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Advanced Mathematical Thinking and the Way to Enhance IT

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Abstract
This journal discuss about Advanced Mathematical Thinking (AMT) and how to enhance it. AMT is ability in representing, abstracting, creative thinking, and mathematical proving. The importance of AMT ability development in accord with government expectation who realize about the importance of mathematical competency mastery for student's life. The advantage in developing AMT among others are: 1) student capable to deliver idea which is possessed in verbal, symbol, table, graphic and diagram. 2) Find deep connection among mathematic branches, 3) know the outcome in one of branch which can trigger conjecture in branch which is related, 4) technique and method from one branch can be applied to prove the outcome in branch which is related, 5) capable to find new idea in problem solving, and 5) capable to understand and construct the proof. One of theory which not only enhance student's learning outcome but also can construct knowledge through student's mental activity, enhance creativity, create class atmosphere which is enjoyable and challenging, and capable to enhance student's AMT ability is APOS approach.
Keywords: Advanced Mathematical Thinking, Representation, Abstraction, Creative Thinking, Mathematical Proving, APOS

Introduction

One way which can be used to create human resource which is ready to face challenge is by developing education program which is more focus on thinking ability development, in this case is advanced mathematical thinking ability. Following is survey result of some experts which state that advanced mathematical thinking ability is still low, among others which is conducted by Davis (Tall, 2002) toward students of Tennesse University of Technology who have passed calculus course, in which they are not given non routine problem.

Find at least one solution to the equation $4x^3 - x^4 = 30$ or explain why no such solution exist.

In fact, there is none of students who answer the problem correctly, generally they cannot do anything. This situation is not different for another four problems. Beside, Davis (Tall, 2002) also had administered test to freshmen who excel when they were in high school. It is found that many of them who has misconception about mathematic concept. According to Tall (2002) there are some reasons which cause this happened, among others: in general, mathematic instruction from elementary school until higher education perhaps teach what is called by ritual: "do this, then do that" and teachers usually will justify student who answer in accord with ritual.

The problem above also happen to students of FKIP Mathematic Education, University of Mulawarman, East Kalimantan. Particularly students who take Algebra course. Based on interview result which is conducted toward six students, information is obtained that algebra course is the course which is difficult to be understood by them. Student admit that they find difficulty in understanding algebra properties in symbolic form, proving algebra properties, connecting between one concept with another concept, and difficulty in solving algebra problem if example is not given by lecture. This show lack of student's Advanced Mathematical Thinking (AMT) ability.

Many models and learning approaches which capable to enhance student's learning outcome, but not all models and learning approaches which capable to design learning in order that knowledge is constructed through student's mental activity, give broad opportunity for students to enhance creativity, and create class atmosphere which is enjoyable and challenging. According to Asiala et al., (1997) through APOS approach, interaction among students occurr and it is hoped that different learning experience exchange occur in order that mental action can continue as expected. Such activity keep continue until student has ability to do reflection on action which has been done, in order that student can reach potential development stage.

Furthermore, Dubinsky & McDonald (2001) state that APOS theory is a learning approach which is generally implemented for mathematic learning in higher education level, which integrate computer usage, discussion in small group and pay attention to mental constructions which are done by students in understanding a mathematical concept. These mental constructions are: action, process, object, and scheme which is abbreviated by name APOS.

Problem Formulation and Problem Limitation

Problem formulation in study which will be researched and whose answer searched focus on the difference of advanced mathematical thinking enhancement after learning process by using APOS approach and conventional learning. Based on thinking above, the problem in this study is formulated in detail based on variable, as follow:

1. How does description of student's Advanced Mathematical Thinking (AMT) ability in algebra course between student who use APOS and use conventional approach?
2. Is there enhancement of student's AMT ability in algebra course after using APOS approach and conventional approach?
3. Is enhancement of student's AMT ability in algebra course who use APOS approach better than student who use conventional approach?

