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# Students' Modelling in Learning The Concept of Speed

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***Abstract***

Previous researchs shows that speed is one of the most difficult in the upper grades of primary school. It is because students must take into consideration of two variables; distance and time. Nevertheless, Indonesian students usually learn this concept as a transmission subject and teacher more emphasizes on formal mathematics in which the concept of speed given as "distance by time"rigorously. A sequence of learning activities with toy cars context was designed based on students' development and Realistic Mathematics Education (RME) principles which are guided reinvention, didactical phenomenology and emergent modelling. Using their own models, students are able to explain a proportion among distance and time in speed as well the relationship of it.

**Key words**: The concept of speed, design research, Toy cars, context, 'ratio table' model

**Abstrak**

Penelitian-penelitian terdahulu menunjukkan bahwa kecepatan merupakan salah satu konsep yang sangat sulit untuk dipelajari oleh siswa di tingkat akhir sekolah dasar karena siswa harus memperhatikan dua besaran yang berbeda dalam waktu yang bersamaan yaitu jarak dan waktu. Akan tetapi, siswa Sekolah Dasar di Indonesia biasanya mempelajari konsep kecepatan ini sebagai subjek yang diberikan oleh guru dan guru lebih menekankan pada matematika formal dengan memperkenalkan rumus kecepatan ; kecepatan diperoleh dengan membagi jarak dengan waktu. Serangkaian aktifitas pembelajaran dengan konteks mobil mainan didesain berdasarkan perkembangan siswa dan beberapa prinsip Pendidikan Matematika Realistik yaitu guided reinvention' (penemuan terbimbing), didactical phenomenology (fenomena yang mengandung muatan didaktik), serta pemunculan/penggunaan model. Dengan model yang dikembangkan sendiri, siswa mampu menjelaskan perbandingan antara jarak dan waktu sekaligus menyelesaikan masalah yang berkaitan dengan hubungan antara jarak, kecepatan dan waktu.

**Kata Kunci**: konsep kecepatan, design research, cerita fabel Indonesia, mobil mainan, konteks, model 'ratio table'

The concept of speed is introduced to the students since in elementary school. It may with the consideration that speed is a fundamental concept for learning kinematics in junior high school as well calculus in the higher education. However, according to Gravemeijer, et al (2007) the concept of speed is one of the most difficult in the upper grades of primary school. Children meet difficulties in understanding the concept of speed because they must take into consideration of two variables; distance and time (Piaget, 1971., Gravemeijer, et al., 2007). In fact, children have the tendency to look primarily at the distance travelled, and lose sight of the time that is required to travel this distance (Thompson&Thompson, 1992). Moreover, Piaget (1971) explains that children might compare properly the objects but still may not recognize the correct relationship between distance and time. From his four teaching experiments, Thompson&Thompson (1992) see a consistent pattern in children's development of the concept of speed in which they differentiate into four levels. However, only a few studies on the teaching and learning process for the concept of speed in elementary schools. Therefore, more study still needed to observe how students think and reason about the concept of speed.

According to Ulfah (2005), most of the teachers in Indonesia teach mathematics in traditional way in which students are not involved actively in the learning process. Meanwhile, from Pujiati (2004) we can see how the concept of speed is given as a transmission subject and more emphasizes on formal mathematics in which the concept of speed given as "distance by time" rigorously. Using the formula, speed is distance divided by time then we can get the distance by multiplying the speed with the time and get the time by divided the distance by speed. Students in upper grade of elementary schools in Indonesia mostly know this rule but it is difficult for them to give reasons behind. Thus, in the researcher point of view it is important to give students the opportunity of constructing their knowledge regard to the concept of speed. One of the supports that the researcher proposed is using a sequence instructional activities based on Realistic Mathematics Education (RME) principles. The researcher uses a design with Indonesian contexts, which is expected facilitate students in developing their own models as one of their reasoning ways to explain the concept of speed and interrelation among speed, distance and time.

RME is the transmission of Freudenthal's idea that views mathematics as a human activity instead of seeing mathematics as a subject to be transmitted (Freudenthal, 1991). It was origin in the 1970s and in Indonesia it is adapted as Pendidikan Matematika Realistik Indonesia (PMRI). The instructional activities that we proposed here more emphasize on 'guided reinvention', didactical phenomenology, emergent modelling.

