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Geometry Students' Self-Determination and Their Engagement in Mathematical Whole Class Discussion

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Abstract

Prevalent characteristics of effective mathematical discourse in the literature include student autonomy, a positive social environment, and a degree of perceived mathematical competence. These characteristics are also found in Self-Determination Theory (SDT), which posits that actions are self-regulated when students' autonomy, competence, and social relatedness are fulfilled. Utilizing SDT as an additional lens for examining student engagement in whole class discussions, this study examined high school students' observed discourse actions and mathematical talk in the context of their self-reported psychological needs of autonomy, competence and relatedness. Results suggest that students' perceived needs correspond with their engagement in making mathematical statements in the context of classroom mathematical discussions. However, students who lack in one or more need can be supported by participation of peers with more fulfilled needs.

Introduction

Mathematical discussion has been characterized as "an essential part of mathematics and mathematics education" (NCTM, 2000, p. 60), which engages students in reflection about mathematics and encourages them to justify their solution strategies (D'Ambrosio, Johnson, & Hobbs, 1995; NCTM, 2000). Student reflection and engagement in the mathematics classroom as a result of discussion has been documented by several qualitative studies (Kazemi & Stipek, 2001; Yackel & Cobb, 1996), as well as some quantitative studies (Gadanidas, Kotsopoulos, & Guembel, 2006; Hiebert, Stigler, Jacobs, Givven, Garnier, Smith, Hollingsworth, Manaster, Wearne, & Gallimore, 2005). Such studies also note that the classroom context influences how students engage in mathematical discussion (e.g., Jansen, 2008; Kitchen, 2004).

Certain characteristics of students and classrooms have been identified by mathematics education literature as facilitative of mathematical discussion, and students' engagement in it. One such factor is a feeling of student autonomy in the discussion process (Kosko, 2012; Kazemi & Stipek, 2001; Krummheuer, 2007). Another often articulated characteristic of effective discussion is a positive social atmosphere amongst students and between the students and teacher (Kazemi & Stipek, 2001; Wood, 1999). Additionally, perceived mathematical competence of students by teachers, peers or themselves interact with how students engage in mathematical discourse (Jansen, 2006; Kotsopoulos, 2008). Taken altogether, these three aspects of effective discussion comprise elements of Self-Determination Theory (SDT).

SDT is a theory of motivation that argues the fulfillment of three innate individual needs allows for the internalization of content and actions (Ryan & Deci, 2000). These needs, autonomy, social relatedness, and competence, seem to align theoretically with characteristics of an effective mathematical discourse setting as outlined by mathematics education literature, and recent findings suggest that perceived fulfillment of these needs relates to students' reported quality engagement in mathematical discussion (Kosko & Wilkins, 2012; Kosko & Wilkins, 2015), as well as aspects of their observed engagement (Kosko, 2012).Such studies provide empirical evidence of a connection, but there is a need to explore in more depth how SDT, as a theoretical framework, explains the nature of students' engagement in mathematical discussions. Thus, it is the primary purpose of this study to examine students' engagement in mathematical discourse by examining how their discourse actions relate with their self-reported needs of mathematical autonomy, mathematical competence, and social relatedness.

Self-Determination Theory and Mathematical Discussion

SDT is a motivation theory that examines an individual's actions in regards to the level of self-regulation it entails (Deci, Vallerand, Pelletier, & Ryan, 1991; Reeve, 2006). According to Ryan and Deci (2000) the development of self-regulated dispositions is facilitated by three interrelated psychological needs: autonomy, relatedness, and competence. Autonomy describes the degree to which the self is in control of initiating and maintaining different behaviors. Relatedness describes the degree to which satisfying social connections are made. Competence describes the degree to which the individual feels able to accomplish different external and internal tasks (Deci et al., 1991; Ryan & Deci, 2000). Additionally, various authors (Deci et al., 1991; Katz & Assor, 2007; Reeve, 2006) characterize each of the three constructs as being interdependent on the other to the point that all three needs must be met in order to facilitate the development of self-regulated dispositions (Deci et al., 1991; Ryan & Deci, 2000). While a cursory glance at SDT would indicate a simple identification of psychological factors that facilitate motivation, the SDT literature contains references to contextual influences in education, such as a teacher's ability to foster and influence the development of students' autonomy, competence, and relatedness (Deci et al., 1991; Ryan & Deci, 2000). Therefore, SDT is concerned with both individual and contextual aspects of student motivation.

