

Mathematics Interventions for Students with Learning Disabilities (LD) in Secondary School: A Review of the Literature

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The purpose of our literature review was to extend and update Maccini, Mulcahy, and Wilson's (2007) review of the literature on mathematics interventions for secondary students with learning disabilities (LD). An extensive search of the research literature netted 15 research studies that focused on mathematics interventions for secondary students with LD. The findings are presented in terms of three main instructional approaches for improving the mathematics achievement of these students: (a) cognitive and metacognitive strategies for solving word problems; (b) use of representations to increase conceptual knowledge and problem-solving skills; and (c) Enhanced Anchored Instruction. Results include evidence of the efficacy of Enhanced Anchored Instruction and Solve It! We discuss additional results, implications for improving classroom practice and professional development, and recommendations for future research.

Keywords: Mathematics intervention, secondary students, learning disabilities, literature review

Introduction

Longstanding concerns about the ability of teachers in the United States to educate our nation's students in ways that enable them to be competitive internationally in mathematics has resulted in a nationwide push to develop more rigorous achievement standards (National Mathematics Advisory Panel [NMAP], 2008; Peterson, Woessman, Hanushek, & Lastra-Anadón, 2011). These concerns, coupled with low student performance in mathematics, have led to the development of more rigorous standards for teaching and learning mathematics (i.e., National Council of Teachers of Mathematics [NCTM], 2000) (Maccini, Mulcahy, & Wilson, 2007; Mullis et al., 2001). More recently, states across the country have implemented Common Core State Standards-Mathematics (CCSS-M) that require students to use a variety of cognitively demanding math skills, procedures, and knowledge to solve cognitively complex mathematical problems (Powell, Fuchs, & Fuchs, 2013). However, rigorous expectations from those standards present considerable challenges for secondary students, particularly those with learning disabilities (LD; Maccini et al., 2007).

Many students with LD lack essential foundational skills (i.e., number fluency, knowledge of fractions, and mathematical reasoning) necessary for them to attain more rigorous standards such as CCSS-M (Powell et al., 2013). As a result, these students continuously struggle to meet expectations (Maccini et al., 2007). For

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example, data from the National Longitudinal Transition Study-2 (NLTS-2) revealed that only 26% of students with LD in secondary schools scored at or above average on math calculation problems, and 15% scored at or above average on math applied problems (Wagner, Newman, Cameto, & Levine, 2006).

Poor performance of secondary students with LD may be linked to teacher-related factors such as ineffective instruction (Fuchs et al., 2005; Gersten, Jordan, & Flojo, 2005; Merritt, Rimm-Kaufman, Berry, Walkowiak, & Larsen, 2011), as well as teacher's inadequate understanding and implementation of research-based instructional adaptations (Maccini & Gagnon, 2006). Moreover, low performance of these students may be related to instruction that is often skill-focused, rather than focused on conceptual and problem-solving knowledge; the foundation for most state assessments (Kurz & Elliott, 2012). Clearly, if students with LD are to perform successfully on assessments that tap CCSS-M, teachers must rely on evidence- and/or research-based instruction focused on basic skills, as well as and the conceptual understanding and problem-solving required by assessments of the CCSS-M.

It is imperative that pre-and in-service teachers are aware of instructional approaches that promote these students' mathematics achievement (Hughes, Maccini, & Gagnon, 2003). However, in almost three decades, only two literature reviews have focused solely on secondary mathematics instructional interventions that included students with LD (Maccini & Hughes, 1997; Maccini, et al., 2007). In the years spanning 1988-2007, Maccini and colleagues found only 43 studies that used either a group or single subject design to assess the efficacy of math interventions. Studies within the two reviews were categorized as behavioral (n=10), cognitive (n=20), or alternative delivery system (n=13). The majority of interventions in these studies were effective in promoting assessed student outcomes. Also, from the first to second review, the number of studies focused on conceptual knowledge or some combination of conceptual and procedural knowledge had increased. There was also an increase in the number of studies addressing algebra, geometry, and problem-solving. The focus on more rigorous content and problem-solving skills reported in the most recent review represents a clear shift away from research on teaching basic computation and algorithmic procedures (see Maccini & Hughes, 1997). Maccini and colleagues concluded that a research base was beginning to emerge that could inform instruction on more rigorous mathematics standards for students with LD.

Also evident in the Maccini et al. (2007) review, was the considerable diversity that existed in terms of the types of interventions and methodologies used, making it difficult to identify specific interventions for teachers' use. For example, in the largest category, cognitive interventions, there were five instructional approaches represented, including: (a) mnemonic instruction to help students remember how to execute a particular algorithm, (b) graduated instructional sequences that taught students to use concrete and visual representations to solve numerical equations, (c) cognitive strategies for solving word problems, (d) schema-based strategies to help students develop the mental structures for solving particular types of word problems, and (e) self-monitoring strategies to improve attention while problem-solving. Finally, across categories, only 13 of the 23 studies used group designs to assess the impact of interventions, and six of these involved random assignment of students to control or treatment. The remaining studies did not account for the nested structure of their data, and two of the studies did not include students with LD. As a result, only tentative conclusions can be drawn about the efficacy of available interventions for helping students with LD to achieve more rigorous mathematics content standards.

Purpose Statement

Clearly, results of the Maccini et al. review (2007) demonstrate a need in special education to conduct more research on mathematics interventions that enable students with LD to achieve more rigorous mathematics content standards. The field of special education first needs to understand the extent to which the research that has been conducted since the Maccini et al. review is sufficient to develop a knowledge base for educating secondary students with LD on more rigorous content standards. Therefore, the purpose of our literature review is to examine the research that has been conducted since the 2007 review by Maccini and colleagues. Unlike Maccini et al. (2007), we focus our review only on those studies that include students with LD, so that we can better understand how to intervene effectively with these students. Research questions guiding our review include:

1. What strategies promote the math achievement of students with LD and, how do these strategies support previous literature?
2. What new practices have emerged as a result of this review?
3. To what degree are studies promoting interventions that would improve the mathematics performance of students with LD on rigorous content standards (e.g., CCSS-M)?
4. To what extent do studies adhere to indicators of high quality research?

In sections that follow, we: (a) describe methods; (b) describe inclusion criteria, (c) summarize individual studies; (d) discuss limitations; and (e) discuss implications for classroom practice, future research, teacher preparation, and professional development.

Method

Criteria for Inclusion

To be included in this review, studies had to meet these criteria: (a) published in a refereed journal between June 2006 and October 31, 2014; (b) included students with LD in secondary schools (i.e., grades 6-12); (c) involved an academic intervention as an explanatory variable (e.g., math instructional program, a problem-solving strategy, computer assisted instruction, etc.) and measures of math performance as the dependent variable(s); and (d) utilized a single-subject or group design (i.e., quasi-experimental and randomized controlled trial).

A study was excluded for the following reasons: (a) included students with disabilities, but those students were not clearly identified as students with LD, (b) did not include students with documented LD (e.g., students with or at-risk for mathematics difficulties), (c) conducted outside the U.S., (d) investigated test-taking strategies or the use of calculators on mathematics achievement, (e) compared participants' response on a measure of academic performance without implementation of an intervention, (f) utilized a research design (i.e., correlation, descriptive and case studies with no quantitative results) that could not establish causal relationships between dependent and independent variables, and (g) was not published in a refereed journal (e.g., dissertations, government reports, technical reports).

Search Procedures

Three standard literature search procedures were utilized to locate pertinent articles for this review. First, a Boolean logic was employed to conduct a systematic search of key electronic databases including EBSCOhost, OMNIFILE Full Text Mega (H. W. Wilson), Academic Search Premier, Education Full Text (H. W. Wilson), and PsychInfo. The following Boolean logic was used: *"learning disabilities" or "math disabilities" or "high incidence disabilities" or "math difficulties" or "math* learning disabilities" AND "intervention*" or "math* program" or "math* instruction" or "math intervention*" AND "middle school*" or "high school" or "secondary school" or "secondary settings" or "adolescents" or "grade 6" or "grade 7" or "grade 8" or "grade 9" or "grade 10" or "grade 11" or "grade 12"AND "math performance" or "fractions" or "math academic achievement" or "problem-solving" or "computation" or "algebra" or "geometry" or "arithmetic"*. This search method resulted in 167 articles, 30 of which were selected for further eligibility assessment. The first and second authors independently screened each of the 30 articles and were in 100% agreement that five met the criteria for inclusion in the final sample for the review. Next, a hand search was conducted in relevant special education journals, including *Exceptional Children, Learning Disabilities Quarterly, Journal of Learning Disabilities, Learning Disabilities Research & Practice, Remedial & Special Education, and the Journal of Special Education.* Eight additional studies were identified using this process. An ancestral search of the references of included studies was conducted and yielded two additional studies. The 10 studies found using both the ancestral and hand search were checked independently by the first and second authors using the inclusion criteria. There was 100% agreement that all eight met the criteria. Overall, it was determined that 15 studies met criteria for inclusion in this review.

