo construction of mathematical concepts could be explored.

3. Results and Discussion

In the following paragraphs, we will describe the types of mathematical construction that have been done by the subjects. The description is based on all of the subjects' answers in the instruments. All possible answers and reasons given by the subjects were identified and recorded. The coding was then done on the subjects with similar answers and reasons. Subjects with the same codes were used as one group. Next, the groups were selected for further interviews and think aloud.

3.1 Pseudo Construction Concept of Integer Number Operation

The following table presents the students' answers categorization based on the main instrument and tracer instrument.

Table 3. Summary of students' reasons about the problem –4 – 3 = –7

Statement	Reasons	True			False	Consistence	
		True	Pseudo	Clarification		Yes	No
–4 –3 = –7	–4 –3 = –7because it has a debt 4 and debt again 3 so that its debts becomes 7	0	81	15	8	92	12
	because negative minus positive is negative	62	10	13	22	97	10
	Because –4 –3 = –7 is the same as –4 + (–3) = –7	36	0	3	14	42	11
	Because negative number minus positive number to be plus operation and it be negative	19	22	5	12	51	7
	Repeat the problem	0	0	15	22	30	7
	Did not give reason	0	0	17	15	28	4
Total		117	113	68	93	340	51

Table 3 shows that 93 out of 391 (23.7%) students answered the questions incorrectly. Students with pseudo thinking reached 38% (i.e. 113 of 298 students who gave true answers). The student answered correctly, but could not give a logical reason.

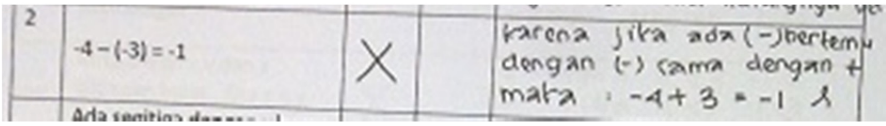
Table 4. Summary of students' reasons about the problem –4 – (–3) = –1

Statement	Reasons	True			False	Consistence	
		True	Pseudo	Clarification		Yes	No
–4 – (–3) = –1	Because negative number is larger than positive number then we must add the result	41	16	7	12	73	3
	–4–(–3)= –4 + 3 (because negative meets negative to be positive, negative times negative results in positive	17	72	12	19	103	17
	because it has a debt 4 and paid 3, the result is –1	20	31	4	12	52	15
	Because negative number minus negative number, the result should be reduced	19	22	5	12	51	7
	Because negative number in front is larger	17		15	4	25	11
	Repeat the problem	0		15	8	17	6
	Did not give reason	6		0	5	11	0
Total		120	141	58	72	332	59

Table 4 shows that students who were pseudo thinking reached 44% (i.e. 141 out of 319 students gave true answers). The result of mathematical construction by students often looks different from what they wrote on the answers. When we look up and down their work, their answers gave the effect of true, but when we trace it back by interviews and think aloud, what the thought was different form the essential concepts. Their construction is impressively true but actually is wrong. This kind of construction is named by pseudo construction.

The fault of students thinking scheme formation was captured when they constructed integer number operation concept. They were faced with problem of giving judgement and reason of the statement "True or false the statement of –4 –3 = –7 and –4 – (–3) = –1." Most of students gave true answers of the statement. The students evaluation seemed true, but when their reasons were traced, almost all students did a assimilation process by interpreting negative number (–) and minus operation by "debt" or "obligation". They mean the number –4 by 4 debt. "Minus 3" was also assimilated by in debt 3. They could not distinguish between negative number symbol and minus operation, both of them were assimilated by "in debt". They did not understand that both symbols of (–) in case of –4 –3 were different. The symbol (–) of –4 constitutes a negative number symbol, but the symbol (–) of –3 means a number operation.

Students start to have dis-equilibration when they confront to the statement of –4 – (–3) = –1. Their representation of debt on the negative number and minus operation could be used to the problem –4 – (–3). Minus of negative 3 could not be represented as in debt (in debt 3), because there is no concept of in debt (in debt). Dis-equilibration causes the subjects do accommodation by changing their thinking structure. The subjects did accommodation by making a justification reason: (1) negative meets negative resulting in positive; (2) minus meets minus resulting in plus; (3) negative times negative resulting in positive; or (4) minus times minus resulting in positive. The following picture depicts the students' justification reasons.



