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Research on Group Learning and Cognitive Science: A Study of Motivation, Knowledge, and Self-Regulation in a Large Lecture College Algebra Class

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At a research University near the east coast, researchers restructured a College Algebra course by formatting the course into two large lectures a week, an active recitation size laboratory class once a week, and an extra day devoted to active group work called Supplemental Practice (SP). SP was added as an extra day of class where the SP leader has students work in groups on a worksheet of examples and problems, based off of worked-example research, that were covered in the previous week's class material. Two sections of the course were randomly chosen to be the experimental group and the other section was the control group. The experimental group was given the SP worksheets and the control group was given a question-and-answer session. The experimental group's performance was statistically significant compared to the control on a variety of components in the course, particularly when prior knowledge was factored into the data.

The enrollment in College Algebra has grown recently to the point that nationally there are an estimated 650,000 to 750,000 students per year (Haver, 2007) and has surpassed the enrollment in Calculus. In *A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education* (2005), the authors write that "nationally 22% of all college freshman fail to meet the performance levels required for entry level mathematics courses and must begin their college experience in remedial courses" (p. 6). Although there

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are almost three fourths of 1 million students enrolling in College Algebra, it is estimated that 45%, and as high as 60% at some institutions, of College Algebra students fail to receive a grade of A, B, or C (Haver, 2007). To address this non-success of students at a large research university in the eastern part of the United States, faculty members teaching College Algebra implemented a new structure in the course that emphasized active learning through Supplemental Practice.

The Interactive, Compensatory Model of Learning

The Interactive, Compensatory Model of Learning (ICML) provides a framework for understanding and improving classroom learning (Schraw & Brooks, 1999). Schraw and Brooks (1999) referred to a wide range of literature that reinforced ICML. Figure 1 shows ICML, which consists of five main components that affect learning: cognitive ability, knowledge, metacognition, strategies, and motivation. Schraw, Brooks, and Crippen (2005) defined cognitive abilities as “a general capacity to learn” and knowledge base as “individuals organized domain-specific and general knowledge in long-term memory” (p. 637). In other words, cognitive ability is the ability to learn (how much an individual can learn). Knowledge base is what the individual “knows.” This “knowing” includes both correct and incorrect information.

Schraw et al. (2005) also stated that metacognition “includes knowledge about oneself as a learner and how to regulate one’s learning” (p. 637). Metacognition refers then to what students know about how they think (how information is organized, stored, and retrieved in their minds, how connections between ideas are formed) and learn (what strategies or techniques they use). Strategies “refer to procedures that enable learners to solve specific problems” (p. 637). Simply put, strategies can be defined as what the students do. Finally Schraw et al. claimed motivation referred to the beliefs one has about their potential success at a given task. Motivation also included the goals set by the learner. If a student feels that he or she can succeed, but needs to work at it, then the student will put forth the effort. Conversely, if the student does not need to put forth an effort or does not believe that he or she can succeed, then the student will not do the work. The key to motivation, based on this definition, is to increase the student's self-confidence in his or her abilities, but still provide a challenge.

Cognitive ability impacts the knowledge base, and influences metacognition and the strategies that are used. Because knowledge, metacognition, and strategies are closely connected, they are combined together into one area (See Figure 1). A more in depth discussion of the empirical research that justified combining these three areas into one can be found in Schraw and Brooks (1999). This combined area is referred to as the knowledge-regulation component. The ICML captures the interactions between these four components (Cognitive Ability, Cognition, Regulation, and Motivation) that affect learning and describes how one component can compensate for deficiencies in others. The ICML model shows that learning can be affected directly by cognitive ability, motivation, and the knowledge-regulation components, and indirectly through the knowledge-regulation and motivation components.

Based on this model, learning can be affected by knowledge-regulation, which is shown by the arrow going from knowledge-regulation to learning. Learning can also be indirectly affected by knowledge-regulation through the motivation component, which is shown in the model by the arrow to motivation and the arrow from motivation to learning. Schraw and Brooks (1999) established and gave reasons for the direct and indirect connections between the components. Schraw and Brooks claimed ICML "is an empirically-based model that provides a *comprehensive* approach to learning. It includes all of the main components known [from the literature] to affect learning. More important, it provides a tentative basis for evaluating the relationships among these components" (p. 8, emphasis added). Therefore, the ICML framework provided insight on the components that affect learning, how they are connected, and how one stronger area in the ICML framework can compensate for a weaker area.

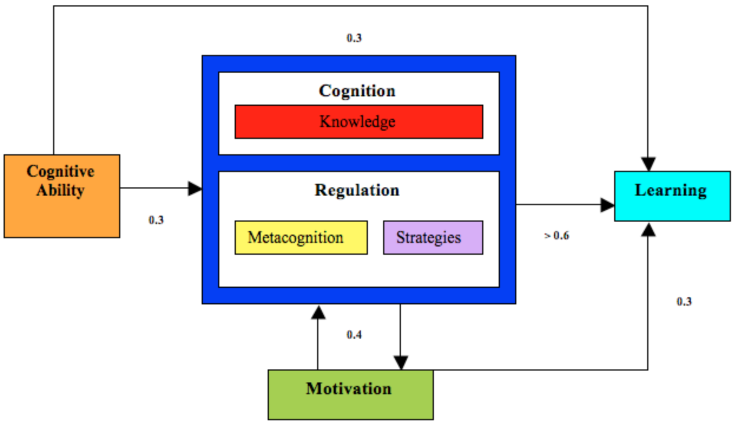

Figure 1. Interactive, Compensatory Model of Learning (with correlation coefficients)

