Students' Errors in Solving the Permutation and Combination Problems Based on Problem Solving Steps of Polya

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Abstract

This article was written based on the results of a study evaluating students' errors in problem solving of permutation and combination in terms of problem solving steps according to Polya. Twenty-five students were asked to do four problems related to permutation and combination. The research results showed that the students still did a mistake in understanding the problems related to permutation and combination, the students still made a mistake in planning problem solving related to permutation and combination, and the students still had a mistake in rechecking the problems given. Besides, there were some students who did the combination problem by using the permutation problem.

Keywords: students' errors, problem solving, permutation and combination

1. Introduction

In studying permutation and combination, students need real-world problems in the form of more relevant one to stimulate learning and retaining knowledge (Busadee & Laosinchai. 2013). Garfiel and Ahlgren (1988) reveal that the permutation and combination are the important parts of statistics introductory course in some universities. Besides, the permutation and combination are the materials that become the basis in learning discrete mathematics topics. It is reflected in the book on discrete mathematics written by Susanna (2004), Rosen (2004), and Rosen (2012) that always contains the material about permutation and combination. It shows that the materials of permutation and combination are still needed very much at the college level and the problems in the form of the real world are also needed so that students can maintain their knowledge.

Some researchers had conducted researches related to the matter of permutation and combination, including Abrahamson (2008) that gave problems to the students related to how many ways to arrange letters HTHT. The students actually only counted the letter H and T. the students did not relate to the concept of permutation or combination. Aisya et al. (2014) says that knowing the sequence of the combination of an event is an important step to solve problems involving opportunities. Other researchers, Lockwood and Gibson (2014) gave the task to the students to find the number of words that could be arranged from the word CATTLE. Garfiel and Ahlgren (1988) reveal that permutation and combination were difficult matters and a prerequisite to learn the advanced mathematics, especially those which related to the material opportunity and statistics.

The concept of permutation and combination underlie topic of combinatorial analysis, so that the concept of permutation and combination are the important concepts in mathematics (Abrahamson & Cendak, 2006). In addition to the field of combinatorics, permutation and combination are also useful in other fields, for example Wraughton and Nolan (2012) discusses the game of poker as an activity of permutation and probabilities. Garfiel and Ahlgren (1988) reveal that the permutation and combination were the important parts of the statistics introductory course in some universities. In fact, students still make a mistake in solving the problem of permutation and combination. Meanwhile, according to Lockwood et al. (2013) permutation problem that was stated in the context provides additional wealth of students' mathematical thinking, but the problem was very difficult for students.

Students' abilities to solve problems in the forms of words problems were still weak. Problems of permutation and combination presented in the form of context are usually in the word problem forms. To solve the words problems, it needs problem solving capabilities appropriate to the context (Salman, 2002). In the process of

problem solving, critical thinking is necessary, that is after students understand the problems, and they make plans to solve them and in such planning, bright ideas are needed in order to be able to find the solutions effectively and accurately. The bright ideas may be obtained if critical thinking is always employed in viewing every problem, where creative thinking is gained through thinking (In'am, 2014). Problem solving ability, according to Polya (1973: xvi), was identified as the ability to (1) understanding the problem, (2) devising plan, (3) carrying out the plan and (4) looking back. Therefore, this research will identify the students' errors in solving the problems of permutation and combination through the stages of problem solving according to Polya as described above.

2. Methodology

This research was conducted in one class at Department of Mathematics Universitas Negeri Malang for 25 students pre-service training. Participants were voluntarily and agreed to engage in research. The student in this research include 8 male (32%) and 17 female (68%). This research was conducted by giving 4 permutation and combination problems in the form of words problems. The four problems consisted of two questions about the permutation and two questions about combination. The implementation was held on February 12th, 2014.

Polya (1973, xvi) set out four steps that could be done to make students focus more on solving mathematical problems, namely the ability to (1) understanding the problem, (2) devising plan, (3) carrying out the plan and (4) looking back. In'am (2014) reveal that understanding the problem was an activity that should be done before making activities of problem solving, devising plan was to make a direction to plan appropriate strategies to solve the problem, carrying out the plan was implementing the problem solving in line with the chosen approach, strategy and model and looking back was an effort that should be made in solving a problem to review the obtained answers. Based on the purpose of this study, the results of student works were identified based on the types of mistakes in problem solving of permutation and combination using the troubleshooting steps according to Polya.

