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Model-Eliciting Activities (MEAs) as a Bridge Between Engineering Education Research and Mathematics Education Research

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ABSTRACT

This article introduces Model-Eliciting Activities (MEAs) as a form of case study team problem-solving. MEA design focuses on eliciting from students conceptual models that they iteratively revise in problem-solving. Though developed by mathematics education researchers to study the evolution of mathematical problem-solving expertise in middle school students, MEAs are increasingly used in undergraduate engineering at the introductory course level, and are the subject of several NSF grants to expand their implementation. A primary implementation challenge involves finding appropriate blends of MEAs with other pedagogies. Current research and development efforts include five areas of expanding the theoretical and empirical scope of the MEA construct. These include development and use of *Reflection Tools*, a device to nurture problem-solving personalities; implementation of current and futuristic learning technologies; elicitation and repair of misconceptions among undergraduates; development of engineering students' ethical frameworks; and implementation of the elicitation model in higher level engineering courses.

Keywords: Model-eliciting activities, problem-solving, conceptual development

INTRODUCTION

The purpose of this article is to discuss an area of growing common interest between engineering education researchers and mathematics education researchers, *Model-Eliciting Activities*, or MEAs. MEAs and their variations form a type of case-study problem that small groups typically solve over one or two class periods. The article discusses the increasing visibility and use of MEAs and the modeling processes on which they rely in undergraduate engineering programs. Broader implementation of MEAs will require successful use and testing at multiple institutions. Several NSF grants currently support research and development of an evidentiary base that such broader implementation will require (e.g., [1, 2]). This article is prepared from the vantage of mathematics education researchers, and begins with our outsiders' envious observation that the engineering education research community has progressed rapidly in the past few years, and has done so with far less contention than our field of mathematics education. Some markers of this progress are well-known among engineering education researchers, and include the increasing readership of *Journal for Engineering Education (JEE)* [3, 4], the advent of new schools of engineering education [5–7], and the formation of the Center for Advanced Study of Engineering Education (CASEE) [8] at the National Academy of Engineering (NAE). NAE's *Engineer 2020* [9] and similarly themed literature [10, 11] are widely understood to present of a re-imaging of the future engineer that obligates not only revitalized or improved but also often fully transformed approaches to engineering education [12, 13]. Yet *Engineer 2020* and related elaborations on future professionals also speak to other scientific disciplines. Across the science, technology, engineering and mathematics (STEM) spectrum this literature calls for sustained and cumulative scholarship in the form of advancing theory, innovation and experimentation. The ascendancy of research within engineering education is a salutary development that promises to produce findings that will benefit other STEM fields, including mathematics education.

In part, this is because the natural problem-solving culture of *engineering* appears to give *engineering education research* inherent strategic benefits. These benefits, which also characterize MEAs as we discuss below, include a capacity to function in complex design or other task settings with competing constraints that frequently are unrelated to underlying science or technology but instead involve human preferences, values, and social dynamics; a pragmatism that welcomes multiple approaches in designing and testing solutions rather than rigid adherence to a single paradigm; and an ethos of continuous field-testing and improvement cycles.

An engineer may find such characterizations of his or her field more obvious than insightful, yet the engineering education community should value these traits as distinctive contrasts to some other forms of research in the social sciences. In particular, the broader field of educational research is a cautionary tale, one with a difficult and uneven history that has been chronicled in countless articles, books and policy documents [e.g., 14, 15]. Many of these fits and starts have also been recounted by the National Research Council's *Scientific Research in Education* [16]. As calls increase for the engineering education research community to look to education research for approaches and models [17], it is important to realize that a large portion of the education research community is still dealing less with generating knowledge than with foundational questions about *how* to generate knowledge. Many of the most constructive trends in education research, such as design experimentation [18–21], originate in observation of the common-sense and common-place characteristics of engineering above.

The MEA research area within mathematics education research capitalizes on and emphasizes some of these main principles of engineering practice while operationalizing key conjectures about strategies to expand complex reasoning. The similarities between MEAs and the professional practice that they are designed to simulate have helped build an engineering education research community focusing on MEA use in undergraduate education [22–28]. This partnership of mathematics education researchers focusing on modeling and counterparts in engineering education research has developed in part through the Twelfth and Thirteenth International Conference on the Teaching of Mathematical Modeling and Applications (ICTMA) conferences that have promoted engagement of engineering education researchers in both and most recently was largely organized around mathematical modeling in engineering [29, 30]. Such partnership is a promising example of cross-disciplinary collaborations such as those proposed by Olds et al [17].