The benefit of using SDT as a lens for explaining student engagement in mathematical discussions lay primarily in the notion that each of the three needs posited by SDT (autonomy, competence, relatedness) are innate, or naturally occurring, within an individual but simultaneously are malleable by the context the individual is nested within. To better explain how SDT can be used in examining student engagement in mathematical discussion, I provide a series of examples from the literature on mathematical discussion. With few exceptions, such studies tend not to explicitly incorporate SDT as a means for examining mathematical discussion. Yet, the language used in these studies, and the findings articulated by their authors, suggest a conceptual link between SDT and student engagement in mathematical discussion.

Various studies in mathematics education literature have focused on discourse as "the production of social actors" (Ryve, 2011, p. 171), or in other words, as a means of forming identities in reference to social contexts (e.g., Bartolini Bussi, 1996; Herbel-Eisenmann & Wagner, 2010; Hufferd-Ackles, Fuson, & Sherin, 2004; Lo, Wheatly, & Smith, 1994; Yackel & Cobb, 1996). One of the most prevalent characteristics in the literature that focuses both on formation of identity and influential contextual factors is the notion of students' autonomy in mathematical discourse (Kosko, 2012; Kosko & Wilkins, 2015; Krummheuer, 2007; Yackel & Cobb, 1996), otherwise described in terms of authority (Goos, 2002; Herbel-Eisenmann & Wagner, 2010) discursive role (Tatsis & Koleza, 2004), or various other derivations. Autonomy has been described as both being influential to and influenced by mathematical discourse (Hufferd-Ackles et al., 2004; Lo et al., 1994). For example, Hufferd-Ackles et al. (2004) found that an essential element in the evolution of the discourse community in a third grade classroom was the role both the teacher and students took in the discourse. As the teacher ceded the role of determiner of discursive exchanges, she scaffolded the students' development of such roles. As a result, students became more engaged in and reflective of the mathematics they focused on. Kazemi and Stipek (2001) analyzed the same lesson as taught by four different fourth and fifth grade teachers. While each class appeared to have similar levels of discussion and a positive social environment for students to learn in, Kazemi and Stipek (2001) found that teachers who emphasized a "high press for learning," which included support for student autonomy, focused more on asking students why certain math procedures are conducted while teachers of" low press" classrooms focused more on how the procedures are done. Observing whole class discussions in Australia, Goos (1995) noticed that "by ceding control of the debate the teacher provided [an] opportunity for students to ask for, and receive, explanations from each other until they were satisfied that they understood" (p. 16). Kosko (2012) observed high school geometry students' hedging in class discussions and found that the relative frequency of their hedging correlated with higher self-perceived mathematical autonomy. Therefore, it is evident from these and other examples from the literature that students' mathematical autonomy is a facilitative factor in the nature of their engagement in discussions. What is less evident is how the fulfillment, or lack, of competence and relatedness interrelate with such engagement when autonomy is, or is not, fulfilled.

As with autonomy, various examples can be found in the literature showing that the degree to which social relatedness (Hoffman, 2004; Wood, 1999; Yackel & Cobb, 1996) and mathematical competence (Jansen, 2008; Kitchen, 2004; McGraw, 2002; Van der Aalsvoort, Harinck, & Gosse, 2006) are facilitated influences how students engage in mathematical discussions. Regarding social relatedness, Hoffman (2004) provides a useful example from her interviews with middle school students. Students who expressed fears of social penalties for speaking in their mathematics class were more likely to describe the need to obtain knowledge from their teacher and to reproduce it accordingly than students who did not express a fear of social penalties.

