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# Supporting Fifth Graders in Learning Multiplication of Fraction with Whole Number

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***Abstract***

The meaning of fractions with integer multiplication is something that is difficult to understand by students. They tend to think that the product it produces a larger number, while the multiplication of fractions with integers, the result can be any number larger or smaller. This study is a research design that aims to develop a local instructional theory to support the students expand their understanding of the meaning of multiplication of fractions with integers. By applying the characteristics of realistic mathematics education (Realistic Mathematics Education), the researchers designed a series of instructional activities related to daily life, such as Indonesia prepares dishes and equitable distribution. Participants of this study were Grade 5 students from an elementary school in Surabaya, along with a mathematics teacher of that class. Some students of the class participated in the first cycle, in order to see how the design of the hypothetical learning trajectory (Hypothetical Learning Trajectory) is running. After going through several revisions, HLT is then implemented in all the other students in grade 5. The results showed that students' prior knowledge affect their learning process. The fractions solve multiplication problems with whole numbers, some students convert the integers to fractions and then use a fraction by a fraction multiplication procedure. The learning process begins with students exploring the contextual situation of fair division, where students extend their understanding that the fraction associated with the division and multiplication. One indicator that the student has broadened his understanding is the more varied representation of the given problem.

**Keywords**: multiplication of fraction with whole number, RME, daily life situations, extend the understanding, initial knowledge, design research

**Abstrak**

Makna perkalian pecahan dengan bilangan bulat adalah sesuatu yang sulit dimengerti oleh siswa. Mereka cenderung untuk berpikir bahwa perkalian itu menghasilkan bilangan yang lebih besar, sedangkan dalam perkalian pecahan dengan bilangan bulat, hasilnya dapat berupa bilangan yang lebih besar atau lebih kecil. Penelitian ini adalah suatu design research yang bertujuan untuk mengembangkan suatu local instructional theory untuk mendukung siswa memperluas pemahaman mereka tentang makna perkalian pecahan dengan bilangan bulat. Dengan mengaplikasikan karakteristik dari pendidikan matematika realistik (Realistic Mathematics Education), peneliti merancang serangkaian aktifitas instruksional yang berhubungan dengan

kehidupan sehari-hari, seperti mempersiapkan menu masakan Indonesia dan pembagian adil. Peserta dari penelitian ini adalah siswa kelas 5 dari suatu Sekolah Dasar di Surabaya, beserta seorang guru matematika dari kelas tersebut. Beberapa orang siswa dari suatu kelas ikut serta dalam siklus pertama, dengan tujuan untuk melihat bagaimana rancangan hipotesis dari trayektori pembelajaran (Hypothetical Learning Trajectory) berjalan. Setelah melalui beberapa revisi, HLT tersebut kemudian diimplementasikan pada semua siswa kelas 5 yang lain. Hasil penelitian menunjukkan bahwa pengetahuan awal siswa sangat mempengaruhi proses pembelajaran mereka. Dalam menyelesaikan permasalahan perkalian pecahan dengan bilangan bulat, beberapa siswa mengkonversi bilangan bulat ke bentuk pecahan dan kemudian menggunakan prosedur perkalian pecahan dengan pecahan. Proses pembelajaran siswa dimulai dengan mengekplorasi situasi kontekstual tentang pembagian adil, dimana siswa memperluas pemahaman mereka bahwa pecahan berkaitan dengan pembagian dan perkalian. Salah satu indikator bahwa siswa telah memperluas pemahamannya adalah dengan semakin bervariasinya representasi dari permasalahan yang diberikan.

**Kata Kunci**: perkalian pecahan dengan bilangan bulat, pendidikan matematika realistik, situasi dalam kehidupan sehari-hari, memperluas pemahaman, pengetahuan awal, design research

The algorithm for multiplication of two fractions seems easy to be taught and to be learned, since we only have to multiply numerator with numerator to get the numerator of the product, and multiply denominator with denominator to get the denominator of the product (Reys et al, 2007). Multiplication with fraction itself is a difficult idea for students as they tend to associate multiplication with making something bigger (TAL Team, 2008). Meanwhile, in multiplication involving fraction, the result can be smaller. For instance, when we multiply $\frac{1}{2}$ by 3, the result is $\frac{3}{2}$, which is, smaller than 3. In addition, we tend to differentiate the word of multiplication symbol "×" (Streefland, 1991), we use word *"kali"* (times) for the amount greater than one, and for the amount less than one we tend to use the word *"dari"* (of).