Theoretical Framework

1. Definition of Advanced Mathematical Thinking (AMT)

Some experts (Dreyfus in Tall (2002); Harel & Sowder (A. Gutierez, 2006); and Sumarmo (2011) explained about definition of Advanced Mathematical Thinking (AMT). Dreyfus (Tall, 2002) state that that AMT process comprise: 1) representation process, 2) abstraction process, 3) relation between representation and abstraction. Further, Tall assert that beside the process above, mathematical creative thinking is included in AMT. This is the same with what delivered by Harel and Sowder (A. Gutierrez, 2006) which define AMT as mathematical thinking process such as representation process, abstraction, relation of representation and abstraction, creativity and mathematical proof. Sumarmo (2011) define AMT tentatively as ability which comprise: representation, abstraction, relating representation and abstraction, mathematical creative thinking, and arrange mathematical proof. Futhermore (Tall, 2002) explain about syllabus of AMT, which comprise process of: representation, analyzing, synthesis, and abstracting or formalizing. From some definitions of AMT above, thus definition of AMT is mathematical thinking process which comprise processes of representation, abstraction, mathematical creative thinking, and mathematical proving. AMT process can also occur in elementary school mathematic problem solving, for example representation process (real world object representation, concrete representation) but definition, abstraction process and formal proving is one of factor which distinguish it with ATM.

2. Aspects of Advanced Mathematical Thinking

a. Mathematical Representation

There are some experts who suggest the definition of representation namely Davis (in Janvier, 1987); Kalathil and Sherin (2000); Goldin (2002); Rosengrant (2005); Hwang (2007). According to Davis (Janvier, 1987) a representation can be combination of something written on the paper, something existed in form of physical object and array of ideas which is constructed in one's thinking. Furthermore, Kalathil and Sherin, 2000 state that representation is something which is done by students to externalizing and showing his/her work.

Furthermore (Rosengrant, 2005) state that some representations is more concrete in nature and functioned as reference to more abstract concepts and as aid in problem solving. While in mathematic education physiology, representation is description of relation between object and symbol, Hwang (2007). In this writing, definition of representation I used is definition according to Goldin, because from some definitions the property is more general. Mathematical representation which is generated by student is expression of idea or mathematical idea which is displayed by student in effort to understand mathematical concept or to search solution from problem which is faced. Therefore, it is hoped to has access to representation or idea which they display, they have set of tool which is ready significantly will broaden their capacity in thinking mathematic (NCTM, 2000).

From some definitions of representation above, it can be concluded that mathematical representation are expressions from mathematical ideas (problem, definition, statement, etc) which is used to show (communicating) his/her work outcome with certain way as result of interpretation in his/her thinking.

Example of mathematical representation problem:

1). Let f: $R^{+} \rightarrow R$ defined by $f(x) = x^2$
   a) Interpret f by using graphs
   b) Give a verbal explanation, is the f surjektive?
   c) Prove formally that f injective.

2). Let U (15) Group
   a) Interpret the words, what is U (15).
   b) Interpret the meaning of the symbol U (15).
   c) Is the $7 \in$ U (15)? if yes, find inverse of 7.

b. Abstraction

Proclus (2006) define abstraction in mathematic as process to obtain the essence of mathematic concept, omit the dependence on real world objects which initially perhaps related each other in order to has broader application or appropriate with explanation of another abstract for equal symptom. While Dreyfus (1991), Sfard (1991, 1992), and Dubinsky (1991) (in White, P & Mitchelmore, M.C, 2010) explain that abstraction is a shift from concrete operational model to structural (abstract) model.

There are two processes which are requisite in abstraction process, namely generalizing and synthesizing.

1) Generalizing

According to (Tall, 2002), generalizing means generating or inducting from specific to identify similarities. The example of abstraction as generalization:

> Suppose that Zn are the set of integers modulo n. n$\in Z^+$
> For n = 2 (Zn, +) group
> For n = 3 (Zn, +) group
> For n = 4 (Zn, +) is not a group
> For n = 5 (Zn, +) group
> For n = 6 (Zn, +) is not a group.
> What can be concluded from the above statement?