The numbers in the figure refer to the estimated correlation coefficient between two components. Each correlation coefficient is the estimated value of what has been measured in a number of empirical studies (Alexander, Carr, & Schwanenflugel, 1995; Brody, 1992; Garner, 1987; Glenberg & Epstein, 1987; Graham & Weiner, 1996; Herrnstein & Murray, 1994; Schunk, 1996; Zimmerman, Greenberg, & Weinstein, 1994). For example, cognitive ability is correlated to learning with correlation coefficients ranging from 0.3 to 0.4. Hence, the correlation coefficient of 0.3 relates these components. The other correlation coefficients are shown in Figure 1. Schraw and Brooks (1999) stated, "most experts agree that knowledge and regulation exert a strong direct effect on learning that is greater than the effects of either ability or motivational beliefs" (p. 9). This connection is the strongest correlation with correlation coefficient of 0.6.

The compensatory part of the model refers to how students can compensate for a weakness in one component with the strength of another component. For example, students who have weaker cognitive abilities (also referred to as intelligence) can compensate by having a stronger knowledge-regulation component. That is, students can regulate their learning while they work diligently to increase their knowledge about a particular topic. Through this iterative process, as they go from one topic to another topic in the course to gain knowledge, they successfully compensate for their weaker knowledge in a subject as compared to other students. The notion of compensatory processes is supported by other literature (Gardner, 1983; Perkins, Segal, & Voss, 1987; Steinberg & Wagner, 1988). Schraw and Brooks (1999) state the following compensations may occur:

(i) Ability compensates in part for knowledge and regulation

(ii) Knowledge and regulation compensate for cognitive ability and motivation, and

(iii) Motivation compensates for ability, knowledge, and regulation.

The compensation part of the ICML could explain why we see some students do better in a course than other stronger students, even though it would appear from earlier performance that they should not. Through hard work, the students strengthen their knowledge-regulation component and become more motivated as they become more successful, compensating for their weaknesses. The compensation component of the ICML becomes a powerful lens to view student learning in a course (or numerous courses). The next section discusses the history of Supplemental Practice and briefly covers literature on worked examples.

Background and Literature Review

Supplemental Practice Structure

At our institution, we first implemented Supplemental Practice (SP) during the Fall 2004 and originally modeled it after Supplemental Instruction (SI) (Arendale, 1994). The prior structure of the college algebra class consisted of three lectures a week morphed into a structure of two lectures a week in a large lecture room, and an active laboratory class once a week in computer classrooms where students met in smaller groups. The lab class, which was overseen by the instructor and several teaching assistants, was held on Tuesdays while the lecture class was held on Mondays and Fridays. The SP days on Wednesdays were originally added to the schedule to help lower-achieving students. Lower-achieving students (those who scored lower than an 80 on a placement exam or scored lower than a 70 on any regular exam) were required to attend the SP sessions. Starting in the Fall 2006 semester, the SP sessions have morphed into active problem-session days modeled after the cognitive science “worked-out example” research (Carroll, 1994; Cooper & Sweller, 1985; Tarmizi & Sweller, 1988; Ward & Sweller, 1990; Zhu & Simon, 1987). .

The worked-out example research, henceforth denoted as worked examples, has students study a worked example for a particular topic, ask questions about anything in the example that they do not understand, and finally work a similar example without reference to the worked example, nor other outside sources. The SP sessions and worksheets have been developed based on the worked example research. The literature on worked example has shown many benefits including: (a) reduces cognitive load and in turn allows cognitive resources to be employed towards problem solving (Atkinson et al., 2000; Cooper & Sweller, 1987; Sweller, 1988), (b) builds problem solving knowledge and schema (Sweller & Cooper, 1985; Sweller, 1988), (c) helps with rule automation (Cooper & Sweller, 1987), (d) helps with flexible problem solving transfer to different problems (Catrambone & Holyoak, 1990; Reed & Bolstad, 1991), and (e) students are more likely to focus on deep structures and more sophisticated problem solving strategies (Atkinson et al., 2000) .

Worked Example Research

The discipline of cognitive science deals with the mental processes of learning, memory, and problem solving. Worked example research was developed based from Sweller’s (1988) cognitive load theory. The total load on working memory at any moment in time is referred as the cognitive load. Most people can retain about seven “chunks” of information in their working memory and when they exceed that limit, there will be a loss of information in the working memory. In other words, if there is an overflow of information in the working memory, then consequently a cognitive overload occurs. Cognitive overload can be thwarted if one limits information so that it does not exceed the students' working memory. One way this can be done is to transfer information from working memory to long-term memory as information is being processed (or soon after). According to Sweller (1988), optimum learning occurs in humans when one minimizes the load on working memory, which in turn facilitates changes in long-term memory.