According to Armanto (2002), mathematics in Indonesia is taught in a very formal way and teachers merely transfer their knowledge to students in the learning process, they teach with practising mathematical symbols and emphasizing on giving information and application of mathematical algorithm. Students are taught how to use algorithms to multiply fraction with whole number without emphasizing on the meaning behind it.

Meanwhile, if students learn to perform these operations using only rules, they probably will understand very little about the meaning behind them. Students may know how to multiply fraction with whole number as $3 \times \frac{1}{2}$ or $\frac{1}{2} \times 3$ if they have studied the rules, but still not be able to interpret the idea in the real world as basis for solving problems (Copeland, 1976). However, once they forget the rules, students cannot solve problems about multiplication of fraction with whole number (Kennedy, 1980). Further, according to an informal interview before this research conducted, the teacher said that even though the students have already studied about multiplication of a fraction by a fraction, it still uneasy for them to understand the topic.

The need for understanding in learning, teaching and assessing mathematics is very important (NCTM, 1991&1995). Learning with understanding is crucial because something learned by understanding can be used flexibly, be adapted to new situations, and be used to learn new things (Hiebert et.al, 1997). Students need flexible approaches that can be adapted to new situations, and they need to know how to develop new methods for new kind of problems. According to Hiebert and Carpenter (1992), we can understand something if we can relate or connect it to other things that we know. For example, students can understand the multiplication of $6$ by $\frac{1}{4}$ if they can relate it to other things they know about multiplication and the meaning of the fraction $\frac{1}{4}$.

Considering the issues mentioned before, the researcher proposed that it would be better if students learn by understanding about the meaning of multiplication of fraction with whole number, rather than only know how to use the algorithms for it. Consequently, the researcher would like to support students to extend their understanding of the subject. Extend the understanding means broaden the connection between ideas, facts, or procedures to the topic that was not learned yet. Since the students participated in this research already studied about multiplication of a fraction by a fraction, then the students should broaden their understanding to the multiplication of fraction with whole number. One of the indicators that show students' understanding can be seen from the way they explore variety types of computations, such as computing a fraction of some distance. However, it is more important that students can relate it to new situations or problems. Another indicator

is that when students can make representation and give reason about strategies they used to solve problem.

Realistic Mathematics Education (RME) is a theory of mathematics education emerged in the Netherlands in the 1970s that focuses on the importance of students' understanding. Inspired by the philosophy of RME, one group, called *Pendidikan Matematika Realistik Indonesia (PMRI)* Team, developed an approach to improve mathematics learning in Indonesian schools to achieve a better understanding (Sembiring et.al, 2008). One of the principles of RME is the use of contextual situations. According to Kennedy (1980) and TAL Team (2008), many contexts can be used to develop the meaning of fractions multiplication with whole number, for instance recipes with fractions and fair sharing. However, the contextual problems that would be used had to be adjusted to school context and the initial knowledge of students.

## *Research Method*

In the following, some elements regarding to the research methods used in this research will be discussed.

### Design Research Methodology

In line with the aim of this research, the method used in this research was *design research*. According to Gravemeijer and Cobb (2006), design research is defined into three phases, namely preparation for experiment, teaching experiment, and retrospective analysis.

In preparation phase, the researcher studied some literature related to multiplication of fraction with whole number. Then, Hypothetical Learning Trajectory (HLT) consisting of learning goals for students, mathematical tasks to promote students' learning, and hypotheses about the process of students' learning was designed. The sequence of instructional activities designed in the preparation phase was implemented in the teaching experiment phase. In this phase, the HLT was used as a guideline for conducting teaching practices. The teaching experiment itself was based on a cyclic process or (re)designing and testing the instructional activities and the other aspects conducted in the preparation phase.