Translated version of the reason: Because if negative meets negative both of them changed to be positive, hence –4 + 3 = –1

Figure 2. Subjects jugdement and reason of –4 – (–3) = –1

The following text is the conversation between the researcher (R) and the subjects (P2, P3, P4) related to justification of their answers.

R: *Is the statement true? And what is your reason?*

P2: *It is true, because at the beginning, it has debt 4 then is paid 3 hence still has debt 1. Minus meets minus resulting in plus, Mam. The problem says, minus 4 minus negative 3 it will be minus 4 plus 3.*

P3: *It is true, this negative meets negative it will be positive, consequently negative 4 plus 3 is the same as negative 1. Yes, this negative times negative, bracket symbol means multiplication, so negative times negative the same as positive, the same with the previous reason.*

P4: *: It is true, because in problem no 2, negative meets negative to be positive, hence –4 + 3 = –1.*

The accommodation process was not based on the true mathematical concepts. The justification of –(–3) to be 3 by saying negative meets negative to be positive or negative times negative resulting in positive was forced by the subjects to perform their next step. In the concept of number operation, multiplication can only be applied to number. There is no concept of multiplication of negative number with minus operation. Their given results seem to be true, but the construction concept of student is still pseudo, we name it as pseudo construction. Fault in the construction of concepts by students is a fundamental mistake (Bingobali et al., 2011; Brodie, 2010; Gal & Linchevski, 2010). Brodie (2010) explained that fault made by students in mathematics learning occurred when they were building mathematical reasoning, which includes: basic mistakes, appropriate mistakes, missing information, and partial insight. Mistakes in the construction of the concept of integer operations that have been performed by the students can be referred as a basic mistake. Gal and Linchevski (2010) found that there had been a mistake students made in learning mathematics, especially in a geometry representation process that includes: (1) the perceptual organization: Gestalt principles, (2) recognition: bottom-up and top-down processing; and (3) representation of perception-based knowledge: verbal vs. pictorial representation, mental images and hierarchical structure of images. The representation of perception-based knowledge can be extended to a representation of the symbol. Symbol of negative numbers and symbols from operations minus expressed by students as “in depth” is a fundamental mistake. According to Bingobali et al. (2011), students’ mistakes in the concept construction of integer operations are included in the concept abstraction fault. Pseudo construction conducted by the subjects can be described in the cognitive map below.

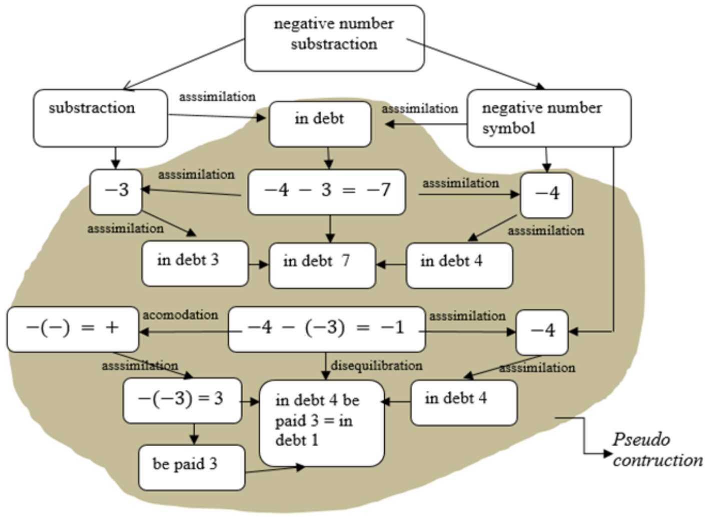


Figure 3. Cognitive map of pseudo construction of negative number subtraction

In constructing the concepts of integer operations, the subjects did not use the number line as a basis for work. They preferred to use an analogy of "debt" as a representation of negative numbers. The subjects also represented the epitome of operation and number as the same thing i.e. "debt". As a result, the subjects could not give reasons when there was a statement "subtract by a negative number". They made justification that negative meets negative result to positive or negative times negative yields a positive. As we know, in integer operations, only numbers can be multiplied. There is no concept of multiplication of numbers with the operation.

