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STUDYING THE ABILITY OF 7TH GRADE STUDENTS TO DEFINE THE CIRCLE AND ITS ELEMENTS IN THE CONTEXT OF MATHEMATICAL LANGUAGE

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Abstract: In this study, it was aimed to study the mathematical language skills that the 7th grade students use in defining the circle and its elements. In the study, the mathematical language skills of students that they use in defining the circle and its elements in a scenario were compared to the mathematical language skills they use in defining them when their names are given. The sampling of the study consists of 138 students of 7th grade. A scenario of 7 questions requiring the students to define the circle and its elements (center, radius, diameter, secant, chord, tangent) was arranged by the researchers and students were asked to respond these questions. Also, the students were asked to define the circle and its elements when their names were given. The students' mathematical language skills were examined in terms of their visual and verbal skills of definition. It was concluded that the students' mathematical language skills they used in defining the circle and its elements in a scenario were at a level that should be advanced, but their mathematical language skills they used in defining when their names were given is at a sufficient level. In addition, it was observed that the students had difficulty in defining the concept of tangent in a scenario at most and in defining the concept of chord when the names were given.

Key words : Circle, Circle elements, Concept, Mathematical Language,

1. Introduction

Language has many functions. In a sense, language is a device that arranges how to think; language performs this through concepts (Tuna, 2006). Vygotsky (1986) states that language is the most important sign or symbolic device because of its mediational function for cognitive development. According to Brodie (1989), language is prerequisite for thoughts and necessary for understanding. Language is important in students' and teachers' sharing and communicating their mathematical thoughts and beliefs (Brown,2000).

Mathematical language is the body of rules where mathematical concepts, operations and symbols are used together that have the feature of stating scientific thoughts easily (Çalıkoğlu Bali, 2003). Mathematics has a peculiar language, a way of expression, terms and words. Just as some words of mathematics are the ones that remain and is used in its inner world, some others can be the words that can be used in social life (Aydın and Yeşilyurt, 2007). The spoken language of mathematics is important in terms of understanding mathematical concepts (Usiskin, 1996). In this context, Jamison (2006) stated learning the language as the utilization of the language as a means of teaching mathematical concepts. One of the major elements of acquiring mathematical concepts and information and attaining mathematical thinking is the correct usage of language belonging to the field. Usage of language plays an important role in students' learning the introduced concepts (Lansdell, 1999).

Martinez (2001) divided the importance of mathematical language into the following three components in the class;

1. We learn through language. This is an important means of our communication
2. Students form a comprehension by directing their thoughts with language
3. We know and assess students' comprehension by listening to the verbal communication and reading their mathematical writing.

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Mathematical language should be meaningful for students and they should feel the necessity for it so that it can be used correctly and effectively. Taking advantage of verbal expression, written expression, pictures, graphs, and concrete models during and after handling mathematics is of great importance (MEB, 2013). Mathematical language is very important in the advancement of mathematical thinking. Poor ability of language causes poor advancement of thinking (Ferrari, 2004).

Lesh (1981) stated that mentioning that a student understands a mathematical concept is the fact that several periods are usable. The conversion of the periods into one another refers to some of the most important periods that are needed when trying to use number concepts for real life problems or basic algebra or geometry (Clement, 1979; Janvier, 1978). These operations are those; the definition of suitable illustration, searching for a similar problem, simplifying the problem, or arranging the problem with their own words. Besides, it includes modelling periods used by skilful problem solvers and simplified version of problem solving (Fuson and Geeslin, 1979).

It can be useful for a student who has difficulty converting real situations into written symbols to start by converting from real situation into spoken words and then from spoken words into written symbols. If a student has difficulty converting two periods into each other, correcting activities may include the opposite of this period. So, a student having difficulty converting real life situations into written symbols may convert written situations into real life situations.

