Geogebra As An Artist's Paintbrush

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ABSTRACT

One of the definitions of mathematics is that it is "a science of patterns and themes". Within the scope of this definition, the current software technology facilitates the creation of visuals and patterns. Thus, GeoGebra software was used. The study was carried out in two stages. In the first stage, the Dynamic Geometry Software and the Discovery of Mathematical Concepts class was taught. In the last stage, the preservice primary mathematics teachers were given an assignment and asked to continue in groups. The groups were told that they were expected to produce aesthetically pleasing visuals and patterns. Therefore this research aimed at investigating how preservice teachers determine the design process and how they solve problems encountered during the process. Thus, as can be seen in the literature, the method and design as a long term study are the new contributions to the literature. Case study was carried out with 39 second-year preservice mathematics teachers. The research data were collected through preservice teachers' finalized projects which included a Microsoft-Word document that described in detail the process, screenshots, and GeoGebra applications. The data were analyzed by using the content analysis technique. As a result of this study, preservice mathematics teachers entered into research and analysis activities, investigated to create original designs, spent more time on the significance of visuals and patterns and so many groups made various creative tasks.

Keywords: *Mathematics Teaching, GeoGebra, Aesthetics, Design, Pattern.*

INTRODUCTION

The common ground of most studies on mathematics education is "how can we teach mathematics better". However, "teaching only with speaking" is called the traditional approach and is not enough anymore, since this approach ignores the cognitive abilities and self-improvement of every student (Dikovic, 2007). It should not be forgotten that the ideal learning environments are encouraging platforms for students to discover the ideas they produced (Battista, 2001). According to Freudenthal, mathematizing is the core goal of mathematics education (Freudenthal, 1973). In this context, Zbiek and Conner (2006) point out that modelling contributes to understanding mathematical concepts thoroughly by demonstrating the applicability of mathematics to real life, and to learn new mathematical concepts, to establish inter disciplinary relations between the conceptual and operational development of the students.

One of the definitions of mathematics is that it is "a science of patterns and themes" (Toluk, 2003). Within the scope of this definition, current software technology facilitates the creation of visuals and patterns and allows us to carry it one step further. Thus, this new technology not only facilitates the calculation and drawing of graphs, but also changes the nature of the important problems in mathematics and the research methods (Baki, 2002). Sendova and Grekovska (2005) pointed out that building a computer model of a construct is motivated to elaborate one's knowledge in mathematics and informatics. Accordingly, such operations as translating, shifting, expanding and reducing can be performed by the students themselves, as part of a dynamic process (Aktumen et al., 2010).

Technological tools enable the discovery of certain new mathematical concepts. This experience of discovery enables formation of powerful geometric intuition and prediction skills, which are essential for problem solving and theory building in any field of mathematics (İsmail and Kasmin, 2007). However, unfortunately, it is determined that teachers frequently use computers for presenting slide shows (Aktumen, Yıldız, Horzum and Ceylan, 2011; Dikovic, 2009; Kabaca, Aktumen, Aksoy and Bulut, 2010) whereas most researchers underline that students enjoy the learning process more actively with computers and use this technology as a tool for exploring (Anabousy, Daher, Baya'a and Abu-Naja, 2014; Gonzàlez and Herbst, 2009; Lachmy and Koichu, 2014; Santos-Trigo and Cristóbal-Escalante, 2008; Van Voorst, 1999). Therefore, in this study, primary preservice mathematics teachers' design processes were examined using software within an assignment in one lesson duration.

One of the most effective methods recommended by researchers for the conceptual grasping of mathematical concepts by students is the use of multiple representations/displays (Brenner et al., 1997; Porzio, 1994, cited in Özmantar et al., 2008). The relationships among representations is strengthened through transitions from one representation to another when working with multiple representations (Akkoç, 2006). It is not always possible to view the relationships among such representations. To clearly view the relationships among multiple representations necessitates the use of dynamic mathematics software. GeoGebra, the dynamic mathematical software employed in this study uses algebra, drawing board, spreadsheet views, mathematical symbols, graphics and table transfers, as part of a dynamic process, to provide rapid transitions between representations, which sets it apart from other dynamic geometry and algebra software (Aktumen et al., 2010). Moreover, GeoGebra software expands the dynamic geometry concepts, and adds a new dimension to mathematics and algebra (Dikovich, 2009). Besides that, GeoGebra's interface has been translated into Turkish and it is a free software; it can be extended and is open-source (Aktumen and Kabaca, 2012; Aktumen et al., 2012; Aktumen et al., 2011; Kabaca and Aktumen, 2010). The advanced modeling capabilities offered by GeoGebra has also been instrumental in its selection as the focus of this study.