2) Synthesizing

Synthesizing means joining or arranging parts in a way in which those parts create wholeness that is totality (Tall, 2002). For example, in linear algebra course there is material which is taught separately about vector ortogonalization, matrix diagonal, basis transformation, linear equation system solution, etc. In learning, all materials which are not related with this is hoped to be joined in a picture in which all materials are interrelated. According to (Tall, 2002), this joining process is called synthesis.

c. Mathematical Creative Thinking

Mann (2005) stated that it is difficult to define mathematical creative thinking clearly, but mathematical creative thinking can be distinguished by characteristics it has. Different from Mann, Welsch, McGregor (2007) define mathematical creative thinking as one kind of thinking which direct to new insight, new approach, new perspective, or new way in understanding something. Mathematical creative thinking can occur when triggered by tasks or problems which are challenging. Furthermore, Coleman & Hammen (in Tenri, et all, 2008) also stated that mathematical creative thinking is the way of thinking which generate something new in form of concept, discovery and work of art.

Example of Mathematical Creative Thinking Skill

1) Fluency thinking skill

> Let G be a group and a a certain element of G. C (a) = {ga | G$\in$ g = ag}. Show that C (a) is a subgroup of G.
> There are three ways to prove the above subgroups, students are expected to able to resolve in a way of his own choosing

2) Flexibility thinking skill

> Prove that a fourth order with respect to the group is abel group. Are there any other finite abel group? Explain your answer.

3) Originality skill

> $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \int \; a,b,c,d \in Q \quad dan \quad ad \neq bc \right\}$ the matrix multiplication is a group. When
>
> $N = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \middle| a,b,c \in Q \quad dan \quad ac \neq 0 \right\}$ and
>
> $H = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| a,b \in Q \quad dan \quad ab \neq 0 \right\}$, is the N and H, respectively subgroup of M? Explain your answer!

4) Elaboration Skill

If a and b are two cycles mutually , what is ab = ba? if so prove it and if it does not give examples.

d. Mathematical Proving

Hanna (Yoo, 2008) state that proof is representation from mathematical outcome to communicating understanding to another mathematic community and accept it as new theorem. Furthermore, Schoenfeld (Arnawa, 2006) stated that proving essentially is make series of deduction from assumption (premise or axiom) and mathematic outcomes which is existing (lemma or theorem) to obtain the important things from a mathematic problem.

According to Selden & Selden (in Tall, 2002), student's mathematical proving ability consist of: (1) ability to construct the proof and (2) ability to validate the proof. Mathematical proving can be functioned as an actual process through proof construction and as final phase. The same with what is delivered by Hadamard in (Tall, 2002) who stated that mathematical proving is final phase in mathematical thinking.

Example of Mathematical Proving:

1) Suppose (G, $\circ$) a group with identity element e. If an element of G has an inverse, then prove that the unique inverse.
2) If p a prime number and $G = \{a + b\sqrt{p} \mid a, b \in Q\}$ then prove (G, +) is a group abel.

3. APOS Approach

According to Suryadi (2012), APOS theory is constructivism theory about how someone learn a mathematical concept. Dubinsky et al (Hawks & Nichols, 1989; Breidenbachetal, 1992; Asiala, Coklat, DeVries, Dubinsky, Mathews & Thomas, 1996) investigate process-object difference and develop it into four kinds of mental construction, namely: Action, Process, Object and Scheme (called APOS).

The following is definition of Action, Process, Object and Scheme according to Asiala (1997) and Suryadi (2012).

Action:

According to Asiala, et al. (1997): An action is a transformation of mathematical objects that is performed by an individual according to some explicit algorithm and hence is seen by the subject as externally driven. The same with definition which is suggested by Suryadi (2012) that action is a mental objects transformation to obtain another many mental objects. Furthermore, Suryadi also stated that someone experience an action of if he/she focus on his/her mental process in effort to understand a concept given. For clarity of explanation about action conception, an illustration is given about group concept. Someone who has not yet capable to interpret something as a group, except if given a finite set toward an operation until capable to determine whether finite set is a group, hence it can be said that he/she has ability to do action toward that group.

