THE FIRST CYCLE OF DEVELOPING TEACHING MATERIALS FOR FRACTIONS IN GRADE FIVE USING REALISTIC MATHEMATICS EDUCATION

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Abstract

There are 3 questions that will be answered in this study, namely (1) what are the contexts that can be used to introduce the meaning of multiplication of two fractions and to find the result of multiplying two fractions, (2) how to use these contexts to help students construct the understanding of the meaning of multiplication of two fractions and find the result of multiplying two fractions, and (3) what is the impact of the teaching-learning process that has been designed by researchers on the process of students' knowledge construction.
Learning approach which was used in developing teaching materials about fractions is realistic mathematics approach. Lesson plan was created for fifth grade elementary school students. The type of research used is development research. According to Gravemeijer and Cobb, there are three phases in development research, namely (1) preparation of the trial design, (2) the trial design, and (3) a retrospective analysis. This paper presents the results of the first cycle of three cycles that have been planned.

Key Words: fractions; realistic mathematics education; design research.

Abstrak

Ada 3 pertanyaan yang akan dijawab dalam penelitian ini, yaitu (1) apa konteks-konteks yang dapat dipergunakan untuk mengenalkan makna dari perkalian dua pecahan dan mencari hasil perkalian dua pecahan, (2) bagaimana menggunakan konteks-konteks tersebut untuk membantu siswa mengkonstruksi pemahaman tentang makna dari perkalian dua pecahan dan cara mencari hasil perkalian dua pecahan, dan (3) apa dampak proses pembelajaran yang dirancang oleh para peneliti terhadap proses konstruksi pengetahuan siswa.
Pendekatan pembelajaran yang dipergunakan di dalam merancang proses pembelajaran pecahan adalah pendekatan matematika realistik. Rancangan pembelajaran yang dibuat adalah untuk siswa kelas V SD. Jenis penelitian yang dipergunakan dalam penelitian ini adalah penelitian pengembangan (development research). Menurut Gravemeijer dan Cobb, ada 3 fase dalam penelitian pengembangan, yaitu (1) persiapan uji coba desain, (2) uji coba desain, dan (3) analisis retrospektif. Dalam makalah ini disajikan hasil yang diperoleh pada siklus pertama dari 3 siklus yang direncanakan.

Kata Kunci: pecahan; pendekatan matematika realistik; penelitian pengembangan.

In 2012, the first writer of this paper had the opportunity to accompany a fifth grade elementary school teacher to implement the realistic mathematics approach. In a discussion, conducted between the lessons, the teacher said that one of the topics in grade five that is difficult to be understood by fifth grade students is fractions, especially on the meaning of multiplication of two fractions, and has to obtain the result of multiplication of two fractions.

According to Lamon (2001, in Ayunika, 2012), the development of understanding of the meaning of fractions in the teaching-learning process is a complex process because the concept of fraction has a number of interpretations, namely (1) fraction as a part of the whole, (2) fraction as the result of a measurement, (3) fraction as an operator, (4) fraction as a quotient, and (5) fraction as a ratio.

There are 3 questions that will be answered in this study, namely (1) what are the contexts that can be used to introduce the meaning of multiplication of two fractions and to find the result of multiplication of two fractions, (2) how to use these contexts to help students construct the understanding of the meaning of multiplication of two fractions and to find the result of multiplication of two fractions, and (3) what is the impact of the teaching-learning process designed by researchers on the process of students' knowledge construction.

According to Gravemeijer (1994), Realistic Mathematics Education (RME) is rooted in Freudenthal's view that mathematics as a human activity. If implemented, the basic philosophy of RME brings about a fundamental change in the process of teaching-learning mathematics in the classroom. The teacher in teaching and learning activities should no longer directly provide information to the students, but he/she provides a series of problems and activities that can be used by the students to build their understanding of mathematical concepts that leads to the formation of formal mathematical knowledge. In other words, in the RME approach, the teacher plays a role as a facilitator to their students. According to Widjaja, Fauzan, and Dolk (2009), to be able to act as a facilitator, the teacher must facilitate students' learning by using contextual problems, asking questions that guide students to develop their thinking processes, and leading class discussions in order to help the students in constructing their understanding of the mathematical concepts that are embedded in the contextual problems.