Quality Analysis

The 15 eligible studies were evaluated using one of two researcher-developed checklists. The checklist used for studies that utilized group designs was based on the quality indicators established by Gersten et al. (2005), and the checklist used for studies that used single subject designs was based on the quality indicators established by Horner et al. (2005). The first two authors independently conducted quality analyses of studies that used group designs (n = 10). The first author reported that 90% of the studies met the criteria for high quality while the second author reported that 100% of the studies met the criteria. After discussion, both authors agreed that 100% of the studies met Gersten and associates criteria for high quality.

The same two authors also conducted quality analyses of the studies that utilized single-subject designs (n = 5). Both authors found that 100% of the studies met over 90% of the quality indicators established by Horner and colleagues (2005). The only quality indicator that was clearly not met by most of the single subject studies was that an atypical agent carried out the interventions (e.g., the instructional agent in the Strickland & Maccini, 2013 and Satsangi & Bouck, 2014 was a researcher)

Article Coding

The first and second authors independently coded the following characteristics of each study: (a) study identification (i.e., author(s) and year), (b) purpose of the study, (c) information on participants (i.e., total number, number of students with LD, gender, grade level), (d) setting, (e) duration, (f) dependent measures (including psychometric properties), (g) fidelity, and (h) significant results (See Appendix 1). Reliability was obtained by determining the number of agreements plus disagreements coded divided by total agreements. Initial agreement was 97%, with additional discussions resulting in 100% agreement.

Results

Fifteen studies were included in this review, with eight including only students with LD. The remaining studies included students with other disabilities (i.e., emotional/behavioral disorders [EBD], autism, attention deficit and hyperactivity disorder [ADHD], speech and language disabilities [SL]) and both low- and average-achieving students without disabilities. The total number of students across the 15 studies is 3,282 (1,682 male, 1,600 female) from 6th to 12th grade. Of these students, 828 were identified as having LD, 593 as having mathematics difficulties, 1,054 as low-achieving, and 693 as average achieving. Students with LD were served in inclusive settings (n = 254), resource rooms (n =357), private schools (n = 35), charter schools for students with LD (n = 14), self-contained classrooms (n = 146), a university tutoring clinic (n =3), a pullout room (n =3), and a technology education classroom (n =16). In the following sections, the 15 studies are described according to the categories outlined below.

It was necessary to organize studies using a different framework than that used by Maccini and colleagues. Interventions in the previous review were categorized as behavioral, cognitive, and alternative delivery systems based on instructional approach used in the intervention. However, this conceptualization would have been unsuitable for grouping studies included in the current review. Thus, we categorize studies according to three major approaches to mathematics instruction used in the interventions: (a) cognitive and metacognitive strategies for solving word problems; (b) use of representations to increase conceptual knowledge and problem-solving skills; and (c) Enhanced Anchored Instruction.

Cognitive and Metacognitive Strategies for Solving Word Problems

Studies were categorized as cognitive and metacognitive strategies for solving word problems if the intervention included students using a reasoning strategy, self-monitoring, think-aloud or mnemonic device (Maccini et al., 2007) to solve word problems. Five (33%) studies in this review used this approach including (1) Cognitive Intervention: *Solve It!* (Montague, Enders, & Dietz, 2011; Montague, Krawec, Enders, & Dietz, 2014; Krawec, Huang, Montague, Kressler, & de Alba, 2013); (2) schematic diagram plus cognitive instruction (van Garderen, 2007); and (3) student-centered instruction plus cognitive instruction (Iseman & Naglieri, 2011).

Cognitive interventions: *Solve It!*. The *Solve It!* intervention was designed for secondary students with LD to improve their problem-solving performance. The intervention teaches students strategies for comprehending, representing, and planning solutions for mathematical problems through explicit instruction (e.g., modeling, verbal rehearsal, and immediate and corrective feedback) (Montague et al., 2014). The goal of the intervention is that students will be able to use cognitive processes and metacognitive strategies (i.e., reading, paraphrasing, visualizing, hypothesizing, estimating the accuracy of their responses, computing and checking their work during the problem-solving process) taught through think alouds to independently solve mathematical problems.

Teachers providing the intervention received three days of professional development and supporting implementation materials. To teach *Solve It!*, teachers were instructed to model strategies while thinking aloud, provide students opportunities for guided and independent practice, provide appropriate cues and prompts during practice, and give immediate and corrective feedback on students' performance. Montague and colleagues have conducted a series of studies to examine the effectiveness of *Solve It!* on the performance and strategies that 7th and 8th grade students, including students with LD in inclusive classrooms, to solve word problems (Montague et al., 2011; Montague et al., 2014; Krawec et al., 2013).

Montague et al. (2011) conducted a cluster-randomized clinical trial to examine the effect of *Solve It*! on the problem-solving performance of students with LD in secondary inclusive 8th grade classrooms. Forty schools paired on the basis of their Florida Comprehensive Assessment Test (FCAT) performance and socioeconomic status (SES) level (i.e., percentage of students who qualified for free or reduced-price lunch) were randomly assigned to either intervention or comparison groups. Intervention teachers (n = 17) had 319 students, and comparison teachers (n = 20) had 460 students. Seventy-eight students were identified as having LD. Students in intervention received three days of intensive instruction and then applied their knowledge in weekly problem-solving practice sessions using the district curriculum and FCAT practice manuals.

Repeated measures data using CBM measures were analyzed using a multilevel growth model to determine difference between treatment and control on one-, two-, and three-step word problems as well as problems drawn from the FCAT. In this model, repeated measures (Level 1) were nested within students (Level 2), and students were nested within schools (Level 3). Students in intervention improved significantly on CBM scores compared to comparison group students. After accounting for student level variance at Levels 1 and 2, school level performance on CBM measures explained 19.8 percent of the variance in those scores. There were no significant differences in the growth trajectories of low-achieving (LA) students, average achieving (AA) students, and students with LD in the intervention group. Additionally, students with LD in the intervention group outperformed their peers in the comparison group across all ability levels (i.e., LA students, AA students, students with LD) on CBMs administered at the conclusion of the school year. The performance of students in the intervention group was significantly different from the comparison group on FCAT items.

In 2014, Montague and colleagues examined the impact of *Solve It!* on 7th grade students' math problem-solving performance. Students were identified as LD, low-achieving, and average achieving. Participating schools were paired based on FCAT performance and SES levels and then randomly assigned to either treatment or control groups. Data were collected on 1,059 students: 664 in intervention and 415 in control. Eighty-six of the students were identified as having LD. Students in intervention received three days of explicit instruction (that included the following instructional practices: structured, organized lessons; appropriate cues and prompts; modeling, verbal rehearsal; guided and distributed practice; immediate and corrective feedback; positive reinforcement; overlearning; and mastery) and weekly problem-solving practice sessions for 8 months. Researchers used CBMs similar to those used in the 2011 study to evaluate the intervention's impact on students' mathematics performance. FCAT reading and math tests were also used to evaluate the interventions' impact.

Data were analyzed using three different analyses. First, a three-level multilevel model with repeated measures (MLMs) was used to analyze nested data (i.e., repeated measures were nested within students, students were nested within schools). Results indicated that students in the intervention group had more pronounced growth trajectories on CBM measures than students in control (Cohen's $d = .613$). A second two-level MLM (i.e., students and schools) showed that students in intervention and those in control did not significantly differ on FCAT measures. Data from 2011 and 2014 studies were also collapsed and analyzed using MLM. This analysis showed a large intervention effect (Cohen's $d = .882$) on the math problem-solving performance of middle school students when measured by CBMs. When ability was entered as a moderator in the MLM analyses, low-achieving students demonstrated greater growth on math problem-solving than their average-achieving peers in the treatment group. The same findings did not hold for students with LD.