Retrospective analysis then was conducted based on the entire data collected during the teaching experiment phase, by using HLT as guideline.

**Research Subjects**

The participants of this research were fifth graders (10-11 years old) and a mathematics teacher from one elementary school in Surabaya, which have been involved in *Pendidikan Matematika Realistik Indonesia (PMRI)* project under the supervision of Surabaya State University.

The teaching experiment was conducted in two cycles. For the first cycle, six fifth graders were involved as subjects and the researcher acted as teacher. Meanwhile, in the second cycle of teaching experiment, all fifth graders from another class that parallel with the first class and the real mathematics teacher were involved as participants.

**Data Collection**

The data collection was collected through interviewing teacher and students, observing activities in the classroom, and collecting students' works.

**Data Analysis**

The progress of students' understanding about multiplication of fraction with whole number was analysed from the data collected during the teaching experiment.

***Reliability and Validity***

In order to gain the reliability of this research, the researcher used data triangulation and inter-subjectivity. The data triangulation in this research involved videotape, students' works, and field notes. This research was conducted in two parallel classes, and the researcher reduced the subjectivity by involving the researcher's colleagues to interpret the data collection.

In order to keep the methodology of this research as valid as possible and to answer the research questions in the right direction, the researcher used the following methods of validity (Wijaya, 2008).

(1) The HLT was used in the retrospective analysis as a guideline and a point of reference in answering research questions, and
(2) The triangulation in collecting data as described before gave sufficient information for the researcher's reasoning in describing the situations and the findings of this research.

***Result of The Research***

Before conducting teaching experiment, the researcher made some refinements to the initial HLT designed in the preparation phase, by changing and making improvement

to the activities and problems. The initial HLT that have been revised then called HLT 1.

The teaching experiment conducted in two cycles. In the first cycle of teaching experiment, the researcher implemented HLT 1 for six fifth graders. Further, based on the data got from the first cycle of teaching experiment, the researcher analysed and improved the HLT 1 to be HLT 2.

The second cycle of teaching experiment was conducted by implementing the HLT 2 for the thirty-one students of another parallel class. The teacher for this teaching experiment was the real mathematics teacher of the class, meanwhile the researcher and some colleagues acted as observer. The scheme of HLT refinement can be seen in Figure 1.

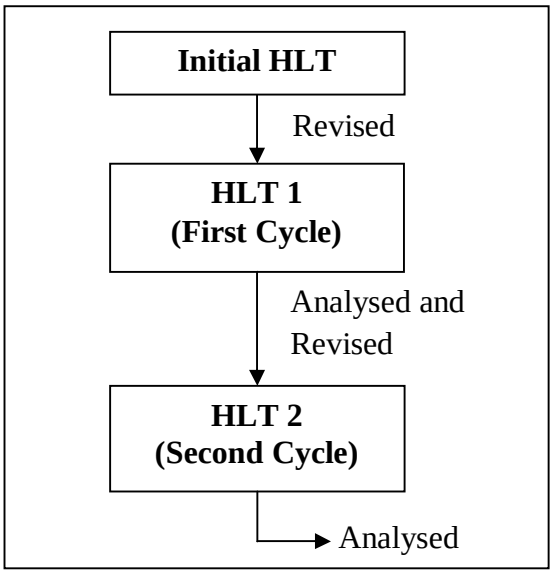



Figure 1. Scheme of HLT Changing

Since the second cycle was the revision of the first cycle, in the following the researcher will focus on the discussion of the result of the second cycle.

**Pre-Test**

Briefly, the pre-test was aimed to know students' knowledge and ability about multiplication of fractions, especially about multiplication of *a whole number by a fraction* and multiplication of *a fraction by a whole number*. In the following, the researcher will describe some remarks got from the pre-test.

According to the students' answers in the worksheets, the researcher observed that the students showed various strategies to multiply a fraction by a whole number. Some of them converted the whole number to be in the form of fraction. Other students directly

multiply the whole number by the numerator and then put the denominator as the denominator of the result. Another strategies appearing was by using repeated addition. However, when the students were asked to relate some situations to its algorithms, most of them relate it to one algorithm only, meanwhile the result can be more than one.