There are three main principles in the RME (Gravemeijer, 1994), namely:
1. Guided reinvention and progressive mathematizing;
2. Didactical phenomenology;
3. Self-developed models.

METHOD

In the first cycle, there were four students involved that came from the fifth grade of a private elementary school in Yogyakarta. The approach used by researchers to develop students' learning materials and teacher's guides in this study was realistic mathematics education approach. The development of the learning materials and the teacher's guide was to be conducted in three cycles. Data analysis was done based on the data of video recording, taken during the teaching and learning process, and the student worksheets. The steps were undertaken in the first cycle following phases of the development research developed by Koeno Gravemeijer and Paul Cobb (in Akker, Gravemeijer, McKeney, and Nieveen, 2006).

RESULTS

The First Phase of the First Cycle

The objective of the teaching-learning process that used the learning materials developed by the first researcher was to facilitate the students so they (1) understand the meaning of multiplication of two fractions, and (2) are able to determine the result of multiplication of two fractions.

Before students experienced the learning process designed by the first researcher, the students had learned about fractions in fourth grade, involving (1) the meaning of fractions, (2) ordering fractions, (3) simplifying fractions, and (4) adding and subtracting fractions.

The Second Phase of the First Cycle

The contextual problems were explored and solved by students for four meetings, i. e.:

The First Meeting

  a. The first problem

Yesterday afternoon during school recess, the teacher saw two groups of children who were sharing bread. The first group consisted of two students who were sharing a piece of bread. The second group consisted of four students who were sharing two pieces of bread. Do you think that each student in the first and second groups got the same amount of bread?

  b. The second problem

Yesterday afternoon during the school break, the teacher also saw two groups of other children who were sharing bread. The first group consisted of two students who were sharing a piece of bread. The second group consisted of three students who were sharing two pieces of bread. Do you think that each student in the first and second groups got the same amount of bread?

  c. The third problem

The third problem consisted of four questions. In each question, there were two groups of children who were sharing the bread. The number of the children and the amount of the bread of each group were different. Students were asked to choose whether they would be a member of the first or the second groups and the reason why they determined their choice.

First question: there were four children sharing two pieces of bread at the first group, while there were six children sharing two pieces of bread at the second group.

Second question: there were four children sharing two pieces of bread at the first group, while there were six children sharing three pieces of bread at the second group.

Third question: there were three children sharing two pieces of bread at the first group, while there were four children sharing three pieces of bread at the second group.

Fourth question: there were four children sharing one piece of bread at the first group, while there were five children sharing two pieces of bread at the second group.
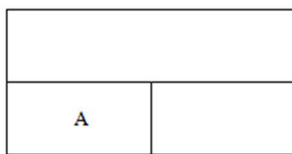
The Second Meeting

a. The first problem

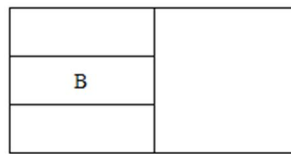Mr. Hongki shares a cake for his friends in the following way:
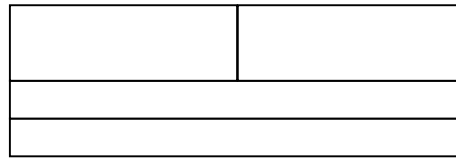
Does every friend of Mr. Hongki get the same portion of the cake?
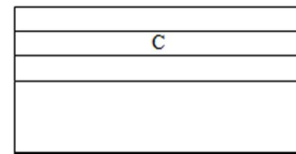
b. The second problem

Among the three pieces of bread, namely A, B, and C, which piece is the biggest?
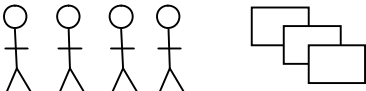

