Supporting prospective teachers to notice students' mathematical thinking through rehearsal activities

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In recent years there have been calls for ambitious mathematics teaching which places student thinking and reasoning at the centre of instruction. Drawing on a larger study concerning implementation of practice-based pedagogies within our initial teacher education mathematics programme, this paper examines the range of opportunities for prospective teachers to practise and explore the role of professional noticing within rehearsal activities. We illustrate how the rehearsal process served to highlight components of professional noticing, namely, making students' thinking visible, eliciting and responding to student thinking, and connecting to mathematical ideas. In looking at exemplars of each of these components, we illustrate how the mathematics teacher educators' coaching moves—prompted by their professional noticing of prospective teachers' learning—supported learning. We conjecture that the learning community's collective exchanges within the rehearsals affirmed more than the 'desired' teaching orientation towards professional noticing; these efforts also made explicit the 'how', 'why' and to 'what effect' such an orientation has on diverse mathematics learners.

Keywords professional noticing· practice-based pedagogies· coaching· rehearsals

Introduction

Attending and responding to students' mathematical thinking is at the heart of pedagogical reforms in mathematics education. Variously described as "ambitious," "dialogic," "responsive" (Stylianides & Stylianides, 2014), "responsible" (Ball & Forzani, 2011), and inquiry-based (Alton-Lee, Hunter, Sinnema, & Pulegato-Diggins, 2011), reform pedagogies emphasise "building on students' existing proficiencies rather than filling gaps in students' knowledge and remediating weaknesses" (Cobb, in foreword of Anthony & Walshaw, 2007).

Instruction that values and respects all students' mathematical thinking, that places students' mathematical thinking and reasoning at the centre of decision making, is ambitious in both agenda and enactment. In particular, learning to listen effectively and respond to the multiplicity of factors specific to students' thinking "is surprisingly hard work" (Empson & Jacobs, 2008, p. 257). For example, when observing or interacting with a student the teacher might attend to: what the student is thinking about and what is important about it; how the student is interacting with a task; what interests and motivates a student; how the student's understanding is developing; what makes the ideas difficult and what does that student already know that might offer a bridge, and so on. As Burton (2004) noted:

> It is not easy to organise a classroom where the mathematics is not prescribed but is generated through the activities of the students and where it is the responsibility of the teacher to help draw the mathematics out of the activities, help the students to interrogate the many different forms of it which they offer, and expect student involvement in the process of questioning, justifying, challenging and reflecting. (p. 372)

Engaging deeply with students' thinking necessitates complex relational pedagogies centred on the creation of robust learning opportunities for students to be oriented to peers' ideas and to the big ideas in mathematics. Developing communities of mathematical inquiry in which students learn to explain and justify their thinking, communicating with and responding to the arguments of others, requires rich discourse intensive practices (Walshaw & Anthony, 2008). While complex and difficult to enact, many studies have found learning within communities of mathematical inquiry to be particularly productive for mathematics students who have been previously disenfranchised or disengaged (Alton-Lee et al; 2011; Ghousseini & Herbst, 2014; Hunter & Anthony, 2011; Staples, 2008). In light of this finding, the goal of supporting prospective teachers (PTs) to understand and learn to enact instructional practices associated with mathematical inquiry learning communities is particularly urgent.

However, as noted above, specific instructional practices that build on students' mathematical thinking have been found "to be complex and difficult to understand and to enact" (Leatham, Peterson, Stockero, & van Zoest, 2015, p. 89). We, along with others (e.g., Ball & Forzani, 2011; Sherin, Jacobs, & Philipp, 2011) argue that attending, interpreting, and responding appropriately to students' mathematical thinking is a specialised pedagogical skill that needs to be explicitly taught within teacher preparation programmes. In this paper we aim to add to the knowledge base about how mathematics teacher educators can support PTs to learn the work of ambitious teaching. Drawing on a series of rehearsal opportunities within our mathematics methods courses we illustrate how the rehearsal process can afford opportunities for PTs to practise and learn about how to listen and respond to students' mathematical thinking. We have chosen exemplars that highlight how teacher educators, acting as coaches in-the-moment, can link particular teaching moves to the core of our vision of ambitious teaching—being responsive to mathematics and students. We explore how the use of the learning community's (PTs and teacher educator) collective efforts to highlight and justify particular teaching moves associated with professional noticing affirmed more than the 'desired' teaching orientation towards professional noticing; these efforts also made explicit the 'how', 'why' and to 'what effect' such an orientation has on diverse mathematics learners. To situate our discussion, we first review the literature on professional noticing, and then provide a summary of the theoretical framing of practice-based teacher education that informed our practice-based pedagogies associated with rehearsals.