The construction concept of "minus a negative number" should be used by a number line or with a pattern. In terms of pattern, students are expected to find the concept of "subtract by the same negative number which means the opponent is added with negative numbers" as shown in the following pattern.

$4 - 3 = 1$
$4 - 2 = 2$
$4 - 1 = 3$
$4 - 0 = 4$
$4 - (-1) = \ldots$
$4 - (-2) = \ldots$
$4 - (-3) = \ldots$

Based on these patterns

$4 - (-1) = 5$ equivalent to $4 + 1 = 5$

$4 - (-2) = 6$ equivalent to $4 + 2 = 6$

$4 - (-3) = 7$ equivalent to $4 + 3 = 7$

Means "subtract by some number equivalent to add with opponent of its number"

3.2 Pseudo Construction Concept of Algebraic Form

When students were faced with the problem 2x + 3x = 5x, students answered correctly but when explored further, the students' constructions were apparent (i.e. Pseudo construction). Students constructed the variables x and y is not as numbers, but as "objects" (i.e. books or apple or other). Summation 2x + 3x = 5x is assimilated with 2 books plus 3 books equals 5 books. 2x + 3y is assimilated with 3 plates + 2 spoons. Because the objects were different object, they could not be summed. Their construction was not based on the mathematical concepts. Table 5 shows that students who were pseudo thinking reached 63% (i.e. 167 out of 264 students gave true answers).

Table 5. Summary of Students' Reasons about the Problem $2x + 3x = 5x$

Statement	Reasons	True			False	Consistence	
		True	Pseudo	Clarification		Yes	No
$2x + 3x = 5x$	Let x = book, then two books plus three books equals to five books	0	64	12	15	85	6
	Because they have same variables, then it could be summed	52	12	0	27	85	6
	Because they have the same coefficients, then it could be union	0	34	5	12	44	7
	Because 2x + 3x = 5x equal to 2 + 3 = 5	3	47	7	21	72	6
	Repeat the problem	0	10	4	14	25	3
	No reason	0	0	14	0	9	5
	$2x + 3x = (5x)^2$	0	0	0	21	19	2
	$2x + 3x = 5x^2$	0	0	0	17	16	1
Total		55	167	42	127	355	36

The following text represented the argument of students related to clarification of their answers.

P2: *True. Because both contain x, so x can be summed. If both the form of the book entirely into two books plus 3 books that is equal to 5 books*

P3: *True. Because both variables are equal. So it can be summed. If for example x that book means 2 books plus 3 books so there are 5 books*

P4: *Right, so we suppose that the object x, such that the 2 apples plus 3 equals 5 apples*

In the algebraic form, variable x is not declared as an object but expressed as a number, so that the representation 2x + 3x can be operated as x expressed as numbers. If x numbers, then there is a distributive properties which ensure that 2x + 3x = (2 + 3) x = 5x. If x represents the object, then no one can guarantee the nature of the operation can be performed. Because the context variable in the algebraic form is a number, the variable x will be true if replaced with the price of the book or price of apple not as an object of books or apple, because the price is a number.

The subjects also experienced pseudo construction when faced with the statement 2x + 3y = 5xy. The subject stated that the statement 2x + 3y = 5xy is wrong, but when explored further, their reasons were not appropriate.

P2: *Not true, because 2x and 3y have different variables. Suppose we have two dishes plus 3 tablespoons, it is true that the number of objects is five but the plate and spoon cannot be summed together*

P3: *I am not sure the answer is correct Mom, because I think that's variable x equals to y so results remain xy and 2 plus 3 equals 5, so it would 5xy*

Subject P2 constructed the variables x and y is not as numbers, but as objects. P2 gave a reason "cannot be summed of 2x and 3y" not because of the properties of numbers in mathematical operations, but because they are different objects. In this case, the apparent construction occurred in P2 when building knowledge of algebra operations. The process of pseudo construction of students in algebraic form operations can be described in the following cognitive maps.