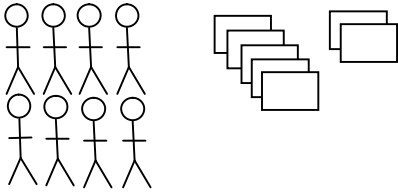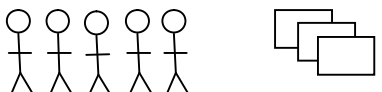Bread 1   Bread 2   Bread 3

The Third Meeting

a. The first problem (inspired by the problems in the book titled "Young Mathematicians at Work: Constructing Fractions, Decimal, and Percents")
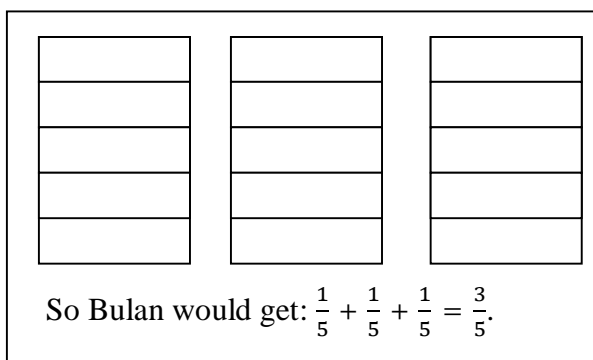
Today fourth grade students of Mekarsari School will make observations at some objects of art and culture in Yogya.

students   bread

The First group will visit Kasongan, Bantul

students   bread

The Second group will visit Affandi's museum.

students   bread

The third group will visit the center of batik art.

students   bread

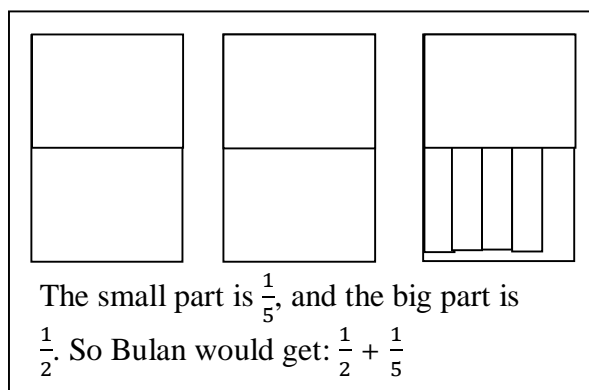The fourth group will visit the silver products.

When students returned from the observation activity, the students began to argue that the bread that was distributed to each student in the group did not have the same amount, because there were some students who got more than other students. Did each student get the same amount of bread?

b. The second problem (inspired by the problems in the book titled "Young Mathematicians at Work: Constructing Fractions, Decimal, and Percents")
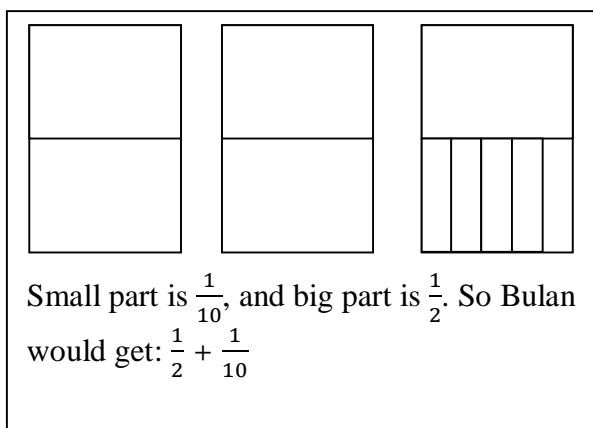
Mrs. Niken gives the following questions to the students. A student, named Bulan, was of five students. The group received three pieces of bread. How much bread was obtained by Bulan? The pictures below were the students' answers. Do the answers produce equivalent fractions? Can you show it?
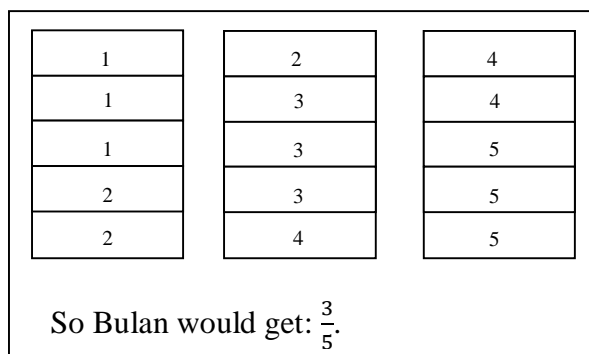
So Bulan would get: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$.

