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Difficulties Encountered by Students in the Learning and Usage of Mathematical Terminology: A Critical Literature Review

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Abstract
This study sought to review literature pertinent to difficulties encountered by students in the learning and usage of mathematical terminology. The need to carry out this study arose from the concern by the Kenya National Examinations Council, and the general public, over the poor annual results in mathematics. Therefore, the objective of this study was to investigate the extent to which the meanings of some mathematical terms are understood and/or confused by students for whom English is a second language. The basis of this study was the constructivist theory by J. Bruner and the cognitive flexibility theory of R. Spiro, P. Heltovitch and R. Coulson which advocates for teaching learners to construct the meanings of mathematical terms. This study's objectives were achieved through the use of document analysis. Data analysis involved document review. The findings of this study showed that students have difficulties in using mathematical terms and their related concepts. Possible ways of teaching these terms so as to generate more meaning to the learners were also suggested. It is hoped that this will assist mathematics teachers, curriculum planners and textbook authors to counter the poor performance in the subject in Kenya.
Keywords: Mathematical Concept, Capability; Confusion, Mathematical Curriculum Mathematical Terminology, Related Concepts, Colloquial Expressions, Second Language

1. Introduction

1.1 Background to the Study

The teaching and learning of mathematics, like any other subject, requires that both the teacher and learner communicate effectively. In Halliday's (1975) view, learning language involves 'learning how to mean'. Thus, the language of mathematics involves learning how to make and share mathematical meanings using language appropriate to the context, which is more than recognizing and responding to words in isolation. This, in turn, demands the use of appropriate language (words and symbols) whose level of difficulty is at par with the cognitive abilities of the learners concerned. Communicating mathematical ideas so that the message is adequately understood is difficult enough when the teacher and learner have a common first language but the problem is acute when their preferred languages differ. A number of studies have clearly indicated that a student's command of English plays a role in his/her performance in mathematics. Souviney (1983) tested students in grades 2, 4 and 6 with various languages and mathematics instruments on eight measures of cognitive development. His results showed that English reading and Piagetian measures of conservation were highly correlated with mathematical achievements.

The primary function of language, in mathematics instruction, is to enable both the teacher and the learner communicate mathematical knowledge with precision. In order to realize the objectives of mathematics instruction, teachers and textbook authors need to use a language whose structure, meaning, technical vocabulary and symbolism can be understood by learners of a particular class level. The communication of meaning frequently involves interpretation on the part of the receiver and this should warn us that messages could be given incorrect interpretations. Donaldson (1978) suggested that:

> When a child interprets what we say to him, his interpretations are influenced by at least 3 things ... his knowledge of the language, his assessment of what we intend (as indicated by our non-linguistic behavior) and the manner in which he would represent the physical situation to himself.

Some of the words and symbols used to communicate mathematical ideas can sometimes be misinterpreted by learners in their attempt to imitate their teachers. Pimm (1987 as cited in Muhandiki, 1992) reported that apart from determining the patterns of communication in the classroom, the teacher also serves as a role model of a 'native speaker' of mathematics. Hence the learners' search for the meaning of whatever they hear can, sometimes, lead to wrong conclusions. An instance of the learners' tendency to change (though not deliberately) the meaning of mathematical words into what they think the teacher intended to say is reported in Orton (1987) as follows:

> A kindergarten teacher drew a triangle, a square and a rectangle on the blackboard and explained each to her pupils. One little girl went home, drew the symbols and told her parents: 'this is a triangle, this is a square and this is crashed angle'.

This observation shows that the little girl's interpretation of 'rectangle' as 'crashed angle' exemplifies a situation whereby the child has a correct symbolic representation of a concept whose technical term she cannot produce due to linguistic problems.

Performance in mathematics has been relatively poor despite the national efforts made in developing a curriculum that is appropriate to the needs of this country. Towards this end, in 1998, the government of Kenya, through the Ministry of Education Science and Technology (MoEST) and the government of Japan initiated a technical co- operation project, known as SMASSE, in Kenya. The whole idea is premised on the realization that the quality of classroom activities is critical for the effective teaching and learning of mathematics.

Studies carried out on factors that affect mathematics achievement in Kenya at the primary level (Kafu,1976; Muriuki, 1991; Munguti, 1984; Omwono, 1990; Eshiwani, 1987), are silent on the role of language in mathematics instruction. Most of these studies have looked at factors such as: qualification of teachers; time spent in lesson preparation; teaching methods; frequency of supervision; students' and/or teachers' attitudes towards mathematics; availability and use of media resources; teaching experiences; class-sizes and in-service training.

This study, thus, explored the difficulties encountered by students in using mathematical terms and their related concepts. Studies carried out with learners for whom English is a first language have shown that learners have difficulties in using mathematical terms. Pimm (1987, as cited in Muhandiki, (1992) observed that:

> It is common place to hear a teacher ... asking pupils if they have understood the meaning of a certain word, and possibly trying to test their understanding of it by requesting either a formal definition or a paraphrase of its meaning!

It is, however, important for teachers to realize that the process of learning definitions of mathematical terms can be complicated by the abstract nature and the consequent difficulty of the words used to refer to them. Since students can find it difficult to comprehend the meaning of some terms even after they have been defined, the teacher ought to discuss various meanings and interpretations of such words and phrases so that each becomes aware of what the other means and understands by particular linguistic forms. Further, Dickson, Brown, and Gibson, (1984), have asserted that many specialized terms have an essential and rightful place in mathematics and it is necessary to incorporate them into the learning and teaching of the subject.