It has been suggested that worked examples reduce the cognitive load on a student and might optimize schema acquisition (Sweller & Owen, 1989**;** Sweller & Cooper, 1985). In addition, worked examples have been researched in a variety of subjects: mathematics (Cooper & Sweller, 1985; Zhu & Simon, 1987), engineering (Chi, Bassok, Lewis, Reimann, & Glaser, 1989), physics (Ward & Sweller, 1990), computer science (Catrambone & Yuasa, 2006), chemistry (Crippen & Boyd, 2007), and education (Hilbert, Schworm, & Renkl, 2004). We highlight a few of these at the end of this section.

Sweller and Cooper (1985) conducted one of the first studies on worked examples with high school-level Algebra students. Through five experiments, they examined the use of worked examples as a substitute for problem solving. Sweller and Cooper's first experiment found that more experienced students had a better cognitive representation of algebraic equations than less experienced students as measured by their ability to recall equations and distinguish between perceptually similar equations. Sweller and Cooper (1985) concluded that there was "evidence that expertise in solving algebra manipulations problems is, at least in part, schema based." (p. 67). During this experiment, the students were asked to read and to make sure they understood the worked examples.

The other experiments integrated an alternating pattern between worked-out examples and conventional problems. The alternating pattern increased the motivation of the students to read and understand the worked example if they had to solve a conventional problem immediately after the worked example. Throughout their experiments, Swellers and Cooper also found that worked examples aided in reducing the acquisition time and improved achievement on the test phase of the experiments. However, when it came to dissimilar problems, the students in both the control and experiment groups struggled. As Sweller and Cooper (1985) concluded, "worked examples are of assistance to students when faced with similar problems, the advantage does not extend to dissimilar problems" (p. 83).

Zhu and Simon (1987) demonstrated the feasibility and effectiveness of teaching Chinese middle school students mathematical skills through chosen sequences of worked examples. In the first experiment, 20 Chinese middle school students were chosen for the experimental group where half of the students learned using ten worked examples and the other half learned by working a sequence of ten carefully arranged problems on factoring quadratics. After students spent approximately thirty minutes working examples or completing problems, they worked through the test questions. All 20 experimental participants solved the test questions correctly.

In subsequent experiments, Zhu and Simon (1987) replicated the results of their first experiment on factoring with another group of students and expanded the research to other tasks devoted to various topics: exponents, geometry, and ratios and fractions. This was followed up with retesting the students after a year on two of the tasks. Zhu and Simon found that students retained the material at a very high level and the experimental groups retained material at a slightly higher percentage than the control group.

Chi, Bassok, Lewis, Reimann, & Glaser (1989) investigated how 10 students studied worked examples on applications of Newton's laws of motion and how this learning transferred. The study was broken down into two phases: a knowledge acquisition phase and problem solving phase. During the problem solving phase the students needed to transfer what they had learned from the worked examples. Chi et al. (1989) found the more successful students, labeled as good students, (a) verbally generated more self-explanations while studying the worked examples, (b) verbally generated more accurate self-monitoring statements while studying the worked examples, (c) referred to the worked examples less during the problem solving phase, and (d) reviewed only specific parts of the worked examples when they referenced worked examples, when compared to less successful students (labeled as poor). Chi et al. (1989) demonstrated that while students studied worked examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations, while "poor" students did not generate sufficient self-explanations or monitored their learning inaccurately.

Ward and Sweller (1990) examined the effect that worked examples would have on high school physics students in Australia when the worked examples required students to simultaneously attend to multiple sources of information at a time (coined split-attention). Ward and Sweller (1990) established that students who used worked examples, formatted to reduce split-attention, achieved test performances superior to those exposed to worked examples that required split-attention. The worked examples that required split-attention, forced a higher cognitive load on students and therefore less working memory to process the examples. Therefore, worked examples that do not require students to integrate multiple sources of information are optimal during learning.

Excluding Zhu and Simon (2007), the research on worked examples in mathematics has been conducted in a laboratory setting. The research was not done on a particular course in high school or college, but students volunteered to be part of the study and meet with the researchers in a designated space. The current research was conducted in a large lecture classroom setting and concentrated on determining if worked examples helped promote success in the course. In addition, past worked example research has dealt very little with college mathematics courses (Sweller & Cooper, 1985; Ward & Sweller, 1990; Zhu & Simon, 1987). In addition, the previous studies have not investigated how worked examples can help students learn mathematics in large lecture classes, or during a supplemental day of class.

The results of our study could be valuable to other researchers that are working to promote student success in large lecture classes. Our guiding research question was: Do students in the experimental group earn better grades on exams and quizzes and more points in the course than students in the control group?

Prior Data

Data has been generated for all students in College Algebra, including SP sessions attended since Spring 2007. Students have historically performed better in the class as the number of SP days they attended increases. Additionally, the success rates—the number of students that receive at least a C divided by the total number of students—exceeded 65% range for students that attended 9 or more SP days in a given semester. Figure 2 shows success rates versus number of SP days attended in the Spring 2007.