Krawec et al. (2013) also conducted a randomized control trial study to evaluate the effectiveness of *Solve It*! on the strategic knowledge of 7th and 8th grade students with LD and those who demonstrated average achievement (AA) in mathematics. Seventy-eight students with LD were assigned to intervention (n=42) or comparison (n=36) groups. Eighty-three AA students were assigned to intervention (n = 46) or comparison groups (n=37). Students in intervention received explicit instruction for three days and then participated in 30-minute weekly practice sessions from October to the end of the school year. A modified version of the Math Problem-Solving Assessment (MPSA) was used to assess students' gains in strategy use during problem-solving from pretest to posttest.

Repeated measures ANOVA was used to evaluate the intervention's impact on students' strategy use while solving mathematical word problems. Students in intervention reported using significantly more strategies than those in comparison (Cohen's $d = .52$). AA students used significantly more strategies than students with LD (Cohen's $d = .68$).

Schematic diagram instruction plus cognitive strategy instruction. Van Garderen (2007) examined the effectiveness of teaching three 8th grade students with LD to generate and use diagrams combined with problem-solving strategies in *Solve it!* to solve one- and two-step word problems presented in different contexts. Researchers utilized a multiple probe across participants design that included baseline, intervention, generalization and maintenance phases. In addition, a pretest and posttest was used following the first instructional phase to determine students' progress in generating diagrams.

Intervention was implemented in three instructional phases: diagram instruction, strategy instruction for one-step word problems, and strategy instruction for two-step word problems. At each phase, students were provided with explicit instruction that included teacher modeling and demonstration, feedback, independent practice, rehearsal, and reinforcement. During the first phase, students were given a general overview of diagrams and how they could be used to solve different word problems. In Phase two, students were taught a visualization strategy adapted from Montague's (1997) problem-solving strategy. The strategy addressed five cognitive processes combined with metacognitive strategies and was memorized through a mnemonic aid. Then, students were expected to use the visualization strategy combined with instruction received in Phase 1 to solve one-step word problems through drawing and arranging of diagrams. In Phase three, students were taught to visualize and solve two-step word problems using backward chaining.

Results indicated that students increased their performance in generating and using schematic and pictorial diagrams to solve one- and two-step word problems. Moreover, students' average number of word problems correctly solved improved from baseline (x = 37.6% for both one- and two-step problems), to intervention for one-step (x = 78.6%) and two-step (x = 79.2%) word problems. Student performance on mixed word problems improved from baseline to intervention. Average percent increases for each student were 45.8%, 43.7%, and 35.0%. Analysis of maintenance probes indicated that two of the students were able to maintain their performance 1 and 3 weeks following the intervention. The third student did not participate in maintenance testing. In contrast, students scored poorly on the generalization probe, with scores ranging from 12.5% to 50.0%.

Student-centered approach to solving calculation problems. Only one study focused solely on a student-centered approach to solve numeric equations involving basic operations, fractions, and algebraic expressions. Iseman and Naglieri (2011) conducted a randomized control trial to examine the impact of planning-based cognitive strategy instruction on the mathematical calculation of 29 students (grades 6 to 10) with comorbid attention deficit hyperactivity disorder (ADHD) and LD. Seventeen classes of 5 to 8 students from a private school for children with learning problems were randomly assigned to either treatment (n=14) or comparison groups (n=15). Students in both groups were required to complete two daily math worksheets. Worksheet problems were developed based on the school curriculum, and included calculations with whole numbers, fractions and simple, linear algebraic expressions. Students in intervention participated in 10 days of 30 minutes sessions that included 10-minute teacher-facilitated discussions between completing worksheets. Teachers encouraged students to consider and verbalize multiple approaches to complete mathematical problems accurately. Students were encouraged to focus their discussions on strategies they used to tackle problems on math worksheets while teachers provided clarification and feedback on the usefulness of the students' strategies. The comparison group received business as usual mathematics instruction.

A three-factor MANCOVA, using pretests as the covariate, was used to analyze mean differences between intervention and control on the worksheets, as well as standardized measures including Math Fluency subtest of the *Woodcock-Johnson Tests of Achievement (WJ-III 3rd edition)* (Woodcock, McGrew, & Mather, 2001), and

Numerical Operations subtest of the *Wechsler Individualized Achievement Test (WIAI-II 2nd edition)* (Wechsler, 2001). Results indicated that the intervention group performed significantly better than their peers in the control on all measures. Effect sizes for students in the intervention group were 0.85 on computed problems correctly, 1.17 on the Math Fluency subtest, and 0.40 on Numerical Operations subtest. Students in the treatment also maintained their learning as evident by their performance on a Math Fluency probe (Woodcock et al., 2001) that was administered one year after intervention. The effect size for the students in the intervention group on *Math Fluency* subtest was 0.85.

Summary. Five studies focused on cognitive interventions that employed mnemonic devices to guide students through different cognitive strategy routines. Strategic routines were explicitly taught through teacher modeling and think alouds, guided practice, independent practice, and generalization. Overall, results of these studies showed that cognitive and metacognitive strategies could be taught through explicit, teacher-directed instruction, and that students' ability to solve one-, two- and three-step word problems would improve. Moreover, implementation of the Solve It! intervention could improve students' problem-solving performance and use of effective strategies by a half of a standard deviation (SD) to nearly a SD. Effect sizes for individual studies ranged from .52 to .88 (Krawec et al., 2013; Montague et al., 2014).

The Solve It! intervention, however, was not effective in helping students use their newly acquired knowledge to successfully solve problems on the one standardized exam employed in this study (Montague et al., 2011; Montague et al., 2014). We do not know from this research if explicit instruction of cognitive strategies is more effective than implicit teacher-facilitated instruction, or if the need for one or the other depends on students' prior mathematical knowledge and/or cognitive abilities. Additional research is needed to improve our understanding of how entering achievement and cognitive abilities might be used to influence the type of instruction provided.

Using Representations to Increase Conceptual Knowledge and Problem-Solving Skills

Studies that examined interventions where students learned to use visual and concrete presentations to acquire concepts and solve problems were categorized as using representations to increase conceptual knowledge and problem-solving skills. Four studies (27%) implemented this strategy. Three intervention studies (i.e., Scheuermann, Deshler, & Schumaker 2009; Strickland & Maccini, 2013, Hunt & Vazquez III, 2014) utilized concrete and visual representations along with some combination of explicit instruction (i.e., modeling or demonstration, ongoing instructional support through scaffolding, progress monitoring, cueing, prompting, and feedback) and teacher-facilitated instruction. The dependent measures included mathematics problems involving algebra, ratios and proportions. The other study (i.e., Satsangi & Bouck, 2014) examined the effect of a virtual manipulative program on student performance in geometry (i.e, area and perimeter).

Representations combined with problem-solving. Scheuermann and colleagues (2009) used a multiple-probe-across-subjects design to examine the effects of a teaching routine (i.e., the Explicit Inquiry Routine: EIR) on students' performance on one-variable equations embedded in word problems. Fourteen 6th to 8th graders between the ages of 11 and 14 years from a charter school specializing in educating students with LD participated in the study; all students selected had LD in math. A teacher taught EIR to students during the regular math period (n=11) or writing period (n=3) in an unused classroom.

The instructional routine included a combination of explicit content sequencing, scaffolded inquiry, and systematic use of various modes of representation (i.e., concrete, representational, and abstract). EIR required teachers to break up essential concepts into self-contained units and present units to students to ensure mastery of prerequisite skills before learning more challenging skills. Next, teachers engaged students in scaffolded inquiry through prompting, demonstrations, and feedback to help students develop conceptual understandings of word problems that involved one-variable equations. Finally, teachers provided students with direct instruction utilizing the concrete-representation-abstract (CRA) model to solve word problems. In the CRA sequence, students learned how to solve problems using concrete manipulatives and visual representations before they solved problems with numerical equations.