From the foregoing, it can be seen that language is critical to many of the processes of learning and instruction and it confers many benefits in terms of enabling us to articulate, objectify and discuss the problems which the field of mathematics presents. Yet language brings its own rules and demands, which are not always in perfect correspondence with the rules and demands of mathematics; it presents ambiguities and inconsistencies which can mislead and confuse.

1.2 Statement of the Problem

For many people, the mention of mathematics is met with downcast eyes (Tankersley, 1993; Too, 1996). Fear of mathematics is learned somewhere around the 4th grade (Too, 1996). In Kenya, this problem starts in the upper classes at the primary level (standard 7 & 8) and becomes acute at the secondary level (forms 2 and 3) (Eshiwani, 1983; Too, 1996]. This has resulted in dismal performance in the subject, persisting over the years. In fact, the performance in the subject at the K.C.S.E level has been estimated on average to be below 20 percent. The problems developed by learners at the primary level are at times carried over to the secondary level. Some of the weaknesses in the K.C.S.E. exam should have been discovered in the lower classes and remedial action taken. The fact that these weaknesses have persisted for a long time requires drastic changes in the teaching of mathematics (K.N.E.C., 1991).

This is exemplified by the following question which was performed poorly in the K.C.S.E. exam:

> A train moving at an average speed of 72 km/h takes 15 seconds to completely cross a bridge that is 80 m long.
>
> a) Express 72 km/h in metres per second.
>
> b) Find the length of the train in metres.

The concept of distance, time and speed taught in upper primary and form 1 was being tested. However, many candidates were unable to do the conversion and relate the three variables to find the length of the train, prompting the council to add that:

> The most glaring weakness is that of the learners' lack of knowledge in elementary techniques and their ignorance of simple algorithms and processes ... it is extremely worrying that inability to perform basic processes as multiplication and division is common feature in candidates work (K.N.E.C., 2004, p. 45].

As seen in the foregoing discussion, issues on the role of language in mathematics instruction have not been dealt with, yet the studies done in other countries show that learners have difficulties with the language of mathematics (Muhandiki, 1992). This suggests that, if the teacher tries to force new ideas that cannot be related to those already learned and mastered, the new ideas can only be learned by rote and remembered in arbitrary and disconnected manner.

This paper sought to investigate the extent of confusions and/or difficulties encountered by students in learning and using mathematical terminology by reviewing literature pertinent to the subject.

1.3 Significance of the Study

The findings of this study would be useful to the inspectorate unit of the Ministry of Science and Technology (MoEST) which is responsible for curriculum development, interpretation and implementation.

It is hoped that these findings would also be beneficial to teacher trainers in Teacher Training Colleges (TTCs) and Universities. They may have to reorganize their units in the teaching of mathematics so as to give the language of mathematics a special consideration.

Mathematics teachers will also benefit from the findings as this study has identified the shortcomings in the teaching and learning of mathematics. From these findings, the author has suggested ways of teaching the mathematics terminology so as to generate more understanding to the pupils. This would enhance the teaching and learning process and, subsequently, improve performance in the subject. The findings would also form a data bank for reference and an area for further educational research.

1.4 Scope and Limitation of the Study

This study confined itself to investigating the extent to which students encounter difficulties understanding and using mathematical terminology and their related concepts. It was conducted in public primary schools within Eldoret municipality. The population of the study was standard eight pupils from the selected primary schools. One limitation was that pupils tended to consult from one another, but this was remedied by strict supervision by the researcher and the subject teacher.

2. Materials and methods

2.1 The Role Of Language In Mathematics Instruction

The primary role of language is to enable both the teacher and learners share mathematical knowledge with precision. A teacher needs to use language suitable for the cognitive development of learners. According to Ishumi (1994), language is a powerful instrument in the formation of concepts, acquisition of particular perspective abilities and the transfer or communication of such concepts. Klein (1998) argues that language serves three important functions: first, language allows people to communicate with each other; second, it facilitates the thinking process; third, it allows people to recall information beyond the limits of memory. This assertion shows that language is not only important for communicating meaning but also because it facilitates thinking. The language used for thinking is most likely the first language, thus mathematics communicated in one language might need to be translated into another language to allow thinking and then translated back in order to converse with the teacher. Errors and misunderstanding might arise at any stage of this two-way inner translation process (Orton, 1987).

Berry (1985) contrasted the progress in mathematics of a group of university students in Botswana and a similar group of Chinese university students in Canada. The former group claimed they had to do all their thinking in English because their own language does not facilitate mathematical proofs and they did not find this easy. The Chinese students, on the other hand, claimed that they carried out their proofs in Chinese and then translated back to English and they were able to do it successfully. Therefore, it can be concluded that the more severe problems would probably be attributed to students trying to learn mathematics through the medium of an unfamiliar language which is very different from their own.

Gagne (1970) classified concepts into 'defined concepts' and 'concrete concepts'. According to him, a teacher is required to know what the learner needs in order to learn new concepts. A child is ready for a new concept when all the sub-concepts that are prerequisites to the concept are mastered. He suggested that children learn an ordered additive sequence of capabilities and that each new capability being more complex than the prerequisite capability on which it is built.