Finding somewhat similar actions of seventh-grade students, but in regards to their reported conceptions of their competence, Jansen (2008) observed students "mentioned wanting to participate only when they were certain that they were correct" (p. 416). In both of these studies is a tacit acknowledgement from the students towards some other authority (e.g., the teacher, other students, the discipline of mathematics). However, these connections are not elaborated upon in these or other similar studies of such phenomenon. Kosko & Wilkins (2012) provide some preliminary evidence of the interrelation between students' perceived autonomy, competence and social relatedness with their engagement in mathematical discussion. Examining survey responses regarding geometry students' perceived quality engagement in mathematical discussions, Kosko & Wilkins (2012) found that individual comparisons between student-reported mathematical autonomy, competence and relatedness each had statistically and meaningfully significant correlations with students' reported quality engagement in mathematical discussions. However, when using structural equation modeling to combine these factors into a single latent trait of 'self-regulation towards mathematics,' a stronger relationship was observed. While Kosko & Wilkins (2012) analysis is helpful in establishing that such an interrelationship may exist, there is a strong need to understand the nature of students' engagement in mathematical discussion when some needs are fulfilled and others are not. It is with these goals and considerations that I seek to answer the following research question:

> *How do students' psychological needs of autonomy, competence, and relatedness relate to their observed actions in whole class mathematical discussion?*

Methods

Participants and Context

Data include observations of 10 lessons from a high school geometry class in the Southeastern U.S. in fall 2008. The class teacher was described by administrators as one who regularly implemented mathematical discussion in their instruction. It was believed that this characterization increased the likelihood that variation of student engagement in mathematical discussion would be observed. Rather, to better understand how mathematics students' perceived autonomy, competence and social relatedness influence their engagement in mathematical discussions, there was a need to conduct observations of a teacher who facilitated discussions that were dialogic in nature, while acknowledging that univocal discourse would emerge from virtually any teacher that would be observed. In this regard, Lotman's (1988) depictions of univocal and dialogic functions of text are helpful. Univocal discourse is a discussion where the teacher is the dominant voice and student participation in the direction of the discussion is limited. Dialogic discourse is described as a discussion that includes both the teacher and students in orienting the direction of content within a discussion (Kitchen, 2004; Lotman, 1988). Given that dialogic discourse seems intuitively more supportive of mathematical autonomy, and univocal discourse seems likely to possess the potential for limiting it, finding a classroom context that would include both forms of discourse was essential for the purposes of this study.

The observed class included 20 students. The majority of these students were Caucasian (*n=16*), which is representative of the school and community population. There was also one African American student, one Asian, one Hispanic, and one of multicultural ethnicity. The class was evenly divided in terms of gender (*10 male, 10 female*). The participating teacher was a white, female, veteran teacher with 25 years of experience teaching both high school and middle school mathematics. She frequently vocalized her support for discussion and writing in the mathematics classroom and her use of both in her teaching. Additionally, the participating teacher regularly pursued positive relationships with her current and former students as a means of developing a positive social atmosphere.

Data Analysis and Collection

The present study used an embedded mixed methods research design (Creswell & Plano Clark, 2007). An embedded mixed methods design collects both quantitative and qualitative data within the same phase of study. Data is then analyzed separately using appropriate quantitative and qualitative methods, but findings from such analyses are used to inform each other. Depending on the particular design, one form of analysis may have a stronger role than another; qualitative analysis took precedent in the current design. Within the present study, I used an adapted version of the *Basic Psychological Needs* survey (Deci & Ryan, 2008) to assess students' perceived mathematical autonomy, mathematical competence, and relatedness with peers (see Table 1), which was validated in a previous investigation (Author & Colleague, 2012). Descriptive statistics for each student provided the quantitative component of this mixed methods research.

Table 1. *Descriptive Statistics for SDT Needs and Mathtalk*

	Range	Score
Autonomy	4.00 – 7.00	$\bar{x} = 5.11$ $SD = 0.85$
Competence	2.33 – 6.17	$\bar{x} = 4.54$ $SD = 1.05$
Relatedness	2.75 – 7.00	$\bar{x} = 5.52$ $SD = 1.18$

Note: $n = 20$; the available responses for items ranged from 1 to 7.