Titin 's answer

The small part is $\frac{1}{5}$, and the big part is $\frac{1}{2}$. So Bulan would get: $\frac{1}{2} + \frac{1}{5}$

Rudi's answer

Small part is $\frac{1}{10}$, and big part is $\frac{1}{2}$. So Bulan would get: $\frac{1}{2} + \frac{1}{10}$

1	2	4
1	3	4
1	3	5
2	3	5
2	4	5

So Bulan would get: $\frac{3}{5}$.

Susi's answer   Andi's answer

The fourth meeting was used for an evaluation activity. The following were the questions given to students in the evaluation process:
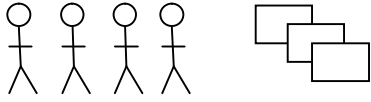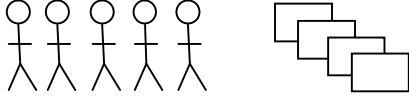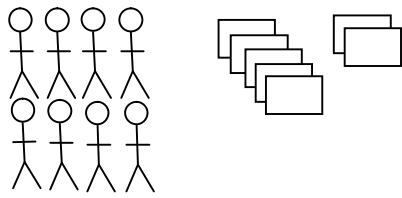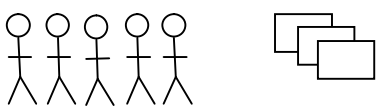
The First Question

Bu Vivi makes a pan cake. Bu Vivi will divide the cake to 8 neighbors, namely Bu Dina, Bu Suci, Bu Mekar, Bu Bulan, Bu Sinar, Bu Bintang, Bu Rosna, and Bu Rini. Bu Vivi cuts a pan cake for the neighbors in the following way:

Bu Dina	
Bu Suci	
Bu Mekar	
Bu Bulan	
Bu Sinar	Bu Rosna
Bu Bintang	Bu Rini

What is the portion obtained by each of Bu Vivi's neighboard? Do Bu Dina and Bu Rini get the same amount of bread?

The Second Question

Today, the 4th grade students of Karya Elementary School will make observations at some objects of art and culture in Yogya. The students are given some bread by the school, to be eaten during lunch time. Here is the place to visit, the number of students in every group, and the amount of bread in each group.

students   bread
The first group will visit Kasongan, Bantul

students   bread
The Second group will visit Affandi's museum.

students   bread
The third group will visit the center of batik art.

students   bread
The fourth group will visit the silver products.

At the time of the distribution of the bread, the bread for group 1 is combined with the bread of group 3, while the bread of group 2 is combined with the bread of group 4. After that, each combination of groups is asked to share the bread for its members fairly. Does each student of that grade, who takes part in the visit, get the same amount. In other words, is the distribution of bread in that way fair?

The Third Question

Mbah Joyo had two children, namely Mr. Jono, and Mr. Jino. Mr. Jono had four children, namely Bulan, Bintang, Sinar, and Surya. Mr. Jino had two children, namely, Wawan and Niki. When Mbah Joyo died, he bequeathed a piece of land of 10,000 square meters. Before he died, Mbah Joyo had written a will of how the land was to be distributed as follows: Mr. Jono got a half of the land, while the rest was for Mr. Jino. The portion of the land obtained by Mr. Jono was evenly distributed to Bulan, Bintang, Sinar, and Surya. The portion that belonged to Mr. Jino was evenly distributed to Wawan and Niki.

a. What was the portion of land acquired by each of Bulan, Bintang, Sinar, dan Surya?

b. What was the portion of land acquired by each of Wawan and Niki?

c. Did Bulan and Wawan get the same portion of land?

d. What was the area of the land acquired by Mr. Jono?

e. What was the area of the land acquired by Mr. Jino?

f. What was the area of the land acquired by each of Bulan, Bintang, Sinar, and Surya?

g. What was the area of the land acquired by each of Wawan and Niki?