Dienes (1960) believes that mathematical concepts are properly understood only if they are presented through a variety of concrete, physical representations. He classified these concepts as pure mathematical concepts, notational concepts and applied concepts. His systems of teaching emphasized mathematical laboratories where he commended the use of MAB to provide suitable early learning environment enabling the construction of place-value concept. He postulated 6 stages through which the teaching of mathematical concepts must progress: free play; playing games; searching for communalities; representation; symbolism and formalization.

Ausubel (1960) expressed the same view in that concept development proceeds best when the most general, most inclusive elements of a concept are introduced first then the concept is progressively differentiated in terms of detail and specificity.

Choat (1974) stresses the close interdependence of language and conceptual development by stating that:

> Even if the learner interacts with the physical aspects of the learning situation, i.e. objects, the verbal element is necessary both as a means of communication and as an instrument of individual representation … in the acquisition of mathematical knowledge, a new conception, a child will not understand the word: without the word he cannot as easily assimilate and accommodate the concept (p.

ll).

This reflects the views of psychologist Vygotsky (1962) in that thought and language are interdependent. Even Piaget, in his later work, accepted that there might be a parallel development in the linguistic and cognitive strategies for making sense of the world.

The acquisition of language and concepts is a dynamic process. The child's understanding and use of language varies with the involvement of the child in the situation in which it is used and the relevance it holds for him. Thus, it is essential that the child and teacher discuss various meanings and interpretations of words and phrases so each becomes aware of what the other means and understands by particular linguistic forms. Pimm (as cited in Muhandiki, 1992) observed that:

> It is commonplace to hear a teacher ... asking pupils if they have understood the meaning of a particular word and possibly trying to test their understanding of it by requesting either a formal definition or a paraphrase of its meaning! (p. 69).

It is, however, important for teachers to realize that the process of learning definitions of mathematical terms can be complicated by the abstract nature of some, and the consequent difficulty of the words used to refer to them. Since students *can* find *it* difficult to comprehend the meaning of some terms even after they have been defined, the teacher ought to provide suitable learning experiences through which students can generate their own definitions. Blandford (1908, as cited in Harvey, Kerslake, Shuard, & Torbe, 1982) deplored the practice of giving students ready-made definitions by stating:

> To do this is ... to throw away, deliberately, one of the most valuable agents of intellectual discipline. The evolving of a workable definition by the child's own activity stimulated by appropriate questions is both interesting and highly motivational (p. 85).

Further, Dickson, Brown, and Gibson (1984) assert that many specialized terms have an essential and rightful place in mathematics and it is necessary that they are incorporated into the learning and teaching of the subject.

2.2 The Vocabulary of Mathematics

Bell (1970) listed a basic vocabulary of 365 words in common use outside mathematics as well as within, which even our slowest learners need to comprehend in dealing with elementary topics in mathematics. This mathematical vocabulary ranges from simple words, like 'link' 'find' and 'sort' to more sophisticated words, like 'bilateral' and 'quadratic'. Rothery (1980) distinguishes three broad categories of words:

i. Words which are wholly specific to mathematics and not usually encountered in everyday life. Such words include: hypotenuse, parallelogram, coefficient, isosceles, locus etc. Many of the difficulties caused by such words occur because children only come across them for the first time in their mathematical lessons where they may be defined once and never again. Children do not usually have easy access to looking up their meanings as few textbooks have indexes. It is suggested that the onus should be on the teacher to repeat definitions, discuss them and index them in some way. Further, Spencer & Russel (1960) also claim that the difficulties in reading mathematics are due to specialized language for expressing fractions, ratios and decimals. That the names and terms are unique to mathematics and reading about computations requires some specialized procedures.
ii. Words which have different meanings in mathematics and ordinary English. These are words which are used in everyday language and have different meanings in mathematics from their meaning in ordinary English. Words which are used in everyday language and have different meanings when used in mathematics can be a source of difficulty for children. These are words such as product, volume, count, odd, prime, power, mean, root, field, group etc. The two meanings of these words may cause confusions for children.
iii. Words which have the same (or nearly the same) meanings in both mathematics and ordinary English. These are words such as between, similar, gradient, relation etc. Rothery (1980) points out that, because many words used in ordinary English have different meanings when used in mathematics, the main problem with words which have the same meanings in both contexts is knowing that they do, in fact, mean the same. Sometimes, children may think that an ordinary English word takes on some mystical meaning when used in a mathematical setting or they may not fully understand its everyday meaning any case.

Bell and Freyberg, (1990) carried out studies which revealed some problems encountered in science lessons. They investigated pupils' meanings of words commonly used in science lessons such as animal, consumer, plant etc. the results revealed that pupils' meanings of these words are different from those of scientists. Similarly, Claessen and Stephens (1986) observed verbal interactions in some Kenyan secondary classes during science lessons. He noted that the teachers did not allow students to use their own language to explain learnt concepts but were to use terms that the teacher used. Although his study was in science lessons, it revealed that teachers did not encourage creativity and negotiation of meanings with pupils but controlled what the latter said.

Since successful use of mathematical terms requires the learner to be aware of their variant usage, this study sought to investigate the extent to which any learning difficulties could be attributed to one or more of the above categories of words used in mathematics.