Results indicated that average participant scores improved from baseline to post- intervention for both instructed and uninstructed word problem probes. Results also revealed that students were able to maintain their performance on both prompted and unprompted versions of the word problem maintenance tests. Percentage of non-overlapping data points (PND) for instructed and uninstructed word problem probes were also calculated. The PND for instructed problems was 93%, an acceptable percentage according to Scruggs and Mastropieri (1998). The PND for uninstructed problems was 63%, which could be considered a questionable effect.

A Friedman Two-Way Analysis of Variance by Ranks found a treatment effect for a concrete manipulation task ($p < .001$). A follow up Wilcoxon Signed-Ranks test with a Bonferroni correction was used to evaluate differences between students' median pre-test, post-test, and maintenance scores on the concrete manipulation test. Results of this analysis revealed a statistically significant difference for pre- and post-test scores ($p < .001$), as well as pretest and maintenance scores ($p < .001$). Effect sizes for students' score were also calculated using Glass's D. The effect size, comparing pre- to posttest mean scores, was large ($\Delta = 2.32$), as was the effect size comparing pretest to maintenance mean scores (effect size $\Delta = 1.93$). However, students did not significantly improve their scores on the concrete manipulation test from posttest to maintenance ($p = .024$). A Wilcoxon Signed-Ranks Test showed that posttest scores were significantly greater than pretest scores on a Far Generalization task and Key-Math Revised. Further, the magnitude of the differences were large for the Far-Generalization task (Cohen's d $\Delta = .67$), and moderate to large for the Key Math Revised tests (Cohen's d $\Delta = .54$), suggesting that the EIR was effective for improving students' performance on one-variable equations embedded in word problems, and transfer of learning to other types of novel problems.

Concrete-representational-abstract integration. Strickland and Maccini (2013) used a multiple-probe-across-subjects design to evaluate the effectiveness of the concrete-representational-abstract integration (CRA-I) strategy on the performance of students with LD on word problems involving linear expressions. Three males, ages 13 to 15, attending a private day school for students with LD participated in the study. Participants received one 30-minute introductory lesson and three 40-minute intervention lessons. Lessons incorporated the following instructional sequence: (a) advance organizer (i.e., linking the current lesson to previous lessons, presenting the objective and the rational of the importance of the lesson); (b) teacher demonstration of how to solve an algebraic task using the CRA sequence and think-alouds; (c) guided practice where instructor prompts were used to help students solve an algebraic task similar to that used in the demonstration; and (d) independent practice.

Researchers taught students how to solve linear equations using: (a) algebra blocks during the concrete stage, (b) sketches of the blocks at the representational stage, and the (c) box method for the abstract stage. The box method was a graphic organizer that students could use to set up and solve linear equations. While solving problems, students were encouraged to make connections between equations, word problems, and data tables. They were also asked to explain and justify answers using mathematical language. Three similarly constructed probes were used to measure change from baseline to intervention 3 to 6 weeks after the intervention including a domain probe comprising computation and word problems, data tables that students used to organize features of word problems and a probe made up of word problems. A transfer probe with algebraic equation and word problems was administered after intervention.

Students increased on probes that measured computation and word problem performance from baseline (0% to 17% correct) to intervention (78% to 93% correct), and met or exceeded mastery criteria on lesson probes (88% to 100%). Performance on maintenance probes, collected four to six weeks after instruction, was 96%, 98%, and 52% separately. Percent correct on transfer probes, administered immediately after posttest probes, was 50%, 67%, and 83% separately, indicating that students were also able to transfer their knowledge to similar problems. The PND from baseline to intervention was 100%, indicating that the intervention was highly effective. On a social validity survey, students indicated they benefited from and enjoyed the intervention.

Ratio strategy instruction. Hunt and Vasquez III (2014) conducted a multiple baseline across participants design to examine the effect of an intervention designed to develop conceptual understanding of ratio equivalence problems. Participants included 3 students enrolled in a university-sponsored tutoring clinic for students with LD. The intervention included 15 lessons that were based on Battista and Van Auken Borrow's (1995) developmental progression of conceptual milestones and the CCSS-M. Three 25-minute lessons were delivered individually to students.

During each instructional session, the teacher first presented and read problems (usually three) to an individual student, and constantly evaluated strategies and representations used by the student to solve problems. If the individual student came up with a wrong solution and did not appear to use a strategy (i.e., build-up or unit rate strategy) or representation, the teacher would demonstrate, using a think aloud, how to use a strategy or representation to solve the problem. The teacher would also provide a series of prompts until the student correctly solved the problems. Prompts guided individual students to notice troublesome aspects of his or her solution strategy and identify an alternative way to correctly solve problems.

Students' performance on forty-five probes, comprising 20 ratio-equivalence problems, were graphed and analyzed through visual inspection and by calculating the PND. The mean percentage of correct problems increased from 15.7% at baseline to 86.7% at the intervention's conclusion. The PND from baseline to instruction was 70%, indicating the instruction has a moderate effect on students' performance. Researchers collected students' written descriptions of how they solved problems and analyzed them to determine how their responses evolved. The result of this analysis showed that participants' strategy use changed during the intervention, moving from simpler, less sophisticated strategies to more sophisticated strategies for developing multiplicative thinking and proportional reasoning. Simultaneously, students exhibited a decrease in the use of less efficient strategies.

Virtual manipulative instruction. Satsangi and Bouck (2014) employed a multiple baseline design to investigate the effects of virtual manipulative instruction on performance of students with math LD on concepts of area and perimeter. Three 9^{th} -11^{th} grade male students from a private charter school participated in the study. One of the researchers provided each student with 40 minutes of explicit instruction on concepts of area and perimeter, and then trained students to use virtual manipulatives to compute area and perimeter of regular and irregular shapes. During training, students were provided with additional instructional support (i.e., re-teaching) if they demonstrated difficulty on previously taught concepts. Instructional support continued until students demonstrated mastery by independently answering four out of five perimeter and area word problems. Additional teacher assistance provided after this point was recorded as a prompt on the student's event recording sheet.

To determine the intervention's effectiveness, the PND was calculated to measure the between-phase performance difference between baseline and intervention scores for each student. The PND for area and perimeter for all the students was 100%, indicating a large effect of treatment. In addition, the researchers also calculated the Tau-U, a non-parametric statistical measure of effect size that combines non-overlap data between phases with trend data from the intervention phase, for each of the participants and the combined weighted average. The individual Tau-U scores were at least 1.0 and the combined weighted average Tau-score of the group was 1.0, indicating a large effect size for the intervention (Parker, Vannest, Davis, & Sauber, 2011). These combined results indicated that virtual manipulatives were effective in increasing the performance of all three participants on solving area and perimeter mathematics problems.

Summary. Four studies employed the CRA sequence or virtual manipulatives to promote students' conceptual knowledge and problem-solving approaches. The CRA sequence combined with problem-solving approaches and virtual manipulatives were effective in improving students' performance problems involving algebra, geometry, ratios and proportions. Further, two studies showed that such instruction could be used to improve students' underlying conceptual knowledge (Scheuermann et al., 2009), and their strategic approaches to solving problems that involved concepts about ratio equivalence (Hunt & Vasquez III, 2014).

Moreover, the Scheuermann et al. study was able to establish that their intervention had moderately large to substantially large effects on the different posttests, far generalization task, and the Key Math-R (Connolly, 1998), suggesting that embedding CRA strategies inside an explicit intervention for teaching problem- solving could be effective. Findings from these studies support conclusions from the Maccini et al. review, which showed that manipulatives and representations could be used to develop conceptual knowledge and improve students' strategic approaches to problems. However, additional experimental studies are needed to replicate the reported findings.

Enhanced Anchored Instruction (EAI)

EAI is an instructional approach that uses computer-based interactive lessons, videos, and hands-on applied projects to help learners solve anchored problems with the intention of improving their problem-solving and computation skills (Bottge, Rueda, & Skivington, 2006). Anchored problems incorporate several subproblems presented in real-world contexts, such as building a house or designing the fastest car. Students are required to solve these problems using a sequenced problem-solving procedure (i.e., defining and understanding the context of the problem, identifying the necessary knowledge and procedures, and using the information to solve the problem). Six studies included in this review used quasi-experimental and experimental designs to assess the impact of EAI on students' performance on problems that involved ratios and proportional relationships, fractions, statistics and probability, and geometry.