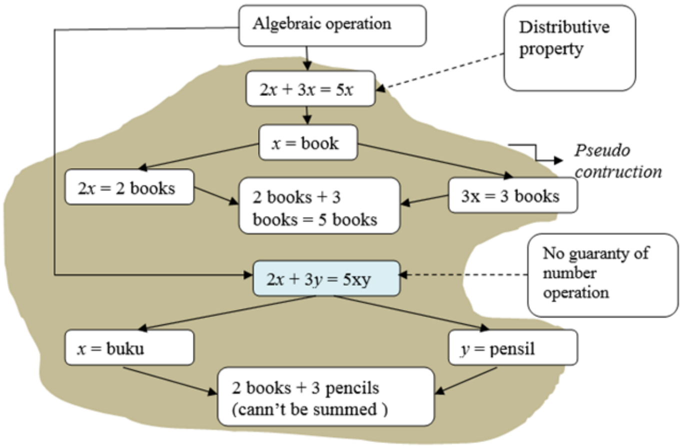


Figure 4. Cognitive map of pseudo construction of algebraic form operation

The properties of the algebra operations derived from the properties of number operations, such as addition, subtraction, multiplication, division, and root or exponent. Statement 2x + 3y actually states the representation of a number to the value of x and y, and 2x + 3y representation as a number is not owned by the student. So, they constructed 2x + 3y as a set of objects, then a mistake occurred when two books and three pencils plus, as they collected two books and three pencils and there were five objects such as pencils and books.

Fault of representation of algebraic form may inhibit the transition from arithmetic to algebraic thinking (Elizabeth, 2003; Trygve and Barbro, 2006). Elizabeth (2003) emphasized the importance of the mathematical properties: associative, commutative, and distributive in the transition from arithmetic to algebraic thinking. When students represent the variables x and y as an object, it cannot use the mathematical properties and this could hamper subsequent learning algebra. Therefore, the representation 2x + 3y as numbers are very essential in learning algebra. Trygve and Barbro (2006) asserted that in the last decade there was a shift from behavioristic perspective to a thorough analysis of the cognitive competencies that are involved in learning algebra. Tracing the thinking of students in constructing algebraic concepts is very important in order to know the mistakes and make efforts to repair. The importance of student math error/mistake correction was also confirmed by Shein (2012). In learning mathematics, the construction process of the math concepts students need to be constantly monitored in order to know immediately if there is a mistake. Assessing pseudo thinking of students in constructing mathematical concepts as steps can be used to explore students' thinking mistakes.

3.3 Pseudo Construction Concept of Area Concept

Pseudo construction also occurs when students constructed concept of an area. Students could calculate area and could write unit extent by m^2, but the process of constructing was pseudo. The concept of area has not been constructed, yet only the procedures were successfully constructed. It was characterized by a statement of the student m^2 unit area resulting from m x m instead of the unit square with sides of 1 m. The research instrument asked students to "assess the following statement"

"A rectangle with a size of 6 m x 5 m. The area of the rectangle is 30 m^2. A unit m^2 is derived from m x m"

Table 6. Summary of students' reasons about the area problem

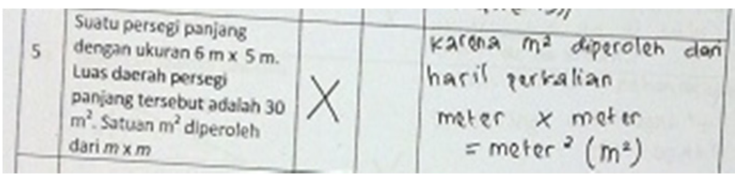
Statement	Reasons	True			False	Consistence	
		True	Pseudo	Clarification		Yes	No
A rectangle with a size of 6 *m* x 5 *m*. The area of the rectangle is 30 m^2. A unit m^2 is derived from *m* x *m*	L = p x l = 6 *m* x 5 *m* = 6 x 5 *m*x*m* = 30 m^2	0	198	4	0	85	117
	Because unit area ism^2	31	23	5	12	23	48
	Because, if unit times unit then the result is unit square	0	42	5	9	37	19
	Unit $m^2$is obtained from square of number	0	0	0	4	4	0
	Because*m* x *m* = $m^2$and$m^2$is unit area	12	23	9	9	12	5
	There are two *m*, so to be m^2	0	0	0	5	14	1
Total		43	286	23	39	175	190

Table 6 shows that students who were pseudo thinking reached 81% (i.e. 286 out of 352 students gave true answers). When the researchers examined them further, it turned out that the students thought that multiplication m x m was assimilated with multiplication in the algebra i.e. $a\ x\ a = a^2$. Students did not think about the concept of area but thinking about algebra as shown in the following statement.