Geometry is the branch or mathematics dealing with point, line, plane, plane figure, space, space figures and the relationship between them, and the measures of geometric shapes such like length, angle and scope volume (Erol, 2008). The student who encounters a new expression in geometry first tries to interpret this concept within his/her current knowledge. If the student cannot make connection between the new concept and the ones he/ she knows, he/she seeks to memorize this new concept (Akkurt, 2010: 2). Definition is another way of developing a concept. Concepts are the thoughts present in the individual's brain, while terms are the names of the concepts (Akgün, 2004). Concepts are defined with the expression that tells a concept with words.

In geometry, visualization is very important in learning and understanding the concepts. According to Arcavi (2003), visualization is the ability, period and product of creation. In order to define the knowledge and form connection between them, it is the reflection or interpretation of pictures, shapes and schemes in the mind on the paper or technological device. It is the thought of developing the formerly-unknown ideas and of improving the understanding. Zimmerman and Cunningham (1991) made the definition of visualization as; "Visualization is the process and ability of making the knowledge more understandable, and interpreting and reflecting it by using visual designs such as pictures, images, diagrams to form previously-unknown thoughts and to develop the understanding of these thoughts". According to the definition made by Zazkis, Dubinsky and Dauterman (1996), visualization is the individual's behavior of forming a strong connection between his/her inner construction and feelings and what he/she manages to gain.

It can be seen that visualization is a process helpful for developing the understanding and in this process many two-dimension visual teaching element can be utilized such as pictures, drawings, shapes, schemes, graphs, images, diagrams, and tables. At the same time, visualization is one of the ways of students' revealing their knowledge of mathematics. That the student can explain his/her knowledge through visual models as well as through algebraic models is one of the contemporary gains of mathematical teaching (Duval 1999; NCTM 2000).

In view of all this knowledge, this study aims to examine 7th grade students' mathematical language skills that they use in defining the circle and its elements. With this study, it was intended to show how the students could define the concepts they learnt in mathematics classes within daily life situations. In this sense, the mathematical language skills the students used in defining the circle and elements in a scenario was compared to the mathematical language skills they used in defining them when their names were given.

2. Methods

In the research, descriptive methods used because it was aimed to examine the students' mathematical language skills they used in defining the circle and its elements. Definitional study is a method trying

to define and explain the similarities and differences of phenomena with other phenomena (Gall, Borg and Gall, 1996). Also in the study, relational screening model was used in the process the comparison of the mathematical language skills the students used in defining the circle and elements in a scenario with the ones they used in defining them when their names were given. Relational screening models are the research models that aims to determine the presence and/or degree of covariance among two or more numbers of variables (Karasar, 2002).

Sampling of the Study

In the selection of the sampling of the study, cluster sampling model was used that is included in probability based sampling method. Cluster sampling is used in the event that there are different groups resembling in themselves in terms of definite characteristics, which are naturally formed in the space thought to study in and which are made to be formed artificially for different purposes (Yıldırım and Şimşek, 2008). Three schools were determined for the study and these schools were classified as village, town and city center schools. The sampling of the study consists of 138 secondary school students attending 7th grade in Turkey. The personal information for the students having participated in the study is shown in Table 1.

Table 1 : Personal information of the study group

	f	**%**
Gender		
Female	76	55,1
Male	62	44,9
Grade average		
1	28	20,3
2	20	14,5
3	18	13,0
4	40	29,0
5	32	23,2

Data Collecting Tools

Two data - collecting tools arranged by the researchers were used in the study. First, a scenario was prepared composed of 7 question requiring the students to define the circle and its elements (center, radius, diameter, secant, chord and tangent), and the students were asked to answer these questions. This activity was named as "Let's map". In the process of preparing a scenario, a situation was selected among daily life situations and the circle and its elements were hidden in this scenario by making their definitions without giving their names. The opinions of two lecturers and three teachers of mathematics were taken in the process of preparing a scenario. The reliability coefficient of this data-collecting tool was calculated to be 0,875. In the second data-collecting tool, the students were asked to define the circle and its elements by giving them the manes.