2.3 Concepts Associated with the Four Arithmetic Operations

Studies relating to students' use of the four arithmetic operations have been reported by, among others, Hart, Otterburn and Nicholson (as cited in Muhandiki, 1992). According to Hart *et al.* (Muhandiki, 1992), this aspect of number operations was the first of the C.S.M.S. (Concepts in Secondary Mathematics and Science) project investigations whose aim was to find out:

> ... to what extent children recognize which operation to apply in order to solve 'word problem' set in the 'real' world, and supply an appropriate context for a formal computation 'sum' (p. 22).

Foster (1994, as cited in Nickson, 2000) investigated children's difficulties with what appear to be simple addition tasks. He considered variations of language used in connection with mathematical operations which children meet. For example: 'add 5 and 3'; 'add 3 to 5'; 'find the sum of 5 and 3' and '5 + 3'. Thus, he concluded that children need to be able to interpret these apparently different instructions and attach them to the symbolic form '5 + 3'.

Anghileri, (1995) studied the importance of language on children's learning of division: how they read and interpret the division symbol. She concluded that whereas there is evidence that children's learning is so clearly related to the language they use to interpret the symbols of arithmetic, teachers and researchers need to reflect on the ways that the classroom interactions may facilitate these two processes in order that true understanding will result.

These results suggest that students had some conceptual understanding of the four operations. This study, therefore, sought to investigate students' ability to use and produce the technical terms 'sum', 'difference', 'product' and 'quotient' and the corresponding concepts (addition, subtraction, multiplication, division). It was anticipated that the students' performance on the relevant test items would reveal their knowledge of the use of each term (and also the ability to distinguish between the four terms). Difficulties related to this latter aspect were also noted by Otterburn (as cited in Muhandiki, 1992) who reported that:

> The usual errors were to confuse the word 'product' with 'sum' (addition) ... or 'difference' (subtraction) ... quite a number of pupils described its (product) everyday use and gave 'something produced' or an equivalent expression (p. 20).

Thus, in this study, the need was felt to determine how widespread this confusion, similar to the above among standard eight pupils, was so as to suggest suitable ways of remedying them.

2.4 Concepts Associated with Fractions

Behr *et al.* (1992, as cited in Nickson, 2000) observe that there is a great deal of agreement that learning rational number concepts remains a serious obstacle in the mathematical development of children. Part of this difficulty may be due to the fact that the idea of a fraction is one of the earliest abstract ideas with which children have to cope since there is no 'natural' context in which they automatically arise (Booker as cited in Nickson, 2000). It is generally accepted that a lack of successful development and understanding of fractions in the earliest stages of children's learning will result in that will follow them through secondary years.

In a study, (Muhandiki, 1992) asked students to: identify the 'numerator' and 'denominator' in a given fraction; generate 'equivalent' fractions; formulate fractions representing shaded/unshaded regions in some shapes and to draw shapes and shade regions corresponding to given fractions.

Students' responses to the relevant test items revealed that they had difficulties in identifying the 'top number' and 'bottom number' of a fraction as representing on the other test items on 'equivalent' fractions and formulation of fractions representing shaded/unshaded regions of some regular shapes, it was felt that teachers and textbook authors probably should use the 'top number' for numerator and 'bottom number' for 'denominator'. However, (Shuard & Rothery, 1984) cautioned that omitting (such) technical words is a short-term policy, which makes the text easier to read but may bring long-term disadvantage to the pupil.

The researcher felt that an analysis of students' responses to these three parallel tasks would lead to an understanding of the mastery of the use of the terms 'numerator' and 'denominator'. This study also required students to give 'equivalent' fractions. The results showed that students had a fair grasp of the meaning of 'equivalent' fractions. However, given the students' limited perception of the domain of fractions, it became necessary to include it in this present study in order to identify the specific errors which students' were likely to make. Hart (1981) reported the following types of erroneous or inadequate strategies students employed:

- Some students, faced with ${}^{2}/_{7} = {}^{4}/_{14}$, did not multiply both the denominator and numerator by 2 but, instead, said, "7 and another 7 make 14, so it is 2 and another 2". Hart (1981) has cautioned that this method becomes cumbersome when the question is: ${}^{2}/_{7}={}^{10}/_{15}$ since the number of sevens to be added together has to be remembered.
- Desiring to find a number pattern: ${}^{2}/_{7} = {}^{10}/_{15}$ 'because 2 is 5 less than 7 and we want a number 5 less than 15'. This strategy is also indicative of the view that a fraction is simply two whole numbers which can be treated

separately.

- Looking at the sizes of the numerator and denominator and not at the ratio of the two numbers. 20% of each of the 12 and 13 year old groups denied the equality of $^5/_{20}$ and $^1/_4$ and 20%, likewise, said that $^4/_8$ was larger than $^2/_{10}$.

The concept of equivalence of fractions is often required in the items on representing fractions and skills needed in ordering fractions. Thus, it became necessary for this study to further investigate literature on the same concept.

2.5 Concepts Associated with Number Properties

The concepts associated with number properties whose understanding by students that were investigated by the author were: 'square of a number', 'square root', 'even', 'odd', 'prime numbers', 'divisor', 'factor' and 'multiple'. Otterburn as cited in Muhandiki, (1992), Nicholson (1977) and Muhandiki (1992) have reported students' difficulties with the concepts of 'multiple', 'factor', 'prime number' and 'square root'. Students' responses to the test items revealed several confusions hence lack of understanding of the use of each term and the distinction between them. With respect to the word 'multiple', Otterburn (Muhandiki, 1992) and Nicholson (1977) reported that the test item was poorly attempted not so much in the number of blanks but in the very large number of confused responses. Those who did not muddle it thought it was a misprint or synonym for 'multiply' and others thought it meant 'factor'. Of the 103 muddled responses, 85 muddled with 'factor'.