Professional Noticing

Noticing student thinking is an active and intentional act (Mason & Davis, 2013) that is considered essential for effective teaching (Miller, 2011). In mathematics education, professional noticing involves two main components: attending and making sense (Philipp, Jacobs, & Sherin, 2014). Jacobs, Lamb, and Philipp (2010) contend that teacher noticing actions progress through three interrelated phases: *attending, interpreting*, and *deciding*. Attending involves noting actions most significant to the mathematical learning at hand—be it in terms of gestures or representation. Interpreting involves coordinating the observed actions with what is known about mathematical development in the area of focus. Deciding refers to the response to the interpretation of students' mathematical thinking. Others descriptions of professional noticing frameworks include *eliciting*, *supporting,* and *extending* (Fraivillig, 2001) or *eliciting* and *interpreting* students' mathematical thinking (Sleep & Boerst, 2012). Irrespective of the terminology to describe the phases of noticing, the assumption is that teachers who notice student thinking and make decisions based on what is noticed will occasion learning grounded on what students know and need to learn.

The valuing of professional noticing in mathematics education is matched by a growing area of research focused on understanding how and what is noticed and how noticing can sustain or generate professional learning. In all cases, research findings link professional noticing with teacher expertise, specifically with adaptive expertise that enables teachers to "innovate where necessary, rethinking key ideas, practices, and values in order to respond to non-routine inputs" (Lampert, 2010, p. 24). For example, Choppin (2011) examined what teachers noticed as they observed and reflected on enactments of challenging tasks. Those teachers who attended closely to students' thinking were better able to adapt tasks in order to maintain the task complexity associated with students' opportunities to engage with the mathematical concepts. Moreover, Choppin, like others (e.g., Anthony, Hunter, & Hunter, 2015; Kazemi & Hubbard, 2008) found that when teachers in the classroom involved others (peers and mentors) in their inquiry into student thinking they were able to develop teacher knowledge that helped transform their practice.

Another major strand of research has focused on the development of professional noticing practices of inservice and preservice teachers. Multiple studies (e.g., Davies & Walker, 2005; Jacobs et al., 2010; McDuffie et al., 2014; Star & Strickland, 2008) concern the development of an orientation towards professional noticing through the use of observation frameworks that press teachers to focus their attention on more complex aspects of teaching and learning. Most early studies involved teachers using these frameworks to view videoed episodes of teaching. These efforts to conceptualise and study teacher noticing have contributed to the more recent attempts "to decompose the practice of teaching into specific components that might be studied and learned" (Philipp et al., 2014, p. 465). Philipp et al. are careful to point out that professional noticing differs from constructs such as knowledge and beliefs—noticing is "an interactive, practice-based process rather than a category of cognitive resource" (p. 466). These researchers claim that the conceptualisation and study of teacher noticing has contributed to recent efforts to decompose the practice of teaching into specific components—often referred to as core or high-leverage practices (Forzani, 2014)—that might be studied and learned.

In terms of decomposition of practice, Leatham et al. (2015) offer an important and significant piece of research that focuses our attention, not so much on the process of professional noticing but on the 'what' is noticed—in terms of understanding how teachers might learn to notice and engage with instances of students' thinking that "have considerable potential at a given moment to become the object of rich discussion about important mathematics ideas" (p. 90). They offer a research-based framework that highlights those instances of students' thinking that are linked to significant mathematics and supports us to recognise when this thinking creates a pedagogical opportunity to build on that thinking to support student learning. In doing so, they make the important caveat that listening to students' thinking must be accompanied by "thoughtful consideration of whether a particular idea or comment is worth pursuing in the limited amount of instructional time that is available" (p. 121).

Practice-based approaches to learn the work of professional noticing

The complex relational practices and goals of ambitious teaching, and associated professional noticing, have implications for the process of learning to teach. In looking at beginning teachers' disposition to notice students' thinking, Van Zoest, Stockero, and Kratky (2010) found that even when teachers were able to utilise pedagogical practices that made students' thinking visible, in most cases the teachers did not then "engage the students in the kind of mathematical thinking that would transform their learning" (p. 48). Rather than using professional noticing to engage students in active inquiry and sense-making, the beginning teachers were more likely to use professional noticing to make classroom instruction more efficient. For professional noticing to become part of beginning teachers' repertoire of instructional strategies, these researchers recommend that teachers educators must become more transparent in their implementation of this practice "so that our prospective teachers are aware of what they are experiencing" (p. 49). This goal is all the more significant given that the communal classroom practices and inquiry learning opportunities associated with eliciting and responding to students' thinking are unlikely to be in the histories of PTs' school experience, nor readily evident in many of their field-based experiences (Anthony et al., 2015).