The Third Phase of the First Cycle

Table 1 show the results of students' work in the first, second, and third meetings, and the results of the evaluation
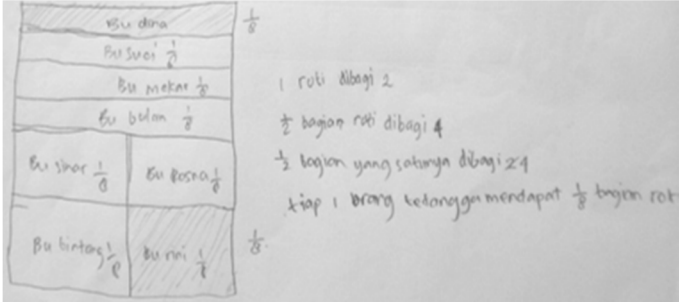
Meeting	Students Answer	Notes
1	**First problem:** The students in the first group divided the bread for the first group into two equal parts. Then the students in the second group divided each of the first and the second pieces of bread for the second group into two equal parts. After that, they gave each part to each student in the second group. The conclusion is that students in the first and second group received the same portion, i.e. half of bread. **The second problem:** **Method 1:** In the first group, the students divided the bread for the first group into two equal parts and each student obtained a half of the bread. In the second group, the students divided each of the first and second pieces of bread for the second group into three equal parts and each student obtained $\frac{2}{3}$ of the bread.	• The number of students who participated in the first meeting was four. • The first problem was done individually. • For the first problem, the four students answered in the same way. • The second problem was done individually. • For the second problem, there were two students who answered using method 1, and there were two students who answered using method 2.

Method 2: In the first group, the students divided the bread for the first group into two equal parts and each student obtained half of the bread. In the second group, the students divided each of the first and second pieces of the bread for the second group into two equal parts. Then every student in this group was given half of the bread. Then they cut the remaining bread into three equal parts. Students wrote that the bread slice was $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$. So, the portion obtained by each member of the group was $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$. The students said that the portion obtained by each student in group 2 was more than that obtained in group 1 because each student in group 2 got extra bread from the rest of the half part of bread that was divided by three.

The third problem:

The first question:

Students divided each pieces of bread for the first group into two parts, and for the second group into three parts. Students chose group 1 because they wolud get more bread than each student in group 2.

Second question:

Students divided each bread for the first group into two parts, and for the second group into two parts. Students said that the portion for each student in group 1 was equal to the portion for each student in group 2.

The third question:

Method 1: Students divided each pieces of bread for the first group into three parts. Students divided each of the first and second pieces of bread for the second group into two parts, while the third pieces of bread was divided into four parts. Students did not give an answer, concerning which group they chose.

Method 2: Students divide each piece of bread for the first group into three parts, and for the second group into four parts. Students said that they select group 1,

- From the second problem, there had been students who brought about the notion of multiplication of two fractions, i.e. $\frac{1}{3}$ of $\frac{1}{2}$. Statement $\frac{1}{3} \times \frac{1}{2}$ means $\frac{1}{3}$ of $\frac{1}{2}$.

- The third problem was done in pair.

- For the first and second questions, the students' answers were the same.

- For the third question, the first group answered using method 1, while the second group answered using method 2.

- For the fourth question, the method of dividing the bread was the same, but which group was selected by the each group was different.