The data were collected by correcting the students works in doing permutation and combination problems. To correct the students' works, the researcher used rubric that identified the students' errors in solving the problems of permutation and combination.

Instruments validated by two lecturers who are experts in their fields. The instruments is used to identify the students' errors in problem solving of permutation and combination consisting of 4 about the words problems were as follows; 1) In one organization containing four members, namely A, B, C and D, it would be elected a chairman and vice-chairman. How many alternative arrangements of the chairman and vice chairman could be selected? 2) There were 4 persons in a family. In the family, 3 people were chosen to have an interview. How many ways of the family members could be selected to have an interview? 3) In a meeting, there were 4 people who did not know each other. To get to know each other then they shook hands. How many handshakes did happen? 4) How many words were formed from the word "SUSU"? The problems number 1 and 4 dealt with the problems of permutation while number 2 and 3 related to the problems of combination.

3. Findings and Discussion

The percentage of students who made mistakes in problems solving of permutation and combination based troubleshooting steps according to Polya can be identified by the following Table 1.

Table1. The percentage of students who answer the problems wrongly those are appropriate to the troubleshooting steps according to Polya

Students' Mistakes	Percentage (%) of Students per Item				Average (%)
	1	2	3	4	
Mistakes in understanding the problem	52	52	56	44	51
Mistakes in making plans	56	60	60	52	57
Mistakes in implementing the plan	56	60	6	60	60
Mistakes in noticing the problems given	64	60	64	60	62

Based on Table 1 it is obtained a description that the biggest mistake of students is a mistake in looking back at

the given problem, this happens because when students made mistakes in understanding the problem then automatically student made a mistake in making the decision and also made a mistake in applying and looking back on a given problem.

The findings of this research indicated that there were students who had a difficulty in solving the problem of permutation and combination. The findings of this research supported the opinion of Lockwood, et al. (2013) who said that the problem of permutation and combinations that were expressed in terms of the context provided additional wealth of mathematical thinking of students, but the problem was very difficult for students. Difficulties experienced by students could not be separated from the fact that the topic permutation and combination are always associated with the real context. Thus, if the students were given the problems of permutation or combination in their real context, they were confused to solve the problems; whether the problem was solved by permutation or combination.

The percentage of students misunderstanding in permutation problems (question no 1 and 4) was 48% and the percentage of students' errors in understanding the combination problems (question no 2 and 3) was 54%. Comparison of students' errors in understanding the problems of combination was higher than the students' errors in understanding the problems of permutation. If related to the implementation of permutation and combination done by the teacher, learning was usually begun by discussing the permutation then combination problems. Because the topic of permutation and combination were close, when a student did a combination problem, they were confused with the problem of permutation, so there were combination problems done by using permutation for example when the student did the problem: in a meeting, there were 4 people who did not know each other. To make them know each other then they shook hands. How many handshakes did happen?. The students did such combination problem using the concept of permutation as follows.

Translate in English

n = 4

$^4P_2 = \frac{4!}{2!} = \frac{4.3.2!}{2!} = 12$

So, there are 12 handshakes that happen.

It showed that the concept of permutation give affect the concept of combination. According to Makovski and Jiang (2008) was called proactive interference. Meanwhile, according to R. Stemberg and K. Stemberg (2012) proactive interference was old information disturbing recalling of the new information.

On the whole, there were 57% students who were still wrong in making plans on permutation and combination problems. One example of a student who did an error in making plan of problem solving No. 3 was "In a meeting, there are 4 people who do not know each other. To make them know each other then they shake hands. How many handshakes can happen". This is the result of the students' work of in solving the problem.

