

# Teaching and Learning Calculus in Secondary Schools with the TI-Nspire

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## ABSTRACT

Technology can help develop understanding of abstract mathematical concepts through visualisation and graphic representation. The teaching and learning of calculus can be challenging as it involves abstract and complex ideas. The purpose of this study was to investigate how students and teachers attempt to use TI-Nspire, the latest graphing calculator from Texas Instruments, in teaching and learning calculus. The study also explored the use of TI-Nspire technology and how it affected students' motivation, interest and self-confidence in mathematics. This study used qualitative techniques such as interview and observation for data collection. The participants of this study were three Form Five (Grade 11) students from a mixed ability classroom in a public secondary school. Results of the study indicated that TI-Nspire reinforces students' understanding of calculus concepts besides stimulating mathematical thinking. Students perceived that TI-Nspire had a positive effect on their mathematical knowledge and ability.

**Keywords:** *Calculus, TI-Nspire, Teaching & Learning*

## INTRODUCTION

Research has indicated that the teaching and learning of calculus can be challenging as it involves abstract and complex ideas (Gordon, 2004; Zachariades, Pamfilos, Christou, Maleev, & Jones, 2007). This means that students face difficulties in learning the key concepts of calculus (Artigue, Batanero, & Kent, 2007). It does not help that in traditional calculus classes the emphasis is on computational procedures without understanding the concepts (Gordon, 2004). Many studies have demonstrated that students' difficulty in understanding calculus are caused by their weak understanding of functions (Dubinsky et al., 1992; Tall & Vinner, 1981; Williams, 1991) and the inability to use functions to reason and represent relationships (Carlson et al., 2002; Monk & Nemirovsky, 1994; Thompson, 1994).

The teaching strategies in calculus have become merely list of procedures to follow and results only in practicing usual routine in algebraic manipulations. Weaker students get frustrated easily over the manipulations required in calculus (Tucker & Leitzel 1995). Students expect a lengthy algebraic solution for problems which is common in calculus (Gordon, 2004). In teaching calculus, teachers focus more on the procedures rather than understanding of the underlying concepts (Zachariades et al., 2007). According to Axtell (2006), teaching calculus using the traditional approach fails to help students understand the basic concepts of calculus. Gordon (2004) and Axtell (2006) concluded that the calculus curriculum should be improved by focusing on the conceptual understanding of calculus in particular, balanced with the use of graphical, numerical, algebraic and verbal representation in the teaching and learning of calculus. Visualization and visual thinking is pertinent in improving the approaches in teaching calculus. Hughes-Hallett (1991) stated that the equilibrium of the graphical, numerical and analytical method is necessary because it enhances understanding of calculus concepts. Gordon (2004) suggested that students choose the right tools such as the graphing calculators to assist in learning calculus and apply the balanced approach in solving problems.

New learning technologies such as the graphing calculator have gained acceptance in the mathematics education field. Technology can help develop understanding of abstract mathematical concepts through visualization and graphic representation. This will increase students' competence in obtaining sufficient knowledge of mathematics. Bert Waits, co-founder of T<sup>3</sup> (Teachers Teaching with Technology) mentioned that graphing calculator is a great pedagogical tool as it offers multi-representational approaches in teaching and learning of mathematics. In Malaysia, the implementation

of graphing calculator is still at the minimum level although the government has suggested the usage of graphing calculator in the upper secondary mathematics curriculum.

### Review of Related Literature

Graphics calculators are hand-held, battery powered devices equipped with functions to “plot graphs, give numerical solutions to equations, and perform statistical calculations, operation on matrices and to perform more advanced mathematical functions such as algebra, geometry and advanced statistics” (Kor & Lim, 2004, p. 69). The National Council of Teachers of Mathematics (NCTM) encouraged the usage of graphical calculators in the teaching and learning of mathematics. This was implemented to enhance students’ experience with multiple representations of mathematical ideas (NCTM, 1989). The graphics calculator is a powerful tool because it empowers students in solving mathematical problems by actively engaging them in doing the mathematics (Noraini et al., 2010). Several studies have shown the benefits of using graphing calculators in teaching mathematics (Lyublinskaya & Zhou, 2008; Sang, 2003). Students form a better understanding of the mathematics involved and are motivated to solve more complex problems (Noraini et al., 2003). More specifically, the use of graphics calculators in classroom teaching improves ‘spatial visualization skills’, ‘critical thinking ability’ and ‘understanding of connections among graphical, tabular, numerical and algebraic representations’ (Rich, 1991 as cited in Noraini, 2004, p. 46).