The studies above raise the question of whether the teaching of the corresponding terms is done through definitions, examples (relevant and non-examples) or a combination of both. It would be appropriate for the teacher to plan suitable learning activities for learners so that they can generate their own 'workable' definitions of such terms instead of being given tight definitions which is likely to lead to confusion. This also occurs when a term is defined differently by different authors. Orton (1987) has made a similar observation by noting that:

> We all know what a triangle is, but do we know what a natural number is? ... to many professional mathematicians, the natural numbers are: 0, 1, 2, 3 … The definition of prime numbers at one time included the number '1' and may still do for some people but, nowadays, most definitions ... exclude the number '1'.

2.6 The Concepts of Perimeter and Area

The terms 'perimeter' and 'area' are examples of words that are unique to mathematics and which (Shuard, H. & Rothery, A. (Eds.). (1984). refer to as 'Mathematical English' words. This means that the student first meets these words in mathematics class. What then, are the challenges that the student faces when trying to interpret these concepts? Dickson, Brown and Gibson, (1984) observed that students performed much better on the test items requiring them to find 'distance all the way round' than on the item that required them to find 'perimeter' and that some interpreted 'distance all the way round' to mean 'adding the measurements of the two labeled sides of the rectangle'. The results also showed that the concepts of 'area' and 'perimeter' are confused by many pupils who may give one when the other is asked for, with more pupils giving 'perimeter' instead of 'area' than vice versa (Dickson, Brown & Gibson, 1984).

The author, therefore, further sought to investigate the difficulties and errors associated with the use of the terms 'perimeter' and 'area' and the extent to which they exist among standard eight pupils. In this respect, the tasks required a demonstration of the correct use of the terms 'perimeter' and 'area' and the corresponding colloquial expressions.

3. Results and discussions

3.1 Influence of the Language of Primary Mathematics Textbooks

The analysis of the responses to the written test items presented in the previous chapter has shown that difficulties associated with the learning and use of mathematical terminology and the related concepts may be attributed to either the student's inadequate grasp of the language of mathematics or the fact that some terms cannot be expressed explicitly in ordinary language. However, since the students' greatest difficulties relate to the production of the technical terms, it seems that the latter are either avoided during mathematics instruction or they are not linked to the ordinary English expressions. An examination of the language used in the primary mathematics booklets, shows that, although direct reference is made to a few technical terms, the omission of the rest is deliberate.

Going through the booklets, one notices that a number line is used to introduce the concept of addition and subtraction. For example, addition is introduced by applying the ideas of 'adding on' and 'how many more ... altogether'. Similarly, for subtraction, the ideas of 'how many more' and 'how many', 'how much are left' are used but no reference is made to the concept of 'taking away'. However, the term 'difference' does not appear anywhere and students are simply asked to do 'these subtractions'. Similarly, although the term 'sum' appears in these booklets, no attempt has been made to define it explicitly as 'the result of addition and students are just asked to do these sums'. Furthermore, although 'division' is introduced as 'sharing equally', with the words 'divide/divide by' also used the term 'quotient' has been omitted. On the other hand, 'multiplication' is introduced (using multiplication tables) using the words 'multiply/multiply by', 'multiplied by', multiplications and answer, the latter is eventually given the technical term 'product'. However, the term 'product' is not introduced until children start learning about 'factors'. But, the use of the term 'total' in word problems, e.g. 'find the total number of...', where either addition or multiplication can be used, is likely to create the impression that the result of multiplication is 'total', a confusion that was evident in this study.

This, therefore, suggests that, rather than avoiding or delaying the use of technical terms like 'sum', 'difference', 'product' and 'quotient', they should be introduced soon after the corresponding colloquial expressions have been understood. The avoidance of the use of technical terms is also evident in the work of fractions as a part of a whole with no reference made to the terms 'denominator' and 'numerator'. Instead, the words 'bottom number' (for denominator) and 'top number' (for numerator) are used, including fraction names and their representation both in words and symbols (e.g. one-third, $^1/_3$; two-thirds, $^2/_3$; a quarter, $^1/_4$).

By using rectangular strips, circles and other visual aids, the child is made to realize that, to form a fraction the 'parts' that make up the 'whole' must be equal in size so that the bottom number will be the 'total number of parts' while the 'top number' will be the number of parts having some common characteristics. However, the same visual aids are used to show how to compare the sizes of fractions and mention is made of the fact that without such materials it is not easy to tell which of any two given fractions is bigger or smaller than the other unless they have the same bottom number. Furthermore, the concept of 'equal' fractions is also introduced using similar materials (paper strips of various unit lengths such as halves, thirds etc), but the term equivalent is omitted. At a later stage, the procedure of formulating 'equal' fractions is explained in the following example:

> … you get a fraction 'equal' to $^3/_5$ by multiplying 'top and bottom' numbers by the same number, e.g. $^3/_5$, $^6/_{10}$ you get back $^3/_5$ by dividing 'top and bottom' numbers by the same number.