Translate in English

to choose 3 people in a group

$$^nC_r = \frac{n!}{(n-r)!}$$
$$^4C_3 = \frac{4!}{(4-3)!} = \frac{4 \times 3 \times 2 \times 1}{1!} = 24 \text{ ways.}$$

Based on the student's work, it showed that the student had understood that the existing problem was that which related to combination. It was reflected in the formula that would be used to solve the problems namely ^nC_r. However, when planning the problem solving, the student made a mistake in determining the combination formula. He used permutation formula. It showed that the student, in thinking the problem solving, only used one system that was not supported by the system two of the dual process theory. The finding of this research supported the research conducted by Kahneman (2002) which gave the characteristics of information processing model called the dual-process consisting of system 1 (S1) and system 2 (S2).

Students' error in implementing the plan of permutation and combination problem solving was still relatively high because there were 60% students who still made a mistake in implementing the problem solving plan. One example of students' error in solving problem No. 3, namely "In a meeting, there are 4 people who do not know each other. So that they know each other then they shake hands. How many handshakes do happen?". This is the result of the students' work in solving the problem.

Translate in English

$$_4C_2 . 2 = \frac{4!}{2!2!} . 2 \text{ (because the handshake is done by 2 people)}$$
$$= 6. 2$$
$$= 12 \text{ ways.}$$

Based on the result of the work, it seemed that the student had already understood the problem and was able to plan problems well. However, when he did the problem solving, he was distracted by a real context that there were two men shaking hands, so it was multiplied by two. It showed that if the student did not master the topic well and the problem was stated in the form of context, so the student found the difficulty. The finding of this research supported the opinion of Lockwood et al. (2013) who said that the problem of permutation and combination stated in the form of context provided additional wealth of mathematical thinking of students, but the problem was very difficult for students.

Students' error in looking back problems given was in the highest rank of the troubleshooting steps according to Polya, namely 62%. This difficulty, according to Lee et al. (2007) was because the students tended to do the problems in the form of symbols and formulas so when they were given the problems in the form of context,

they tended to do in the form of symbols or formulas, and never returned the problem in the form of context. Here's one example of the work of students in doing problem no 4 "How many" words "are formed from the word 'susu'?"

Translate in English

SUSU

The number of words formed from the word SUSU is $\frac{4!}{2!2!}$ = 6 ways.

Based on the result of the student's work, it could be seen that the student had understood the problem, had been able to devise a plan, and had been able to implement the problem solving, but the students did not look back the problem given. The given problem was to look for the number of words, but the answer of the student still used the formula. It showed that the student still tended not to recheck the problem already solved.

Based on all things mentioned above, it could be seen that the Students' errors in solving the problems of permutation and combination in terms of troubleshooting steps according to Polya was due to the lack of students 'ability in understanding the context problems related to permutation and combination. Students tended to be ambiguous on these two topics. The problem of combination was considered as the problem of permutation or vice versa. Therefore, a lecturer, in implementing the permutation and combination should always combine these two topics, do not convey the material separately because the two topics were close, and teachers may always integrate teaching of the permutation and combination in real context.

4. Conclusions

The ability of students to understand the problems of permutation and combination is low. It is characterized by more than half of students made mistakes in understanding the problems of permutation and combination. The ability of students in making a plan of the problems solving of permutation and combination is low. In addition, although the students had understood the problem, but they found a difficulty in determining what the formula should be used in planning the problem solving. The ability of students in implementing the problem solving plan is low. Although the students had already understood the problem and were able to arrange the problem solving, but the students found a difficulty when implementing the problem solving related to the real contexts. The ability of students to look back the problems given is low. Students also had a difficulty when they were given the problems in the form of a real context related to permutation and combination. The tendency of students always was solving the problems in the form of symbols and it was considered having been solved the problems.

There were problems related to permutation that were solved by the students using combination. It showed that the students' understanding in the material of permutation give affect the students' understanding in the combination material. The distraction was happen because the topics of permutation and combination were close, so the permutation and combination problems that were presented in the form of a real context made the students confused to solve them.

Because the topics of permutation and combination are close, a lecturer, when teaching these topics should implement them in integrated way, not separated. It is done to provide the stabilization of the permutation and combination concepts as a whole and in integrated way, so the students can distinguish the concept of permutation and combination simultaneously.

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