In moving to support ambitious teaching practices one significant strand of teacher education reform is represented in efforts to ground professional learning in practice by structuring experiences "around the critical tasks and problems that permeate teachers' daily work" (Ghousseini & Herbst, 2014, p. 2). In supporting PTs to learn about these complex practices, and more importantly to learn how *to do* these complex practices, initial teacher education has witnessed a turn towards practice-based approaches that "view teaching not only as a resource for learning to teach but as a central element of learning to teach" (McDonald et al., 2014, p. 500). According to Grossman, Hammerness, and McDonald (2009) practice-based instruction draws on three pedagogical approaches: representation of teaching (e.g., modelling, examining video or written case exemplars); decomposition of practice (e.g., focus on core/high–leverage practices); and approximation of practice (e.g., microteaching, rehearsals). A combination of these approaches, researchers claim, can provide opportunities to occasion shifts in PTs' beliefs (professional visions) about teaching and support productive dispositions for teaching and learning, while simultaneously providing opportunities to learn the work of ambitious teaching practices, including professional noticing.

In mathematics education, research associated with the *Learning in, from, and for Teaching Practice* (LTP) project (see Lampert et al., 2013) provides us with what is arguably the most sustained study of practice-based initial teacher education to date. A key feature of the LTP project is the use of rehearsals as a designed pedagogy for approximating the complex relational character of ambitious teaching. Linked to professional noticing, these researchers found that rehearsal activities provided opportunities for teacher educators and PTs to work together on specific aspects of practice while simultaneously attending "to the variations of the practice as it relates to particular students and mathematical goals" (p. 238). Analysis of rehearsal exchanges confirmed that eliciting and responding to students' ideas characterised a large portion of the collaborative work of the PTs and teacher educator within each rehearsal activity. Moreover, the LTP researchers argued that the very nature of the rehearsal process—the modelling of practice and the consideration of specific and alternative moves about aspects—acted to scaffold adaptive expertise development associated with professional noticing.

Other studies also utilise practice-based approaches to focus on key components of mathematics pedagogy that involve professional noticing. For example, Ghousseini and Herbst (2014), in the context of a secondary mathematics methods course, investigated how PTs learnt to perform instructional practices associated with orchestrating mathematical discourse while being responsive to the mathematics and students' thinking. They provided an example of how working with a constructed dialogue sequence provided opportunities for PTs to make sense of students' thinking and identify the mathematical ideas that were critical to the agenda of steering the discussion towards multiplicative thinking. When invited to insert teacher dialogue into the sequence, PTs' choices of moves were linked explicitly to their understanding of students' thinking and potential mathematical trajectories that they were opening up. Using a different focus, Santagata and Yeh (2014) studied PTs' use of video-records to assess their teaching. They noted that those PTs who systematically attended to students' thinking were more likely to use evidence of students' learning in the analysis of their own teaching effectiveness—a practice that they claimed would support continued professional growth.

In all of these studies the researchers noted that the practice-based learning opportunities supported the PTs to solidify their understanding of the instructional moves associated with classroom discussion centred on students' thinking and also to articulate the dispositions that guide their use. That is, the pedagogies of practice (inclusive of representations, decompositions, and approximations of practice) created opportunities to learn "not only knowledge of content and students, but also specific techniques and routines to manage that work, elements of the professional vision of teachers, and a sense of the complexities involved in the work of teaching" (Ghousseini & Herbst, 2014, p. 23).

Study Context

Our design study (Kelly, Lesh, & Baek, 2014) involved the trial and evaluation of new practice-based instructional strategies and tools that would support PTs learning the work of ambitious mathematics teaching. Our project took as its starting point those practices identified as key to the principles and vision of ambitious mathematics teaching. Most notably, pedagogical practices that place students' mathematical thinking and reasoning at the centre of instruction and support equitable engagement of diverse learners in rich mathematical activity: teaching towards a big mathematical idea, eliciting and responding to students' ideas, and positioning students as competent.

In designing our pedagogical and curriculum intervention we took our lead from the *Learning in, from, and for Teaching Practice* (LTP) teacher education project (see Kazemi, Franke, & Lampert, 2009; Lampert et al., 2013). The LTP practice-based approach involved opportunities for PTs to engage in "approximations of practice" (Grossman et al., 2009, p. 238) activities within cycles of enactment and investigation. A key innovative feature of the design—and the context for discussion of professional noticing within this paper—was the use of public rehearsals. In a rehearsal, the PT is responsible for teaching an Instructional Activity (IA) (see Kazemi & Waege (2015) for descriptions of the IAs) to a group of peers acting as students, with the teacher educator acting as coach. The rehearsal activity was supported by first having PTs observe and analyse the teacher educator's teaching of an Instructional Activity. This modelling/observation phase provided opportunities for the 'representation' and 'decomposition' of practice (Grossman et al., 2009).