The analysis of the data

The application of the study was performed in 2014-2015 academic year three months after the students received the education of the topic on circle. First, the activity "Let's map" was performed. One week after this activity, they were asked to define the circle and its elements by giving their names. The responses given by the students to data –collecting tools were assessed in three steps. The total scores of students were calculated by giving (3) to both visually and verbally correct responses, (2) to partially correct responses and (1) to completely wrong or no answers. The following score ranges were taken into consideration in determining the 7th grade students' mathematical language skills they used in defining the circle and elements both in a scenario and when their names were given:

0,00-3,50: has significant deficiencies
3,51-7,00: should be advanced
7,01-10,50 : prospering
10,51-14,00 : quite prospering

Related sampling t test was used to study the relation between the students' mathematical language skills they used in defining the circle and the elements in a scenario and when their names were given. Independent samplings t test was used in studying the students' mathematical language skills according to their gender, and frequency and ratio were used in their skills of defining the circle and elements visually and verbally. Also, quotations were made from students' answers and they were shown in the part of findings.

3. Findings

The findings obtained in the study are given in this part in accordance with the aims of the study. The figures about the scores of 7th grade students' mathematical language skills they used in defining the circle and its elements in a scenario are shown in Table 2.

Table 2. The figures about the scores of 7th grade students' mathematical language skills they used in defining the circle and its elements in a scenario

	N	Mean	sd
The scores of students' mathematical language skills they used in defining the circle and its elemens in a scenario	138	4,24	4,42

When the data in Table 2 are studied, it was found out that the average of the scores that they students got proved to be 4,24. In this sense, the students' mathematical language skills they used in defining the circle and the elements in a scenario were seen to be at the level to be advanced.

The statistical data about the students' skills of visually defining the circle and its elements in a scenario are shown in Table 3.

Table 3. 7th Grade students' skills of visually defining the circle and its elements in a scenario.

	Correct responses		Wrong responses	
	f	**%**	**f**	**%**
Circle	72	52,2	66	47,8
Center	58	42	80	58
Radius	30	21,7	108	78,3
Diameter	36	26,1	102	73,9
Secant	30	21,7	108	78,3
Chord	28	20,3	110	79,7
Tangent	20	14,5	118	85,5

According to Table 3, it is seen that the students could visually define the circle at most in the scenario. It is determined that the concept which the students had most difficulty defining visually was the tangent.

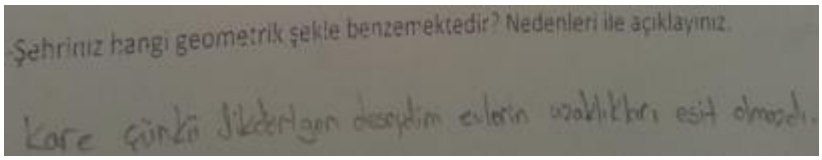
The statistical data about the students' skills of verbally defining the circle and its elements in a scenario are shown in Table 4.

Table 4. 7th Grade students' skills of verbally defining the circle and its elements in a scenario

	Correct responses		Wrong responses	
	f	**%**	**f**	**%**
Circle	60	43,3	78	56,5
Center	68	49,3	70	50,7
Radius	36	26,1	102	73,9
Diameter	32	23,2	106	76,8
Secant	32	23,2	106	76,8
Chord	34	24,6	104	75,4
Tangent	26	18,8	112	81,2

According to Table 4, it is seen that the students could verbally define the center at most in a scenario. It is determined that the concept which the students had most difficulty defining verbally was the tangent.

Within the activity "Let's map", it was seen that a great majority of the students were able to define the circle correctly, while some of them gave the answer of square or rectangle in the part in which the students were asked to name and draw the expression circle defined without giving its name. Figure 1 provides the answer of a student who thought the circle to be square.