The qualitative component stems from a micro-ethnographic analysis of episodes of univocal and dialogic discourse. Micro-ethnography examines the minute by minute, action by action aspects of observational data to study interactions within dialogue (Streeck & Mehus, 2005). Because of the detailed analysis necessary for micro-ethnography, audiovisual recordings are used and reanalyzed until saturation of data has occurred. This saturation occurred in the present study when the actions and aspects within the recordings became common enough to be termed themes. A common practice in micro-ethnography is the selection of excerpts from transcripts so that a more detailed examination of subtle actions can be examined (Craig & Sanusi, 2003). Before episodes of discussion were selected in the present study, an analysis of all classroom observations was conducted to evaluate their overall discourse features. This included categorization of different discourse settings within each lesson, evaluating which students participated more actively in discussion and what types of mathematical talk were produced (e.g., procedural and/or conceptual comments). Each class contained episodes of both univocal and dialogic discourse that were very brief; a few lines of transcript in many cases. Of the observed class sessions and episodes analyzed, four episodes were selected for presentation: two from univocal discourse settings and two from dialogic discourse settings. Episodes were evaluated based on how well they were observed to fit with the definitions of the different discourse settings as presented in the literature. The role the teacher took in leading class discussions was a primary indicator of whether discourse was univocal or dialogic, with dialogic settings including the teacher's ceding of control in the course of discussions.

During analysis, special interest was paid to the type of talk the students and teacher produced as well as the roles the students and teacher played in the discourse. The two main types of mathematical talk of interest were procedural and conceptual. Given the focus on procedural and conceptual talk, spoken statements and physical gestures were the unit of analysis when analyzing episodes of discourse (Bloome et al., 2005; Streeck & Mehus, 2005). I used a synchronic analysis approach to examine students' gestures (as outlined by Arzarello, Paola, Robutti, & Sabena, 2009). Such an approach "considers the relationships among different semiotic resources simultaneously active by the subjects at a certain moment" (Arzarello et al., 2009, p. 100). Thus, gestures made by students were paired with spoken dialogue when examining video recordings and transcripts.

Individual scores obtained from the survey for perceived mathematical autonomy, competence, and relatedness were used in analysis of students' actions within a discourse setting. Specifically, once students' roles and actions in an episode were defined, I used scores from the survey data to help explain these roles and actions. As will become obvious from the results that follow, I found this approach particularly useful in explaining why students either failed to take on specific roles in a discourse setting and engaged or disengaged from the discussion at particular points. This latter facet of the analysis prompted the selection of the episodes shared here, as well as the representativeness of these particular episodes for various interactions in the class.

Results

Frequency and Quality of Mathematical Talk

Prior to examining episodes of univocal and dialogic discourse, a surface analysis was conducted of each observed lesson. Certain students were categorized as more generally engaged in discussion and others as less generally engaged. This classification of engagement was based on their observed participation in different discussions. Specifically, for each observation, the number and length of students' comments were identified for each student. Students were then divided into two groups relative to each other as more engaged and less engaged. While this classification is limited in that it assesses only observed comments, thus not accounting for less observable engagement such as mathematical listening (Kosko, 2014), it provides a useful surface-level metric for student participation within particular observations and across all 10 observations. An additionally indicator included the quality of contributions made by students. For example, certain students were observed to produce only answers, which I refer to as simplistic talk, while others would make procedural and/or conceptual statements. For the purposes of space, only the results from the second observation are presented here, and displayed in Table 2. Students who produced at least one procedural statement were categorized as producing procedural talk. A similar criterion was used for conceptual talk and simplistic talk. Students were assigned to only one talk category; the category they produced their most sophisticated statement in the class session. Such a distinction does not fully account for the quality or depth of various student contributions, but was not meant to do so. Rather, this process, repeated for each observation, was used as a preliminary indicator for how various students engaged within particular and across various observations. Thus, it provided generalized information in the more in-depth analysis of particular episodes within certain observations.

Table 2. *Trends in Frequency and Quality of Math Talk in Observation #2.*

	More Engaged	Less Engaged
Simplistic Talk	*n* = 0	*n* =8 *Autonomy = 4.69* *Competence = 4.17* *Relatedness = 5.99*
Procedural Talk	*n* = 4 *Autonomy = 5.14* *Competence = 4.33* *Relatedness n = 5.44*	*n* = 2 *Autonomy = 5.43* *Competence = 5.57* *Relatedness = 6.19*
Conceptual Talk	*n* = 5 *Autonomy = 5.51* *Competence = 5.03* *Relatedness = 5.33*	*n* = 0

Note: 1 student absent during observation.

Interestingly, the surface analysis revealed a general trend across various observed lessons which is illustrated within Table 2. Specifically, while individual students would vary to some degree in their relative engagement in discussions, most students were generally either in one classification or the other across most observations, and produced similar forms of statements.

Further, more engagement and more sophisticated talk was typically exemplified by students with higher autonomy and competence scores. This trend appeared across all observations.