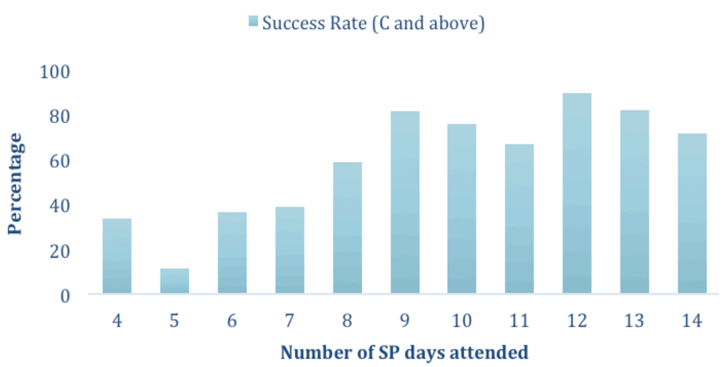

Figure 2. Percentage of grades of C or above versus supplemental days attended

The overall course averages in the spring 2007 College Algebra course are shown below (See Table 1). Table 1 gives the overall course average of students that attended 1 to 13 supplemental days. We see that students that attended eight or more SP days earned overall course averages in the 70% range with a maximum course average of 77.8% for students that attended 12 SP days. Also, all but the students who attended 8 and 11 SP days earned overall course averages above 75%. The data looked very promising in showing that SP helps students in the course. However, another reading of the data could be suspect because students voluntarily attended the SP sessions and perhaps only the motivated students attended the majority of the sessions. To be able to investigate if SP sessions were beneficial to the students, the author designed an experiment prior to the Fall 2009 semester to give the experimental group the treatment (worked example worksheets) and the control group an alternative question-and-answer session.

Table 1
Course Averages and SP Attendance

Number of Supplemental days attended	Course Average	Number of students
0	10.615	2
1	16.185	10
2	22.615	4
3	20.893	3
4	35.630	3
5	45.078	9
6	63.376	22
7	67.040	54
8	71.746	34
9	75.354	27
10	74.963	29
11	73.771	15
12	77.838	19
13	76.625	11
14	75.579	7

Methodology

Course Description

Three different arrangements of College Algebra courses were offered at the University. The first arrangement was a 3-day large lecture College Algebra course that was comprised of two lectures a week in a large classroom setting and one day a week in the lab where students actively work in smaller-group on mathematical tasks. The second type was a 4-day College Algebra course, which had the same format as the 3-day College Algebra course, except the fourth day is spent in SP. The final type was a 5-day College Algebra course was comprised of five lectures a week in a class size of approximately 40 students. All College Algebra courses required specific placement exams scores. A course coordinator was responsible for ensuring that all sections of the 4-day College Algebra course's structure, labs, quizzes, and exams were the same from section to section. The same instructor taught all three sections during the semester the study was being conducted.

The quizzes were given online outside of class and students were given up to three attempts on each quiz. Laboratories were completed during the lab day, once a week in groups of two or three students. The labs had interactive applets, mostly a graphing applet, that students worked on to complete the lab. The graphing applet could be used on the exams in place of a graphing calculator. The exams and final were common online exams consisting of 20 multiple-choice questions. They were administered through the online management system. The exams and final were taken during lab time approximately once every three or four weeks. Students were given a list of suggested homework problems, but no homework was collected. The quizzes and exams were based on the homework. The course consisted of 100 points for attendance, 200 points for eight labs, 100 points for six online quizzes, 100 points for each of four computerized exams, and a 200 point common online final.

Participants and Setting

The setting for the research was a large lecture 4-day College Algebra course with an annual enrollment of around 1000 students at a research University near the east coast. The participants for this study were part of the 4-day College Algebra and in one of three sections. One section was taught in the morning and the other two in the early afternoon. The Monday class was the SP class time where students worked in small groups on the worked example worksheets. There were 497 students enrolled in college algebra with 177 students in the control group and 320 students in the experimental group, with the demographics shown in Table 2. Students were from a variety of majors due to College Algebra being a general elective course and many majors required it for their degree requirements.

Table 2
Demographics of the Control and Experimental Groups

Demographic	Control	Experimental
Male	58.19%	52.83%
Female	41.81%	47.17%
Freshman	68.93%	64.78%
Sophomore	21.47%	27.04%
Junior	7.34%	5.35%
Senior	2.26%	2.52%
Graduate	0%	0.31%

Worked Example Worksheets

During the SP days, worked example worksheets were handed out to the students to work on in groups. SP days were moved from Wednesdays to Mondays in Fall 2008 so that students would have SP days before the exams that occurred periodically on Tuesdays. Also it was decided that we would not do SP during the first week since students can add the class late and this would allow us to spend more time reviewing basic algebra. Students formed groups with peers near them as they saw fit because the class was in a large lecture classroom setting with theatre-seating structure. Usually, students worked with one to three other students seated close to them. The worked example worksheets consisted of a solution of a College Algebra problem followed by a problem for the students to work out. For example, the following worked examples (see Figure 3) were given on worksheets 3 and 11 during the 5th and 13th SP sessions.

Worked-out Example (2.2):
Find the slope and y-intercept for the equation $2y + 3x = 1$.

To find the slope and y-intercept we need to rewrite the equation into the form $y = mx + b$. That is, solve for y. Thus rewriting $2y + 3x = 1$ we have $2y = -3x + 1$ which simplifies to $y = -\frac{3}{2}x + \frac{1}{2}$. So the slope is $-\frac{3}{2}$ and y-intercept is $\left(0, \frac{1}{2}\right)$.