S3: *True. Because the formula of rectangle area is length times width. Yes, then we multiplied the meter times meter, becoming meter squared. Yes like that. This is the same as in algebra. If a multiplied by a will produce a squared, multiplied by the same thing m times m to be m squared*

The students constructed the area by assimilating the multiplication of two numbers and unit area obtained from the multiplication between the units.

S3: *True. The area of rectangle is width times length 6m × 5m = 30m^2, 6 × 5 = 30 and m × m = m^2*



Translated version of the reason: Because m^2 is obtained from multiplication of meter x meter = $meter^2$ (m^2)

Figure 5. Subjects jugdement and reason of area problem

They constructed the concept of a rectangular area by a pseudo manner. From the responses, the students were able to calculate the area and could write unit by m^2, but the process of constructing was still a pseudo. Their constructed concept of area was not true area concept. Pseudo thinking processes of students in constructing the concept of the area can be described through a cognitive map in Figure 5.

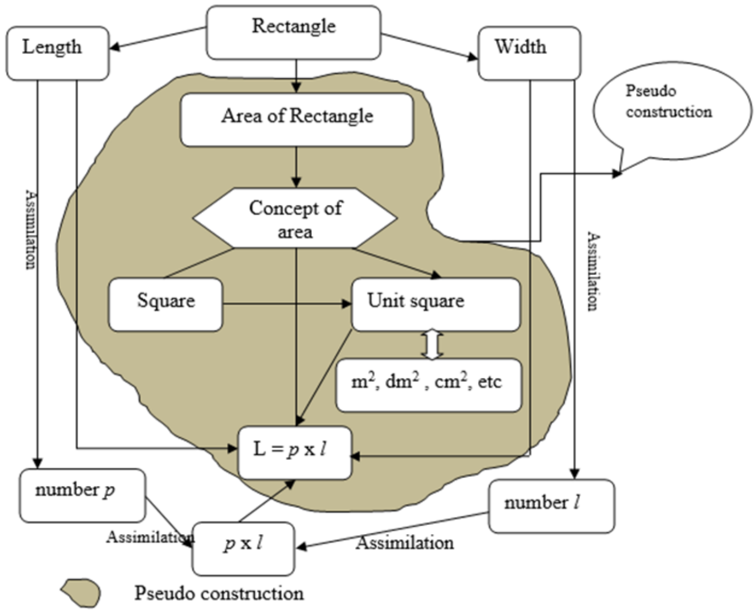


Figure 6. Cognitive map of pseudo construction of area concept

The pseudo process experienced by students in constructing the area concept is classified as a true pseudo thinking (Vinner, 1997; Subanji, 2011, 2015). Students "seem properly construct" the concept of the area, but when explored in depth, they constructed the wrong thing.

3.4 Pseudo Construction Concept of Triangle Concept

The following are details of the reasons given by students when assessing the statements of the triangle.

Table 7. Summary of students' reasons about the triangle problem

Statement	Reasons	True			False	Consistence	
		True	Pseudo	Clarification		Yes	No
There is a triangle with sides length is 6 cm, 7 cm, and 14 cm	Because there are 3 sides, so it can be made a triangle	0	64	3	43	102	8
	Because, does not match with Pythagorean theorem ($6^2+7^2\neq14^2$)	0	51	4	47	85	17
	Because the contiguous sides are equal	0	6	5	12	23	0
	Because 6 cm, 7 cm, and 14 cm are sides of right triangle	0	23	7	11	37	4
	Because it does not fit with right triangle	0	10	4	14	25	3
	Because 6, 7, and 14 are not triangle property	54	6	12	8	9	5
	No reason	0	2	0	6	14	1
Total		54	162	35	141	295	38

Table 7 shows that students who were pseudo thinking reached 65% (i.e. 162 out of 251 students gave true answers). The construction pseudo was observed in students from the absence of triangle requirement. In the concept of triangle, there is a requirement that sum of any two sides length should be greater than length of another side. When students were faced with the statement “there is a triangle by sides measure 6 cm, 7 cm, and 14 cm”, the students considered that the statement was “true”. Students did not pay attention to that 6 + 7 = 13 < 14, this condition was certainly not compatible with the requirements of triangle. Students did not know or did not pay attention to these conditions and immediately concluded that the triangle could be made, because there were three sides.