PTs' teaching of the IAs was enacted in two different designed settings—rehearsals in the mathematics methods course and small group teaching in a school-based setting. These settings enabled the PTs to teach under conditions of controlled complexity and receive feedback from peers and teacher educator. Importantly, the in-the-moment coaching by the teacher educator was used to scaffold the learning of practice. This was achieved in multiple ways: stepping in and modelling aspects of practice; suggesting alternatives moves to retry; prompting teacher or peer group reflection related to students' thinking, learning, and participation; asking for teacher explanation of teacher moves in order to highlight effective practice; or putting an idea on the table as a student that the teacher has to address. In a nutshell, teacher educator scaffolding was intended to press the rehearsing teacher and PT peers to think more deeply in an evidence-based way about what he or she was doing in relation to opportunities for each student's learning and participation.

The changing nature of the IAs that were used (e.g., choral counting, quick images, strings) and the sequential timing of the rehearsals in terms of the PTs' learning progression meant that rehearsal afforded opportunities to practise and discuss differing aspects of ambitious pedagogy (Hunter, Hunter, & Anthony, 2013).

Data collection and analysis

The aim of the larger study was to examine how practice-based pedagogies and curriculum could support PTs learn the work of ambitious teaching. In seeking to understand how the rehearsal process can support opportunities to learn our focus in this paper is on the practice of professional noticing. We draw on a subset of our data set involving PTs in the first semester of a one year graduate primary teacher education programme. The data set includes 16 video records of in-class rehearsals and 16 audio records of post rehearsal interviews.

In this paper it is not our intention to present detailed coding of the substance of the rehearsals. Rather, we have reviewed each of the videotaped rehearsals with the goal of sourcing exemplars of a range of ways that professional noticing played out within rehearsal, including analysis of how the rehearsal process—with input from peers and the teacher educator coach—supported PTs to learn the work of professional noticing. Specifically, our analysis concerns the identification of: (i) aspects of professional noticing that arose within rehearsals and (ii) coach moves associated with collective exchanges centred on professional noticing.

Multiple analytic passes of the videotapes resulted in the following broad categorisation of instances of professional noticing episodes: making students' thinking visible; eliciting explanations and building on the reasoning; unpacking incomplete or erroneous student responses; and making connections to the mathematical ideas. Our findings present exemplars of each of these categorisations. In each case, the discussion of the exemplars serve to illustrate how the community interactions concerning aspects of professional noticing was supported by specific coaching moves. We have notated the participants within the rehearsal interactions as follows: mathematics teacher educator acting as a coach (C); the prospective teacher taking the teacher role in the rehearsal (RT); the prospective teacher taking the role of a student in the rehearsal (RS); and participants inclusive of observing peers (PTs).

Findings

Making students' thinking visible

Making a student's thinking visible and able to be understood by peers is an important aspect of ambitious mathematics teaching. As teacher educators we were aware that, for the majority of our PTs, ambitious teaching with its explicit focus on building on students' mathematical thinking would collide with PTs' beliefs constructed through their past experience. Therefore, it was important that the IAs selected for the initial rehearsal phase provided multiple opportunities for the PTs to experience the act of making mathematical thinking visible, both in the role of a rehearsing student (RS) or rehearsing teacher (RT). Quick Images, selected as our first instructional activity, supported PTs to learn to both provide and to elicit a range of different mathematical explanations representing their count of the dots within an image. In addition to providing explanations, the act of 'collecting' explanations enabled PTs to learn about a wider range of possible explanations that they had thought of in their planning phase.

More importantly, analysis revealed that the early rehearsals occasioned multiple opportunities for PTs to learn 'how to' intervene to make students' explanations accessible to other members of the community through the use of talk moves and recordings of students' thinking as evident in the following episode from a Quick Image (see Figure 1) activity. We pick up the rehearsal with a RS's explanation for the count of 12 dots:

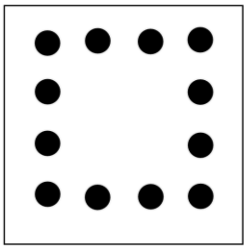

Figure 1: Quick Image

RS: I saw a square and I went two, two, two, two and that was eight and then I went around the corners and added the rest.

C: Pause. So RT this would be a good time to say to RS so how did you get to eight? Because now you want to know, because she is saying two, two, two, two, but how do you suddenly get to eight, so you want to clarify that.

RT: [to RS] Yeah okay, so you went, which twos did you go? These two? [goes to image and indicates middle dots].