Zachariades et al. (2007) reported that graphing calculator is a useful tool in teaching calculus as it integrates the graphical, numerical and symbolic functions. In addition, the findings of Tiwari (2007) indicated a better connection between algebraic representation and graphical representation when using the calculator. Tiwari (2007) also found that students using graphing calculator were more likely to achieve conceptual understanding and enhance their problem solving abilities in learning calculus. This is due to the positive effect of the numerical and graphical functions in the graphing calculator when used as a supplementary instructional tool.

Alkhateeb and Wampler (2002) found that students who used the calculator performed better in achievement tests compared to those who did not in the understanding of derivatives. At the same time, Crocker (1991) revealed that students utilizing the graphic calculator had higher possibilities of trying different methods of solving calculus problems. Students’ response to questions improved significantly when using the graphing calculator in learning calculus as highlighted by Harvey, Waits, and Demana (1995). Consequently, Jones (2005) reported that the graphing technology provides students with alternatives in approaching problems by the graphical, numerical and algebraic way. In particular, various problem solving methods enable students to visualize the problems and support them to obtain the solutions. Indeed, this encourages students’ confidence in exploring more challenging calculus problems and applying various problem solving techniques.

### RESEARCH PURPOSE

The purpose of this study was to investigate how students attempt to use the graphic calculator TI-Nspire handheld technology in learning calculus. Specifically, this study aimed to answer the following research questions:

1. What insights do students provide in the teaching and learning calculus using the TI-Nspire handheld technology?
2. How does the use of TI-Nspire technology affect student’s motivation, interest and self-confidence when working with mathematical instructional activities?

### METHODOLOGY

#### Design

This study employed a case study design to elicit students’ experience of the learning environment using TI-Nspire, the latest graphing calculator from Texas Instruments. The teacher used the software version on the laptop while the students utilized the handheld version. This study mainly focuses on the qualitative study to provide answers to the research questions. The researcher played the role as the teacher during this study. The teaching and learning process occurred within a small tutoring group in a school. Regardless of what setting the researcher uses to gain information about what students know, “it is absolutely essential that the researcher keep in mind that what he sees as ‘the’ problem imposes nothing of necessity upon the problem solver” (Thompson, 1982, p. 29).

## Materials and Instruments

The materials used involved the following main parts: Six instructional activities were designed to achieve the following learning objectives based on the Additional Mathematics Curriculum Specification.

Table 1 Learning objectives on instructional activities

Instructional Activities	Concepts	Learning Objectives
1	Limits and Continuity	Determine the value of a function when its variable approaches a certain value.
2	Limits and Continuity	Understand the concept of discontinuity. State and explain the limit at a particular value based on a graph.
3	Differentiation	Find the first derivative of a function $y = f(x)$ , as the gradient of tangent to its graph. Determine the gradient of tangent at a point on a curve.
4	Differentiation	Determine coordinates of turning points of a curve. Solve problems involving maximum or minimum values.
5	Integration	Find areas under curves as the limit of a sum of areas.
6	Integration	Calculate and analyze Riemann sums

The first, third and fifth instructional activities were designed to promote opportunities for students to explore and generalize mathematical concepts. This enables them to communicate mathematically. The second, fourth and sixth instructional activities were designed as an exposure of the concepts to the real-life application. One student was chosen randomly to be interviewed in a structured form directly after the teaching of the sixth instructional activity. She was interviewed about her experience in using TI-Nspire while solving the task. This interview was recorded and transcribed.