The interesting case happened when the students were asked to write suitable situations for fraction, addition of fraction, and multiplication of fraction with whole number. Out of 26 students who took the pre-test, fifteen students gave clear and proper situation to represent a proper fraction, for instance, they represented $\frac{2}{3}$ as *"two cakes were divided to three children"*. Other three students seemed to understand fraction in a formal level, since they gave formal algorithms as $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. Meanwhile, the rest students gave unclear or even did not write answer.

Related to addition of fractions, most of the students directly add the fractions by using algorithms, rather than giving situations. When some students used two same entities as *"Father bought $\frac{1}{4}$ gram of candies, then he bought another $\frac{1}{4}$ gram"* to represent $\frac{1}{4} + \frac{1}{4}$ , other two students used different objects, for instance *"$\frac{1}{4}$ wheat flour and $\frac{1}{4}$ tapioca flour"*. Perhaps, the two students did not fully understand about the meaning of addition itself, that we cannot add two different entities.

Regarding to multiplication of fraction with whole number, only eight students gave clear situations, for instance by writing *"There are three children. Each child gets $\frac{1}{2}$ part"* as situation for $3 \times \frac{1}{2}$. However, from the answers, it seems that the students considered a fraction times a whole number $\left(i.e\ 3 \times \frac{1}{2}\right)$ similar to a whole number times a fraction $\left(i.e\ \frac{1}{2} \times 3\right)$ since they gave the same idea for both multiplications.

**Teaching Experiment**

There were four main mathematical ideas that was expected to be achieved in this research, namely *fractions are related to multiplication and division*, *repeated addition as multiplication of whole number by fraction*, *inverse property of unit fractions*, and *commutative property of multiplication of fraction with whole number*. In the following, some remarks got from the observations during teaching experiment will be discussed.

***Fractions are Related to Multiplication and Division***

In order to achieve the learning goal that fractions are related to multiplication and division, the researcher offered a fair sharing activity. The students were asked to share some number of cake to some people fairly. As the result, when the number of cake is one, it seems that most of the students could relate fractions to division, they answered it as $\frac{1}{\text{the number of group members}}$.

Giving different strategies of sharing a cake to three people seemed more difficult to the students. Out of our conjectures, two groups used their knowledge of equivalent fractions to show various ways of dividing a cake to three people. They partitioned the cake as many as the multiplication of three and came with equivalent fractions (see Figure 2).

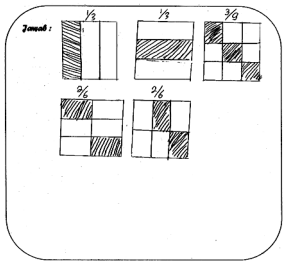
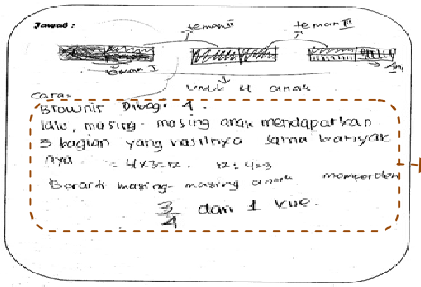
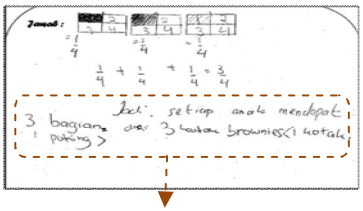


Figure 2. One of Student's Different Strategies to Share a Cake for Three People

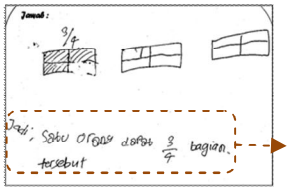
However, when the number of cake was increased, for instance sharing three cakes to four people, students gave different answer each other. They got struggle to notate the part each person gets, to determine what is the whole of the fraction, as can be seen in the Figure 3.