RS: Okay, so I saw the top two in the middle [RT points to dots]. And then the two at the sides

In this instance, the coaching action [suggestion to press for clarification] provided the rehearsing teacher and other members of the community with real-time opportunities to see the effect of RS being helped to make clear and accessible mathematical explanations. Following the coach's modelling the rehearsing teacher took the opportunity to rewind her teaching and incorporate this new instructional move. The result was that RS expanded her original mathematical explanation thus making her reasoning more visible to other listeners. Importantly, we see that the rehearsing teacher was visually linking the RS's explanation through the use of gestures—evidence of practising active listening and interpreting the student's thinking.

Eliciting mathematical explanations and building on the reasoning

Typically in the earlier rehearsals, the rehearsing teacher, after eliciting a student's solution, would spend additional time engaging in a discussion with that individual, or would quickly gather more solutions without responding to any of the given solutions or exploring the connection between solutions. The following episode from a Quick Image activity illustrates how coaching was used to surface discussion about how to enrich the eliciting process and press for reasoning from all participants. We pick up the rehearsal immediately after the rehearsing teacher has elicited two different students' solution strategies. After recording the second rehearsing student's solution strategy of adding four groups of two dots the rehearsing teacher addressed the whole group as follows:

RT: Does everyone understand how RS got that? Were there any other ways?

C: Pause. When you say, "Does everyone understand" how can you ensure that everyone understands?

RT: [long pause] Get them to repeat it?

C: Yes, repeating is one way. What is something else that RT could do [directed to all the PTs]?

PT1: Talking to a partner.

PT2: Asking if they disagree or agree.

In this episode we see that while the rehearsing teacher was aware of the value of all students understanding the discussion, the coach move pressed the PTs to reflect on ways that a teacher can make this understanding visible. Wait time within the rehearsal prompted PTs to consider the possibilities of talk moves associated with turn-and talk and repeating (Kazemi & Hintz, 2014)—instructional moves that press students to engage in mathematical explanations. Incorporating these moves as the rehearsal continued provided PTs with opportunities to see how they could elicit students' thinking in ways that enabled the explanations to act as reflective tools for both the student and their peers.

In another rehearsal, later in the sequence, we see how the eliciting process is extended from having peers engage with a particular response, towards using the response as a building block to extend the discussion. We enter the rehearsal, based on a choral count which involved counting in fives beginning from one (see Figure 2), immediately after the rehearsing teacher records RS1's suggested pattern of "55 being added to each number":

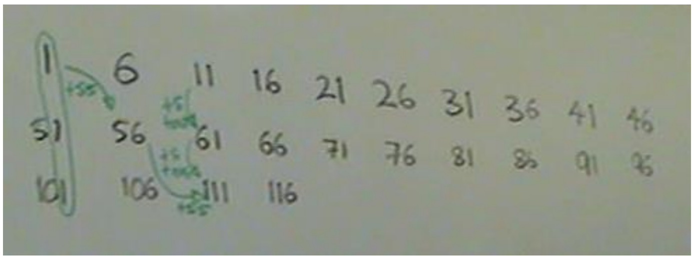

Figure 2: Choral count pattern

RT: That's good. Does anyone have another pattern?

Noticing that the rehearsing teacher was focused on recording a list of observed patterns devoid of an explicit press for others to engage with and build on their peers' reasoning, the coach suggested that the teacher consider how to draw other students into the discussion:

C: Pause. That's quite a complex idea and it might be one which you want to throw back to them and say does everyone agree? Like, "let's look at what RS1 said; he said that they increase by 55. Do you agree, why or why not"?

RT: Right, I would like you all to have a think about what RS1 just shared with us because that is quite a complex idea and think about what RS2 said at the start about how he adds five and somebody else said that when we are going down we are adding five tens so think about that, adding five [pause]. *Oh I am giving it away aren't I?* Have a chat to your neighbour about how that works.

After the rehearsing students had talked for a few minutes the rehearsing teacher asked them to share their ideas:

RS3: If you go across it is plus 5 and then going down is five tens so 5 times 10 is 50 so the 5 plus the 50 is 55 [RT notates the explanation with arrows].

RT: So that way is the same as those two? Is that what you are saying [notating the explanation with arrows]?

RS3: Yes you can add them together.

RT: Great.

C: Pause. You know you said *I am kind of giving it away* but actually what I think RT did was you really structured it so they could work out why that pattern was because if you had just said just look at it, with Year Fours they may not have seen it. You didn't say what you need to do is…, but you said look at that idea, and look at that idea, and that gave a foundation for them to then see that and use that, so that was a good thing to do.

We see in this vignette how the use of the coach's suggestion enabled the rehearsing teacher to facilitate students to engage with each other's reasoning. The rehearsing teacher was initially concerned that she was providing her students with too much guidance. However, as evidenced in the vignette, by drawing on the student generated explanations she was able to scaffold students' structural understanding of the pattern.