All students had to answer a set of survey questionnaires to elicit their insight into the learning environment when using TI-Nspire. Furthermore, the opinions on the quality of the teaching and how it has affected their motivation, interest and self-confidence were obtained. The questionnaire set consists of 15 questions. The TI-Nspire tasks were based on the problem-solving task and designed to investigate students' ability to perform and apply the technology in a versatile way. A document of the task was submitted to the researcher once it was completed.

## Study Development

This section describes on the study development which includes the activities performed and their objectives. The case study was conducted over two weeks. Three Form Five students of equal gender (female) studying in a public secondary school were selected as the sample of the study and they attended all the after school meetings. They were briefed on the purpose of the study at the beginning and researchers stressed on their commitment during the study. Prior to using the TI-Nspire, students were familiar with the non-graphing scientific calculator.

Six instructional activities on exploration and problem solving tasks were designed using the TI-Nspire technology. These calculus activities were developed to enable students to learn the concepts of limit and continuity, differentiation and integration. Consequently, each of these activities was followed by exercises used to detect students' ability to perform tasks without guidance from the teacher.

This study took place over the course of two weeks. Initially, students were trained on some commands and features in the TI-Nspire to familiarize themselves with the handheld technology. Training sessions were conducted to introduce the interface design and use the algebraic, graphical and numerical functions of the TI-Nspire. The first week of the study involved a lesson on the concepts of limit, differentiation and integration. Students were taught how to calculate and find the limit, to differentiate and to integrate. Lessons were delivered through PowerPoint presentations and involved notes, directed examples and "drill and practice". Graphing calculators were allowed throughout the lesson but the researcher never stressed their use during the first week.

During the second week, in order to observe the use of graphing technology, the researcher taught a similar lesson but stressed more on the use of TI-Nspire. Each student used a TI-Nspire handheld to enhance calculus learning. To establish a better learning environment, the researcher displayed the calculator by using the TI-Nspire students' software on the LED monitor screen. At the end of every session, students worked on the exercises related to the concept learnt by using TI-Nspire technology. Responses recorded in the activity sheet and the file containing the answers were collected and analyzed. The students were allowed to cooperate during the problem solving task and they could also ask the researcher if they encounter difficulty in the process.

In the second meeting, the students completed a questionnaire related to the activities solved by using the handheld technology. This was an attempt to collect data on their insights into the quality of the teaching and how it has affected their motivation, interest and self-confidence. During the study, students' attitude toward the lesson were observed and recorded. Based on the analysis, a structured interview was conducted with one student aiming to obtain further information on her experience involving the use of TI-Nspire. The interview was carried out and recorded.

## RESULTS AND ANALYSIS

In this section, the main results of the various methods in the study are reported. Some of the data shown here is in the form of verbalizations from students.

### Students' experience of a learning environment with TI-Nspire

The combined data from the interviews and the questionnaires gave an interesting picture of the advantages and the disadvantages using TI-Nspire technology. Below are some of the elicited opinions of students in learning mathematics using the hand-held technology.

Table 2 Advantages with using TI-Nspire technology

Advantages	Students
Colourful and clear screen.	X
Fast and flexible to work with.	
Easier to present new concepts and demonstrate in the class	X
Manage more difficult tasks, on a higher level.	X
Learn more and understand mathematics better.	X
Apply several ways to solve a problem.	
Focusing on understanding instead of making a lot of calculations	
Mathematics is more interesting and fun working with TI-Nspire.	X
Participate actively during teaching and learning session.	
The use of TI-Nspire has changed individual perceptions on working with mathematics.	

The interview provided many useful insights from the students. Certain extracts from the interview are provided.

*S: It's a very smooth and interesting gadget in mathematics. It's like a mini computer and I can even save my documents in it. So, instead of doing boring exercises in the book, now I can use it as my note book and exercise book.*

*S: I don't understand what mathematics concepts are because we were always asked to do calculation on the paper without knowing why we are learning it. In this class, I learned mathematics in a simpler and more interesting way.*

*S: TI-Nspire class is very fun, less calculation and it can be my guiding light.*

Generally, students endured challenges in using TI-Nspire technology. Table 3 presents the common difficulties faced by students and researchers in using the graphing calculator.

Table 3 Common difficulties faced when using TI-Nspire technology.