It would appear from the way fractions are treated that the exclusive use of the concept of 'equal' fractions may have contributed to its production in this study by more students than those who produced the expected term 'equivalent'. It would be appropriate if the technical terms denominator, numerator and equivalent are introduced when formulating equal fractions. Although the terms 'factor' and 'divisor' have the same meaning, the latter is not used in the primary math's books while the former is seen in terms of division than multiplication as the following examples illustrate: 15 can be divided exactly by 3 we say 3 is a factor of 15, 3 is a factor of 30; when you divide 30 by 3, you get 10.

It would be more appropriate if such examples were given with respect to the term 'divisor' so that the corresponding examples for 'factor' would be as follows: 3 can be 'multiplied exactly' to give 15 we say 3 is a factor of 15; 3 is a 'factor' of 15; when you 'multiply' 3 by 5, you get 15.

Therefore, the non-use of the term 'divisor' in the booklets may also have contributed to the production of 'factor' (instead of divisor) by several students in this study. The simultaneous use of the two terms can make students realize that they have the same meaning.

Furthermore, the concept of factor as described above is applied when performing divisibility tests, leading to the concept of 'prime' number. Thus, a 'prime' number is defined as a number, which has no other factors except itself and '1'. It is, however, evident that although students apparently learn how to perform divisibility tests, starting with that of the number 2, no mention is made of the terms 'even', 'odd' and 'multiple'. This task seems to be left for the classroom teacher to do. In spite of what appears to be a deliberate attempt to avoid or minimize the use of the technical terms in the books, the concepts of 'square' and 'square root' of a number are introduced directly by their definitions as illustrated below:

- When we multiply a number by itself, we call it 'squaring the number'. So when we 'square' 3, we get 3x3, which gives 9.
- 4x4 can be written as 4^2 [which is read '4 squared'].
- The square of 4 is 16.
- We say that 4 is the 'square root' of 16.

It can be seen here that the colloquial meaning of the concept of 'square root' [number which can be multiplied by itself to give another number] is not given. This is likely to lead to difficulties in distinguishing between the technical terms 'square' and 'square root'. The confusion between the two terms may, therefore, be attributed to the problem of linguistic interpretation.

Although, the concept of 'perimeter/circumference' is not included in the booklets there are students' activities which involve measuring and calculating lengths of lines. But then, there is no indication in these activities to suggest any reference to 'perimeter', thus implying that it is through the classroom teacher and/or other texts that students learn about 'perimeter'.

It would be appropriate for the term 'perimeter' to be introduced just before 'area' in order to highlight the distinction between the two terms. On the other hand, the concept of 'area' is introduced as 'the size of (flat) shapes' and use is made of square and triangular grids of unit lengths. Thus, shapes of same lengths are said to have the same 'area'. Including the units of measurement, restriction of its application to square and triangular grids does not bring out its colloquial meaning (the amount of space covered) clearly. Therefore, the term 'area' seems to be more meaningful to students than its ordinary English expression (as was also revealed in the students' responses to test item 9 in this study).

3.2 Summary of the Errors Made

It was assumed that any errors made could be attributed mainly to affected students' inadequate grasp of the language of mathematics. The following observations were, therefore, made:

i) Some terms (e.g. quotient, numerator, equivalent) were either less meaningful (hence more difficult to use) than others or missing from the affected students' registers. This led to the production of either meaningless or blank responses.
ii) Some terms were given colloquial rather than mathematical interpretations.
iii) Some terms were confused with others possibly on the basis of the wrong assumption that they implied the same mathematical operation.
iv) Some terms were confused with their mathematical 'inverses'.
v) A few technical terms seemed to embody the corresponding conceptual meaning better than the colloquial expressions. However, production of technical terms from given definitions (in form of examples) proved more difficult than the recognition of the use of the same terms while the interpretation of the corresponding colloquial expressions were relatively easier.

Therefore, there is need for mathematics educators to address the issue of language of mathematics in order to find ways of improving students' proficiency in using mathematical terms.

4. Possible remedial measures

1. Students should be helped to acquire the vocabulary and correct phraseology of mathematics, appropriate to their age and ability, if they are to succeed in the subject. By administering suitable diagnostic tests, the teacher can get an idea about students' language difficulties and appropriate remedial measures taken. Thus, similar to Richards (1978) observation regarding the language of science, students' acquisition of the vocabulary of mathematics can lead to their understanding of mathematical ideas. However, apart from technical terms, teachers should use linguistic structures (e.g. quantifiers) which are simple enough since the latter can also be another source of difficulty (Austin & Howson, 1979).
2. Given the several confusions observed in the students' responses, the researcher felt there was need for students to be shown as many instances of a given term/concept as possible. Thus, after students have understood the colloquial language for a given concept, they should be gradually introduced to other versions of that concept, culminating in the relevant technical term(s). For example, since children will initially tend to associate 'subtraction' with 'taking away', they need to know (at some stage) that the term 'difference' refers to the same concept. But then, what is the most appropriate moment of introducing technical terms? According to Bulman (1985) "… [for] a secondary school science teacher ... it is extremely difficult ... to know the right time to introduce the scientific term and even more difficult to know if and when to insist on it being used." A similar observation in science education has been made by Carre (1981) as well in noting that "... specialized vocabulary of science must be used; what is of importance is that insisting on the premature use of a register may be a hindrance and can interfere with pupils' learning." However, a solution to the issues raised by Bulman (1985) and Carre (1981) is offered by Prestt (1980) a science educator, who says that: "... the introduction of a technical word is inappropriate if the word does not encompass ideas ... which the child has developed from his own experience and exploration of those experiences through his own personal language." Therefore, teachers must always be aware that they can communicate with their students if the package of ideas is the same as that which the students understand by that word.
3. Students' understanding of a given concept can be developed further by considering non-examples of that concept. Skemp (1971) has also suggested that the learning of mathematical concepts is comparable and that children are not expected to through definitions. There is also need to use examples and counter-examples (Skemp, 1971). For instance, after students have learned the concept of 'even numbers', they should be shown examples of 'non-even numbers'. This technique is likely to enhance students' ability to distinguish between concepts, particularly those that are 'subsets' or 'inverses' of others. Similarly, it is important, particularly when teaching less able children, to repeatedly use a new term to which they have been introduced to enable them to become completely familiar with it (Hinson, 1980).
4. Teachers need to be aware that it is not easy to define some mathematical terms, very precisely, in ordinary English and that such terms are best understood in their technical forms. However, an alternative approach