Literature Survey

Literature review shows that articles on GeoGebra software fall under four main headings. The first category is composed of some studies in which a lesson, constructed with GeoGebra, and some variables are examined (Dikovic, 2009; Saha, Ayubb and Tarmizi, 2010; Tatar, Kağızmanlı and Akkaya, 2014). The second is represented by studies in which mathematics teachers and pre-service teachers' views are examined (Aktumen et al., 2011; Kutluca and Zengin, 2011). The third consists of studies in which some variables are examined in the problem solving processes with GeoGebra (Baki, Yildiz and Baltaci, 2012; Yıldız, Baltacı and Aktumen, 2012). The last is composed of the studies in which a certain geometric locus is discussed with GeoGebra software (Antohe, 2009; Baki, Çekmez and Kösa, 2009). Although mathematical patterns, tessellations and designs allow for appreciation of the relationships between mathematics and other domains, as can be seen in the literature, after a long education term, and after a comprehensive assignment given the preservice teachers, they are not assessed on the designs nor the design procedures.

Purpose and Importance of the Study

In the assignment, stated as extracurricular, the aim of this research is formed as: How do preservice teachers determine the design process and how do they solve problems encountered during the design process.

Due to the GeoGebra dynamic mathematics software, the problems related to real life can be modelled for the students as improving the motivation about mathematics (Price and Stacey, 2005). On the other hand, as can be seen in the previous literature survey, the method of this study is one of the new contributions of the study to literature as well as it is a long term study. Thus, according the results of this study show that computers are not only used for providing visuals.

METHODOLOGY

Research Method

In this research, we used the case study method as it enabled us to examine a particular group in depth and to assess the data obtained through data collection tools without being concerned about generalization.

Participants

The study sample consists of 39 second-year primary preservice mathematics teachers from a state university enrolled in primary school mathematics instruction, during their fall semester. The participants were 17 males and 22 females. The average age of the participants is 20 years. Their mathematics proficiency level is good enough and their general attitude to mathematics is positive.

When taking courses on basic ICT skills in their first year at university, primary preservice mathematics teachers began interacting with GeoGebra in two courses. They were guided in these courses by a lecturer, who had organized various in-service training sessions on using GeoGebra in mathematics education. Table 1 presents some information on the content of these courses.

Table 1. The Lessons Where GeoGebra was Used Before the Application

Course Details	The Method of Using GeoGebra
General Mathematics First Year 6 Hours	In this course, GeoGebra is used only for the purpose of presentation by lecturer when needed: For example; drawing the trigonometric functions' graph by using unit circle, drawing function, the relation between the function and inverse function, the construction of limit concept etc.
Geometry First Year 3 Hours	In this course, GeoGebra is used only for the purpose of presentation by lecturer when needed: To test the theorems belonged to Oklit geometry (menelaus theorem, pisagor theorem, basic rate etc.) in order to make it more understandable, provide an argument environment etc.
Calculus I Second Year 6 Hours	In this course, GeoGebra is used only for presentation by lecturer when needed: For example; To examine the derivation of function concepts in table, graph and algebra representation, Rieman integral, the method of rectangle, to draw parametric curve etc. This lesson was instructed at the same term of application.

Procedure

The study was carried out as part of the Dynamic Geometry Software and Discovery of Mathematical Concepts course. It was designed in two stages. In the first stage, the Dynamic Geometry Software and Discovery of Mathematical Concepts course was taught by the researcher for a 9 week period for 2 hours per week. As part of this course, the basic features of GeoGebra were described to the preservice mathematics teachers and they were given the opportunity to develop GeoGebra applications corresponding to algebra and geometry concepts. At the end of this stage, the preservice teachers were given an assignment and asked to continue in groups, and 13 groups of three students each were formed. The groups were told that they were expected to produce aesthetically pleasing visuals and patterns using their knowledge of mathematics, and their GeoGebra skills. The groups were then left alone and no limitations were applied. As proposed by Kalaycı (2008), the patterns were determined by the groups themselves and guidance was provided by the authors. The groups were given a four week period for the pattern creation process.

Data Collection Tools

The groups were asked to turn in to the authors, as their finalized projects which included a Microsoft-Word document that described in detail the process used in creating the patterns, screenshots of the pattern samples, and GeoGebra applications containing their patterns. From the Microsoft-Word document, it was decided how the teachers determined designs, what they did, how they overcame the problems, since the preservice teachers were asked to write this duration in detail. It will help us to reflect the process better by writing the opinions of the preservice teachers in the findings part. The research data were collected via these tools.

Data Analysis

The data from the documents were analyzed using the content analysis technique. The content analysis process involves compiling the similar data under certain concepts and themes and to comment on these in a more comprehensive way for readers (Yıldırım and Şimşek, 2005). Additionally Bali and Ramadan (2007) have stated that opting for the content analysis technique works well for small scale samples or for samples determined using a specific criteria (cited in Yenilmez and Ersoy, 2011). In this context, each student's documents were analyzed in depth and themes were determined. The themes were then specified by the authors and an expert independently from each other and were brought together and put into their final forms. At the end of discussions, the agreed themes were selected to answer the research problem. The reliability of the research was ensured in this way.