The term emergent modelling has its roots in the description of an instructional design heuristic within the domain-specific instruction theory for RME. Gravemeijer (1994) describes that emergent modelling is an activity of students that describes how *model-of* a certain situation can become *model-for* more formal reasoning. The levels of emergent modelling are illustrated in the following figure:

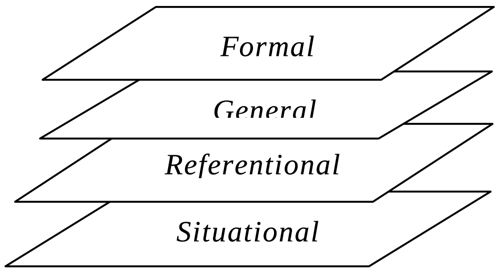


**Figure 1.** Levels of emergent modelling

Situational level is the first level of emergent modelling. In this level, interpretations and solutions depend on understanding of how to act in the task setting. In referential level, models and strategies refer to the situation which is sketched in the problem. In this level, model-of situation is brought forward and used to organize and solve a problem situated in daily life. In general level, a mathematical focus on strategies dominates the reference to the context. In this level, model-for reasoning emerge by changes from referential to general level In general level, model-for make possible a focus on interpretations and solutions independently of situation-specific imagery (Gravemeijer, 1994).In the top level, students do all kinds of operations within the mathematical system itself are no longer dependent on the support of the model. In this research, students are in the formal level when they do procedural computation.

The concept of speed is a type of imaginary situation. Here students need materials, visual sketches, schemes, and so on to organize the situation. In present research, the researcher proposed two models, namely, paper tapes (bar model) and ratio table model. Paper tapes model is a model which is appeared as the improvement of the instructional activities. Meanwhile, ratio table model is a mental model which commonly use to explain proportional reasoning. According to Gravemeijer (2007), ratio table plays an important role in the curriculum on proportion. Ratio table is an ideal aid for making handy calculations and gaining insight because the table invites students to write down intermediate steps. Moreover, they explain that the strength of the ratio table is that students can reason with number relationship that they already know.

### *Research Method*

This article is a part of a thesis with the title “Supporting Students in Learning The Concept of Speed”. Design research is chosen as the research method when conducting the research. A design research which is used consists of cycles of three phases ; preparation and design phase, teaching experiment, retrospective analysis. There are two cyclical process in doing this research. These cycles involved two different group of teachers and students.

The first is a tryout of the Hypothetical Learning Trajectory which is involved a teacher and seven fourth graders in SD Muhammadiyah GKB – Gresik who join the ‘mathematics club’, an extra lesson for students who are interested in mathematics. Hence, it runs only once a week. They were chosen according to their achievement in the classroom. Two students represent high achievers, three students are average students, and the other two are low achievers. After conducting the first cycle, the researcher did retrospective analysis and revised the instructional sequence to be implemented in the second cycle. In the second cycle ; the teaching experiment was applied in SD Al- Fatah Surabaya which is involved a teacher and twenty five students in SD Al – Fatah Surabaya.

The data of this research was collected by videotaping of the learning processes, interviews with the teacher and students. There were also collection of written data from students’ posters, students’ work in solving the problems during the class activities, and some notes gathered during the research.

### *The Results of The Study*

During the teaching experiment phase, we implemented the sequence of  instructional activities based on RME principles as the researcher explained before. It consists of three different contexts with several activities and tools. It was implemented for eight meetings including the tests before and after the lessons. In this article, the researcher will focused on the use of models instead of explain the whole students’ learning process.

In the lesson series, paper tapes at the first time were used at the second activity when the students are asked to compare the speed of two toy cars. At this activity, students got to work on the task easily. Naturally, students use this paper tapes as the track for the toy cars. Students run the car on it and by measuring the time (students use the same distance in their group) of each car so that they can determine which car is faster.

On the next activity, students are given different toy cars with different speed and different length of the track. They are asked to investigate the speed of toy cars in their group. Students did the investigation outside the class as shown in the following figure

**Figure 2.** Students measured the time and the distance of the toy cars

None of the group of students got difficulties in measuring the distance and the time. Below is the result of their investigation:

**Table 1.** Students' Investigation results

| NO | GROUP | DISTANCE | TIME |
|---|---|---|---|
| 1 | Group 1 | 150 cm | 3 seconds |
| 2 | Group 2 | 250 cm | 5 seconds |
| 3 | Group 3 | 350 cm | 6 seconds |
| 4 | Group 4 | 450 cm | 6 seconds |
| 5 | Group 5 | 500 cm | 9 seconds |
| 6 | Group 6 | 600 cm | 13 seconds |

After that, teacher asked the students to find the way to compare the speed of all of the toy cars so that they can determine the fastest car among them. For around 8 minutes, no group gave the answer. It seems very difficult for students to compare the speed of the whole cars although the teacher come to the some groups and try to help them by relating with their previous activity in comparing two-toy cars use either the same distance or the same time. Realizing students' difficulties, teacher proposed his way. He explained that he would use the same time for all and the time that he chose is one second. Hence, he asked the students to predict the distance of each toy car in one second.