Bottge, Rueda, Laroque, Serlin, and Kwon (2007) used a pretest-posttest control design with switching replications to examine the effects of EAI on the problem-solving and computation performance of 100 6th through 12th grade students with math LD served in self-contained classrooms. Researchers randomly assigned four trained teachers, and their students, to two instructional sequences (A and B). Both groups received 50-60 minutes of instruction for 21 to 30 days using EAI and the regular curriculum. Students in Sequence A received EAI using *Kim's Komet*, one videodisc in a series of video-based anchors known as *The New Adventures of Jasper Woodbury* (Cognitive and Technology Group at Vanderbilt University, 1997) while students in Sequence B received typical instruction. Then, students in Sequence B and A switched and students in sequence B participated in *Kim's Komet*, providing an opportunity to replicate the treatment effect.

A two-way repeated measures ANOVA was used to analyze students' performance on computing rate and distance problems. Results indicated that students in sequences A and B improved their ability to compute rate and distance problems as a result of treatment and the treatment effect was large (Cohen's d = 1.08 and 1.42, respectively). Moreover, students in Sequence A maintained their learning, and the mean difference between pretest and maintenance was significant and large (Cohen's d = 1.08). Two-way ANOVAS (time of test by instructional sequence) were also used to analyze the impact of Kim's Komet on students' scores on The Iowa Tests of Basic Skills (ITBS; Form A; University of Iowa, 2001) subtests. Results revealed the *Kim's Komet* had a significant and large effect on students' mean scores on the math computation subtest (Cohen's *d* = .74), and a significant and moderately large effect on the math problem-solving and data interpretation subtests (Cohen's *d* = .57).

In a replication study, Bottge, Rueda, Serlin, Hung, and Kwon (2007) used a repeated-waves nonequivalent dependent variables design to investigate the effects of EAI on the mathematics performance of middle school students. The study involved 128 students who were high-achieving, average-achieving, or LD. Twelve students with LD were placed in one class with 13 typical students and the remaining students were in advanced pre-algebra or regular classes. Teachers were trained to implement the intervention in small-mixed ability groups of two to four students. Students received one EAI unit (*Kim's Komet*) for 13 days and a second EAI unit (*Fraction of the Cost*) for 11 days. During the period between the two EAI units, students were taught geometry and proportional reasoning using the regular curriculum, *Connected Mathematics* (Dale Seymour Publications, 2004).

Two-way split plot analyses of variances (ANOVAs) with repeated measures showed a moderate effect of *Kim's Komet* on students' performance on problems involving estimation and computation of rates ($\eta^2 = .59$). Additionally, there was an instruction by group interaction, such that students in the inclusion class performed better on estimation and distance and rate problems than students in typical instruction. There was also a small to moderate effect of Fraction of the Cost on students' performance ($\eta^2 = .53$) on problems that involved measurement, estimating and computing combinations using whole numbers and fractions, interpreting and recording data in tables, and calculating costs of materials. There was, however, no interaction between instructional group and treatment. All groups benefited from treatment.

Bottge, Grant, Stephens, and Rueda (2010) employed a pretest-posttest randomized cluster design to investigate the effects of two versions of EAI (i.e., explicit and embedded) compared to typical classroom instruction on the math skills of 303 middle school 6th and 7th grade students. Sixteen of the students were identified as having LD. Three EAI modules, *Fraction of the Cost, Fractions at Work, and Hovercraft,* were implemented in the EAI explicit group. One EAI module focused on helping students solve problems using metric conversions, interpreting schematic plans, and adding and subtracting mixed numbers. This approach was used in the embedded treatment group. Students in the explicit treatment group also received teacher-directed instruction as a class or in small groups to develop conceptual understanding of equivalent fractions, improve computational fluency, and develop fraction problem-solving skills. Students in the embedded instruction group received similar instruction to the explicit group, but were not provided teacher-directed instruction. Instead, they received instruction from the teacher on an as needed basis as they worked to solve video-based problems.

Results of a hierarchical linear model (HLM) analysis showed that there were no significant differences on student performance between the three types of instruction. However, the embedded EAI treatment had the largest effect on problem-solving performance (ES = .53), and explicit instruction had the smallest effect (ES = .32). Typical instruction had a moderate effect on problem-solving performance (ES = .44). Assessments of problem-solving performance included items that measured students' performance in interpreting bank statements and calculating 10 to 20% of the balance, and on problems that required students to work with fractions to build a bookcase. Students in each group were also assessed on a fraction computation test. HLM results showed a significant difference from pretest to posttest for the embedded group compared to the typical group on fractions computation, and the effect size was moderate (ES = .42). There was no significant difference from pretest to posttest for students in the explicit group compared to those in the typical group.

In a similar study, Bottge, Rueda, Grant, Stephens, and Laroque (2010) utilized a pretest-posttest cluster randomized experiment design to examine the efficacy of two versions of EAI (i.e. formal and informal) on the problem-solving skills of a group of students with disabilities. Fifty-four 6^{th}-8^{th} grade students who received math instruction in self-contained classrooms for students with disabilities participated in the study; 46 of these students were classified with LD. Students were randomly assigned by school to either informal (informal + EAI) group or formal (formal instruction + EAI) group. Students in the formal group received a blend of explicit instruction (i.e., constant progress monitoring and scaffolding), as well as technology-based instruction in three EAI units. In contrast, those in the informal group received technology-based instruction from three EAI modules, and assistance from teachers on an as needed basis. The intervention was implemented in small groups for 24 sessions that averaged 53 minutes per session. The same outcome measures (i.e., researcher-developed measures and subtests of the ITBS) used in the Bottge, Grant et al. (2010) study were used to collect data on students' performance.

Results of hierarchical linear modeling (HLM) analysis revealed that all groups of students made learning gains from pretest to posttest on the *Problem-Solving Test.* This test required students to solve problems involving percentages, measurement conversions, fractions, and monetary quantities. Informal and formal instruction had higher effects on student performance (d = 1.14 and 0.81, respectively) than typical instruction (d = .44). Analysis of students' performance at the subtest level showed that explicit instruction and embedded instruction were differentially effective, depending on the skills tested. Both conditions were more effective than typical instruction and effects of the two treatments were moderate to large (i.e., .52 to 1.19). On the Fraction Computation Test, the typical instruction group did not make significant gains, but the informal instruction group showed a statistically significant mean gain compared to the typical group (ES= .42). There was no significant difference between the formal instruction and typical instruction groups, or the formal and informal instruction groups. Finally, the treatment did not have the desired effect on the ITBS Problem-Solving and Data Interpretation and Computation subtests, a finding that contrasted with Bottge, Rueda, Laroque et al. (2007).

Bottge and Cho (2013) used an experimental design to compare differential effectiveness of EAI and typical instruction on the problem solving-performance of 308 middle school students with math LD. Students in the EAI group were taught how to compute fractions and solve problems using four EAI modules. Accompanying activities were developed around the CCSS-M for measurement and data, number and operations (i.e. fractions, ratios and proportional relationships), and graphing. Intervention lasted for 74 days and was conducted by special education teachers who provided instruction for approximately 45 to 60 minutes at a time. Control group students were taught using a math curriculum that was aligned with the Kentucky

Department of Education Combined Curriculum Document (2012). Objectives of this curriculum were consistent with the EAI curriculum.

A paired t-test was used to analyze students' performance from pre- to post-test. Results revealed that students' overall and subscale posttest performance scores were significantly different from pretest scores for both groups. A multi-level longitudinal item response model was calculated to determine change scores from pre -to post-test for EAI and typical instruction. The mean change score, based on a t-test with unequal variance, was higher for EAI than typical instruction. Fifty-six percent of EAI students had change scores greater than 1 SD; whereas only 33% of typical students had change scores greater than 1 SD. Further, IRT results revealed that students in the EAI group outperformed students in the typical group on some of the most difficult items related to measurement and data, number and operations with fractions, ratio, and proportions, and graphing.