2(b) Find the slope and y-intercept for $\frac{1}{2}y + 5x = -3$.

Worked-out Example (6.6):
Solve $\log_4 x + \log_4 (x-3) = 1$ for x.

Using the property $\log_b x + \log_b y = \log_b (xy)$, we have $\log_4 (x(x-3)) = 1$. Changing this from logarithm to exponential form, we have $4^1 = x(x-3)$. So

$$4^1 = x^2 - 3x$$
$$0 = x^2 - 3x - 4$$
$$0 = (x-4)(x+1).$$

So $x - 4 = 0$ or $x + 1 = 0$. Therefore $x = 4$ or $x = -1$. But we can not include $x = -1$ as a solution because when we substitute it back into the original equation it yields $\log(-1)$ and $\log(-4)$ which are undefined since we can not take a logarithm of a negative number. The solution is $x = 4$.

1. Solve $\ln(x+1) - \ln x = 2$ for x.

Figure 3. Sample Worked-out Examples from several Worked-out Example Worksheets

The first SP occurred on the third Monday of the semester. These two problems are stated exactly as they were on the worksheet, which was always given to the students as one sheet in a two-column format with headings on all worked examples. Also, each of the examples were followed by the section in the textbook (Sullivan & Sullivan, 2006). Student could then reference the text outside of class. There were approximately 8 to 10 worked examples and problems on each worksheet.

The material on the worksheets reviewed some of the content covered during the previous weeks lecture. Due to time constraints, not all of the topics from the previous week's lecture were covered on the worksheets. The worksheets comprised of problems directly from or derived from the problems in the textbook with no new material being presented. Finally, the worksheets were modeled after worked example research (Cooper & Sweller, 1985) because it presented an expert's solution to a problem followed by a problem for the student to work out. Unlike previous worked example research, it was not plausible to ask the students not to reference the worked example while completing another problem. Furthermore, most studies on worked examples stated that the student should be given a similar problem. However, the SP problems varied in how similar they were to the worked examples.

Experiment

Of the three 4-day College Algebra courses, the researchers randomly designated one course section as the control group (n = 177) and the other two as the experimental group (n = 320). In the experimental group, the students were given a "worked-out example" worksheet at the beginning of each of the 13 SP days and asked to work in groups to complete the worksheet. Two to three undergraduate class assistants and one graduate student circulated around the room to answer any student questions about the worksheet. In the control group, a graduate student organized a question-and-answer session during the extra day instead of giving a worksheet to the students. Students were able to get any question answered, but the graduate student only answered student questions. Quantitative data including course scores on exams and quizzes, supplemental days attended, class attendance, and total points were collected for all students, and analyzed at the end of the semester. There were similar demographics in both the control and experimental groups.

Data and Results

In this section, we will compare the control and experimental groups as whole groups and in terms of levels of prior knowledge. We will then go into more detail by comparing the control group versus groups of students that were extremely motivated, as measured by attendance of SP days, and comparing specific groups in the experimental group. We begin by looking at the control and experimental groups as a whole.

Control and Experimental Groups

Data from the experimental and control groups were compared on a variety of levels by using a t-test with equal variances. This test (t-test with equal variances vs. t-test with unequal variances) was deemed appropriate because the F-test used to measure unequal variances resulted in p-values that were greater than 0.05, meaning that the variances were not significantly different. There was one occurrence where the p-value was close to 0.05 and we have made a note of this in Table 4. The control and experimental groups had similar levels of retention, the number of students that completed the course, at 84% and 80.5% for the control group and experimental group, respectively.

At the beginning of the semester, all students were given an old ACT math exam (ACT, 2002-2003) that consisted of 60 questions. Students were given extra credit points for the ACT exam on a sliding scale. This ensured that the better a student performed, the more extra credit, up to 10 points, he or she earned. The ACT exam provided a measure of students' prior mathematical knowledge. Students were given 60 minutes to complete the ACT exam, with each question graded as right or wrong. The number of correct answers out of 60 was recorded. Table 3 shows the topics covered by the ACT, what section of the course's textbook covered those topics, and the number of questions related to those topics. There was some overlap among the sections. For example, laws of exponents were covered in both the review and sixth chapter.

Table 2 shows that 27 out of 60 questions are on review topics that students should have covered in high school algebra. An additional 17 questions would be more than likely covered in high school algebra (linear equations, inequalities, absolute value, distance, midpoint, basic graphing, and slope). Furthermore, one could argue that the Chapter 3 topics on the ACT dealing with the concept of functions are covered in high school algebra. Therefore, around 75% of the ACT questions address concepts that are covered in typical high school algebra, so it was seen as a valid measure of students' prior knowledge entering the class. This estimate is conservative because there are also questions on the ACT that come from Chapter 6 (composition of functions) and Chapter 12 (systems of equations), topics that are covered in many high school algebra classes. Butler, Pyzdrowski, Walker, and Butler (2012) reported the same breakdown (75%) after analyzing the ACT on topics covered in the Sullivan and Sullivan (2006) College Algebra textbook. With all of this information and the fact that ACT is used for college admission, we believe the ACT provided a good assessment of prior knowledge and was a valid assessment for this purpose.