Difficulties	Students
Hard to start using TI-Nspire.	X
Takes time to learn how to use TI-Nspire.	X
Difficult to use different tools.	X
Sometimes difficult to know how to start solving a problem.	X
It is essential that you also practice solving tasks with paper and pencil. You must do both.	
When you work with paper and pencil you understand better.	X
Technology often brings problems of technical nature.	X

The interview provided several insights from the students. These are some important excerpts from the interview.

*S: The calculator was a little difficult to learn at the beginning. Maybe, I need one month to two months to get use to the technology itself before applying it in the class.*

*S: During the studying of limit, it was very hard to move the cursor to get the answer.*

From the interview, the student explained how complicated using TI-Nspire was for the first time. But she answered that, after a short while, when she was familiar with the technology, it did not seem so complicated at all. The students' versatility in using TI-Nspire progressed substantially during the study and by the end of it many of the challenges begin to disappear.

### Effects of TI-Nspire technology on students' motivation, interest and self-confidence

Students were asked about these effects, both in the interviews and in the questionnaires. One important point was the introduction and implementation of the handheld technology.

*S: I can explore more advanced tasks. I realized that when I do the exercises with TI-Nspire, it taught me to help each other during the discussion. All this while I had been sitting down and solving the problems on my own.*

Students' general skills in using the handheld units were put to the test in the final task of this study. The researcher's observation showed that the students handled TI-Nspire in a satisfying way but they spent some time in finding the keystrokes and choosing the suitable functions in the calculator. Their problem-solving skills with TI-Nspire were good.

Most students feel that it is more fun learning mathematics with TI-Nspire because mathematics appears in a new form where you are able to manipulate objects. This affected their beliefs about calculus and the instructional activities. In particular, the students believed that TI-Nspire brought a positive effect on their calculus knowledge and ability. The usefulness of this handheld technology was also explained by the student.

*S: And later in life when you have to do calculation then you still require a calculator. We will not sit and calculate mentally the difficult sums. So I would still need a calculator. It seems unnecessary not to have calculators especially this TI-Nspire technology.*

## DISCUSSION

The purpose of this study was to investigate how students attempt to use the graphic calculator TI-Nspire in learning calculus. The instructional strategy using TI-Nspire technology was implemented as to teach concepts in calculus such as limit and continuity, differentiation and integration. Students usage of the graphing technology were

determined from the interview and observation.

Students expressed common advantages with the usage of TI-Nspire. Some of the physical features of the calculator mentioned include a clear, colourful screen and is flexible to work with. But more importantly are the mathematical features of the graphing calculator, for instance easier working with complex functions, new ways to work with problem-solving tasks and managing more difficult tasks. They like learning mathematics with TI-Nspire as it improves the understanding of concepts and the focus is not only on doing calculations. These results are similar to those in the Tiwari (2007) study on students' achieving conceptual understanding when using the graphing calculator.

The researcher found that the learning of mathematics had changed to some extent. Fully utilizing the graphical, numerical and symbolic functions enhances the graphing calculator as an important tool in the teaching and learning of calculus (Zachariades, 2007). Lessons that focus on the exploration of concepts and using problem-solving strategies encouraged group discussion in the class. The graphing tool acts as a calculator, a problem-solving tool, and a mathematical concepts discovery tool as well. With these features, students developed a better understanding of calculus and were motivated to solve challenging problems (Noraini et. al., 2003). Among the common obstacles to high-quality teaching were that students might have some difficulties in handling the technology. Student discussion increases and they become active participants with the usage of graphing calculator; the graphing calculator is a useful tool in teaching calculus as it integrates the graphical, numerical and symbolic functions.

## CONCLUSION

Based on the findings, graphing calculator reinforces students' understanding of calculus concepts. In addition, the learning of calculus with this tool also stimulates mathematical thinking. TI-Nspire handheld offers a number of applications in calculus that helps visualize the abstract and complex concepts. With the use of instructional activities, students can learn calculus concepts by exploration and discovery. There are numerous resources available on the Texas Instrument websites for students to explore. Using TI-Nspire technology for a mathematics classroom is highly recommended to make teaching and learning of mathematics more fun and meaningful.

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