Saban (2007) has stated that to increase self-confidence and validity, student opinions may be frequently quoted. Part of the expressions from the documents created by the preservice teachers have been quoted in their original form in this study. When using the quotations, preservice mathematics teachers have been assigned ID numbers from 1 to 39; as an example, the preservice mathematics teacher corresponding to 1 has been shown as Ö1.

FINDINGS

As a result of content analysis on documents created by the groups, seven themes are found to emerge. In the following, the themes established in the design process are exemplified.

Preservice mathematics teachers were able to connect algebraic and geometric representations when inputs were changed

Preservice teachers expressed their thoughts as part of this theme as follows:

Now we have made various changes to the functions and the curves, to come up with different shapes. For example; we added multiplication of cos(x+b) to function f. (Ö1)

We realized that for half of the intervals where c(x) = sin(x)^sin(x) and b(x) = cos(x)^cos(x) have definitions on the x axis, these intervals are common. Later we simultaneously displayed the graphics for b(x) = cos(x)^cos(x) and e(x) = cot(x)^cot(x). We noticed that the e(x) function begins at the start and end points of the b(x) function and continues, and that it is defined for a smaller interval compared to the b(x) function.

(Ö 37) *(Fig. 1.)*

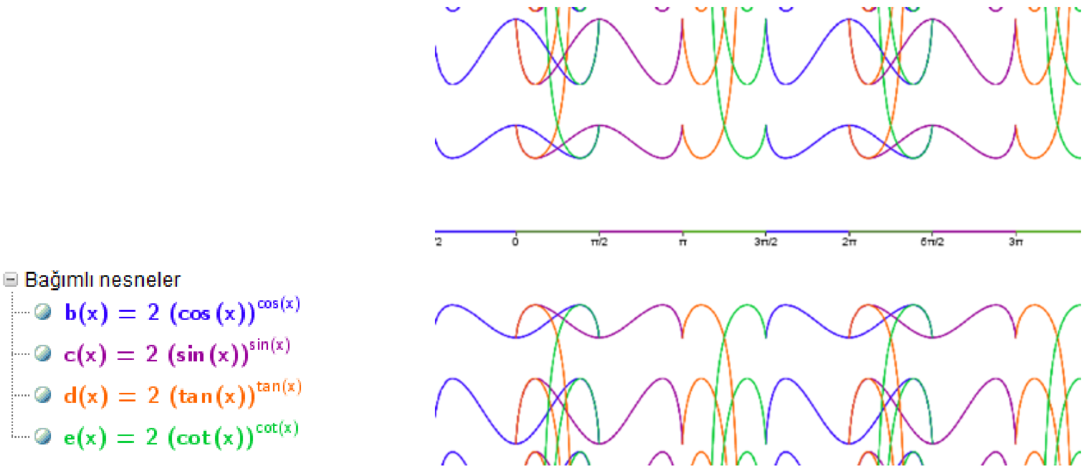

Fig. 1. Algebra and its graphical representations

As seen from these representations, GeoGebra is able to simultaneously reflect to the graphical window changes made by the preservice mathematics teachers to the algebraic expression, and conversely, reflect to the algebraic expression any changes made in the graphical window; in this manner, it enables the preservice teachers to establish relationships between the algebraic and geometric representations when inputs were changed.

Some preservice mathematics teachers were able to select the proper mathematical concept

The following are some preservice teachers expressions assessed as part of this theme:

Following a discussion, we decided that this shape not only resembled a ghost, but also the ear and the eye of a mouse. So we decided to do additional work on it. We tried to make this pattern resemble the head of a mouse. We decided to craft a mouth for the mouse and felt that it could be accomplished with the help of an ellipse. We generated an ellipse and placed it below the eyes of the mouse. We formed a nose with the help of a circle and placed it above the ellipse. Then, with the help of a function, we created two whiskers for the mouse. Finally we graphed teeth for our mouse using lines and line segments. (Ö 1) (Fig. 2)

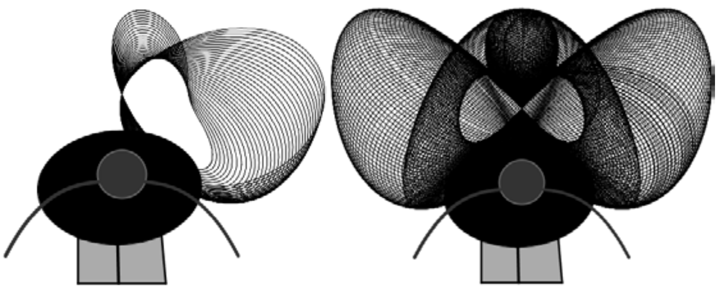

Fig. 2. Angry and crazy mouse.