Process:

According to Asiala, et al. (1997) as follow: When individual re-acts on the action and constructs an internal operation that performs the same transformation then we say that the action has been interiorized to a process. The same with Asiala, et a., Suryadi (2012) also stated that when an action is repeated, then reflection on action which is done occur, then subsequently will enter into process phase. Different with action which can be done through object manipulation or something which is concrete, process occur internally under control of individual who do it. Someone experience a process about a concept, if his/her thinking limited on mathematical idea which is faced and marked by the appearance of ability to do reflection toward that mathematical idea. For example in group concept, someone experience a process if he/she has capable to manipulate some sets and toward arbitrary operation, hence it can be said that he/she has already had ability to do process on that group concept.

Object

According to Asiala et al. (1997), object is defined as follow: When it becomes necessary to perform actions on a process, the subject must encapsulate it to become a total entity, or an object. In many mathematical operations, it is necessary to de-encapsulate an object and work with the process from which it came. Suryadi (2012) stated that someone is said that he/she has had object conception from a mathematical concept, if she had capable to treat that idea or concept as an cognitive object which comprise the ability to do action on that object, and give explanation or reason about its properties. Beside, that individual has also capable to do decomposition again an object to become process as at the time properties of object intended will be used. This can be illustrated in group concept. Someone is said that he/she has had object conception in group concept if she had capable to do group grouping and also capable to explain properties of each group.

Scheme:
According to Asiala et al. (1997): A scheme is a coherent collection of processes, objects and previously constructed schemas, that is invoked to deal with a mathematical proble situation. As with encapsulated processes, an object is created when a scheme is thematized to become another kind of object which can also be de-thematized to obtain the original contents of the scheme. Suryadi (2012) stated that a scheme of certain mathematical material is a collection of action, process, object and another scheme which is related each other in order to create framework which is interrelated in one's thinking. The indicator that someone had possessed a scheme is that if he/she had possessed the ability to construct examples of a mathematical concept in accord with properties which is possessed by that concept. In group concept, someone is said that he/she had possessed a scheme if he/she had capable to construct examples of group in accord with properties which are possessed by group concept.

Methodology

This study is conducted through two stages, that are 1) preparation stage, and 2) implementation stage. Research design which is used is Nonequivalent control group design. This study is conducted in University of Mulawarman Samarinda with the reason that research about advanced mathematical thinking has not been yet conducted in UNMUL and UNMUL is the one and only public university in East Kalimantan.

This study consist of two classes of sample, namely experiment and control class. In experiment class and control class learning is implemented in succession by APOS approach (X) and conventional approach. Before given the treatment, pretest is administered in two classes to find out student's AMT ability before given the treatment

Data processing of student's AMT ability is done in accord with following stages:

1. Test all statistic requisites which are needed as basic to test the hypothesis. Before student's AMT ability test is used, requisite which is tested in advance is data distribution normality test. Statistic test which is used is: Shapiro-Wilk test or Kolmogorov-Smirnov test. In this study, Levene test is used for homogeneity test and Shapiro-Wilk test is used for normality test.
2. If all data is normal distributed, continue with variance homogeneity test. Homogeneity test is done by using Levene test.
3. Determine statistic test and test criteria in accord with problem in order to test hypothesis which has been formulated. If data is normal distributed and has homogeneous variance, then:
   a. Test-*t* is used to compare the difference of two paired sample, and test-*t'* is used for difference test of two independence samples, whereas one-way ANOVA is used for difference test of more than two independence samples.
   b. Two-way ANOVA and General Linear Model (GLM) is used to test the influence of interaction.
   c. Contingency coefficient from cross tabulation is used to test association among variables.
   To see its quantitative picture: if one-way ANOVA show difference, then follow-up statistic test is continued by using Scheffe test.
4. If one of data or all of data tested are not normal distributed or normal distributed but its variance is not homogeneous, then hypothesis test is done by using nonparametric statistic principle, namely:
   a. Kruskal-Wallis test is used to compare more than two independent samples.
   b. Mann-Whitney test is used to compare two independent samples.
   c. Friedman test is used to compare more than two paired samples.
   Hypothesis test is done by aid of SPPS-17 for Windows software.

Result of Study

Based on data of AMT ability test result, students are described and analyzed based on learning approach. Description of study result about AMT is described in following table.