Bottge et al. (2014) employed a pretest posttest cluster-randomized design to investigate the effect of EAI on fraction computation and problem-solving skills of 335 middle school students with disabilities including 49 students with LD. Students' resource classrooms were randomly assigned to either EAI or typical instruction. EAI instruction was compared to typical instruction based on the Kentucky Department of Education Combined Curriculum Document (2012). Objectives of this curriculum were closely related to those covered in EAI instruction. The intervention was implemented in 94 daily sessions for approximately 45 to 60 minutes.

A three-level hierarchal linear model (HLM) was employed to analyze students' scores on two researcher-developed measures and three subtests of the ITBS. Results indicated students in the EAI group demonstrated greater mean gains than to students in the control group on combined parts of the Problem-Solving Test, and the treatment effect was moderate (ES = .39). The problem-solving tests measured students' understanding of measurement and data, number and operations-factions ratios, proportional relationships, and geometry. Students in the EAI condition also made significant gains compared to students in the control on the researcher-developed fractions computation test, and the effect of the treatment was large (ES = 1.00). EAI instruction had less of an impact on the ITBS subtests for computation, problem-solving, and data interpretations. There was only a significant and moderate effect of the EAI treatment for computation (ES = .44).

Summary. Bottge and colleagues extended the work that had begun prior to the previous review (Maccini et al., 2007). The six studies described in the current review used videodisc instruction combined with various instructional approaches, some which were more explicit than others, and involved more teacher-facilitated instruction to develop students' conceptual knowledge and their application of that knowledge to solving problems in real-world contexts. Our review confirmed what Maccini et al. (2007) established – that EAI instruction has a positive impact on students' ability to solve mathematical problems, and this impact is larger than that achieved through typical instruction. Effect sizes for the different EAI studies were moderate to large. Further, in two studies, EAI instruction with a teacher-facilitated component outperformed EAI with an explicit component.

Additionally, EAI interventions were effective for low-performing students in these studies and students with LD, even though instruction did not always contain an explicit component. Finally, in the Bottge, Rueda, Laroque et al. (2007) study, EAI instruction had a large effect on the ITBS subtests for computation, and problem-solving and data interpretation subtests. These findings did not hold for the Bottge et al. (2014) study. Thus, researchers can conclude that EAI instruction has an effect on the problem-solving and computational skills of secondary students with LD. However, more research is needed to better understand exactly how EAI instruction might be crafted for different types of learners.

Discussion

The purpose of this review was threefold: (a) to extend and update a previous review of the literature (i.e., Maccini et al., 2007), (b) identify effective interventions for improving the mathematics performance of secondary students with LD, and (c) discuss recommendations for future research, implications for practice, and teacher preparation and professional development programs. Specifically, we wanted to determine: (a) which math interventions promote the math achievement of students with LD; (b) how findings in our study complement findings from previous literature reviews; (c) if new practices emerged as a result of this review; (d) the degree to which studies are promoting interventions that hold potential to improve the performance of students with LD on rigorous content standards; and (e) the extent to which studies adhere to the indicators of high quality research established by Gersten et al. (2005) and Horner et al. (2005).

The current review of the literature included 15 studies; this represents a 35% and 25% decrease in comparison to the reviews conducted in 2007 and 1997. This decline might be due to variation in inclusion and exclusion criteria across reviews. Unlike previous reviews, we only included studies where students with LD were included in the analysis. Like previous reviews, 100% of the studies in our review included mathematics interventions that were effective in improving students' mathematics performance. Moreover, the number of studies that focused on specific key effective strategies has remained constant or increased, in some instances. For example, the number of studies in our review that supported cognitive strategy instruction as means of improving students' performance on problem-solving was five compared to the 2007 and 1997 reviews that included five and six, respectively. Moreover, this review found six studies that provided evidence for the efficacy of EAI in supporting the math achievement of students; this represents a 50% and 500% increase over the 2007 and 1997, reviews, respectively. In this review, four studies supported the effectiveness of concrete and visual representation instruction in promoting mathematic achievement. In contrast, the 2007 review included six studies that supported these interventions and the 1997 review did not include studies that focused on these types of interventions.

In terms of the second objective, our review of the literature did not yield new research-based practices. In fact, studies that focused on some interventions (e.g., schema-based instruction and explicit instruction) previously identified as effective were not found during the current review. This is due mainly to our inclusion criteria that excluded some studies focused on schema-based instruction. These studies did not specify the categories of students with disabilities that participated. Our review includes studies where the targeted interventions improved the performance of students with LD on skills relate to rigorous content standards, such as the CCSS-M; and thus, adds to the studies reviewed by Maccini et al. 2007 which showed that effective interventions were being developed that could help students with LD learn more rigorous content and approaches to problem-solving. Our review included intervention studies that focused on developing students' conceptual knowledge (e.g., Hunt & Vasquez III, 2014; Satsangi & Bouck, 2014; Strickland & Maccini, 2013), their ability to problem solve (e.g., Scheuermann et al., 2009, Montague et al., 2014, Montague et al., 2011) and their ability to solve problems that required more advanced knowledge of mathematics (e.g., Bottge, Grant, Laroque et al., 2010; Bottge and Cho, 2014; Bottge et al., 2014).

In particular, EAI studies that included large samples of secondary students with LD produced significant gains for these students in problem-solving and computation in experimental studies. For example, Bottge, Grant, Laroque et al. (2010) found that on measures of fraction computation, the group of students who received informal instruction and EAI had significant improvement from pretest to posttest, but those in the group that received formal instruction and EAI group scored higher than those in the informal instruction and EAI group. They also reported that on measures of problem-solving, students in the informal instruction and EAI groups had significant improvement from pretest to posttest and those in the formal instruction and EAI groups also had significant improvement from pretest to posttest. Similarly, Bottge et al. (2014) found that students in the EAI group outperformed those students who received traditional instruction using the prescribed curriculum for problem-solving. Specifically, EAI students scored significantly higher than their peers on ratios and proportional relationships and geometry.

In terms of the quality of the studies, all 15 studies included in the review adhered to the indicators of high quality research. The 10 studies that utilized grouped designs were evaluated using a checklist created using Gersten and associates (2005) quality indicators for group research. The results of this analysis showed that all 10 studies met all of the essential quality indicators and demonstrated at least four of the quality indicators identified as desirable. The quality of the remaining five studies was evaluated using a checklist created from using Horner et al. (2005) quality indicators of single subject research. The results of this evaluation revealed that all of the studies had at least 90% of the quality indicators.

Limitations of the Research

Although intervention studies included in our current review were conducted with rigor (according to Gersten et al., 2005; Horner et al., 2005) and yielded positive results, there are several limitations that must be considered. First, the intervention studies that used concrete and visual representations combined with teacher-directed and teacher-facilitated instruction were conducted in private or charter schools for students with LD or tutoring clinic, and they relied on multiple baseline designs. We do not know, therefore, if strategies can be implemented successfully in typical public school settings, and if findings from these studies can be replicated in larger, carefully controlled experimental studies. Moreover, several of the larger, group studies did not disaggregate data for students with LD. Thus, it is not always clear if interventions had an impact on these students' performance. It is important to recognize that the cost of oversampling students with LD in these large-scale studies can be prohibitive. Additionally, studies in this review, with few exceptions, measured treatment adherence for the intervention only, and did not examine differentiation between treatment and control group instruction. Thus, it is not always easy to determine how different treatment and control conditions were in terms of the type of instruction provided.

Finally, all of the studies included in this review have relied on researcher-generated measures. Some of these measures were used in multiple studies, and students changed in expected ways on these measures as a result of participating in similar interventions, thus some validity evidence was provided. Only a few studies, however, examined the degree to which increased performance on researcher-generated measures resulted in more favorable performance on mathematics measures that reflected states' content standards. Although there are numerous problems associated with using standardized assessments to assess the progress of students with disabilities, not the least of which is their sensitivity to change, there needs to be some assessment of how well interventions focus on the mathematics' knowledge and skills that will be needed to enable students with LD to make progress towards achieving more rigorous content standards. Thus, it will be imperative for researchers to develop measures that while sensitive to changes in student understanding, can predict the performance of students with LD on standardized assessments.