Brownies is divided by 4
Then, each person gets 3 parts with the same result.
$4 \times 3 = 12$ $12 : 4 = 3$
It means, each person get $\frac{3}{4}$ of 1



Therefore, each person gets 3 parts of 3 boxes brownies (1 box 1 piece)



Therefore, one person gets $\frac{3}{4}$ part

Figure 3. Different Wholes of Fractions

Furthermore, even though one group wrote $\frac{3}{4}$ of 1 cake (see the first answer in the Figure 3), which actually means $\frac{3}{4} \times 1$, but the students seems did not realize it. In addition, in the class discussion, the teacher only gave reinforcement that fractions are division by giving some questions such as "*Bu Mar has ten cakes. The number of students in this class is thirty-one. How much part got by each student?"* As the consequence, the students tended to relate it to division as $\frac{\text{the number of cakes}}{\text{the number of people}}$. The idea that fraction also related to multiplication have not appear yet.

In order to achieve the idea that fractions are also related to multiplication, thus the students were given problem to find fraction from a number of pieces of cake. One interesting case found in one group when they were solving the answer. The worksheet of the group showed that they gave two different approaches to solve two problems that have similar idea. The researcher then looked to videotapes and found that the groups used different strategies because they got some guidance from the researcher for the first question. Because they did not have enough time, then the answer of second question still the same. The answer of one group before and after the guidance can be seen in Figure 4 below.

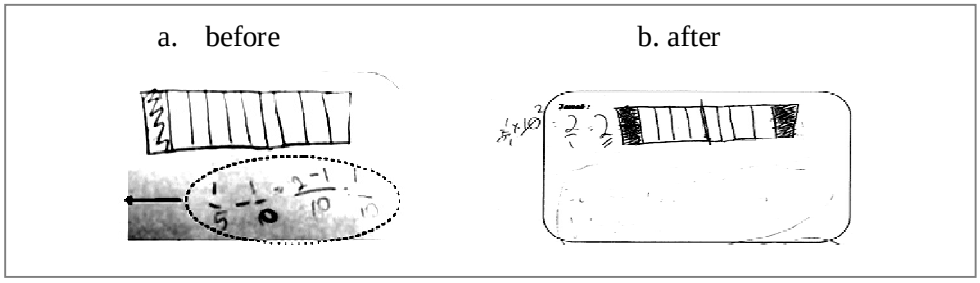



Figure 4. Students' Answer before and after some Guidance

When the researcher tried to ask clarification from the students in the group about their answer, the researcher interpreted that the students did not have a clue how to find one-fifth of ten pieces. They messed up some procedures to answer it. Their confusion might be happen because they did not familiar with this kind of problem, where they had to find fraction of a collection of something. After more than fifteen minutes, the group then came to the answer as in the right side. However, the answer appeared because one of the students remembered that the word 'of' means multiplication. Once they recognized the word 'of' means multiplication, they will use the algorithms to solve problems consisting word 'of' in it.

***Repeated Addition as Multiplication of Whole Number by Fraction***

The researcher offered the activity of preparing a number of menus to draw the mathematical idea that repeated addition of fractions as multiplication of whole number by fraction. Since the idea of multiplication as repeated addition already emerged when learning about multiplication of whole numbers, then the students did not need much support to relate it to multiplication of whole number by fraction. However, the use of rice to make *lontong* in the beginning of the meeting seems could draw students' enthusiasm to learn.

In solving the problems in the worksheets, all of the answers were in line with the conjectures made. Some students used repeated addition and the other students directly used the algorithm of multiplication of fraction with whole number. Figure 5 below shows strategies used by the students.