All of the students divided the distance by the time in every group. When they finished the calculation, teacher then asked them to put their car in the track as if the cars travel for one second. First, students measured the distance that they want using a measuring tape. After that, they put the car on it. Without request from the teacher Riska's group made a sign in the paper tape for the distance travelled by the car in one second.

**Figure 3.** Ifah and friends made signs for the position of the car in every second

Although the paper tapes appear only as the track for the car at the first time, at this activity the length of the paper tapes indicate also the total distance travelled by the car.

Observation during the next activity shows that students are more advance in using the paper tapes than of those in previous. While being asked to determine the distance travelled by the car for every second, students made segments separated by lines on the paper tapes as many as the number of seconds which is needed by the car. If the car needs two seconds to travel the total distance then the number of segments are two. If the car needs five seconds to travel the total distance then the number of the segments are five and so on. As an example, we can see one of the group of students (Riska's group) measured that the total distance is 250 cm and the total time is 5 seconds. Hence, they made five segments on the paper tapes. They measured 50 cm from the initial side then made the line on it. In making the second line, they start to measure from the first line instead of the initial side. For the students, every segment that they made became a way to show the distance travelled for every second besides the calculation that they made before. Hence, the paper tapes are used not only as the total distances but also the distance travelled by the car for every second.

We may say that at this stage this model starts to meet the bridging function between the informal and the formal level of thinking. The bar model here constituted a context-specific model of the situation; 'model of'. Later it could be generalized over the situations and becomes then a model that can be used to organize related and new problem situations and to reason mathematically, 'model for'(Van den Heuvel-Panhuizen, 2003).

The following step was shown in the class discussion when teacher displayed two paper tapes of the students which visualize constant speed on the whiteboard. Having experience with the previous activities, students explained that one segment in the bar shows the speed if it is measured for every second. Since, all segments in the first bar are of the same length, students concludes that the first bar shows constant speed meanwhile the second bar shows variable speed. Moreover when the teacher gave an example ; if the length of the first bar is 280 meters and the toy cars travelled along it within is 7 seconds, students can calculate that the distance travelled for every second is 40 meter so that 80 meters in 2 seconds, 120 meters in 3 seconds and so on; thus students did not only show the distance based on the consecutive seconds. We cannot say exactly whether this was a moment of a shift from a 'model of' to a 'model for'. However, it seems that students start to use the model for their reasoning because based on van den Heuvel-Panhuizen (2003) the real shift of course is made in students' thinking.

The ratio table models at the first time emerge in the class during 'converting the unit of speed activity'. Teacher made a ratio table in horizontal form as a structured way to write the proportion between the distance and the time. After that, teacher asked the students to make their own table based on their investigation of their paper tapes in the previous meeting. Using the table, students had to show the speed in other units without difficulties, for instance in m/sec, km/minutes, and km/h.

Although teacher proposed the ratio table models, most of the students participate actively in the process of model building. Both bar model and ratio table model here show the proportion between distance and time in speed. However, ratio table forces the students to develop the higher order thinking because it shows the growth or the proportion of the distance and the time in a more abstract way. Using the bar model, the times are present in a consecutive number and the growth of the distance as well; from one second into two seconds and so forth. Meanwhile, in ratio table students can show the distance in one second then jump to the distance in one hour for instance. Below are students' work in converting the unit of speed.

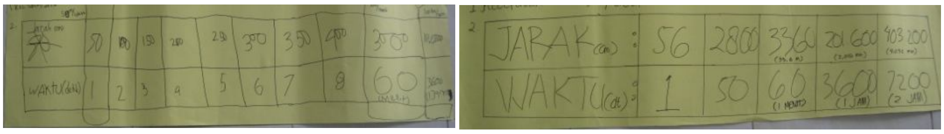


**Figure 4.** Students' work in converting the unit of speed

Through the ratio table that students made, we can see how the table as a tool for thinking and reasoning for them. Students' poster above show what the students think and what the intermediate steps that the students used to convert the unit of speed. Candra and Leo at the second column wrote the distance is 50 while the time is 1 second. After that, they continue the table by showing the distance in consecutive times until 8 seconds then jump to the 60 seconds or one minute, In other words, the distance is 50 cm for 1 second, 100 cm for 2 seconds, 150 cm for 3 seconds, and so on. It seems that Chandra and Leo are adding the distance by 50 every time they move to the next column except in the last two columns. Different from Candra and Leo, Nadya and Ifah jump immediately from one second to the 50 seconds, then 60 seconds (1 minute), 3600 seconds (1 hour), 7200 seconds (2 hours). Nadya and Ifa probably use multiplication instead of addition.