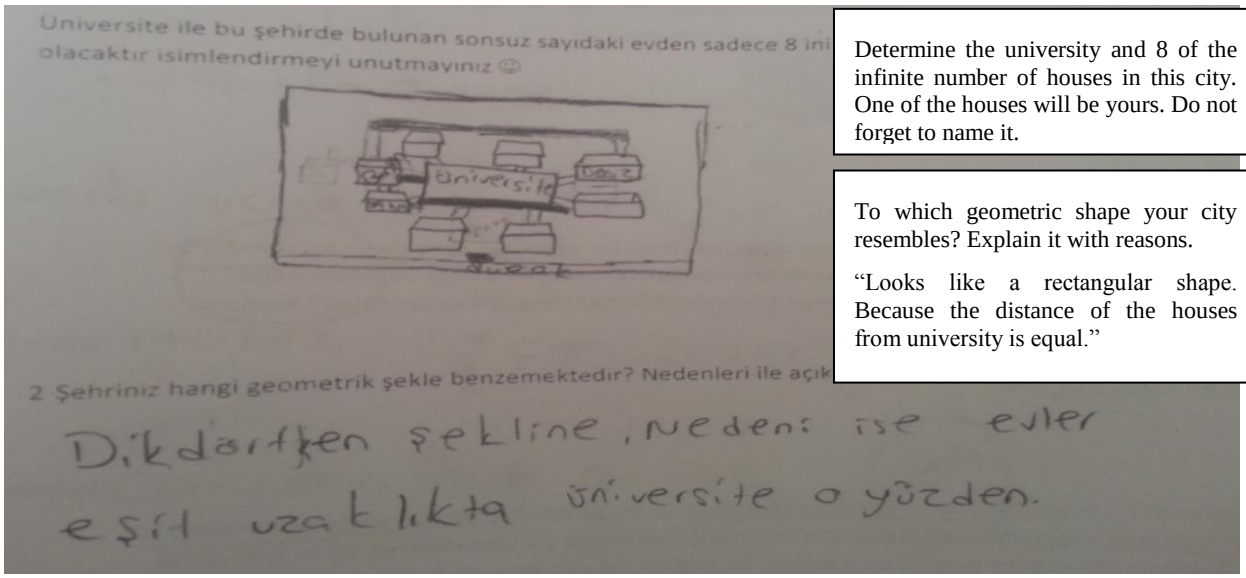


To which geometric shape your city resembles? Explain it with reasons.

"It' s square. Because if I said rectangular, the distance of houses would not equal."

Figure 1. The answer of the student who thought the circle as square in a scenario.

The answer of the student who thought the definition of circle in a scenario as rectangle is given in Figure 2.



Determine the university and 8 of the infinite number of houses in this city. One of the houses will be yours. Do not forget to name it.

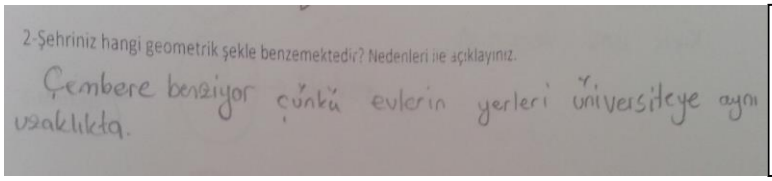
To which geometric shape your city resembles? Explain it with reasons.

"Looks like a rectangular shape. Because the distance of the houses from university is equal."

Figure 2. The answer of the student who thought the circle as rectangle in a scenario.

When Figure 2 is examined, the student gave the answer that he/she resembled the shape to rectangle because of its rectangular shape he/she drew separately to the outside although he /she could form the circular shape. Here, we can say that the student couldn't envisage the concepts of either the rectangle or the circle.

The answer of the student who gave the correct answer of circle in a scenario is provided in Figure 3.



To which geometric shape your city resembles? Explain it with reasons.