Level KAM	Data Statistik	Pendekatan Pembelajaran					
		APOS			Konvensional		
		Pretes	Postes	Rerata N-*Gain*	Pretes	Postes	Rerata N-*Gain*
Atas	N	5	5		11	11	
	Rerata	10,60	40,40	0,65	10,73	38,64	0,61
	SB	3,21	3,13	0,08	4,80	4,54	0,11
Tengah	N	27	27		22	22	
	Rerata	8,15	25,44	0,36	10,27	24,68	0,31
	SB	1,94	6,18	0,13	4,08	2,01	0,08
Bawah	N	7	7		7	7	
	Rerata	6,43	21,00	0,29	9,86	19,43	0,20
	SB	1,51	2,31	0,05	5,37	3,41	0,09
Gabungan	N	39	39		40	40	
	Rerata	8,15	26,56	0,39	10,32	27,65	0,37
	SB	2,31	7,73	0,15	4,40	7,77	0,18

Based on description of data in table above, data analysis result of student's AMT ability enhancement is obtained as follow.

1) In a whole, AMT ability enhancement of student who get APOS approach is higher compared to student who get conventional learning. This is showed by acquisition of student's N-Gain mean, in APOS class is 0.39 an in conventional class is 0.37. AMT ability enhancement in two classes is included in medium category.
2) Based on KAM level, for upper level of KAM, AMT ability enhancement of student who use APOS approach is higher compared to student who use conventional learning, but the difference is not significant. This can be seen from N-Gain mean of APOS class that is 0.65 and N-Gain mean of conventional class that is 0.62. AMT ability enhancement of student for upper level of KAM in two classes is included in medium category.

   As for middle level of KAM, AMT ability enhancement of students who get APOS approach is also higher compared to student who use conventional learning, but the difference is not significant. This can be seen from N-Gain mean of APOS class that is 0.36 and N-Gain mean of conventional class that is 0.31. AMT ability enhancement of student for middle level of KAM in both classes is also included in medium category.

   Next, in lower level of KAM, AMT ability enhancement of student who use APOS approach is also higher compared to student who use conventional learning, but the difference is also not significant. This can be seen from N-Gain mean of APOS that is 0.29 and N-Gain mean of conventional class that is 0.20. AMT ability enhancement of student for lower level of KAM in both classes is included in low category.
3) For student in two classes (APOS, Conventional), AMT ability enhancement of student from upper level of KAM is higher compared to student from middle and lower level of KAM. This can be seen from N-Gain mean in APOS class, for upper level of KAM , N-Gain mean is 0.65, whereas N-Gain mean of middle and lower level are 0.36 and 0.29, respectively. In conventional class, for upper level of KAM, N-Gain mean is 0.62, whereas N-Gain mean for middle and lower level of KAM are 0.31 and 0.20, respectively.

Description of data toward student's AMT ability enhancement based on learning approach show that there is enhancement in APOS class as well as in conventional class. If it is seen, student's AMT ability enhancement in two classes (APOS, conventional), then the difference is not significant.

That description of data can be clarified by using bar diagram in Figure 4.1 as follow.

Data description toward enhancement of student's AMT ability based on learning approach show that there is enhancement in APOS class as well as in conventional class. If it is seen, the difference of student's AMT ability enhancement in two classes (APOS, conventional), then the difference is not significant. That description data can be clarified by using bar diagram in Figure as follow.