We had to create a body for the tulip. We had to define a function to flesh out the body. The formula for the first function we tried is: Function [10x³, -2, 0]. The resulting shape was as follows. Seeing this output, we sensed that there was no need to try the mirror image of the body. Now we had to try another function, but what could we try that would give us a flower-like body with the bowl contacting the petals at the correct location and would look like as if it was a part of the tulip? We started to try other functions. After a few trials, we found a body as follows, with the formula: Function [-(x - 1)³ 2, 0.75, 2.25]. This was the shape that came out and we really felt like we could use the application. (Ö4)

Later, as the shape that was forming, started resembling a butterfly's wings, we used the same formula to create additional wings by modifying the axis and wrote it as shown below. (Ö 20)

- *To form the butterfly's antennas;*

f(x) = -5 sqrt(x)

g(x) = 5 sqrt(x)

h(x) = -5 sqrt(-(x))

k(x) = 5 sqrt(-(x))

After writing out the functions, in order to describe the antenna shape, we entered the functions in the input field as follows..."

As shown, during the process for creating an envisioned pattern, it was observed that a portion of the preservice teachers displayed the skills to select the proper mathematical concept among several available.

Some of the preservice teachers' abilities to predict and generalize were exhibited during the process

Some preservice teachers expressed their thoughts as part of this theme:

This pattern, which resembles an egg, came about as a result of adding an angle, and captured my interest. If one angle could be the source of so much change, the addition of a function into the curve could . . . generate better result. So we decided to create the function and add it into the curve. (Ö 25)

I had researched the homework you gave us for the Analysis class. The homework was for finding the derivatives of arctanh(x) and arccoth(x). So I researched it and showed you the result. When you looked at it, you connected the graph to the sequence using arctanh(x) and arccoth(x). We were impressed with that figure and so we connected the graph of the derivative of arctanh(x) to the sequence. Sequence [i Derivative[arctanh(x)], i, -10, 10, 0.5]. (Ö22) (FIG. 3.)

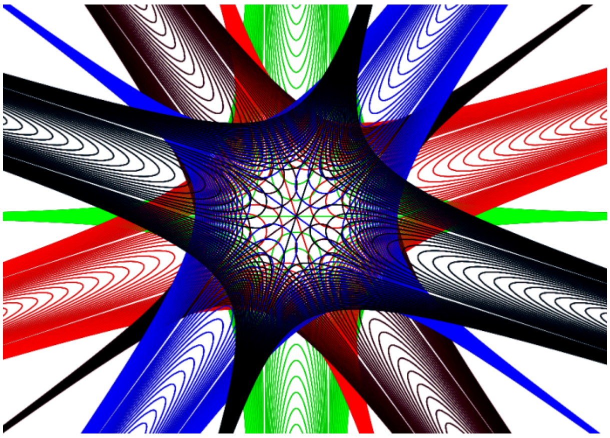

Fig. 3. Polygon and a water drop

As can be seen above, some preservice teachers predicted and were curious about the results, so they formed the design process in this way. On the other hand, some of the preservice teachers took the essential steps after they generalized the former information, as can be seen in the example above.

The preservice teachers showed the skill of using mathematical content in their designs by using different information sources

Preservice teachers expressed their thoughts as part of this theme in the following excerpts:

First we searched for 'trigonometric functions' using Google. (Ö35)

We browsed our notebook to see whether there was a topic we had not thought of. When I reviewed the notebook, I saw that it actually contained exactly what I had been thinking of. I realized I had to learn the *topic of sequences.* (Ö 2)

We formed this pattern using an internet search. We wanted to create patterns related to cyber spaces (nested labyrinths), fractals, the Fibonacci series and the golden ratio, but we could not really get a lot done. We again consulted a book on the fundamentals of mathematics; additionally we examined such patterns as spider webs and honeycombs. We also tried some functions we retrieved from basic mathematics books. (Ö 8)

We examined the formula for the ellipse in the section for conics from the mathematical geometry formula booklet published by the X training center. (Ö 19) (FIG. 4.)

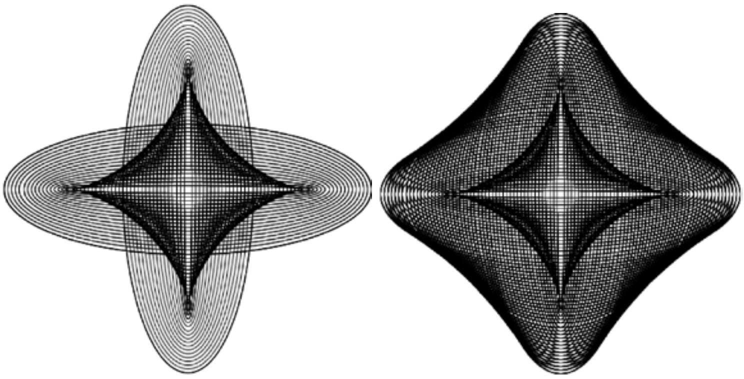

Fig. 4. Ellipses

As seen from above, GeoGebra environment has enabled preservice mathematics teachers to try other sources of information. These resources are university text books, the internet, lecture notes and supplementary text books.