Univocal Discourse Setting

The episodes selected from the univocal setting were taken from the third and fifth observations. Each episode was situated during portions of lessons where the teacher was the dominant voice and provided mostly procedural instructions. The discussions, up to that point, followed the familiar IRE (initiation-response-evaluation) pattern (Schleppegrell, 2007). The first episode, from the third observation, took place in a lesson on geometric constructions. The teacher was prompting students to determine how they would construct a 22.5 degree angle, given their current construction involving a perpendicular line. Because the task is of a lower cognitive demand[1], it does not press for meaning from the students (Kazemi & Stipek, 2001) and thus does nothing, on its own, to shift the discursive environment from univocal.

> **Teacher**: Okay, so I'm thinking that we need to construct a 22 ½ and a 22 ½ that share a common side, . Now, let's do some thinking. We're not going to use any protractors, so how am I going to get a 22 and ½
> **Liz**: Do one fourth.
> **Teacher**: Hmm?
> **Liz**: A fourth. One fourth of 90.
> **Teacher**: One fourth of 90 works beautifully. Cause half of 90 is what?
> **Liz**: 30.
> **Robert**: 45.
> **Teacher**: 45, and half of 45 is?
> **Greg**: Twenty two point five.
> **Teacher**: 22.5, okay. But I need to be up here. Cause if I use these two sides, my angles will go this way. So, guess what I need to do.
> **Liz**: Gonna have to draw it there.

As stated earlier, the focus of this conversation was on the procedures for constructing angles, and not on the concepts underlying their properties. Therefore, participants are predominately using procedural and simplistic talk, with Liz providing procedural responses "Do one fourth…Gonna have to draw it there," and both Robert and Greg providing answers-only. Interestingly, Liz had relatively high scores for autonomy (6.67), competence (7.00) and relatedness (5.67). While Robert and Greg both had comparable perceptions of social relatedness (5.17 & 5.33 respectively), their reported perceived autonomy (3.00 & 4.00 respectively) and competence (4.71 & 4.86 respectively) were lower. Recall a score of 4.00 is considered mid-range for

[1] The task corresponds to Stein, Grover, & Henningsen (1996) description of procedures without connections because it solicits a procedure without making connections to the underlying concepts at hand for the given class.

each of these variables, with the possible range being from 1.00 to 7.00 (see Table 1). Relating the quantitative data to the dialogue presented here, it appears that Liz's higher perceived autonomy and competence allowed her to provide a procedural comment in a univocal discourse setting, whereas both Robert and Greg maintained the expected IRE structure typical of such settings. While the teacher presents the task of constructing a 22.5 degree angle, Liz's hand went to the side of her head and her brow scrunched as she visually appeared to concentrate, preempting the teacher's comment "now, let's do some thinking," by a fraction of a second. Since this concentration appeared to have been prompted by the teacher's starting of the task, we can reasonably assume that Liz had taken it upon herself to solve this task. As soon as Liz heard the teacher's prompting question "how," Liz voiced the procedure which she had appeared to have come up with when presented with the mathematical task. By contrast, both Robert and Greg were working on their constructions during this point in the episode. Greg did scrunch his brow around the same time as Liz, but after looking briefly at his paper and twirling his pencil, he looked back toward the teacher. Greg's gesturing seemed to indicate his mental examination of the problem, but he appeared content to wait for the teacher's explanation, rather than initiate the development of his own.

While observations of all three students across various lessons showed that each was very competent, mathematically, Liz had much higher perceived competence. This may have contributed, along with her perceived autonomy, to her confidence in presenting a procedure that departed from the IRE process that was ongoing in the episode. By contrast, Robert and Greg's lower perceived autonomy and competence appears to correlate with their acceptance of the present IRE structure, providing a potential reason for why their actions tacitly endorsed the univocal structure that was in place. Further, this episode characterizes the nature of these students' actions in other univocal episodes, which also appears to relate to their reported quality of engagement in discussion.

The topic of the lesson in fifth observation was angle relations. The teacher had passed out a sheet with different figures that students were to work on together to find the values of missing angles. After students worked on the problems, the teacher began reviewing the problems with the class. The following episode occurred during this review, with the figure displayed on the overhead projector.