"Looks like a circle. Because the distance of houses from university is equal."

Figure 3. The answer of the student who gave the correct answer of circle in a scenario.

In figure 4, on the other hand, there is the student's answer who correctly defined the circle and elements in a scenario visually.

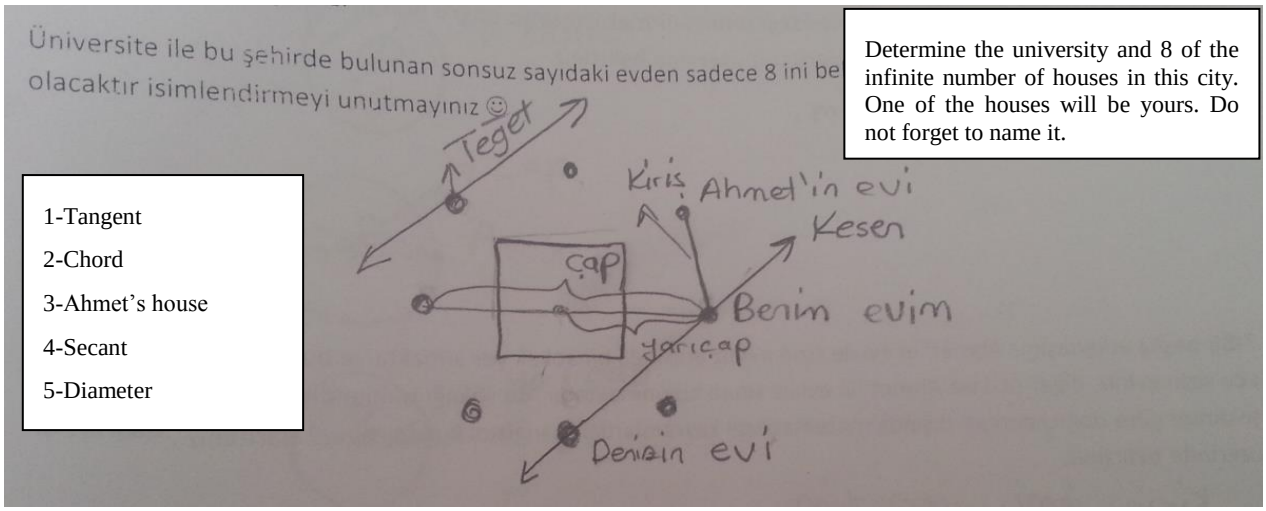


Figure 4. The student's answer who correctly defined the circle and its elements in a scenario visually.

Table 5 includes the data about whether the 7th grade students' mathematical language skills they used in defining the circle and elements in a scenario differs according to gender or not.

Table 5. Independent samplings t test results about the difference among the 7th grade students' mathematical language skills they used in defining the circle and elements in a scenario according to gender

Gender	N	Mean	sd	t	p
Female	76	5,13	4,78	2,728	0,007
Male	62	3,16	3,70		

As the result of the t-test performed, it was concluded that the students' mathematical language skills they used in defining the circle and its elements in a scenario significantly differed according to gender ($t=2,728; p=.007<.05$). Accordingly, considering the averages, the mathematical language skills of girls were seen to be higher than those of boys ($Mean_{girl}$:5,13; $Mean_{boy}$:3,16).

The values about the scores of 7th grade students' mathematical language skills they used in defining the circle and its elements when their names were given are shown in Table 6.

Table 6. The values about the scores of 7th grade students' mathematical language skills they used in defining the circle and its elements when their names were given

	N	Mean	sd
Students' mathematical language skills they used in defining the circle and its elements when their names were given	138	8,37	4,15

When the findings are examined, it was concluded that the 7th grade students' mathematical language skills they used in defining the circle and its elements when their names were given was at a sufficient level with the average of 8,37.

The statistical data about the 7th grade students' skills of defining the circle and its elements when their names were given are shown in Table 7.