Preservice mathematics teachers sought aesthetically pleasing results

Preservice mathematics teachers expressed their thoughts as part of this theme:

At first, we define an "f" function. After that, by using function we define a "g" function. The gap of the "g" function is determined from 1 to 10. The functions are as in the following:

f(x) = 1 - 2sin(x) sin(x) sin(x)

g(x)= Function[x^4, 1, 10]

We thought to use it in curve by adding some trigonometric functions.

Curve [f(a) cos(a + α), f(a) tan(a + α), a, 0, 10]

In this figure, by bounding a angle to cursor in the gap (0°, 360°) we complete the increment 5°. We bounded a to cursor and minimum 0.1, maximum 5, the increment was completed as 0.4. Then, we pushed to Enter. After opening the vitalization and trace the figure was emerged as below (Fig. 5.).

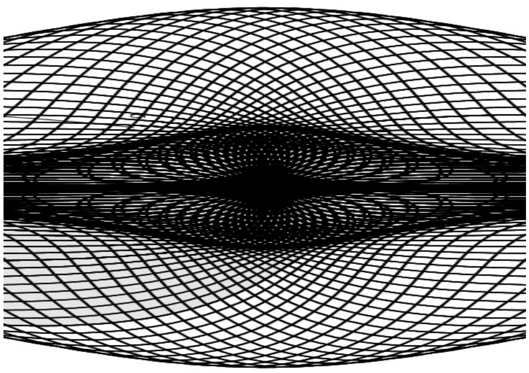

Fig. 5. Outer space

As can be seen above, when we look inside of the figure, the pattern is getting more identical to outer space. But we do not like this pattern. Because we were not satisfied with that. We wanted to find more different patterns… (Ö 2)

After adding some trigonometric functions randomly, we found the formula below. We define b as (0, π),and a bounding to cursor in the gap (0°,360°), with 0.1° increment. And the formula is:

$$Curve\ [g(b + \alpha) \sin(b + \alpha), f(\alpha + b) \tan(b + \alpha) \sin(\alpha + b) \cos(b + \alpha), b, 0, \pi]$$

After writing this function and pushing enter, and opening the vitalization and trace, we got Fig. 6.

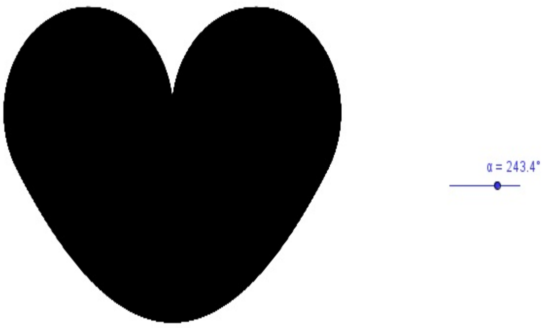

Fig. 6. Heart

The shape above resembles a heart. We felt very happy when we first saw this pattern. We thought we were done, but then we felt it might not be good enough, so we decided to try new functions. (Ö 11)

Working on this topic was going to be exciting for us, as we were thinking that we would be getting new patterns each time. (Ö 21) (Fig. 7)