to defining technical terms would involve the use of suitable relevant examples already known to students. An observation similar to this is implied in what Skemp (1971) calls 'first principles' in learning mathematics which are:

(i) Concepts of higher order than those which a person already has cannot be communicated to him by definitions but only by arranging for him to encounter a suitable collection of examples.

(ii) Since, in mathematics, these examples are almost invariably other concepts, it must first be ensured that these are already formed in the mind of the learner. Nevertheless, whether the best way of communicating the meanings of technical terms and concepts is (in the teacher's opinion) by using suitable examples and/or giving formal definitions ways of enabling students to remember these meanings should be sought. Dickson, Brown, and Gibson (1984) have suggested that the onus should be on the teacher to repeat definitions, discuss them and index them in some meaningful way. Similarly, as Krulik (1980) has suggested, students can also be encouraged and guided on how to compile their own 'dictionaries' using illustrative sentences and examples. This approach can also assist students to:

a) Identify words that have similar mathematical meanings.
b) Discriminate between words that have:
   - Multiple mathematical meanings.
   - Mathematical meanings, which are different from their colloquial meanings.

This is in line with the strong activity message adopted by Cockroft (1982) where we find that, for most children, practical work provides the most effective means by which the understanding of mathematics can develop'. Related to (b) above, Durkin and Shire, (1991) have suggested that, in preparing materials for use in mathematics teaching (from lesson plans to textbooks), it is advisable to consider whether any words might have a different meaning for the pupil from that intended or assumed in this specialist context. If teachers and authors are aware of the potential confusions that some words can cause, appropriate instruction can be planned. For example, textbooks with a glossary of mathematical terms at the end of each topic would be very useful to learners.

5. In order for the suggestions proposed above to be successfully implemented, it will be necessary for students to be actively involved in the learning activities, with the teacher playing the role of a guide and a facilitator of learning. This observation has also been made by Copeland, (1984) who reported that the first condition for the implementation of an appropriate educational program would be one which would allow and encourage that every new truth to be learned be worked out or rediscovered by the student and not simply imparted to him via an explanation. Teachers should therefore understand that learners are likely at first to use new terms incorrectly but as they are made aware of this, they acquire the capacity to use them appropriately.

5. Recommendations

Due to the limited scope of this work, the author felt that there is need for more extensive research in the same area in order to arrive at conclusions that would be more valid, and applicable, to a larger student population.

1. Further research should be done, preferably covering a larger sample of primary schools, including both urban and rural schools, and at all levels. This will, in effect, necessitate the inclusion in the research instruments of more mathematical terms and concepts than those tested in this study. The subjects in any further research could include not just students, but teachers and other resource persons as well.
2. A similar study should be done, preferably covering a larger sample of secondary school students at all levels.
3. Research on the language of mathematics focusing on:
   (i) Types of classroom interactions among pupils and between pupils and their teacher(s).
   (ii) Language used in mathematics textbooks needs to be done. In this respect, special attention will need to be paid to these two aspects on both the learners' and teachers' use of mathematical language.
4. A similar study on the language of mathematics, focusing on the signs and symbols used in mathematics instruction, need consideration. This will provide an insight as to whether students understand the various signs and symbols and the instructions that they stand for.
5. A comparison of performance in mathematics by students who are good or poor in English as a subject.

References

Anghileri, J. (1995). Division and the role of language in the development of solution strategies. Proceedings of the eighteenth international congress of the Group of the Psychology of Mathematics Education II. In M. Nickson (2000). *Teaching and learning mathematics: A teacher's guide to recent research and application.* London & New York: Continuum.

Austin, L., & Howson, A. G. (1979). Language and Mathematical Education. *Educational Studies in Mathematics*, 10(3): 167-197.

Ausubel, D. P. (1960). *Educational psychology: A cognitive view*. New York: Holt, Reinhart and Winston Inc.

Bell, B. & Freyberg, P. (1990). Learning science. In R. Osborne & P. Freyberg (1988). *Learning in science: Implications of children's science* (pp. 22-39). Heinemann, Aukland.

Bell, P. (1970). *Basic teaching for slow learners.* London: Muller.

Berry, W. (1985). Learning mathematics in a second language: Some cross-cultural issues. *For the Learning of Mathematics* (pp. 18-23).

Bulman, L. (1985). *Teaching Language and Study Skills in Secondary science*. London: Heinemann Educational Books Ltd.