Table 7. The 7th grade students' skills of defining the circle and its elements when their names were given

	Correct responses		Wrong responses	
	f	**%**	**f**	**%**
Circle	94	68,2	44	31,8
Center	94	68,2	44	31,8
Radius	62	45,0	76	55,0
Diameter	74	53,7	64	46,3
Secant	78	56,5	60	43,5
Chord	52	37,7	86	62,3
Tangent	70	50,8	68	49,2

When table 7 was examined, it was seen that the students were able to define the circle and the center at most in defining the concepts when their names were given. It was determined that the students had the most difficulty defining the concept of chord while defining the concepts when their names were given.

The data about whether the 7th grade students' mathematical language skills they used while defining the circle and its elements when their names were given differed according to gender or not are shown in Table 8.

Table 8. Independent samplings t test results about the difference among 7th grade students' mathematical language skills they used while defining the circle and its elements when their names were given according to gender.

Gender	**N**	**Mean**	**sd**	**t**	**p**
Female	76	8,55	4,41	0,557	0,019
Male	62	8,16	3,84		

As the result of the t-test performed, it was found out that the students' mathematical language skills they used while defining the circle and elements when their names were given significantly differed according to their gender (t=0,557; p= .019<.05). Accordingly, considering the averages, the mathematical language scores of girls were found to be higher than those of the boys ($Mean_{girls}$:8,55; $Mean_{boys}$:3,84).

The related samplings t-test results about the difference between the7th grade students' mathematical language skills they used in a scenario and when their names were given while defining the circle and its elements are given in Table 9.

Table 9. The related samplings t-test results about the difference between the 7th grade students' mathematical language skills they used in a scenario and when their names were given while defining the circle and its elements

	N	**Mean**	**sd**	**t**	**p**
in a scenario	138	4,24	4,42	-13,213	0,000
when their names were given	138	8,37	4,15		

According to Table 9, it was revealed that there is a significant difference between the students' mathematical language skills while defining the circle and its elements in a scenario and while defining them when their names were given (t:-14,213; p:.00<05). When the averages were examined, it was concluded that the students were able to define the circle and elements more easily when their names were given ($Mean_{\text{in a scenario}}$:4,24; $Mean_{\text{with their names given}}$:8,37).

4. Conclusion and Discussion

One of the most important problems in mathematics teaching is the learning of mathematical concepts. Each new concept in mathematics is learnt as new knowledge and a new word. Learning through

mathematical comprehension of the language takes place in human brain in a rule of logic and the knowledge becomes more permanent (Sinanoğlu, 2000). Students' correct usage of the language in their written and verbal expressions about mathematics is very important in terms of transforming their mathematical knowledge (Aydın and Yeşilyurt, 2007).

Extensive evaluations of mathematics learning indicate that elementary students are failing to learn basic geometric concepts and geometric problem solving (Kouba et al., 1988; Stigler, Lee, Stevenson, 1990). With this study it was aimed to search for 7th grade students' mathematical language skills they used while defining the circle and its elements in a scenario and when their names were given.

In the study it was concluded that the students' mathematical language skills they used while defining the circle and its elements in a scenario were at a level that must be advanced but their mathematical language skills they used when their names were given were at a sufficient level. When the students' mathematical language skills they used while defining the circle and elements in a scenario and with their names given were compared to each other, the students were understood to be able to define the concepts more easily when their names were given.

It was seen that the students could define the circle visually at most and the center verbally at most in a scenario. The students were determined to have had the most difficulty defining the tangent visually and verbally in a scenario. Also, it was seen that the students were able to define the circle and the center at most in defining the concepts when their names were given. It was determined that the students had the most difficulty defining the concept of chord while defining the concepts when their names were given.