Recommendations for Future Research

Over the past 7 years, only 15 studies have been published that examine effective mathematics interventions for secondary students with LD, and 9 of these were published by two groups of researchers. All but one of the large group design studies were conducted by these two groups of researchers. Clearly, the field of special education is not generating the infrastructure needed to build a research base for mathematics interventions for secondary students with LD. In part, this lack of infrastructure could be due to limited funding available to special education researchers over the past decade. Federal funding for special education research has been reduced by more than 30% in the past 7 years, resulting in a 75% reduction in research projects funded (Council for Exceptional Children, 2014). Thus, any suggestions to increase research in this area must be offered within this reality.

Two additional recommendations for future research concern fidelity of implementation. First, the notion that treatment fidelity consists solely of adherence to the treatment condition must be addressed in future research. Durlak and DuPre (2008) identified 8 components of treatment fidelity that should be considered the standard in high quality research: (a) adherence; (b) dosage; (c) quality; (d) participant responsiveness; (e) program differentiation; (f) monitoring of control/comparison conditions (i.e., treatment contamination); (g) program reach (i.e., participation rates, representativeness of program participants); and (h) adaptation. Second, with regards to adherence, researchers should focus on the collection of data that addresses whether or not components of the intervention were taught by teachers in control group, as well as adherence to underlying instructional principles. For example, if an intervention includes explicit instruction, then data should also be collected on existence of the necessary components of explicit instruction in the control group.

Comprehensive evaluation of fidelity would provide greater clarity when interpreting study results.

Recommendations for Practice

Results of the current review indicate two strategies that have moderate evidence and should be considered by teachers for students with LD in their classrooms: EAI instruction, and cognitive and metacognitive strategy instruction. EAI (Bottge, Rueda, Laroque et al., 2007; Bottge, Grant, Stephens et al., 2010; Bottge & Cho, 2013; Bottge et al., 2014) and the combination of *Solve It*! and schematic diagram instruction (van Garderen, 2007) should be used to develop students' conceptual understanding and procedural knowledge for solving word problems that are presented in authentic and other contexts. Second, teachers should include cognitive and metacognitive instruction to help students with LD enhance their performance in problem-solving (Montague et al., 2011; Montague et al., 2014; Krawec et al., 2013). Furthermore, teachers should employ instruction that incorporates visual representations along with some components of explicit instruction (i.e., modeling or demonstration, on-going instructional support through scaffolding, progress monitoring, cueing, prompting, and feedback) to improve students' performance in word problems (Strickland & Maccini, 2013), ratios (Hunt & Vasquez III, 2014), and algebra (Scheuermann et al., 2009).

One caveat is necessary concerning the recommendations for classroom practice. Although effective, EAI, cognitive and metacognitive strategy instruction are rather complex interventions in which teachers would need extensive time and support to develop related materials and activities. Therefore, integration of these two instructional strategies will require development and/or alteration of existing materials and texts, updates to the format (i.e., move from videodisc to more accessible formats), as well as ongoing and formalized support to teachers to ensure appropriate implementation.

In order for teachers to improve the academic achievement of secondary students with LD, they must receive training during their preparation and professional development to enhance their knowledge of math concepts and procedures and their abilities to implement effective evidence-based practices. Therefore, it is imperative that teacher preparation and professional development programs include a variety of courses that include opportunities for teachers to learn math concepts and procedures, opportunities to implement appropriate research-based instructional approaches (e.g., cognitive and metacognitive strategy instruction, explicit instruction) and programs (e.g., EAI, and *Solve It!*).

Conclusion

Mathematics is an integral component of the academic curriculum and has been viewed as an essential tool for improving the economic competitiveness of the U.S. on the global market. Currently, all students are expected to demonstrate proficiency on mathematics skills and knowledge in order to graduate from high school and access post-secondary educational and vocational opportunities. However, many secondary school teachers across the U.S. struggle to help their students, especially those with LD, to attain proficiency on tests of mathematics achievement. This problem may be exacerbated with the advent of more rigorous mathematical standards, such as the CCSS-M.

The goal of this paper was to review the literature for the purpose of updating a previous review that identified effective instructional practices teachers could utilize to help secondary school students with LD. We identified a number of practices that teachers can use to help students with LD to access the mathematics curriculum and improve their performance. The findings revealed that there was substantial evidence to support the efficacy of Enhanced Anchored Instruction and *Solve It!* These two instructional practices were investigated using large samples and robust group designs; thus these practices yielded the most promise for improving the mathematics achievement of secondary students with LD.

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Appendix 1. Table Showing Math Interventions for Secondary

Study & Purpose	Participants, Setting & Duration	Dependent Measure	Fidelity	Significant Results
Cognitive & Metacognitive Strategies for Solving World Problems				
Montague, Enders, & Dietz (2011) *Purpose*: Randomized control study to assess the impact a cognitive strategy (*Solve It!*)on math problem-solving	N=779; LD= 78; M=359; F= 420; Gr 8; *Setting*: General Education; *Duration*: 8 months	• Curriculum-based measures (CBMs) involving one-, two-, and three-step math word problems calibrated using Item Response Theory (internal consistency ranged from .67 to .80) • Mathematics problems selected from state standardized assessment	Treatment adherence average for treatment (TAAT): 90% & 84%; inter-observer agreement (IOA) average: 96%	• Treatment outperforms control on CBM measures for all groups, contributing 19.8 % of the variance to achievement • No significant difference on FCAT problems
Montague et al. (2014) *Purpose*: Randomized control study to assess the impact of *Solve It!* on math problem- solving.	N=1059; LD=86; M=453; F=606; Gr 7; *Setting*: General Education; *Duration*: 8 months	• CBMs used by Montague et al., 2011 • Florida Comprehensive Assessment Test (FCAT): math and reading subtests	TAAT: 97% & 93% IOA average: 99%	• Treatment outperforms control on CBMs (*d=.882)* • No significance on FCAT math
Krawec et al. (2013) *Purpose*: Randomized control trial study to evaluate the effectiveness of *Solve It!* instruction on students' knowledge of math problem-solving strategies.	N=161; LD= 78; M=82; F=79; Gr 7-8; *Setting*: General Education; *Duration*: 8 months	The Math Problem-Solving Assessment (MPSA) consisting of three word problems, 5 Likert-type items, and 29 open-ended items that measured strategy used for solving problems	TAAT for two types of treatment sessions: 90% & 84% IOA average: 94% & 98%TAAT for comparison group: 2.8% IOA for comparison: 99%	• Treatment outperforms control on MPSA *(d= .52)*

van Garderen (2007) *Purpose*: Multiple-probe design to investigate effect of schema instruction combined with *Solve It!* on problem solving ability.	N/LD=3; M=2; F=1; Gr. 8; *Setting*: Junior High School; *Duration*: Scheduled 40-minute classes 4 times a week	Researcher-developed word problems tests with one- and two-step word problems involving basic math operations	Fidelity established through the use of scripted lessons. IOA: Ranged from 98.6% to 100%	• Average percent of word problems solved correctly increased from baseline (37.6% for one- and two-step problems combined) to intervention (78.6% for one- step and 79.2% for two-step) • Average percent increase for each student on mixed word problem probes was 45.8%, 43.7% and 35.0% respectively • Two of three students maintained performance following intervention
Iseman & Naglieri (2011) *Purpose*: Randomized control study to test the efficacy of a planning-based cognitive strategy instruction on computation fluency.	N/LD= 29; M=21; F= 8; Gr 6 -10; *Setting*: A private school for children with learning problems; *Duration*: 30 minutes per day for 13 blocks	• Cognitive Assessment System (CAS) measure of intelligence • Math Fluency subtest of Woodcock-Johnson Test of Achievement (WJ-III 3rd edition) • Numerical Operations subtest of Wechsler Individualized Achievement Test (WIAT-II 2nd edition) • Researcher developed worksheets for computation	Treatment adherence reported; details not provided	• WJ-III: Treatment *(d= 1.17)* outperformed control on (*d= .09)* treatment *(d= 1.17)* • WIAT-II: Treatment *(d= .40)* outperformed control *(d= .14*) • Worksheets measures: Treatment *(d= .85)* outperformed control *(d= .26)* • WJ-III: Treatment maintained their learning *(d= .85)*
Using Representations to Increase Conceptual Knowledge and Problem-Solving Skills				
Scheuermann, Deshler, & Schumaker (2009) *Purpose*: Multiple-probe study to examine efficacy of Explicit Inquiry Routine on problem solving with one variable algebraic equations.	N/LD=14; M=10; F=4; Gr. 6-8; *Setting*: Charter School; *Duration*: 13 to 23 sessions daily for 55 minutes	• Word Problem Test (WPT) (93.1%-IRR) • Concrete Manipulation Test (CMT) (96.0%-IRR) • Far-Generalization Test (FGT) (97.5%-IRR); • KeyMath-R (Connolly, 1998).	Data not provided	Significant increases from baseline to treatment based on PND and effect sizes: • WPT (93%, high PND) • CMT (Intervention: Δ=2.32; Maintenance: Δ=1.93) • FGT (Δ= .67) • KeyMath-R (Δ= .54)