Teacher: I know that—what's this one right here? Cause that's a linear pair.
Mark: 68?
Teacher: 68 and 68. Okay.
Mark: 62 and 62.
Teacher: 62 and 62 here and what? 112?

Beth: Yeah 112, up (*pointing at a different angle on the projection*).

Mark: No that's 118 (*talking to the teacher*).

Mark: 112 is where it equals, the bold one (*both Mark and Beth point and direct the teacher*) and then you can figure the vertical.

Greg: You can figure out B and S.

The episode includes predominately simplistic statements, but is punctuated by brief procedural statements towards the teacher. As such, the teacher maintains the central role of director common in univocal settings. While Beth, Mark, and Greg did not describe procedures explicitly, they did direct the teacher through gesturing, head nods, and partial sentences to specify which angle should be looked at next. Mark had higher perceived autonomy (5.33), competence (4.71) and relatedness (5.00). Noticeably, he provided the majority of procedural statements and indicators within this episode. Greg, whose autonomy (4.00) was mid-range, also provided a procedural statement, but did so only after his peers had offered such directions. Beth also participated in a similar manner, mainly through gesturing and little vocal direction, following Mark's example. Beth had relatively low perceived autonomy (3.00) and relatedness (3.33), but a mid-range perceived competence score (4.43).

Similar to the previous described episode, a student with relatively higher perceived autonomy, competence, and relatedness provided procedural statements in what was otherwise an IRE sequence within a univocal setting. However, in this latter episode, the student's participation in the univocal setting corresponded with other students providing procedural statements. Both of these students had different levels of perceived autonomy and relatedness, but both reported at least mid-range perceived competence. This suggests that such students might not believe they have the individual volition or social backing to break from an in-place univocal setting, but have sufficient perceived competence to do so when another student who has such perceptions is engaged in the discussion. Yet, it is important to note that such breaks from the univocal setting (both in this and the preceding episode) were generally accepted by the teacher. Therefore, such breaks from univocal settings may transpire differently depending on the social and sociomathematical norms of a given classroom (Yackel & Cobb, 1996). Furthermore, such breaks were not consistent, as the initial episode with Liz, Robert and Greg illustrate.

Dialogic Discourse

In the class sessions observed, dialogic discourse seemed most prevalent when the teacher had students come to the front of the classroom. The typical scenario would include the teacher assigning each pair of students with a problem and having each pair come up to the front of the class and explain their strategy. If the problem was not pre-assigned, such as a homework problem, then the class would have a certain amount of time to prepare themselves before presenting their problems. The teacher would generally stand over to the side or find a place amongst the students to position herself. In either case, the repositioning of herself yielded the physical position that the teacher would typically be found in most math classrooms to the students.

During the ninth observation, the class was studying how to construct proofs. Students were provided a worksheet with mathematical situations they were asked to construct proofs for. After the teacher went over a few problems with the class, she asked for student pairs to volunteer and present different problems. At one point the teacher asked Greg and Robert to explain their problem, in which they were to prove two triangles were congruent (see Figure 1).

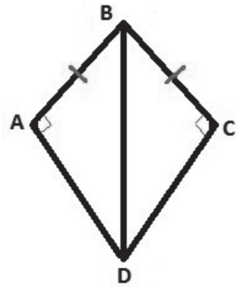

Figure 1. Recreation of Robert and Gregg's figure.

Robert proceeded to explain what he and Greg did to prove the triangles were congruent, while Greg made the occasional comment. At one point, Robert provided his reasoning as to what needed to be done to prove the triangles were congruent:

> **Robert**: Okay. AB equals BC. Now from there, we're trying to figure – we want to have three things. Right now we have side, side, angle, but we know that can't work because sides could just go in any direction. So we have to find the angle here or a side here (*pointing at parts of the triangles*) to prove the triangles are the same.
> **Teacher**: Or?
> **Robert**: Or, an ang-
> **Greg**: Interior angle?
> **Liz**: Hyptenuse leg.
> **Rebecca**: The definition of a right triangle, hypotenuse leg.
> **Teacher** (*speaking at same time as Liz & Rebecca*): What angle do you use if you've got it? (*acknowledging Liz & Rebecca*) Hypotenuse leg.
> **Robert**: Hypotenuse leg. You are absolutely right.
> **Chase**: I was going to say that!
> **Greg**: I don't understand how that works.
> **Robert**: Of course—okay, well I guess you could do that but I also don't understand how it works so I think there's another way to do it.