In Erol (2008)'s study, in which mathematical skills of 8th grade students were researches about the circle and closed circular region, it was concluded that the visual and conceptual skills of students were better than their operational skills. The students were concluded to tend more to solve problems by drawing and depending on visuality and therefore to be more successful. Wu and Ma (2005) explored the geometric concepts of the elementary school students at the first level of van Hiele's geometric though and they found that the circular concept was the easiest for students. As the result of the study made by Clements et all (1999) with middle-class children, it was concluded that the circle was easily recognized but difficult to describe for these children. Because most children described circles as "round". Güngörmüş (2002) determined that middle school students had difficulty remembering the pre-knowledge about the concept of circle. Akuysal (2007), on the other hand, found out that the 7th grade students could not express the geometric concepts, although knowing them, and could not comprehend the relationship between them. In the study of Özsoy and Kemankaşlı (2004), the fact that any point in the circle is accepted as the center is regarded as the misconception in their research about the mistakes and misconceptions of middle school students. As the result of the study made by Kaygusuz (2011) with Primary School 5th grade students, it was seen that the students failed in radius at most and center at least in the field of "circle sub-learning", and there was not a significant difference between girls and boys in learning the concepts. In this study, it was concluded that the students' mathematical language skills they used while defining the circle and its elements in a scenario significantly differed according to gender and it was seen that girls' mathematical language scores were higher than those of boys. Also, it was concluded that the students' mathematical language skills they used while defining the circle and their elements when their names were given significantly differed according to their gender and considering their average scores, girls' mathematical language scores were higher than those of boys.

Within the activity of "Let's map", in the part in which students were asked to name and the draw the circle defined with no name given, a great majority of them were seen to define the circle correctly but some of them were seen to give the answer of square or rectangle. It was concluded that this situation was caused by the expression " at equal distance" in the definition and the students associated the expression equal distance with square.

When the study is evaluated in general, the reason why the 7th grade students had difficulty defining the circle and its elements in a scenario can be said to be the fact that they had trouble in associating mathematical concepts with daily life. The fact that the students were able to define the concepts when their names were given in a thinking system depending on memorization can be caused by the manner

based on memorization in teaching. Teachers should focus on the process of formulation of concepts in students' minds and should plan the teaching for understanding to take place. Teachers should arrange their course teaching on this focus so that students can define the concepts using correct mathematical language visually and verbally. Students should be encouraged to use mathematical language and should be enabled to make the sense of symbols and showing formats in their minds.

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Appendix

Let's Map

You will start to live in another city for your university education and you have never been to this city before. Now imagine a city. But you don't know anywhere and there isn't any map of the city. So, it will be helpful for you to form your own map. You can start to form your map of the city by using the clues here.

Your city was established on a plain soil. Your university is in the middle of the city and there are infinite number of houses in your city. All houses are at an equal distance to your university. There is a street from each house to university.

Draw the shape of your city.

Determine the university and 8 of the infinite number of houses in this city. One of the houses will be yours. Do not forget to name it.

1. To which geometric shape your city resembles? Explain it with reasons.

2. Where is your university situated in your shape? What name do you give to this place mathematically other than point?

3. What name do you give to the street between your university and house among mathematical concepts other than line segment? Show it on your map.

4. According to your shape, if you think of other name from mathematical concepts what name do you give to line segment which is the linear combination of two streets that will pass through university? Show it on your map.

5. Your house and your best friend Deniz's house are far away from each other and your houses are in a street which is of infinite length. If you want to name this street, what other mathematical concepts do you use apart from line? Show it on your map.

6. There is one street between your house and your another friend Ahmet's house, and one end of this street is restricted to your house and other end of it is restricted to Ahmet's house. In giving name to this street which of the mathematical concepts do you use other then line segment according to your shape? Show it on your map.

7. This city, where your university is located, is situated on a linear road of infinite length which comes from the city where you have grown up. The city where your university is located has only one bus stop on this road. The only combination point of this road and the city is this bus stop. In view of this information, which of the mathematical concepts do you use while naming this road apart from line? Show it on your map.

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