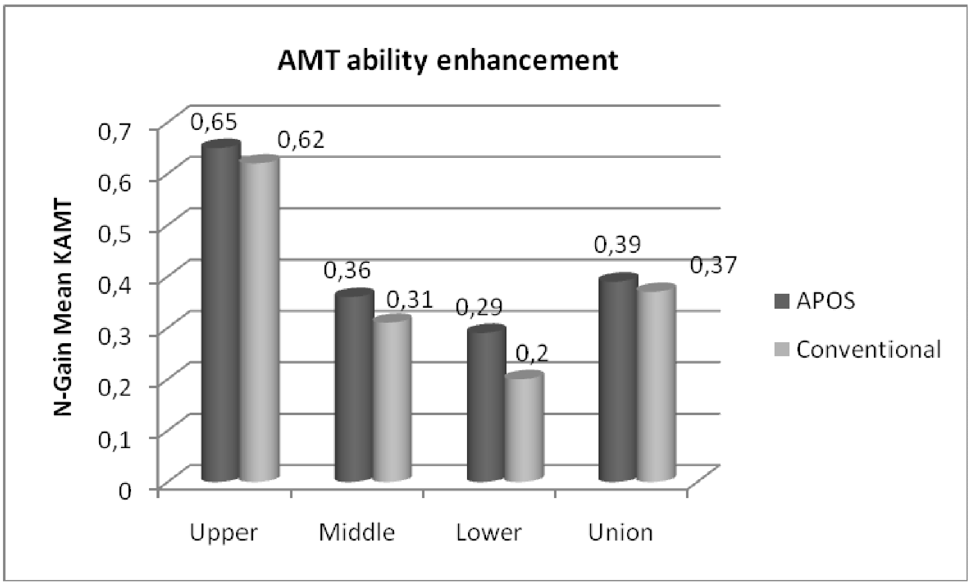

Figure 1.1
student's AMT ability enhancement
based on approach and KAM

Test result toward enhancement of student's AMT ability after get learning by using two learning approaches (APOS and Conventional) is presented in following table.

Table of Test Result of Student's AMT Ability Enhancement Difference in Two Learning Approaches

	Learning	U Mann Whitney	Z	p-value (sig)	Annotation
AMT enhancement	APOS Conventional	751.000	-0.284	0.776	There is no difference

Based on table above, statistic calculation result is obtained in student's AMT ability enhancement after get learning by using two learning approaches (APOS, and Conventional) with Z value of Mann Whitney test as much as 751.000 with p-value (sig) as much as 0.776. If p-value obtained is bigger than 0.05, then Ho is accepted. It means that there is no difference of student's AMT enhancement between students who get learning with APOS approach or conventional learning.

Discussion

One of finding about AMT ability if it viewed based on approach is AMT ability of student who use APOS approach and conventional class is not significantly different. This can be seen on Table 4.7 in which N-gain average of AMT ability score in APOS class and conventional class are 0.39 and 0.37, respectively. According to Hake (1999), AMT ability of students of APOS class and conventional class is included in medium category. This result is also supported by posttest mark, in which student who get learning by APOS approach show posttest average which is higher than students who get conventional learning. But in a whole, posttest average which is obtained by students of APOS class is still low, that is 27.60 and posttest average which is obtained by students of conventional class is 26.56. If viewed in a whole, in both learning, enhancement of student's AMT ability is in the same level, that is in medium level.

This is supported by analysis result statistically by using Mann Whitney test that there is no difference of AMT ability enhancement of students between APOS class and conventional class. This finding is different with findings of Arnawa (2006), Nurlaelah (2009), and Yerizon (2011). Arnawa (2006) in his study in higher education find that student who get abstract algebra learning which use APOS theory has proving ability which is better compared to student who get conventional learning. As for with Nurlelah's finding (2009) who conducted the study toward student teacher, found that student teacher who learn by APOS and M-APOS learning model in general achieve mathematical ability which is better compared to conventional learning model. Whereas for achievement of mathematical creativity, student teacher who learn by using M-APOS learning model in general is better compared to APOS learning model and conventional learning. Arnawa and Nurlaela's findings in parallel with Yerizon's finding (2011) who conducted study toward students and find that learning by M-APOS approach can enhance ability in reading the proof and construct the proof.

At the time class discussion, each group deliver what which had been understood before. This discussion is an opportunity for student to correct misconception which is occurred. Beside this, class discussion also can train student's mathematical representation ability, because many students who cannot be able to understand symbols which contain in teaching material. This is seen when they present discussion result in front of class. This finding in accord with what delivered by Nodding (Polla, 2000) that class discussion can help student to develop social skill and communication skill. Suryadi (2010) also stated that by listening what was invented by others and discuss it, student is enabled to improve strategy which they discover themselves. Therefore, interaction enable student to do reflection which finally will force them to obtain higher understanding than before.