The conversation concerning this proof continued for the majority of the class period, with Robert and Greg being encouraged by the teacher, along with the majority of the class, to explore a conjecture that the triangles could be proven congruent without using hypotenuse leg as a justification. With the exception of Liz, the other students had at least one need (autonomy, competence, relatedness) with lower survey scores. Chase had lower perceived competence (3.80), while Robert (3.00), Rebeca (3.33) and Greg (4.00) had mid to lower scores regarding autonomy. However, the teacher's ceding of control appears to have provided the context that allowed for each of these students to engage in the conversation. This resulted in a mixture of procedural and conceptual statements included in the excerpt above and occurring in the discussion of Robert and Greg's conjecture that followed.

Contrasting the aforementioned episode of dialogic discourse is one in which Cindy, a student with a relatively lower score for autonomy (2.00) and mid-range scores for competence (4.00) and relatedness (4.33) was asked to present a proof. Carol is asked to accompany her to the front of the classroom (*autonomy* = 5.33; *competence* = 4.86; *relatedness* = 2.33).

> **Cindy**: Yeah, she can do it (laughs and hands overhead pen to Carol) Sorry, I don't know what to do.
>
> **Carol**: I don't know what to do either.
>
> **Cindy**: Oh that's not good.
>
> **Teacher**: So, folks. Let's help them out. (*3 seconds*). We're going to use the collective brain and help them out. Say it loudly, please.

Several students then proceeded to provide suggestions on how Cindy and Carol could begin the proof. Cindy was then able to complete the proof. However, when the teacher commented "It wasn't so bad," Cindy responded "Well, I didn't do anything" despite having completed the proof after an initial nudge from her peers, and explained the proof afterwards. Cindy's low perceived autonomy (2.00) likely contributed somewhat to her reluctance to begin the proof, although she was given the floor by the teacher and encouraged by her peers. Her mid-range scores for competence (4.00) and relatedness (4.33) were likely a factor as well. Specifically, Cindy was observed in various lessons as avoiding a central role in discussion, and vocally expressing her lack of confidence in her mathematics. While Carol had higher perceived autonomy (5.33) and competence (4.86), she did not offer much description in the form of procedural or conceptual statements in the episode. However, her lower perceived relatedness (2.33) was likely a factor. In general, Carol was observed to make occasional comments during discussion, but rarely took a central role in explaining any mathematics, unless either directly prompted by the teacher or partnered with a student who had relatively higher perceived relatedness scores (i.e., Robert or Gregg who neighbored her). Thus, both of these students' participation roles in dialogic discourse contrasts other students who had relatively lower scores on certain factors. The key differences regarding the scores appear to be relative autonomy for Cindy, and the relative perceived relatedness for both Cindy and Carol.

Trends Across Discourse Settings

Within univocal settings, students who had the combination of higher perceived autonomy, competence and relatedness were observed to initiate breaks or changes in the IRE exchange typical of such settings, or be participants in episodes where such breaks occurred. Because the teacher typically allowed such breaks, in certain episodes, students with relatively lower autonomy would also contribute to such a break. This suggests that within the observed mathematics class, while students were typically allowed to break away from the teacher-centered, univocal episodes, those students with a higher perceived sense of mathematical autonomy, competence, and relatedness needed to be present for such breaks to be initiated. Within dialogic settings where expectations for student-centered discussion were more explicitly obvious, students with lower scores on one of the three needs initiated procedural and conceptual comments more readily.

In both univocal and dialogic settings, students who had lower scores along one or two factors (autonomy, competence, relatedness) were often observed contributing to mathematical discussions. However, this generally was observed only when students participated who had higher scores on those factors aforementioned students had lower scores on. For example, in the first described episode for the dialogic setting, various students were involved such that the collective scores of these students for autonomy ($\bar{x}$ = 4.48), competence ($\bar{x}$ = 5.25), and relatedness ($\bar{x}$ = 5.40) were higher than the mid-point (4.00). A similar phenomenon is observable for other episodes both within univocal and dialogic settings. By contrast, Cindy and Carol's averaged autonomy ($\bar{x}$ = 3.67) and relatedness ($\bar{x}$ = 3.60) scores were lower than the mid-point (4.00), and they did not explain their mathematical proof until after the teacher solicited other students to help them begin. This is not meant to imply that simply averaging a collective group of students' scores for various needs would predict or explain their participation. Rather, it merely implies that when some participating students in certain episodes had lower perceived needs along some dimension, the participation of a student with higher perceived needs along the same dimension corresponded with an increased likelihood of productive mathematical talk, as well as dialogic breaks from univocal settings.