Unpacking incomplete or erroneous student responses

While being able to explain and justify mathematical reasoning is a fundamental mathematical practice of proficient users of mathematics, we know that in the process of learning students will often provide partial and confusing explanations. Indeed, as PTs experienced more rehearsals the norms associated with engagement in sharing mathematical thinking shifted to the extent that the rehearsal students became more willing to take risks, and in doing so they offered more complex or incomplete, and sometimes erroneous, explanations. Thus rehearsal provided an ideal opportunity for PTs to notice and learn how to use student errors as an important resource (Lampert, 2010).

Our analysis revealed that the rehearsal provided multiple opportunities for PTs to explore a range of teacher actions that can be used to both respect and build on confusing, incomplete, or incorrect student contributions. For example, in the following Quick Image rehearsal the rehearsing teacher asked students to identify the similarities between the different explanations recorded on the whiteboard. The episode starts with a response from a rehearsing student:

RS: Everyone is using four and two.

RT: Yep, so everyone was using four and two.

RS: With the exception of RS1 [reference to another RS's earlier response].

C: Pause. RT I think this would be a good time to get RS to clarify what he means because he said 'everyone is using' so I think you need to say, 'what do you mean by using'?

RT: [to RS] What do you mean by using?

RS: The factors are four and two when you are multiplying, adding is two and one which actually gives us RS1's answer which is four plus three, plus three, plus two.

Noticing the confusion arising from the elicited response, the coach steps in again to suggest a number of alternative moves:

C: Pause. Did you understand what he is saying?

RT: No.

C: Pause. Okay, so if you didn't understand what he is saying then you know that probably everyone else is grappling with it. You need to revoice, or 'can you say that in a different way', or 'do you want to come and show us what you are talking about'? One of those.

RT: RS, do you want to come and show us what you are talking about?

Here the coaching move provided the rehearsing teacher with the opportunity to reflect and select the next action to trial and see the effect in real time In discussing the difficulty in understanding the student's explanation, the coach also took the opportunity to reaffirm that the interaction is not just between the teacher and student, reminding PTs of the expectation all participants in the learning community need to understand the explanation. In this case, the rehearsing teacher's revised move, requesting that the student explicitly link back to the different explanations recorded on the whiteboard, served to prompt further elaboration of his explanation in ways that were accessible to everyone.

As the rehearsal students became more confident in their roles they became more likely to offer partial solutions or conjectures and or simulate student errors. For example, in the following String rehearsal involving a linked set of multiplication calculations the rehearsing teacher asked the students to solve 35 x 5:

RT: Would anyone like to share their answer?

RS: One hundred and fifty-five.

RT: So RS you think it is 155?

At this point, the rehearsing teacher noticing the student error paused indecisively, and the coach intervened:

C: Pause. This is a really good moment to say agree, disagree, not sure. Don't indicate what the answer is.

RT: So does everyone agree, disagree, or are you unsure about the answer?

C: And now you need to say remember if you agree or disagree you have to have a mathematical reason, but RS may first want to say whether he agrees or disagrees with a mathematical reason.

Here the coach deliberately introduced a new alternative to the 'agree/disagree' talk move that had not surfaced in earlier discussion—that of allowing the contributor to disagree with their own response, to change their mind and reconstruct their reasoning. As the rehearsal proceeds RS takes up this option as part of his role play:

RT: So RS do you agree or disagree?

RS: Yes, I disagree with my answer now.

RT: Do you have a new answer or would you like more time to think about it?

C: Well done.

RS: One hundred and seventy five.

RT: And how did you get that answer?

RS: For some reason what I originally did was that I knew that 30 times 5 was 150 and I don't know why but I just added 5.

RT: Because you saw another five there?

RS: Yeah because I saw another five there and then when everyone disagreed I was wondering why. But then I clicked, so it is 5 times 5 and that is 25. So I know that 30 times five is 150 and I know that 5 times 5 is 25 because we did that before, so I just added 150 and 25 together to make 175.

In this vignette, we see how prompting the rehearsing teacher to withhold her evaluation of the response, combined with the offer of more time for the students to reconsider their thinking, resulted in RS reconstructing his thinking process. In this case the PTs were able to see in-the-moment how the teacher's response to the students' thinking meant that the erroneous thinking became a learning tool which supported reconstruction and justification of the reasoning using mathematics as the authority.

Connecting students' mathematical thinking and important mathematical ideas

In attending to students' thinking one of the challenges is how to keep the discussion focused on the *important* mathematical idea. The following episode from a Choral Count rehearsal involving a count in twos recorded, as shown in Figure 3, illustrates how the coach explicitly surfaced the need to connect students' mathematical thinking to a "mathematics point" (Leatham et al., 2015, p. 92).