Strickland & Maccini, 2013 *Purpose*: Multiple-probe study to investigate the effect of using multiple visual representations on students' conceptual understating of quadratic expressions embedded within area word problems and procedural fluency in solving them.	N/LD= 3; M=3; Gr. 8-9; *Setting*: Private separate day school for students with LD; *Duration*: One 30-min introductory lesson and three 40-min target lessons	• Researcher-developed: 1. Domain probes 2. Lesson probes 3. A Transfer probe 4. Social validity measure Content validity established through expert review	IOA average: 100% for treatment only	Participants improved performance: • on domain probes (0% to 17% correct for baseline; and 78% to 93% for treatment) • on maintenance probes (from 52% to 98%) • on transfer probes (from 50% to 83%) • participants enjoyed the intervention
Hunt & Vasquez III (2014) *Purpose*: Multiple-baseline study to evaluate efficacy of ratios Strategy Intervention on ratio performance.	N/LD=3; M=1; F=2; Gr 6-8; *Setting*: University Tutoring Clinic; *Duration*: 45 instructional sessions, 25-per session	Multiple forms of a formative strategy-use probe consisting of 20 ratio-equivalence problems	IOA average: 91%	Intervention resulted in • a mean percent increase on correct problems from baseline to the end of instruction (*PND = 70%).* • students using more sophisticated strategies for developing multiplicative thinking and proportional reasoning
Satsangi & Bouck (2014) *Purpose*: Multiple-baseline study to measure the effect of virtual manipulative instruction on area and perimeter problems	N/LD=3; M=3; Gr 9 -11; *Setting*: Charter School; *Duration*: 8 weeks, 40 min lessons	Researcher-developed probes consisting of area and perimeter problems	TAAT: 100% IOA average: 100%	• Treatment had a large effect on intervention, maintenance, and generalization probes (TAU= 1.0)

Enhanced Anchored Instruction (EAI)				
Bottge, Rueda, LaRoque et al. (2007) *Purpose*: Conduct a pretest-posttest control design with switching replications to examine efficacy of EAI on problem-solving and computation.	N/LD=100; M=64; F=36; Gr 6-12; *Setting*: Self-contained; *Duration*: 50-60 minutes of instruction for 21-30 days	• Kim' s Komet Problem-Solving Test (KKPST) (Cronbach's alpha = .89, IRR= 98%) • Iowa Test of Basic Skills (ITBS)-Computation subtest • Iowa Test of Basic Skills (ITBS)-Problem solving and Data interpretation subtest	Fidelity ways established through several methods but no rate was reported. IOA average = 99% for treatment only	Treatment students outperformed control students. Intervention resulted in significant gains on: • KKPST (d= 1.08 for group one and d= 1.42 for group two) Maintenance effect for group one is significant (d = .99) • ITBS measure: Mean scores significantly increased after receiving treatment for both groups (computation, d = .74 and (problem-solving, d= .57)
Bottge, Rueda, Serlin et al. (2007) *Purpose*: Repeated-waves nonequivalent dependent variables study used to examine effect of EAI on problem solving	N=128; LD=12; M=60; F=68; Gr.7; *Setting*: General Education; *Duration*: 24 Days (13 in October; 11 in March), Students received instruction from regular curriculum when not in treatment.	• Kim's Komet Challenge (KKC) problem solving test of rate estimation and calculation • Fraction of the Cost Challenge (FCC) problem solving test of area, calculating percentages, interpreting data, and performing computations using whole numbers and fractions	TAAT: 96% IOA average: 100%	Students in treatment condition made significant gains compared to non-treatment condition: • KKC Test (n^2 = .79) • FCC Test (n^2 = .67)
Bottge, Grant, Stephens et al., 2010 *Purpose*: Randomized pretest-posttest comparison group study to examine effects of two versions of Enhanced Anchored Instruction (EAI; Embedded & Explicit) on problem solving and computation on problem solving.	N=303; LD=16; M= 158; F=145; Gr. 6-7; *Setting*: Information Technology Classroom; *Duration*: Between 20-27 days	• Problem solving test (PST) (α= .90; IRR=97%); • Fraction Computation Test (FCT) (α= .97; IRR=98%); • Iowa Test of Basic Skills (ITBS)-Computation subtest (Kuder-Richardson= .814) • Iowa Test of Basic Skills (ITBS)-Problem solving and Data interpretation subtest (Kuder-Richardson= .842)	TAAT: Average 97%	Students in treatment made significant gains compared to control group on two of the assessments: • PST: Embedded group • (ES = .53) and Explicit group (ES = .32) • FCT: Only Embedded group made significant gains (ES = .42) • No significant differences on ITBS subtests

Bottge, Grant, Laroque et al., 2010 *Purpose*: Pretest-posttest cluster randomized experiment study to examine efficacy of EAI + formal and informal instruction on problem-solving and computation	N=54; LD=46; M=39; F=15; Gr. 6-8; *Setting*: Self-contained; *Duration*: 24 days average 53 minutes per session	• Measures used were the same as those used in the previous study (See Bottge, Grant, Stephens et al., 2010)	IOA ranges from 77% to 100%	Students in both treatments outperformed control group on two of the assessments: • FCT: EAI informal mean gain (ES = 1.14). EAI formal mean gain (ES = 0.81) • PST: EAI informal mean gain (ES = 1.16). EAI formal mean gain (ES = 1.19) • No significant differences for ITBS subtests
Bottge & Cho, 2013 *Purpose*: Experimental study to investigate the efficacy of EAI on common core derived word problems.	N/LD= 308; M=204; F=104; Gr. 6-8; *Setting*: Resource Rooms; *Duration*: 74 days; 45-60 minutes & 90 minute blocks	• Problem Solving Test Part A (PS-A) (α= .76^; .82**); • Problem Solving Test Part B (PS-B) (α= .63^; .73**)	Data used from previous study with high IOA (See Bottge et al., 2014)	Treatment students outperformed control group: • 56 % in EAI and 33% in control had change scores (i.e., pretest to posttest) greater than 1 standard deviation • 24% in EAI and 11% of control group had change scores greater than 2 standard deviation

Bottge et al. (2014) *Purpose*: Pretest/ posttest, cluster-randomized study to determine efficacy of EAI Instruction on computation and problem solving	N=335*; LD= 49; M=223; F=112; Gr. 6-8; *Setting*: Resource Rooms *Duration*: 94 days; 45-60 minutes & 90 minute blocks	• Fraction Computation Test (α = .81^; .96**) • Problem-Solving Test (PS I-PS2) (α = .63^; .73**) • Iowa Test of Basic Skills (ITBS)-Computation subtest (Kuder-Richardson 20= .72^; .78**) • Iowa Test of Basic Skills (ITBS)-Problem solving & Data interpretation subtest (Kuder-Richardson 20= .61^; .58**)	IOA: 94%	Treatment students outperformed control on three of the four measures: • FCT: EAI scored approximately a standard deviation higher than control on all subscales • PST: Effect size for the entire test was. 39; effects were larger for ratios and proportional relationships (ES = .61) and geometry (ES = .57) scales • ITBS Computation: Students in the EAI group made significant gains only on fraction subscales (ES= .56) • No significant differences were found for the ITBS Problem Solving and Data Interpretation subtest

Note: N = total participants; LD = total students with learning disabilities; M = males; F = females; Gr. = grade level; * = all students have disabilities; ^ = reliability of pre-test measure; ** = reliability of post-test measure