The next finding is based on observation result which is done by observer during lecturing go on, that in first meeting and second meeting, it is difficult to create learning atmosphere which is active, student still find difficulty to change the mindset from habitual to receive lecturing material casually become should understanding themselves mathematical concept in advance in order that student has initial knowledge about a concept. Such as case in class discussion, none who response to presentation result of their friend in front of class. In group discussion, only student who has high ability who play role actively, whereas students who has medium or low ability tend to receive and pay attention to explanation from their friend.

In third meeting and the next meeting, lecturing atmosphere which is active started to created. Students are seen serious in understanding concept which will be discussed, so at the time class discussion each group has courage to give opinion. Interaction among students in group started being seen, students who have medium ability and student who have low ability started to participate in giving opinion or idea which they have, thus student has ability to do reflection on what they have understood. This finding in accord with opinion of Asiala et al., (1997) that through APOS approach, interaction among students occur and it is hoped that learning experience exchange which is different occur so mental action occur as expected.

Next at time solving problems which is contained in Student's Work Sheet, in first meeting and second meeting it is only done by student who has high ability, whereas students who has low and medium ability only see and receive their friend work casually. But in third meeting and next meeting, the effort has been seen from each individual even though the answer they given still incomplete. In the final of lecturing, lecture guide students in delivering conclusion about concept that they have been learned.

The next finding which is related with approach that based on study result of Brophy and Good (Suryadi, 2010), direct learning model is the most effective way to develop lower mathematical thinking ability and there is no evidence that direct learning model can enhance low higher mathematical thinking ability. Nevertheless, this finding result show that conventional approach can develop AMT ability.

But if it seen from average score, it is found that average-gain of APOS class is 0.39 and average-gain of conventional class is 0.37. According to Hake (1999), average-gain still included in medium category. Based on this finding, there is very broad opportunity to enhance student's AMT ability in two classes of APOS and conventional.

Conclusion

Based on result of study and discussion above, generally the conclusion about advance mathematical thinking ability can be obtained as follow:

1. Based on descriptive analysis result, it is obtained that:
   a. There is AMT ability enhancement of student who get learning by APOS approach as well as student who get conventional learning. This is based on mean comparison, and standard deviation of student's AMT ability score.
   b. AMT ability enhancement of student in APOS class almost the same with AMT ability enhancement of student in conventional class. This can be seen from N-Gain mean of student's AMT ability in two classes.
   c. There is difference of student's AMT ability enhancement between APOS class and conventional class based on level of KAM. This can be seen from N-Gain mean result of AMT ability score of student from upper level of KAM which is higher than student from middle and lower level of KAM. Furthermore, AMT ability enhancement of student from middle level of KAM is higher than student from lower level of KAM.
2. There is no difference of AMT ability enhancement between student who get learning by APOS approach and student who get conventional learning in a whole.
3. There is difference of AMT ability enhancement between student who get learning by APOS approach and student who get conventional learning, based on level of KAM (upper, middle and lower).

Recommendation

Based on explanation of study result and conclusion above, it is found that there is no AMT ability difference between students who get learning by using APOS approach and students who get learning by conventional approach. In this case, recommendations which are suggested are:

1. Thinking ability is better to be trained started from elementary school until higher education in order that student has mathematical thinking habit.
2. It is hoped that learning by using APOS approach can be developed in field and made to become alternative of lecture's choice in mathematic learning. This is because that learning can enhance AMT ability, even though still in low category.
3. Based on finding of study result that learning by APOS approach is influential toward student's AMT enhancement even though still in medium category. It is hoped that it will attended by lecturer, that during learning process student is not only demanded to solve the problem, but the importance thing is how student find that problem solving. Thus, the more important is student's thinking process, not only his/her thinking outcome.
4. Based on experience during study, it is better that before study is conducted, socialization is done in three times of meeting about APOS approach. This is not only done toward lecturer who teach the course but also important to be done toward students. This is done in order that when study is started, students not feel unfamiliar with approach which is applied.
5. In this study, researcher also examine AMT aspects (representation, abstraction, creative thinking, and mathematical proving). Based on those four aspects, the most difficult aspect for students is creative thinking aspect, and mathematical proving. This can be studied more deeply, so further research is needed to give contribution in learning quality improvement.

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