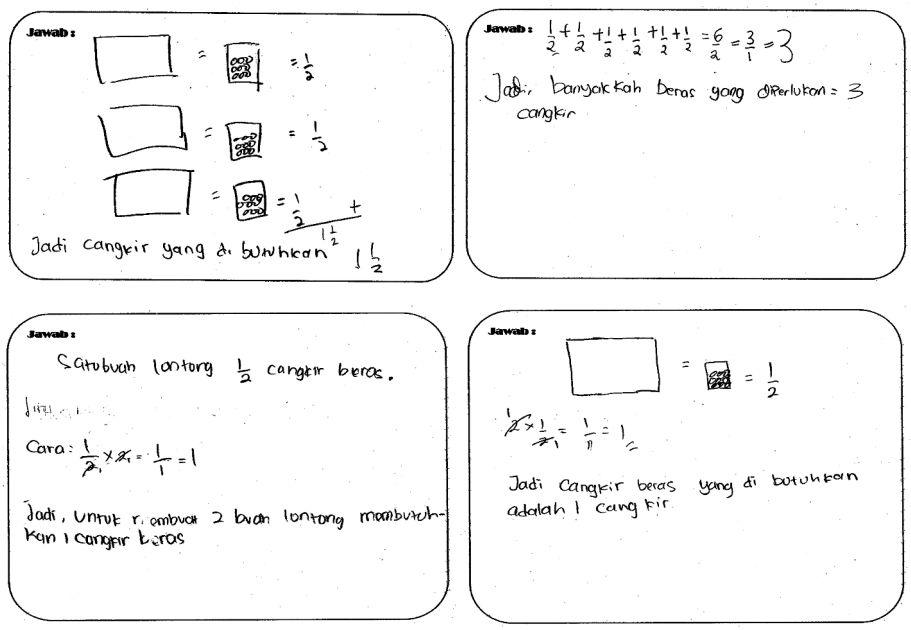



Figure 5. Students' Strategies in Solving Problems about Preparation of Number of Menus

***Finding the Inverse Property of Unit Fractions***

In order to gain the idea of inverse of unit fractions, the researcher offered a context about knitting yarn. The students had to determine how many colours of yarn needed to make 1 meter of colourful knitting. Since the idea of this activity more or less similar to preparing number of menus activity, then the students also did not need much support in solving the problems. One small support might be given for students

in this activity was by giving guidance to make mathematical conclusion, such as 'when a unit fraction multiplied by its denominator, then the result will be one'.

***Commutative Property of Multiplication of Fraction with Whole Number***

In this last activity, the students were expected to gain the idea of commutative property of fractions multiplication with whole number through comparing the length of ribbons activity. As the tool, the students were given *one blue ribbon that has 30 cm of length* and *three pink ribbons that has 10 cm of length.* Unfortunately, rather than came to the idea of commutative property, students in one group needed much more support. The researcher interpreted that the students even could not show a quarter of a ribbon.

**Post-Test**

Post-test was intended to assess to what extend the students' understanding through the learning process. The result of this post-test more or less can give description about students' progress from their initial knowledge. Comparison of the result of pre-test and post-test showed some improvement for some students, but some other also still have struggles. In the following, the researcher will describe some remarks got from the post-test.

In solving multiplication of a whole number by a fraction, the students showed various strategies. Some students used the procedures to multiply a whole number by a fraction; they multiplied the whole number by the numerator and then put the denominator as the denominator of the result.

Regarding to finding suitable situation for multiplication of fraction with whole number, some students still did not differentiate situation for multiplication of a while number by a fraction and a fraction by a whole number. They gave the similar type of situation for both multiplication problems. Different with the pre-test, in the post-test some students gave pictures (for instance picture of cakes) to represent the situations, not only gave situations. It might happen because the one of the contexts given was about sharing cakes.

When the students were asked whether *the total of 3 strands of ribbon with the length* $\frac{1}{4}m$ are same with *the length of* $\frac{1}{4}$ *of 3 meters ribbon*, all students said it was the same. Some of them used formal procedure to solve the problem; they multiplied the whole

number 3 by the fraction $\frac{1}{4}$ and then multiplied the fraction $\frac{1}{4}$ by the whole number 3 to prove that the answer is the same. Most of the students made drawing to answer it. However, since they did not use scale in drawing the ribbons, actually we cannot see that it has the same length (see Figure 6).