Translated version of the reason: Because, there is no information about triangle in the statement; right triangle or arbitrary

Figure 7. Subjects jugdement and reason of triangle problem

Another student’s judgement was based his statement on the Pythagorean Theorem. The student’s statement “there is a triangle by size 6 cm, 7 cm, and 14 cm” was false (not triangle) because it does not meet the Pythagorean theorem.

Translated version of the reason: Because, it does not fulfill the Pythagoras theorem

Figure 8. Subjects’ judgments and reasons on the triangle problem

The student correctly answered the given statement. But the reason given was not right. He gave a reason that it was not a triangle, not because they did not meet the requirements of the triangle, but because it did not meet the Pythagorean triple. Errors construction process of students in the concept of a triangle can be described in a cognitive map in Figure 9.

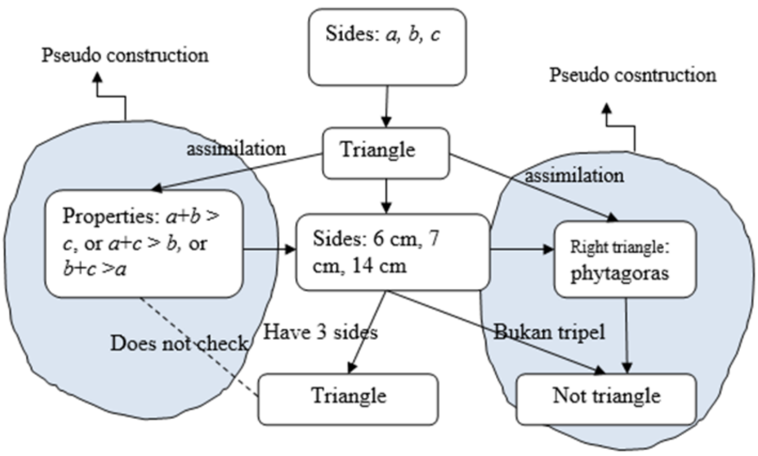


Figure 9. Cognitive map of pseudo construction of triangle

The construction process of students in understanding the triangle is affected by the grouping procedure of right triangle. Students used to check the triangle by checking the size of the three sides meeting the size of the right triangle. The habit of checking the right triangle directly used in this process of assimilation is "checking" with the Pythagorean concept.

4. Conclusion

The process of students' pseudo thinking in constructing mathematical concepts is necessary to get attention. Teachers need to be aware whether or not the students really understand the concepts being taught. Teachers also need to think about how the concept that was presented to the students was strengthened with a variety of exercises, as well as reconsidering the material prerequisites prior to new concepts. As is known, the operation on integers and operations in the algebra is a basic concept in mathematics in secondary schools, which underlies the "almost" all the advanced mathematical concepts. Pseudo construction experienced by students on the concept of integer operations and algebraic forms would impede the process of studying advanced mathematics concepts. This pseudo construction will certainly bring difficulties to hinder the process of learning mathematics and subsequent construction of mathematical concepts. Students' mistakes in constructing the concept of subtraction operation integer that is dominated by the process of assimilation "in debt" as well as for negative numbers would complicate the learning of addition and subtraction of negative numbers. The mistakes in constructing the variables x and y as objects (books, apple, and so on) will complicate the learning about multiplication and rank of the algebra. 2x + 3x assimilated with 2 books + 3 books still make sense, but when faced with the problem of 2x2 + 3x2 will not fit anymore, because there is no concept of "book squared". Mistakes in constructing the concept of area and triangle lead to learning mathematics meaningless

Acknowledgements

This study was funded by the Indonesia Ministry of Education project Tim Hibah Pascasarjana, DIPA UM, No: SP.DIPA-023.04.2.415009/2014.