Table 3
Breakdown of the ACT

Chapter of the Textbook (Sullivan & Sullivan, 2006)/Other Topics	Topics from Chapter on ACT	Number of Questions
Review	Number Systems, Evaluating Algebraic Expressions, using laws of exponents, perimeter	27
1	Linear equations, application using linear equations, absolute value inequalities, inequalities, midpoint, application of slope, absolute value, basic graphing, distance	11
2	Lines	6
3	Properties of Functions	1
6	Composite functions, Logarithms, and Exponential and Logistic Growth and Decay Models	3
12	System of Equations	2
Probability and Statistics	Average, Probability, Median, Weighted Average	5
Trigonometry	Definition & Graphs of Trig functions, Trig Identities, angles, arc length	5

Figure 4 shows the control and experimental groups' prior and post mathematical knowledge measured by the old ACT exam. At the beginning of the semester, the mean control and experimental groups' ACT scores were 28.40 and 26.91 with standard deviations of 6.41 and 6.86, respectively. The difference between the control and experimental groups were statistically significant ($p = 0.012$) on the (pre) ACT test.

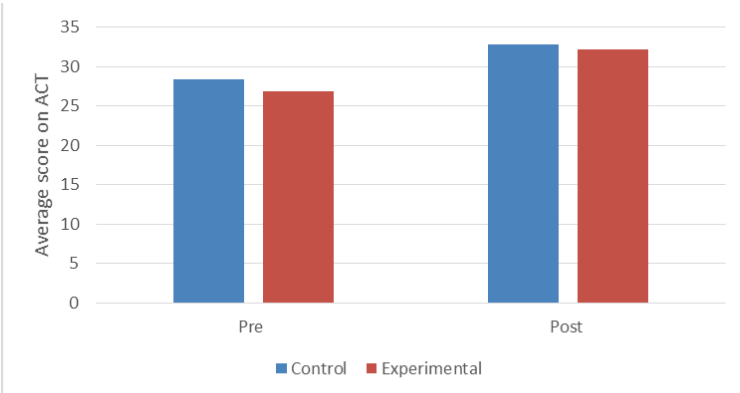

Figure 4. Prior and Post Mathematical Knowledge measured by an old ACT exam

At the end of the semester, the control and experimental groups earned mean ACT scores were 32.81 and 32.16, with standard deviation of 6.46 and 7.22, respectively. There was no significant difference between the mean post-ACT scores of the two groups. Therefore, the experimental groups were able to close the gap on the control group.

Table 4
Means and Standard Deviations for Tests

	Test 1	Test 2	Test 3	Test 4	Final	Quizzes	Current Pts
Control Group (n=177)	68.84 (16.21)	66.86 (19.93)	66.58 (19.61)	67.23 (22.67)	112.20 (48.04)	71.52 (18.89)	453.24 (120.04)
Experiment Group (n=320)	67.52 (16.18)	69.38 (17.99)	70.05 (19.76)	68.66 (23.88)	120.66 (47.28)	74.57 (20.65)	470.82 (119.49)

The two groups were compared with respect to each exam, the final, quizzes, and current points (without any attendance or lab grade). Table 4 shows the exact scores, with standard deviation in parentheses. The experimental groups outperformance on test 3 ($p = 0.031$) and the final exam ($p = 0.029$) was statistically significant. The experimental groups scores outperformed, but not statistically significant, the control group on test 2 ($p = 0.076$), quizzes ($p = 0.052$) and current points ($p = 0.059$). There was no difference between the control and experimental groups with respect to test 1 or test 4. We note that the experimental group outperformed the control group on everything except test 1. There were only two SP days before the first exam and the students were getting used to the worked example worksheets, so it is reasonable that there were no significance on exam 1. For all other exams and the quizzes, the group of students that used the worked example worksheets outperformed the group of students who did not.

Course grade point average was calculated to compare the two groups on the average course grade earned. This was accomplished by assigning a quantitative score for the final grade that each student earned in the course (A = 4, B = 3, C = 2, D = 1, and F = 0). The course grade point average for control group was 1.97 and the experimental group was 2.13 with standard deviations of 1.17 and 1.27, respectively. The experimental group outperformed the control group, although this was not statistically significant ($p = 0.080$).

Experimental Group versus Control Group Based on Levels of Prior Knowledge

Through the ICML framework students can compensate for weaker knowledge by having stronger knowledge-regulation and motivation components. To investigate this, the researcher examined the difference between the control and experimental groups based on levels of prior knowledge. The high prior mathematical knowledge group was defined to be all students with a score of 31 or more out of 60 on the old ACT math exam. The high prior mathematical knowledge control ($n = 54$) and experimental ($n = 88$) groups were denoted as high control and high experimental. The medium prior mathematical knowledge group was defined to be all students with an old ACT math exam score from 26 to 30. The middle prior mathematical knowledge control ($n = 59$) and experimental ($n = 89$) groups were denoted as middle control and middle experimental. The low prior mathematical knowledge group was defined to be all students with an old ACT math exam score of 25 or below. The low prior mathematical knowledge control (n = 49) and experimental (n = 119) groups were denoted as low control and low experimental. These ranges in scores resulted from the researchers dividing the control and experiment participants in similar size groups of students with higher, average, and lower scores. The n values in each group would change quite a bit if the range was changed and hence the chosen ranges yielded approximately similar groups.