	because the part of group 1 more than group 2.	
	The fourth question: Students divided bread for the first group into four parts. Students divided each piece of bread for the second group into five parts. In the classroom, there were two groups of students were worked on this fourth question. The first group's answer was they chose group 2, because each student in group 2 got more bread than each student in group 1. The second group's answer was they chose group 1, because each student in group 1 got more bread than each student in group 2.	
2	**The first problem:** The students in the classroom said that the part obtained by each friend Mr. Hongki is $\frac{1}{4}$. The reason given by students is because after Mr. Hongki divides the bread into two equal parts, Mr. Hongki divides each section into two equal parts. Thus, each Mr. Hongki's friend gets the same portion, namely $\frac{1}{4}$ part, although the form of cake obtained by each person is not same. **The second problem:** **Method 1:** The piece A of bread 1 is $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{4}$. The piece B of bread 2 and the piece C of bread 3 is $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$. So, the biggest piece is piece A. **Method 2:** The piece A of bread 1 is $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{4}$ or $\frac{1}{2}$ divided by 2 or $\frac{1}{2}$ of $\frac{1}{2}$. The piece B of bread 2 is $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ or $\frac{1}{6}$ that comes from $\frac{1}{2}$ divided by 3. The piece C of bread 3 is $\frac{1}{6}$ that comes from $\frac{1}{2}$ divided by 3 or $\frac{1}{3}$ of $\frac{1}{2}$. Students did not say which one was the biggest piece. **The third problem:** **Portion of each student in group 1:** The students divide the first and second pieces of bread into two parts, while the third pieces is divided into four parts. The portion obtained by each student in group 1 $= \frac{1}{2} + \frac{1}{4} =$	• The students who participated in the second meeting were two students. • The first problem for the third meeting was solved also by the students in this second meeting. • For the first problem, the ways and the reasons put forward by both students were alike. • For the second problem, first student used method 1, while the second student used method 2. In the first method, the student got the right answer in saying that the piece of A was $\frac{1}{4}$, but less precise in stating the reason why the amount was one fourth. In the second method, there was part that was less precise,

$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

Portion of each student in group 2: The students divide each of the first, second, and third bread into two. The rest of the half part of the third piece of bread is divided into five parts. The students divide the fourth piece of bread into five parts. The students wrote that the portion obtained by each student from the reminder of the third bread is $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$. So, the portion obtained by each student in group $2 = \frac{1}{10} + \frac{1}{2} + \frac{1}{5} = \frac{1}{10} + \frac{5}{10} + \frac{2}{10} = \frac{8}{10}$.

Portion of each student in 3: The students divide the first to fourth pieces of bread each into two parts, the fifth and sixth pieces of bread each into four parts, and the seventh piece of bread into eight parts. Portion obtained by each student in group $3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$.

Portion of each student in group 4: Students divide the first, the second, and the third bread each into two equal parts. The remainder of the half part of the third bread is divided into five parts. Students wrote that the portion by each student from the remainder of the third piece of bread is $\frac{1}{5}$ dari $\frac{1}{2} = \frac{1}{10}$. The portion obtained by each student in group $4 = \frac{1}{10} + \frac{1}{2} = \frac{1}{10} + \frac{5}{10}$. Both students who worked on this problem in the class did not answer the question of whether every student in those four groups got the same portion of bread or not.

but there was a part that was the right answer as a reason of the student that piece A was $\frac{1}{4}$.

- From the second problem, the students' understanding of the meaning of multiplication of fractions was strengthened. From this problem, the understanding of the meaning of division of fractions with integers also appeared, namely $\frac{1}{2}$: 3, and $\frac{1}{2}$: 2.

- For the third problem, the method used by both students was the same.

- To solve the third problem, especially when dividing the bread for the second and fourth groups, two students had used their understanding of the meaning of multiplication of two fractions. Students could not only use it. But both students could also find the result of the multiplication of two fractions.

3	**The first problem**	- The third meeting was only attended by two students who also
	For Titin's answer:	
	Method 1: Students redrew the picture in the student	

worksheet, and provided shading on the first piece of each piece of the bread. Then the students gave a check on the answer. In other words, the students said that Titin's answer was correct.

Method 2: Students redrew the picture in the student worksheet, and wrote down the names of students who received each part of the piece and each part is $\frac{1}{5}$. Names written by the students for each piece of bread were Candra, Rudi, Adi, Budi, and Bulan. Then, the students shaded three pieces that belonged to Bulan, and concluded that the portion was obtained by Bulan is $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$.