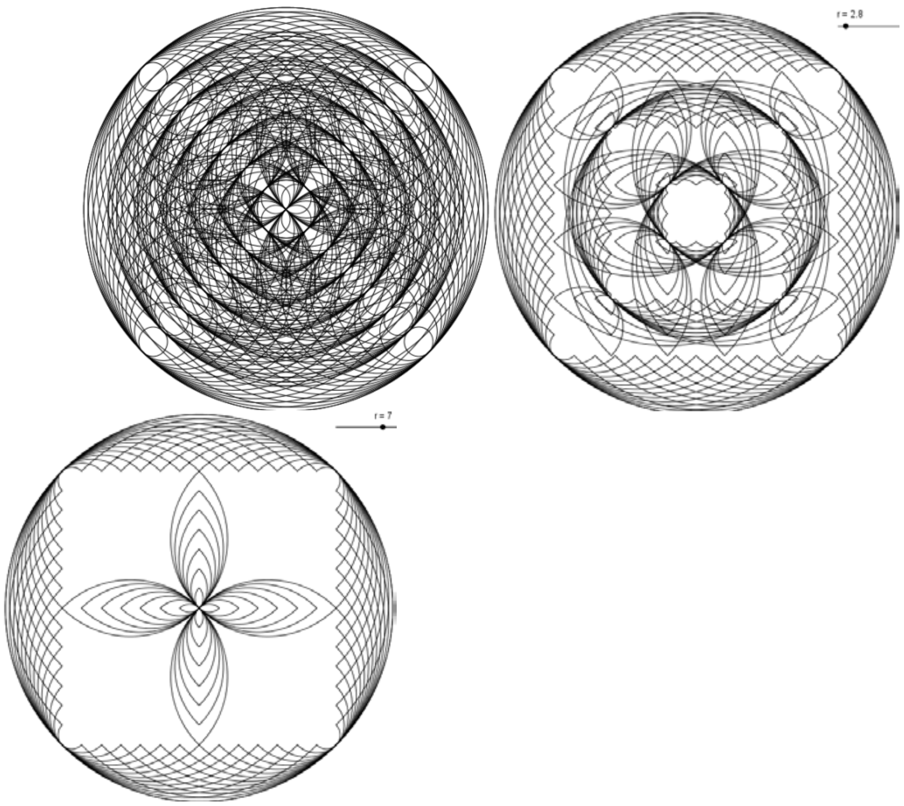

Fig. 7. Designs within a circle

When the documents prepared by the preservice teachers were examined, it was determined that as above in certain cases they did not like the patterns they formed, and in certain cases they did. However, in both cases the process led them to create new patterns.

Some preservice teachers gain self-esteem with improving to use GeoGebra software

Some preservice teachers expressed their opinions as part of this theme in the following:

We set it so that Sequence_1= Sequence[Sequence[(i,j),j,9.9,r,-0.3],i,9.9,r,-0.1]. We set initial values of i and j as 9.9 and we set their increment values as -0.1 and -0.3 such that it would appear to come from the opposite side as the reverse of the first sequence. It starts from 9.9 as we did not want it to overlap with the first sequence. And we created slider r, decreasing from 10 to 1, having slider speed of 0.5 and a decrease rate of 0.01. (Ö 1)

As it is seen from above it can be stated that a project such as this one advances the skills of some preservice teachers for making use of the GeoGebra syntax. This situation facilitates them to create a good pattern with self-esteem.

On the other hand it was observed that a portion of the preservice mathematics teachers researched and used in generating their patterns features of GeoGebra that were not covered as part of the lectures. The following are preservice teachers' expressions assessed as part of this theme:

We decided to issue new commands. We decided to use the circular sector (CircularSector[<Point>, <Point>, <Point>]) found on the help button at the lower right-hand corner of the GeoGebra window. (Ö 36) (Fig. 8)

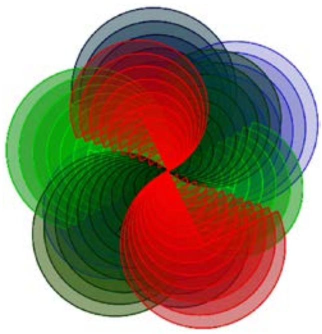

Fig. 8. Color and Pattern

Most of the preservice teachers assimilated the patterns they created to the real life objects

Preservice mathematics teachers expressed their ideas as part of this theme in these excerpts:

The week we learned about the curve command, Bülent had been promised tea and simit (Turkish bagel) by you. I had an idea and shared it with members of my group. We explored how we could end up with the shape of a simit. (Ö 25) (FIG. 9.)

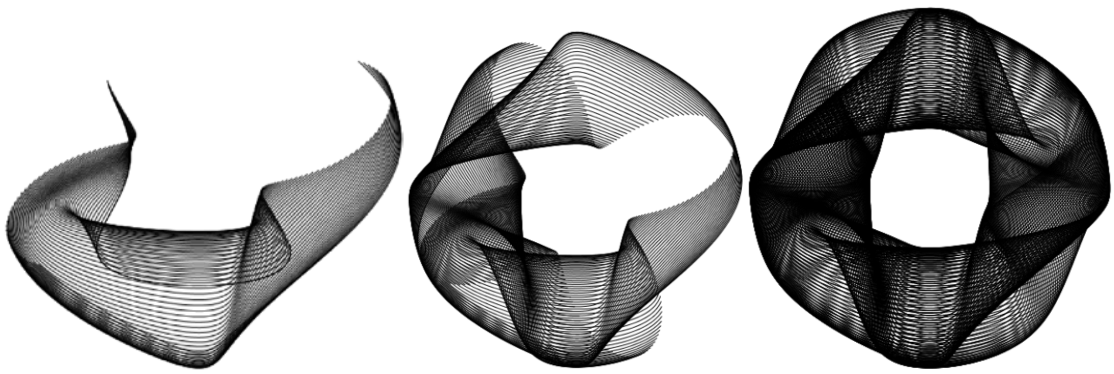

Fig. 9. Simit (Turkish bagel)