Conclusion

The results of the current investigation provide support for a relationship between students' self-regulation and their mathematical discourse actions. These results support Kosko & Wilkins (2012) quantitative findings that such a relationship exists between students' perceived needs and their perceived engagement in discussion, but extends them to a relationship between student perceptions and their observed actions. Students who reported higher perceived autonomy were observed to engage more in dialogic discourse, and were more likely to initiate breaks from IRE exchanges in univocal settings. As such, they typically initiated more conceptual talk and at a deeper level than students with lower perceived autonomy.

While individual students' self-regulation was observed to relate to their observed engagement in discussion, an interesting phenomenon was observed regarding groups of participating students in various episodes of discussion. Specifically, students tended to engage more actively in discussions when the group of participating students within an episode collectively had sufficient autonomy, competence, and relatedness. Whether or not students were consciously aware of it, students were able to pool the psychological resources of their groups needed to engage in mathematical discussion. This tacit pooling of psychological resources, however, appeared contingent on two factors in observed lessons and episodes: the nature of discourse setting and whether particular students chose to engage in particular episodes. Regarding the first factor, a univocal setting was initially more limiting to student discussion and required that a student with all perceived needs fulfilled be participating for a break to be initiated from the univocal setting. However, even when such conditions were met, students did not break from the univocal setting at every given opportunity. Thus, the reasons for when and why such students chose to break from univocal settings remains unanswered and would benefit from future investigation.

As has been suggested in previous literature (e.g., Kitchen, 2004), dialogic settings allowed for a wider range of students to actively engage in the mathematical discussions in the observed class. However, in episodes similar to when Cindy and Carol were the central contributors of the mathematical discourse, such dialogic exchanges either reverted to univocal settings or otherwise required the teacher's facilitation. This suggests that even in dialogic settings for whole class discussion, there is a need for the various students participating to collectively have sufficient autonomy, competence, and relatedness. The findings and conclusions presented here are from a single class over the course of 10 observations. They are limited in regards to their generalizability both across various teachers and mathematics classes. Thus, further study is needed to examine whether similar phenomenon occur in classrooms both with similar and different norms associated with mathematical discussions, as well as at varying grade levels and student populations.

Perhaps the most pragmatic implication of this study is that students' perceptions matter in regards to their engagement in mathematical discussion. Specifically, students' perceptions, both individually and collectively, of their autonomy, competence, and relatedness were found to interact with the actions they took, or did not take, in mathematics classroom discussions. So, while researchers have found that teachers' actions facilitate student engagement in mathematical discussions (e.g., Hufferd-Ackles et al., 2004), it may be that teachers' actions facilitate students' perceived autonomy, competence, and relatedness which in turn interact with their engagement in discussion. If such a relationship is valid, and it seems intuitive that it would be, then examination and implementation of teacher facilitation of discussion should take into account how such facilitation encourages student needs (i.e., autonomy, competence, relatedness). Yet, the findings from this study imply that facilitating only one of these needs may not be sufficient.

The current findings support mathematics education literature that suggests the importance of encouraging students' autonomy in mathematics (e.g. Krummheuer, 2007; Wood, 1999; Yackel & Cobb, 1996), along with perceived competence (Jansen, 2008; Kitchen, 2004; McGraw, 2002; Van der Aalsvoort, Harinck, & Gosse, 2006) and social relatedness (Hoffman, 2004; Wood, 1999; Yackel & Cobb, 1996). In certain contexts, students who are lacking in one or more need are more likely to engage in more meaningful mathematical talk if a peer with more fulfilled self-regulation is also participating. Finally, the major implication in this study is that facilitating only one student need is insufficient for full engagement in mathematical discussions across multiple contexts and discourse settings. Rather, at minimum, the combined aspects of students' autonomy, competence, and social relatedness need facilitation in order for students to become more actively and sufficiently engaged in mathematical discourse.

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