For Rudi's answer:

Method 1: Students redrew the picture in the student worksheet, and shaded the parts acquired by Bulan, i.e. the top piece of the first piece of bread, and the leftmost bottom of the third piece of bread. The students wrote that the small part obtained by Bulan is $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$, and the whole portion that belonged to Bulan is $\frac{1}{2} + \frac{1}{10} = \frac{5}{10} + \frac{1}{10} = \frac{6}{10}$. After that, the students made the cross sign, indicating the students stated that Rudi's answer is wrong.

Method 2: The students redrew the picture in the student worksheet, and wrote down the amount of bread in each piece. Each of the big piece is $\frac{1}{2}$, and each of the small pieces is $\frac{1}{10}$. Students wrote the method to obtain a small piece is $\frac{1}{2}$: 5. Then the students concluded that the whole portion of bread obtained by Bulan is $\frac{1}{2} + \frac{1}{10} = \frac{1 \times 5}{2 \times 5} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$.

Susi's answer:

Method 1: The students redrew the picture in the student worksheet, and shaded the parts acquired by Bulan, i. e. the top piece of first bread, and the leftmost

attended the second meeting.

- An understanding of the students about the meaning of multiplication of two fractions and how to find the result of multiplication of two fractions had been used by the students to solve this problem. Moreover, for the second student, students' understanding of division between fractions and integers was also used in determining the portion of a small piece of cake for Rudi's answer.

- The way of resolve the problem for two students is different.

	bottom of the third piece of bread. Students wrote that the small part obtained Bulan is $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$, and the whole portion of bread obtained by Bulan is $\frac{1}{2} + \frac{1}{10} = \frac{5}{10} + \frac{1}{10} = \frac{6}{10}$. **Method 2:** The students redrew the picture in the student worksheet, and wrote down the amount of bread in each piece. Each of the big piece is $\frac{1}{2}$, while each of the small pieces is $\frac{1}{10}$. Students wrote the method to obtain a small piece is $\frac{1}{5}$ of $\frac{1}{2}$. Then the students concluded that the whole portion obtained by Bulan is $\frac{1}{2} + \frac{1}{10} = \frac{6}{10} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$. **Andi's answer:** **Method 1:** The students redrew the picture in the student worksheet, and put each of the numbers 1, 2, 3, 4, 5 three times consecutively, and shaded the parts acquired by Bulan,i. e. the top three pieces of the first bread. Then students wrote that the whole portion obtained by Bulan is $\frac{3}{5}$. **Method 2:** The students redrew the picture in the student worksheet, and put each of the numbers 1, 2, 3, 4, 5 three times consecutively, and shaded parts acquired by Bulan, i. e. the bottom three pieces of the third bread Then the students wrote that the whole portion obtained by Bulan is $\frac{3}{5}$.	
4	**First question:** **Method 1:** 	

Method 2:

Second question:

Method 1:

Method 2:

Tiap siswa mendapat $\frac{1}{8}+\frac{1}{8}+$

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{7}{8}$

kelompok ketiga

Tiap sisma mendapat: $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{3}{5}$

kelompok keempat

Kelompok 1 dan kelompok 3 mendapat: $\frac{3}{4}+\frac{7}{8}=\frac{10}{8}=\frac{5}{4}$