We felt that the generated shape resembled decorations used for New Year's Day celebrations. The images on the screen of algebra and graph are given in Fig. 10 (Ö 29):

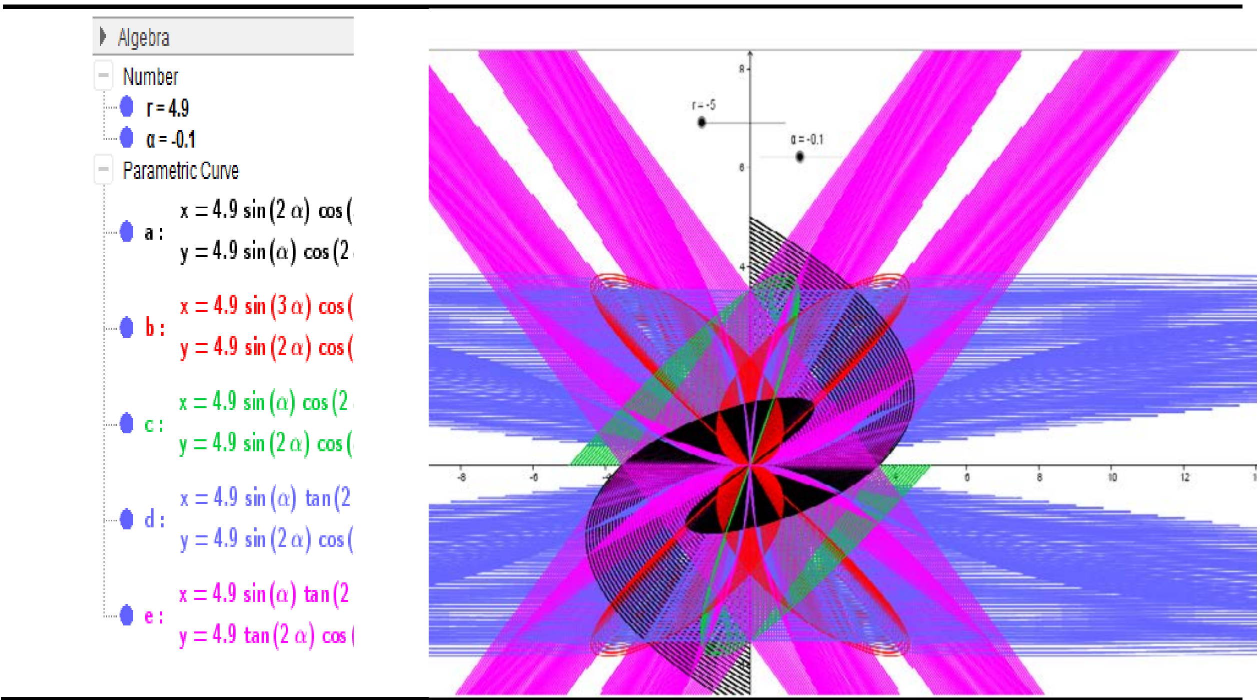

Fig. 10. New Year's Day celebrations.

This pattern resembles the head of a flying dragon.

F(x)=1-sin(x+b). tan(x) (Ö 1) (Fig. 11)

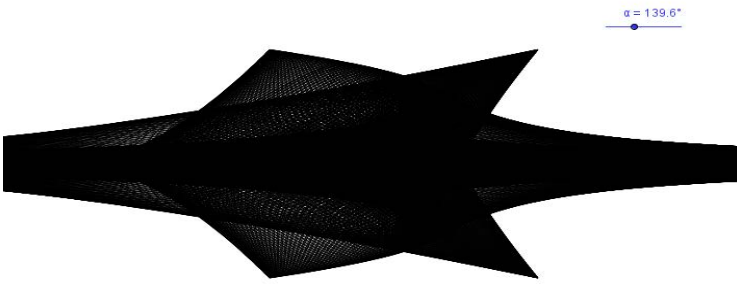

Fig. 11. Flying dragon

When we rotated the curve we obtained this time, we realized that it formed the pattern of a tulip. (Ö4) (Fig. 12)

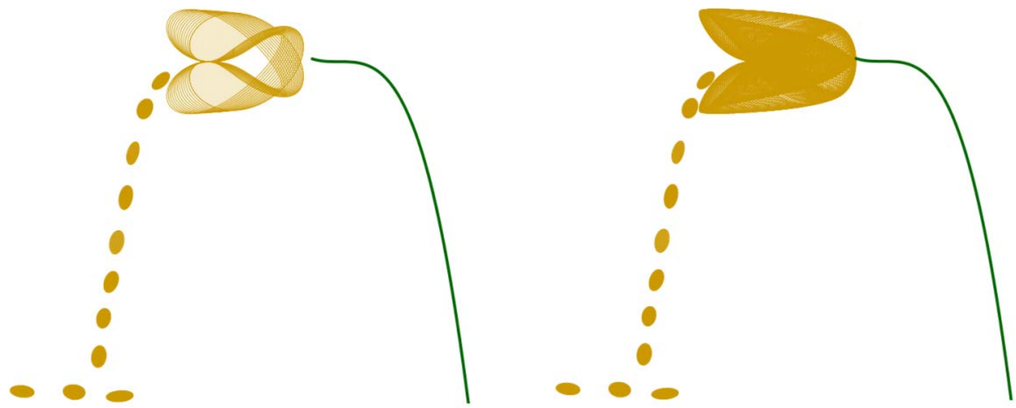

Fig. 12. A tulip.