Figure 5 shows the prior/post mathematical knowledge for the control and experimental groups based on the level of prior knowledge groups. Comparing prior, there was no statistically significant difference in mathematical knowledge for the high control, 35.26, and high experimental, 34.84. Similarly, no statistical significance was seen in the low control, 21.14, and low experimental, 20.45. However, for the prior knowledge score differences between the middle control, 28.14, and the middle experimental, 27.71, was statistically significant (p = 0.041), with standard deviations of 1.48 and 1.43, respectively. Comparing post, there was no statistically significant difference in mathematical knowledge for the high, middle, and low groups.

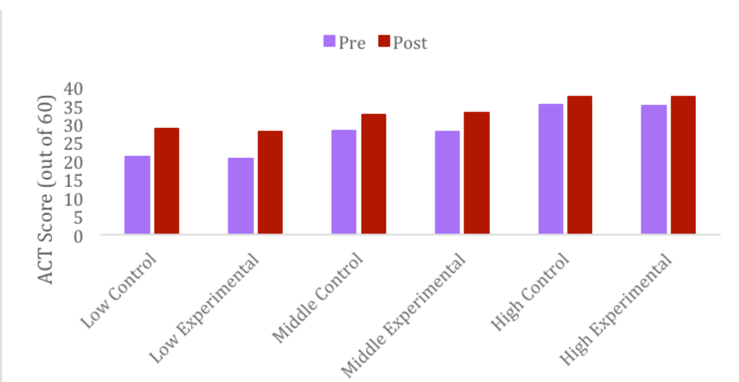

Figure 5. Prior/Post Mathematical Knowledge Based on Prior Knowledge Level

Furthermore, there was no statistically significant difference between the performance of the high control group and the high experimental, but the high experimental consistently outperformed the high control group on the exams, final, quizzes, course GPA, and total points (See Table 5). Similarly, the difference between the performance of the low control and low experimental was not statistically significant These results are expected because the high prior knowledge group comes into the class with a stronger mathematical background than many of the students in college algebra. The low prior knowledge group came into the class with a weaker mathematical background than other students in college algebra and has too much ground to make up to be proficient in college algebra.

Unlike the high and low prior mathematical knowledge levels, the middle prior mathematical knowledge level was where the data was statistically significant. The middle experimental outperformed significantly the middle control on current points, test 3, final, quizzes, and course GPA. In addition, the middle experimental group outperformed the middle control group on test 1, 2, and 4 (See Table 6).

Table 5
Scores for the low and high prior knowledge group

	High Prior Knowledge Group		Low Prior Knowledge Group	
	Mean Control (n=54)	Mean Experimental (n=78)	Mean Control (n=49)	Mean Experimental (n=104)
Current Points	520.03	529.45	421.84	436.57
Test 1	75.83	74.20	64.80	61.30
Test 2	77.69	78.01	59.08	63.91
Test 3	74.07	78.13	61.12	64.62
Test 4	75.37	76.70	64.18	64.20
Final	139.26	142.73	101.84	110.34
Quizzes	77.72	79.68	70.82	72.20
Course GPA	2.52	2.72	1.61	1.80

Table 6
Middle Prior Knowledge Mathematical Level

	Mean Control (n=59)	Mean Experimental (n=89)	Standard Deviation Control	Standard Deviation Experimental	P – Value (p < 0.05 is significant)
Current Points	431.33	474.89	131.46	110.62	0.016*
Test 1	67.88	69.66	15.18	13.87	0.231
Test 2**	64.41	69.49	22.32	15.45	0.065
Test 3	66.19	72.53	19.94	17.44	0.021*
Test 4	63.47	69.33	24.48	24.54	0.079
Final	100.17	117.64	54.63	45.90	0.019*
Quizzes	69.21	76.24	20.09	20.43	0.020*
Course GPA	1.85	2.19	1.26	1.22	0.050*

** The variances were unequal according to the F-test, so the appropriate t-test was used.

Discussion

For the most part, prior anecdotal data had shown that the more days of SP that a student attended, the better the student performed overall in College Algebra. One could argue that the students that were motivated, as measured by SP attendance, came to the SP sessions 8 or more times. Possibly explaining why they were more successful. This research study was developed to investigate whether the SP sessions helped students be more successful on specific course components and overall course success.

We see that the control group started with statistically significant higher prior mathematical knowledge than the experimental group. However, the control and experimental groups ended the semester with no statistically significant difference in post mathematical knowledge. The students in the experimental group added to their prior knowledge throughout the class. Consequently, they had similar post mathematical knowledge when assessed by an old ACT exam. On average students in the experimental group increased their ACT exam score by 5.15 points compared to an increase of 4.32 points for the control group, which was not significant ($p = 0.089$). Many of the students attended the SP days regularly. We took this as evidence of their motivation to learn the college algebra content. We see the above data as showing the experimental group increased, notable but not significant, in their prior knowledge more than for the control group. One possible explanation for this would be that students in the experimental group had a stronger motivation component that compensated for a weaker prior knowledge component. This is very important because the ICML framework takes into consideration that students can compensate a weaker knowledge component with a stronger motivation component

Examining things a little further, we see that the high and low level prior mathematical knowledge groups started with similar prior knowledge but the middle prior mathematical knowledge control group was higher than the middle prior mathematical knowledge experimental group. However, all three experimental groups (high, middle, and low) ended the semester with similar post mathematical knowledge as the three control groups. The middle group started with lower prior knowledge, but ended with similar post knowledge in the course. We can conclude that it was the middle group's mathematical knowledge that was affected the most by the worked examples. An explanation for this is two-fold and based on the ICML framework. First, students compensated for a weaker prior knowledge component with a stronger motivation component. Second, the worked examples worksheets helped the students in the middle experimental group to accumulate the knowledge needed to bring their mathematical knowledge to a similar level of the middle control group.