Kelompok 2 dan kelompok 4 mendapat: $\frac{4}{5}+\frac{3}{5}=\frac{7}{5}$
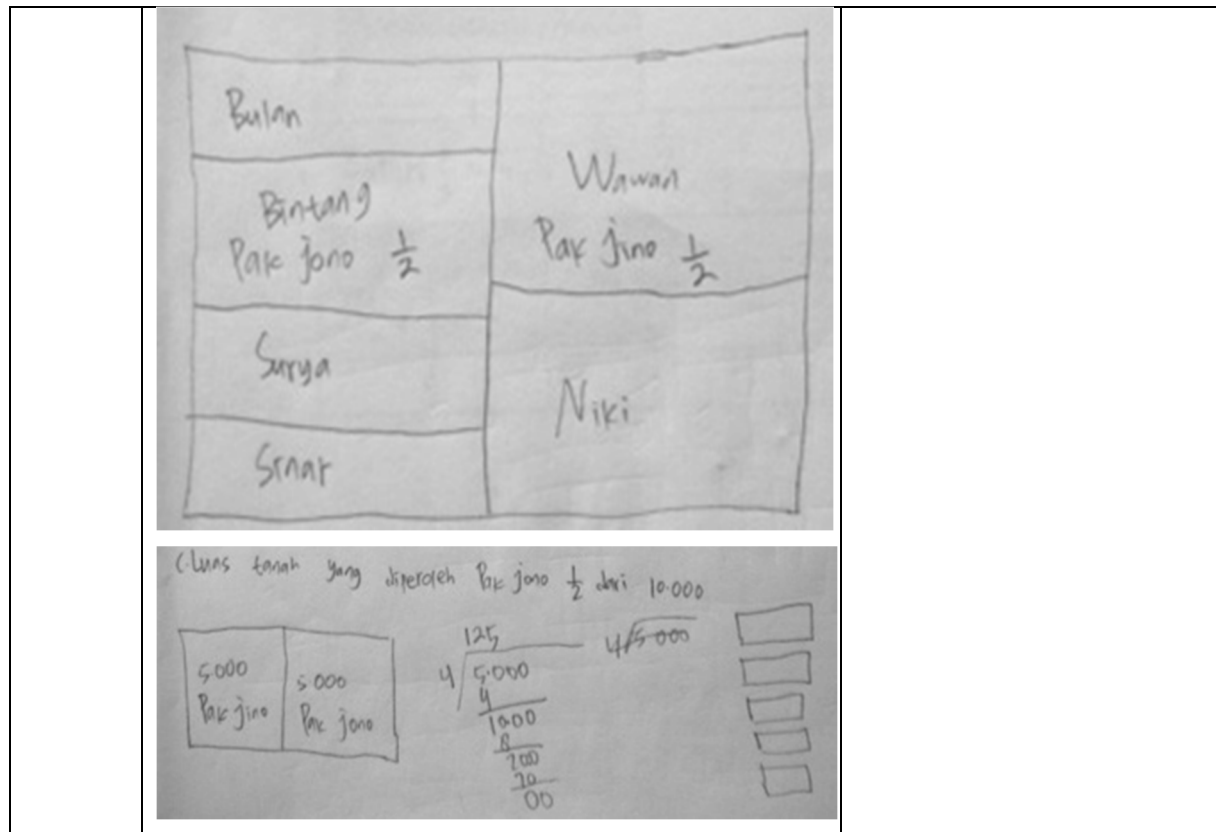
Pembagian roti ini tidak adil

Third question:

Method 1:

Pertanyaan $a=\frac{1}{8}$

$b=\frac{1}{4}$

c: Berbeda yaitu bulan $\frac{1}{8}$, Wawan $\frac{1}{4}$.

$d=5000\ m^2$

$e=5000\ m^2$

$f=5000:4=1250$

$g=5000:2=2500$

Method 2:

3. A Bagian tanah yang diperoleh Bulan, Bintang, Sinabdan Surya:
$=\frac{1}{4}$ dari $\frac{1}{2}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$

B Bagian tanah yang diperoleh Wawan dan Nei: $\frac{1}{4}\frac{1}{2}$ bagian dari Pak Jino=$\frac{1}{2}$ bagian dari $\frac{1}{2}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

CONCLUSIONS AND RECOMMENDATIONS

There are four things that can be inferred from the results of the exploration and responses of the students to the problems and evaluation given by the researchers:

1. From the first problem in the first meeting, the students built a model for a half to solve the problem. This model was also used by the students to solve the second and third problems. Due to the use of the model of a half in solving the second problem, the students could build an understanding of the meaning of $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$.

2. From the students' understanding of the meaning of $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$, the students could develop an understanding about $\frac{1}{2} : 2$ or $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ for the second problem in the second meeting, $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$ for the third problem in the second meeting and the first problem in the third meeting, and $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ and $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ for the third question in the evaluation.

3. The students were able to build up an understanding about the multiplication of two fractions and how to find the result of multiplying two fractions.

4. To further strengthen the establishment of formal knowledge of how to multiply two fractions, students need more experiences through exploration activities and solve other problems in addition to the ones that are already given.

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