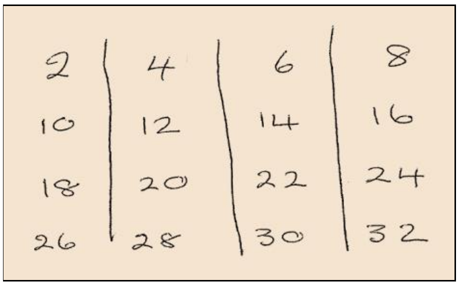

Figure 3: Choral count pattern

We enter the episode with the rehearsing teacher eliciting different patterns, supported by revoicing and students' elaboration of the solution strategies. A rehearsing student noted that the pattern increased by eight when looking down the columns:

RS1: [responding to RT's request to justify where the eight came from] It was ten take away two.

RT: Okay, so you say ten take away two and that's eight [recording the calculation in the first column of the choral count].

C: Pause. Try to think at this point about getting other students to agree or disagree. You are getting some interesting patterns here.

RT: Okay does anyone disagree with RS1's observation there? What do you think RS2?

RS2: I can see the same thing.

RT: You can see the same thing, so you agree with RS1.

RT: What do you think RS3?

RS3: Yes, and the second row seems to be the same, like 28 – 20 is 8.

RT: So you see it in the second row as well [marking the calculation up on the choral count].

C: Pause. So thinking about your questioning here, rather than just "do you agree or disagree", try a more structured approach. For example, taking what RS1 said, you could have said, "RS2 can you have a look at what RS1 said and see if that works in the fourth column?"

Here we see the coach prompting the PTs to reflect on what might be the bigger picture in getting students to disagree or agree. Noting that the rehearsing teacher's immediate response was to attend only to the previous student's single instance, rather than expecting all students to explore and make connections across the strategies, the coach pressed the PTs to consider how they could use this opportunity to link the rehearsing students' thinking to the generalisation of the pattern across the rows. In this instance, the coach engaged PTs in practice and reflection on how they could use talk moves to support students to "articulate a mathematical idea that is closely related to the student mathematics of the instance" (Leatham et al., 2015, p. 92).

Concluding a discourse rich task (such as Instructional Activities) activity typically provides a further opportunity to build on students' mathematical thinking by way of engaging with and making connections between the different elicited explanations and the key mathematical ideas and mathematical practices of the lesson (Smith & Stein, 2011)—a practice we termed as 'making the connect'. Our experience with the trajectory of rehearsals was that RTs found the exposition of the mathematical point of the lesson based on students' thinking difficult to construct in-the-moment. In looking for opportunities to surface occasions to focus on making a connect the following episode from a Choral Count rehearsal illustrates how the coach facilitated PTs' reflection on the 'connect' in action. We join the rehearsal mid stage at which point the rehearsing teacher has asked the students to discuss the similarities between two solution strategies ($4 \times 5 = 20$ and $4 + 4 + 4 + 4 + 4 = 20$). Receiving the response that they were the same except one involved multiplication and the other involved addition, the rehearsing teacher proceeded to ask for additional solution strategies. The coach instigated a pause as follows:

C: When you were using that, what did you want them to draw out?

RT: That this was the same as that [indicates the solution strategies on the board].

C: So just think about the kinds of questions that you could ask because you've kind of got halfway there but I don't think that it is explicit that doing this and doing this is the same. So let's just have a think, what could RT ask to push that a little further?

The coaching pause presses all the PTs to consider the reasons for students connecting strategies alongside the need for the teacher to link the students' thinking to a big mathematical idea. One of the rehearsing students responds as follows:

RS2: Well she could ask, "how many times did you need to add four in order to get the answer?"

C: Pause. So trying to find that connection there. Another thing that you might want to do so if you pull that out what RS said—that you have got 4 times 5 and then you have got (points to each 4 in the repeated addition)—would that work if you had other numbers? So then you are pressing them to generalise.

At this point the RT takes up the suggestion and the rehearsal proceeds:

RT: RS3, how many fours did RS1 have to add together?

RS3: She added five 4s.

RT: So I will give you another moment to talk to each other about how those two equations relate to each other.

Here we see how the coaching pause invited all the PTs to consider alternative teaching moves and provided the rehearsing teacher with space to consider her next move. This resulted in the students needing to engage and build on the two students' (RS1 and RS2) mathematical thinking.

Discussion and Conclusion

In this paper we have provided exemplars of different ways PTs can engage directly with the work of professional noticing within rehearsals. In addition, our analysis of the interactions associated with components of professional noticing highlight two important aspects of the rehearsal process. Firstly, there was a meta-level of noticing by the coaches; what was noticed and responded to in terms of occasioning learning opportunities changed with the enactment of more rehearsals. Secondly, it was evident that the nature of the coaching moves, and associated interactions, changed from initially being almost exclusively directive, to become more inclusive of the learning community. We discuss each of these features in turn.