We also found that, for the most part, the difference in the middle prior mathematical knowledge experimental group performance was greater than the middle prior mathematical knowledge control group was statistically significant. In fact, there were only three components where they only outperformed (not significantly) the control group. The non-significance on test 1 for the middle groups is understandable. There were only two SP days before test 1 and students had to get comfortable with the worked example worksheets and the structure of the SP sessions. Both test 2 and test 4 showed that the middle experimental group outperformed the middle control by a half letter grade and the difference was very close to being statistically significant. Exponential and Logarithmic Functions and Properties make up the majority of the topics on test four and we believe the students were less comfortable with these topics. Therefore, this presented a plausible reason why there was not significance on test four. Finally, the middle experimental group outperformed the middle control group on current points, test three, final exam, quizzes, and course GPA. We see that unlike the lower prior knowledge group, the middle prior knowledge experimental group was able to compensate for their lack of prior mathematical knowledge via motivation to become more successful in the course compared to the middle prior knowledge control group.

Conclusion

We found that the SP sessions benefited the middle experimental group the most. Using the ICML framework, SP sessions helped the students in the middle experimental utilize a stronger motivation component to compensate for a weaker knowledge component. In addition, as their stronger motivation compensated for weaker knowledge, they increased (strengthened) their knowledge through learning in an iterative process throughout the semester. This led to more learning directly from their knowledge-regulation component and indirectly through motivation. We propose that the high experimental group was not motivated any differently than the high control. That is, the high prior mathematical knowledge group would learn the material no matter what intervention was given in the class. Using the ICML perspective there are two components that can compensate for the knowledge-regulation component: ability and motivation. As such, the high prior knowledge group possessed the ability and did not need as much motivation.

It is not completely clear why there was no statistically significant difference in the lower prior mathematical knowledge groups. The research by Chi et al. (1989) might shed some light on this. Perhaps the high prior mathematical knowledge group was comprised of the "good" students that have sufficient self-regulation skills and the low prior mathematical knowledge group was comprised of the "poor" students that do not have sufficient self-regulation skills. We propose that the lower groups started the course with a low prior knowledge base, such that they were not able to compensate for this lack of knowledge through cognitive ability or motivation.

It would be interesting to see if the higher cognitive ability students in the low prior mathematical knowledge group are able to compensate for the weak knowledge-regulation component. This would illuminate whether cognitive ability compensates for knowledge-regulation for the lower prior mathematical knowledge group. Future research will attempt to investigate this and why the middle group seemed to benefit the most from motivation.

Implications for Teaching

Many college instructors teach large lecture sections of introductory mathematics classes and struggle with high percentages of students that earn grades of D or F, or simply withdraw (DFW rate) from the class. It takes resources to offer recitation sessions, out-of-class sessions, or tutoring. These are familiar interventions colleges use to lower the DFW rate and help students be successful. This study demonstrated that carefully designed worksheets modeled after worked examples coupled with active group sessions can be very beneficial in helping students become more successful.

The SP day each week allows an extra active session where students can work on comprehending material in groups. These SP days are like an extra day in class, however, they only emphasize material that has already been covered during the lecture days. Students benefit from asking questions when they do not understand a problem and get individual attention from a class assistant or the instructor. The obligation of the instructor is to have a group of class assistants ready to help students with the material. This extra day of class per week is also important because, for the most part, students will show up for the extra day of class to actively participate compared to an out-of-class session. Because SP is part of their normal class schedules, students will not have any class or work conflicts, which they may have with out-of-class sessions. This eliminates one reason for not attending.

Moreover, students are less likely to visit the instructor in his office during office hours, nor go to a mathematical tutoring center. For the instructor, SP days is the most efficient way to help many students at the same time and can be thought of as an office hour with the whole class. There would be no way for the instructor to help this many students during office visits and reduces the time needed to explain the material many different times to different students during office hours. The worked example worksheets act like a tutor by presenting students with a number of examples and problems to practice.

We showed that the high and low prior mathematical knowledge groups did not benefit much. Therefore other modes or uses of worked example might be designed to help these students. For example, maybe a smaller group session using the worked example worksheets with a focus on helping the students more would benefit the low prior mathematical knowledge group. One would have to be more creative with the high prior mathematical knowledge group. The addition of more challenging problems or tasks in SP session could be beneficial for these students. Perhaps the high prior knowledge group could become peer mentors to help the low prior knowledge group. Both of these suggestions would challenge the high prior knowledge group and help improve their knowledge-regulation component.

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