Carre, C. (1981). *Language Teaching and Learning 4: Science*. London: Ward Lock Educational.

Choat, E. (1974). *Children's acquisition of mathematics*. Windsor: NFER.

Claessen, A. & Stephens, D. (1986). *Language Education: A resource book*. Nairobi: Kenyatta University Press.

Cockroft 1982 Cockroft, W. H. (1982). *Mathematics Counts*. London: HMSO.

Copeland, R. W. (1984). *How Children Learn Mathematics. Teaching Implications of Piaget's Research* (4th ed.). New York: Macmillan.

Dickson, L. Brown, M. & Gibson, O. (1984). *Children Learning Mathematics*. Eastbourne: Holt, Rinehart & Winston (school council).

Dienes, Z. P. (1960). *Building up mathematics.* London: Hutchinson.

Donaldson, M. (1978). *Children's minds*. Glasgow: Fontana Collins.

Durkin, K., & Shire, B. (1991). Lexical Ambiguity in Mathematical Contexts. In Durkin, K., & Shire, B. (Eds.), *Language in Mathematical Education: Research and Practice.* Milton Keynes! Philadelphia: Open University Press.

Eshiwani, G. (1983). *An investigation into the use of programmed materials in teaching probability to Kenyan high school students*. Kenyatta University: Bureau of Educational Research.

Eshiwani, G. (1987). School mathematics and work: A study of external efficiency of primary schools of Kenya. Kenyatta University: BER.

Gagne, R. M. (1970). The conditions of learning and theory of instruction. New York: Holt, Rinehart and Winston.

Halliday, M. A. K. (1975). Learning how to mean-explorations in the development of language. In A. J. Bishop (1988). *Mathematics education and culture*. Netherlands: Kluwer Academic Publishers.

Hart, M. K. (Ed.) (1981). *Children's understanding of mathematics* (pp. 11-16). London: John Murray.

Harvey, R., Kerslake, D., Shuard, H. & Torbe, M. (1982). *Language teaching and learning 6: Mathematics*. London: Ward Lock.

Hinson, M. (1980). Children with Linguistic Difficulties. *Language in Science: Study Series No. 16.* Hartsfield. Herts: The Association for Science Education.

Ishumi, A. G. M. (1994). *30 years of learning: Educational development in eastern & Southern Africa from independence to 1990*. Ottawa: IDRC.

K.N.E.C. (1991). *1990 K.C.S.E Report (Mathematics).* Nairobi: MoE.

K.N.E.C. (2004). *KCSE examination candidates' performance report.* Nairobi: Kenya.

Kafu, P.A. (1976). *Analysis of elementary school teachers' rationale concerning their use of various instructional media in Bungoma District and Eldoret Municipality.* Unpublished M. Ed Thesis. Kenyatta University.

Klein, S. B. (1998). *Learning: Principles and applications.* New York: McGraw-Hill Inc.

Krulik, S. (1980). To Read or Not to Read that is the Question! *The Mathematics Teacher*, 73(4): 243.

Muhandiki, G. L. (1992). *A study of students' capacity to use mathematical terminology and the related concepts*. Unpublished M.A Thesis. Darwin College.

Munguti, K. P. (1984). *A survey of factors affecting the teaching and learning of mathematics in primary schools in Mbooni Division in Machakos District, Kenya*. Unpublished M. Ed Thesis. Kenyatta University.

Muriuki, M. F. (1991). *A study of problems encountered by primary school teachers in some upper classes in Makadara Division, Nairobi*. Unpublished M. Ed Thesis. Kenyatta University.

Nicholson, A. R. (1977). Mathematics and language. *Mathematics in school,* 6(5), 32-34.

Nickson, M. (2000). *Teaching and learning mathematics: A teacher's guide to recent research and its application.* London: New York. Continuum.

Omwono, J. O. (1990). *A study of the availability and use of instructional media in teaching and learning of primary mathematics in Kuria educational division, South Nyanza, Kenya*. Unpublished M. Ed Thesis. Kenyatta University.

Orton, A. (1987). *Learning mathematics: Issues, theory and classroom practice*. London: Cassel Educational.

Prestt, B. (1980). *Towards Precision. Language in Science: Study Series No. 16*. Hartfield, Herts: The

Association for Science Education.
Richards,J. (1978).Classroom Language: what sort? London: John Murray.
Rothery, A. (Ed.). (1980). *Children reading mathematics*. Worcester: College of Higher Education.
Shuard, H. & Rothery, A. (Eds.). (1984). *Children reading mathematics*. London: John Murray.
Skemp, R. (1971). *The Psychology of Learning Mathematics*. Harmondsworth: Penguin Books.
Souviney, R. (1983). Mathematics achievement, language and cognitive development: Classroom practices in Papua, New Guinea. *Educational studies in mathematics*, 14, pp. 183-212.
Spencer, P. L. & Russel, D. (1960). *Reading in arithmetic*. London: Ward Lock Educational.
Tankersley, K. (1993). Teaching mathematics their way. *Educational Leadership*, Vol. 5, No.47
Too, J. (1996). *A survey of the availability and use of media resources in mathematics instruction: The case of secondary schools in Nandi District.* Unpublished Research Thesis. Eldoret: Moi University.
Vygotsky, L. S. (1962). *Thought and language*. New York: MLT Press & Wiley.