References

Bingolbali, E., Akkoç, H., Ozmantar, M. F., & Demri, S. (2011). Pre-service and in-service teachers' views of the sources of students' mathematical difficulties. *International Electronic Journal of Mathematics Education*, *6*(1), 40-59.

Brodie, K. (2010). *Teaching Mathematical Reasoning in Secondary School Classrooms*. Springer: New York Dordrecht Heidelberg London. http://dx.doi.org/10.1007/978-0-387-09742-8

Elizabeth, W. (2003). The Role of Arithmetic Structure in the Transition from Arithmetic to Algebra. *Mathematics Education Research Journal*, *15*(2), 122-137. http://dx.doi.org/10.1007/BF03217374

Gal, H., & Linchevski, L. (2010). To see or not to see: analyzing difficulties in geometry from the perspective of visual perception. *Education Study Math*, *74*,163-183. http://dx.doi.org/10.1007/s10649-010-9232-y

Huitt, W., & Hummel, J. (2003). *Piaget's theory of cognitive development Educational Psychology Interactive*. Valdosta, GA: Valdosta State University. Retrieved December 15, 2015, from http://www.edpsycinteractive.org/topics/cognition/piaget.html

Jacobs, F. L. (2003). The Evolution of the Cognitive Maps. *Brain Behavior Evol*, *62*, 128-139. http://dx.doi.org/10.1159/000072443

Leron, U., & Hazzan, O. (2009). Intuitive vs analytical thinking: four perspectives. *Educ. Stud Math*, *71*(3), 263-278. ME 2009f.00534. http://dx.doi.org/10.1007/s10649-008-9175-8

Lithner, J. (2000). Mathematical Reasoning in Task Solving. *Educ. Stud. Math.*, *41*(2), 165-190. http://dx.doi.org/10.1023/A:1003956417456

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 International Mathematics Report: Finding From IEA'S Trend in International Mathematics and Science Study at the Fourth and Eighth Grades*. United States: International Association for the Evaluation of Educational Achievement.

Pape, S. J. (2004). Middle school children's problem-solving behaviour: A cognitive analysis from a reading comprehension perspective. *Journal for Research in Mathematics Education*, *35*, 187-219. http://dx.doi.org/10.2307/30034912

Pena, S, & Gutierres. (2007). Cognitive Map: An Overview and Their Application for Student Modeling. *Computacion Sistemas*, *10*(3).

Perdikaris. (2012). Using the Cognitive Styles to Explain an Anomaly in the Hierarchy of the van Hiele Levels.

Journal of Mathematical Sciences & Mathematics Education, *6*(2).

Shein. (2012). Seeing with Two Eyes: A Teacher's Use f Gestures in Questioning and Revoicing to Engage English Language Learner in Reapir of Mathematical Errors. *Journal for Research in Mathematics Education*, *43*(2).

Subanji, & Nusantara, T. (2013). Karakterisasi Kesalahan Berpikir Siswa dalam Mengonstruksi Konsep Matematika. *Jurnal Ilmu Pendidikan (JIP)*, *19*(2). 208-217.

Subanji, & Supratman. (2015). The Pseudo-Covariational Reasoning Thought Processes in Constructing Graph Function of Reversible Event Dynamics Based on Assimilation and Accommodation Frameworks. *J. Korean Soc. Math. Educ.*, *19*(1), 55-73. http://dx.doi.org/10.7468/jksmed.2015.19.1.000

Subanji. (2007). *Proses Berpikir Pseudo Penalaran Kovariasional Mahasiswa dalam Mengonstruksi Grafik Fungsi Kejadian Dinamik* (Unpublished doctoral dissertation). UNESA Surabaya.

Subanji. (2013). Proses berpikir pseudo siswa dalam menyelesaikan masalah proporsi. *J-TEQIP Jurnal Peningkatan Kualitas Guru*. Tahun IV. Nomor 2. November 2013

Trygve, B., & Barbro, G. (2006). The Transition from Arithmetic to Algebra: To Reason, Explain, Argue, Generalize and Justify. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, *2*, 225-232.

Vinner, S. (1997). The pseudo-conceptual and the pseudo-analytical thought processes in mathematics Learning. *Educ. Stud. Math.*, *34*(2), 97-129. http://dx.doi.org/10.1023/A:1002998529016