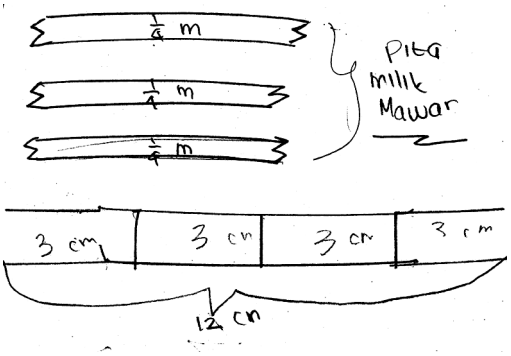



Figure 6. Students' Strategies to Show that Three times Quarter Meter is Similar to a Quarter of Three Meter

***Discussion***

**Contextual Situation as Starting Point**

Supporting students who were already familiar with formal algorithms needed some adjustment in designing activities and problems so that they can extend their knowledge. A rich and meaningful context as the first tenet of RME can be used as starting point in learning multiplication of fraction with whole number. However, working with students who accustomed to use formal procedure need some adjustment in choosing the situations to be used. The problems given should be organized such that the students did not realize it as multiplication of fraction with whole number. For example, since the students did not recognize that finding fraction of some object as multiplication of fraction by whole number, then the students did not use the formal procedure to solve it. Here, the role of teacher was very important. Another support that can be given to extend their understanding of multiplication of fraction with whole number is by asking them to make representation and to give reasons to prove their answer.

**Class Discussion: Teacher's Role and Students' Social Interaction**

Students' social interaction in the classroom was stressed in the fourth tenet of RME, *interactivity*. Working in small group enable students to discuss strategies used might scaffold students understanding. During the teaching experiment, the researcher observed that sometimes it was more effective when the students work in pair, so that the researcher can reduce the number of students that did not work.

Teacher plays an important role in orchestrating social interaction to reach the objectives both for individual and social learning Cooke & Bochholz and Doorman & Gravemeijer (in Wijaya, 2008). Further, the roles of teacher in supporting social interaction found in this research will be described as follows.

In order to stimulate social interaction among the group member, one of the teacher's roles could be by dividing task for each member, when in the group only some students that worked. Further, the teacher could provoke students to discuss their idea so that all member of the group understand it.

Further, the learning process of students can be shortened by, for example, giving different question for each pair and then sharing the strategies in classroom discussion, where the range of social interaction can be broader. The roles of teacher in class discussion of this research could be by offering a chance for students to present their strategies to their friends. In order to stimulating social interaction among all component of classroom, the teacher could pose questions as *'Is it the same with Ale's answer?'* or *'Anyone can help them?'* or *'Before Dinda has said it. What is it?'* etcetera.

The most important goal of a class discussion is transforming students' concrete experiences into mathematical concept as mentioned by Cooke & Buchhlolz in Wijaya (2008). Further, the teacher should ask for students' clarification in order to investigate students' reasoning about their idea or strategies that could reveal both students' difficulty and achievement in their learning process. For example by asking *"Is it just because the question about how meter the journey? Please give clear answer. Why it was multiplied?"*

***Conclusion and Recommendations***

The initial knowledge of students more or less affected their learning process in the teaching experiment. Students who were familiar with formal algorithms seemed to

have a tendency to use the algorithms in solving problems. Once they know that the question is about multiplication of fraction, they will use the algorithms of multiplication of fraction to solve it.

**Recommendations**

Based on the whole process of teaching multiplication of fraction with whole number, the researcher have some considerations to be recommended for further research in this topic. One of the recommendations is about discussions in the learning process. The discussion itself can be separated into two, namely group discussion and class discussion. The number of students in one group should be considered carefully. The finding of this research, when the numbers of group member is quite big, then only few students were active in the group discussion. Therefore, one possible solution to this problem could be by making small group, for example two students in one group.

The teacher who was involved in this research is an experienced teacher who has been involved in *Pendidikan Matematika Realistik Indonesia* for long time. Therefore, she was good in conducting class discussion. One of the strategies she used was by asking students with different strategies to present their work in front of class. Since she did not blame students, who gave incorrect approach, then the students feels free to share their ideas. Therefore, the researcher could adjust the learning process based on the students' understanding.

The last tenet of RME is about intertwinement. It will be better if the learning process of multiplication of fraction with whole number is intertwined with other topic, for example with the learning of percentages. Therefore, the time allocation could be more efficient and effective.

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