In the last activity, students are given problems dealing with the relationship among speed, distance and time. One of the problems as following:

> *A rabbit runs 12 km in 15 minutes. Determine the speed of the rabbit in km/h and explain your answer!*

Although in the pre-test one student wrote the formula of speed, none of the students who solve the problems use the formula as many Indonesian students did while they are facing those kinds of problems. Many approaches appear in students' work. One of them is a use of a ratio table either in an initial or in a sophisticated way. An initial way here is a ratio table as the teacher shows in the figures above. Meanwhile, a sophisticated way here is ratio table which was developed by the students from the initial form. In the following figure we can see two different forms of ratio tables. Ifah made the first ratio table in the same form made by the teacher. Ifah made the table in horizontal way, the distance in the first row and the time in the second row. Meanwhile the ratio table made by Naufal and Dani is in vertical form. They did not write the distance and the time anymore. Only the number showed the distance and the time.

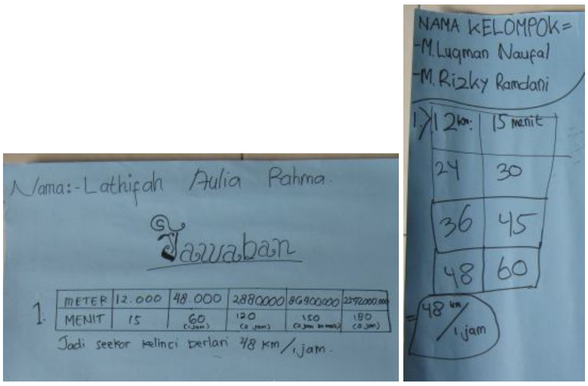


**Figure 5.** Ratio table made by the students

The figure also shows that there is no fixed strategy to solve the problems; the ratio table allows the flexible approach. From the data given in the problem in which the rabbit runs 12 km in 15 minutes, Ifah changes the distance into 12.000 m. On the next step, she tries to determine the distance during 60 minutes (1 hour) by multiplying 12.000 by 60 so that she got 48.000 m of distance and so forth. Different from Ifah, Naufal and Dani did not change neither the distance nor the time from the initial form. It seems that they multiply both the distance and the time consecutively by 2, 3 and 4.

The learning process shows clearly that the models develop more and more throughout the trajectory, not only in the form of the models but also its function for the students. From the bar model which is close to the situation of the speed to the ratio table which shows number relationship.

### *Discussion*

Although most of the students can solve various problems that teacher gives, we cannot guarantee that all of the students understand speed as the proportion of the distance and time since it was a kind of thinking experiment. However, it seems that paper tapes quite effective for students to visualize the distance and the time growth during the car travelled. By visualizing it, students might have enough base to go to the concept deeply.

Giving the students so many mathematical ideals dealing with the concept of speed in six meeting only seems too much for the students. There are aspects of speed which have hardly to touch for the students. What students learn in the concept of speed above is ‘average speed’ which is very difficult to meet in real life situation. Most of the time what they meet is variable speed as shown in speedometer. The big challenge is about making a link between the real situation with the mathematical concept which we want to develop by the students. *Momentaneous* speed is one of the aspects also. *Momentaneous* speed is speed in certain moment in which students see at the speedometer. We might only help the students to read the speedometer but not go deeply to the *momentaneous* speed itself.

## *Conclusion*

The result of this study has shown that the concept of speed could be given not merely as the transmission subject to the students as long they get enough support and opportunity. In learning the concept of speed which means quantity of motion as a proportion between the distance and time, students need support from others, especially from the teacher. We can give the supports by providing contextual situations with the problems that can be organised by the students. By giving such problems, students are given more space to develop their understanding toward the concept based on previous knowledge that they have. We can see how the students solved problems related to the conversion among unit of speed and explained relationship among speed, distance and time not by using the formula but rather by the models which is reinvented by themselves. We can also say that for the students, it is not so easy to develop the models until it works as the tools for their thinking and reasoning. It might needs more time, more guidance from the teacher but sometimes not all of the students come to achieve the learning goals. However, how it would be nice if students get an insight about something that they learned.

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