Our retrospective analysis of the rehearsals provided evidence of multiple opportunities to highlight aspects of professional noticing, be they potential or actual opportunities. As coaches we initially focused on eliciting, then responding, and then building on and connecting students' thinking to the mathematical goals of the lessons. Scheduled early in the rehearsal process, Quick Images, in particular, supported PTs to develop an appreciation of the process of explaining one's mathematical thinking. The expectation that rehearsing students provide an explanation of how they arrived at a count of the dots within an image emphasised the PTs' own struggle associated with providing a clear and coherent explanation for what was often a relatively automatic response.

Also from our experiences in the early rehearsals it was apparent that the need to actively listen to peers' explanations was something new. Moreover, in the initial sharing the PTs expressed surprise at the diversity of responses. To facilitate opportunities to 'share and hear' other students' thinking, coaching focused on teacher listening and recording/revoicing of students' thinking and discussions around the engagement in routines such as 'turn-and-talk'. Real time experience of turn-and-talk provided important learning opportunities for the PTs to practise explanation and justifications. As one PT noted, "rehearsals allowed me to see the types of misconceptions students have as well as hear the way that students think about the ideas being discussed".

Once routines and norms for collaborative participation were established, the focus on professional noticing moved towards helping PTs learn the work of making students' thinking visible through instructional strategies directed at eliciting students' thinking. For example, coaches initiated pauses to consider a range of talk moves (Chapin & O'Connor, 2007) that would encourage students to share their thinking. From there, coaches moved to explicitly encourage PTs to work on how to respond to the students' mathematical thinking in ways linked to the mathematical goal of the activity and the big ideas of mathematics. We see in the following post rehearsal interview how a rehearsing teacher was becoming aware of need to learn how to respond to students' thinking:

> Thinking about what I should or could say every time a student said anything was really important…I think I have learned a little on how to ask questions and lead the discussion without evaluating the contributions right away.

We conjecture that our meta-noticing as coaches, informed by our knowledge of PTs' prior experiences and the trajectory of learning to notice, also influenced the nature of our coaching interactions. One can see in the early exemplars provided in the paper that interactions were typically directive in nature. That is, the rehearsing teachers were frequently directed to practise in real-time specified teaching moves—either as a next move, or in the form of a rewind and retry. However, as the course progressed attending to the work of professional noticing within the rehearsal process was more often supported by the wider communities of learners—inclusive of the teacher educator coach and the PT peers.

The establishment of a community of learners supported the shift in focus towards errors and risk taking. To counter PTs' initial difficulties around responding to non-responses, incomplete or incoherent thinking, errors, and orienting the participants to each other's reasoning coach moves shifted increasingly towards invited reflective commentary. As one PT noted in a post-rehearsal interview, opening up one's practice for public scrutiny involved an element of risk taking, but also was an important part of developing a culture of teaching as inquiry:

> When we first started, even though it's a group and you're teaching, and you're learning so I feel that you are on show, that you're going to be critiqued. But as I've done one or two of the lessons you don't feel like that, you just get in and you just forget about that. My thoughts are that if you make mistakes that's good. I'm here to learn, we're here to learn.

It was as if the process of making one's practice public within the learning community provided a productive way for PTs to engage in a mathematical inquiry community (Alton-Lee et al., 2011).

Building this community required that we took time to know our PTs and learnt to read their body language (such as signals for help, gestures, and worried looks). In our new role as coaches we needed to develop our understanding of when to step in and out during rehearsals; learning to both trust and use our community as a resource for learning. Central to this was a sense of reciprocity where we recognised that the learning was shared.

The rehearsals episodes presented in this paper highlight the complexity and challenges involved in learning to notice—from both the PTs' and mathematics teacher educators' perspective. Ambitious mathematics teaching involving relational practices that value and build on students' thinking while orienting them to each other's perspective requires teachers who are able to respond and adapt their teaching in-the-moment. Our role, as mathematics educators working with future teachers placed us in an agentic position in which we had the opportunity to connect theory to practice. We were able to offer our PTs a vision of ambitious mathematics and its enactment within the structure of their coursework, rather than the model of them learning new ways of teaching mathematics then learning how to implement it within the complex situation of the classroom.

In this paper, our examination of a curriculum and instructional design incorporating rehearsals demonstrates that developing expertise in professional noticing needs multiple opportunities to learn over time. However, in thinking about our professional noticing as educators—our meta-noticing—further research is needed to explore what teacher educators need to notice and build on when working with the PTs in rehearsals. There is a need to build on this emergent work to explore different practice-based instructional designs (McDonald et al., 2014), including assessment of specific IAs, and the range of ways they can support PTs learning the work of ambitious mathematics teaching.

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