Although it was stated to the preservice mathematics teachers that project expectations were for them to create visually pleasing shapes, as seen in their submissions a majority of the preservice teachers were striving to convert the shapes they generated into those that resembled real-life objects.

DISCUSSION AND RESULTS

Mathematics teachers and instructors are attempting to develop various strategies to make mathematics interesting and meaningful (Penas & Guzon, 2011). Several studies indicate that classroom work may be made more enjoyable using dynamic geometry software (Furner & Marinas, 2011; Kutluca & Zengin, 2011; Sendova & Grekovska, 2005; Wakwinji, 2011). Therefore, teachers should be attentive and use GeoGebra efficiently in their classrooms (Baltaci & Yildiz, 2015). In addition, for having more fun, some enjoyable activities designed by students using dynamic geometry software will also help them get a sense of appreciation of mathematics as art (İsmail & Kasmin, 2007). As described in the study findings, the preservice mathematics teachers have carried out a study to prepare aesthetically pleasing visuals and patterns which they found to be interesting. Moreover Penas and Guzon (2011) visualized certain learning, teaching and research activities through the use of applicable technological tools and the utilization of tessellations and designs. In the same study, Islamic tessellations were used as part of establishing connections among arts, history, culture and abstract mathematical concepts. Consequently, they emphasized that the various histories and cultures of the world hold immense potential resources for studies to be conducted in mathematics and other domains of knowledge. Thanks to technological advances, such connections are easily established.

In this study preservice mathematics teachers have been able to enter into intensive effort on algebraic and geometric representations and have been able to observe the relationships between these two representations when inputs were changed thanks to the GeoGebra software features. Thus, the preservice teachers were able to see both graphics and algebra at the same time; when they made alteration in one window, they could see the change in the other window at the same time. Contrary to some other software, the diversity of GeoGebra software is one of its important roles as observed in this study. Because GeoGebra is a free and dynamic software that integrates algebra and analysis in a single package (Baki, Yildiz, & Baltaci, 2012; Tatar, Akkaya, & Kağizmanli, 2014), educators should benefit from this software for a more efficient learning and teaching process.

Cuoco and Goldenberg (1996), Laborde et al. (2006) and Olive (2002) have concluded that the learning process taking place within an environment incorporating dynamic geometry software is not a process where information is provided and is then simply received, but one where individuals form their own knowledge of geometry or where they re-fashion their existing knowledge. It has been observed that this process has also been experienced by the preservice teachers taking part in this study; some preservice teachers were able to pick up the mathematics concepts from the acquired information for their design, and most of them were able to use the acquired mathematical content in their designs by constructing with their own knowledge and accessing different information sources.

Dikovic (2009) and Lachmy and Koichu (2014) pointed out that various mathematical structures can be explored using GeoGebra software. This situation supports the judgements like "some preservice teachers gained self-esteem in using GeoGebra by improving their abilities and the preservice teachers tried aesthetical designs". Most of the preservice teachers gained enough motivation with this kind of application and conditioned themselves to create the best design. These teachers made an effort to use GeoGebra software in parallel with their imagination. In the related literature, it is stated that the students were more eager and did not get bored (Chrysanthou, 2008) and became more motivated by using this kind of software (Bakar, Ayub, Luan, & Tarmizi, 2010; Tezer & Kanbul, 2009).

Arcavi and Hadas (2000), Gonzàlez and Herbst (2009) and İsmail and Kasmin (2007) have stated that in addition to visualization, dynamic geometry software contributes to student learning through experience, and as a result, students perform not only observations but also activities such as measurements, comparisons and modifications to figures. As stated by the preceding studies, preservice mathematics teachers have performed in a dynamic environment such activities as performing measurements, comparisons, and modifying figures. According to Karataş and Güven (2008), computer assisted environments allow students to make assumptions, test such assumptions and make generalizations. Making assumptions and testing them were frequently observed in the applications created by the preservice mathematics teachers in this study. Our results suggest that preservice mathematics teachers facing such creative tasks where constructivist applications are found will have motivation for classwork, will seek to create original designs, will enter into research and analysis activities and will spend more time on the significance of concepts.

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