This article describes MEAs and several of their design features, including the role of elicitation, the express-test-revise cycle that is systems thinking in MEAs, and the phenomenon of local-concept stage development. The article describes a recently formed collaborative of six colleges and universities experimenting with MEAs under support from NSF [1, 31–35] in multiple engineering fields, and one of the challenges such research faces: finding the right blend of modeling activities with other instructional approaches. The article also reviews five areas of active research in expanding the scope of the MEA construct in engineering education. These include the development and use of *Reflection Tools*, a device to nurture problem-solving personalities; fuller implementation of current and futuristic learning technologies; elicitation and repair of misconceptions among undergraduates; development of engineering students' ethical frameworks; and implementation of the elicitation model in higher level engineering courses, in contrast to the lower level courses where they are more commonly used. Figure 1 recaps these features and areas of expansion as an outline for this article. Because MEAs and related approaches are in relatively early stages of development and implementation within engineering education, this article is more of a prospectus for their potential as tools both for engineering education researchers and practitioners. Through this article,

What is an MEA?
- An MEA is problem that simulates authentic, real-world situations that small teams of 3-5 students work to solve over one or two class periods. The crucial problem-solving iteration of an MEA is to express, test and revise models that will solve the problem.

How is an MEA designed?
- There are six design principles (Table 3) developed over several years of testing in mathematics and engineering courses

Beyond the design principles, what are other important findings and premises about the MEA approach?
- Elicitation of models and of systems thinking is emphasized in contrast to imparting ideas to be used in problem solving
- Local concept development: A team's iterations through the "express-test-revise" cycle of model revision can yield new cognitive structures and understandings in the team members more effectively than single iteration application of textbook formula.
- The solution orientation of MEAs enables crucial development of complex reasoning processes, and suggests an alternative balance for how "product" and "process" are emphasized in the curriculum.

Selected areas of expansion of the MEA construct by engineering education researchers
- Heavier use of student reflection tools in assessment; heavier use of learning technologies; elicitation and repair of misconceptions; development of ethical framework; use of model-integration activities in upper level courses

Figure 1. Characteristics of Model-Eliciting Activities.

we invite *AEE* readers to help refine the conceptualization and theory behind these approaches and to find ways to test their implementation.

MODEL ELICITING ACTIVITIES (MEAS)

Model-Eliciting Activities (MEAs) are a class of problems that simulate authentic, real-world client-driven situations that small teams of three to five students work to solve over one or two class periods. While these baseline elements are common to a large swath of the problem-based learning and case reasoning literature [e.g., 36, 37–40], MEAs have a different origin that affects their design and use. MEA theory and practice grew as a means for mathematics education researchers to observe the development of student problem-solving competencies and the growth of mathematical cognition [41–43]. Part of that evolution entailed revising notions of problem-solving as a research-domain, and concluding that the conceptual model and model refinement processes that underlie problem-solving offered more insight into how or why students pursued particular strategies than focusing on the strategies or problem-structure [41].

Different researchers have used various MEAs to illustrate distinguishing characteristics of this tool. These MEAs include the "Volleyball Problem" [44], the "Summer Jobs Problem" [45], and "Big Foot" [46]. The "Paper Airplane" problem [2] is an example of an MEA that has been used with both middle school students and freshmen engineering students at Purdue. Moore, Diefes-Dux and Imbrie [47] review four MEAs in one of Purdue University's introductory courses. Gainsburg [48] examined the connection between modeling activities of structural engineers and those of mathematics students participating in MEAs. A higher level problem for engineering students involves the *Quantifying Aluminum Crystal Size* MEA and is outlined in depth in [49, 50]. A collection of MEAs, including, several that are in development, and a link to MEAs that have been implemented in Purdue's undergraduate curriculum, appears at modelsandmodeling.net. Finally, an MEA based on an engineering problem and carried out with calculus students at the Air Force Academy, appears in Figure 2 and is used to explain six design principles of MEAs that appear in Figure 3.

As the MEA line of research unfolded, what started as a tool for helping researchers understand conceptual models became increasingly documented as an approach that helped students become better problem solvers, especially including students who had not exhibited strong performance in more traditional mathematics curriculum settings [23, 51]. While furnishing an avenue for underperforming students to experience and exhibit success in mathematics, they also proved effective as a diagnostic tool for gifted and highly creative youngsters [52, 53]. MEAs also became a tool for helping teachers become more observant and sensitive to the design of situations that engaged learners in productive mathematical thinking. That is, a device to help mathematics education *researchers* elicit student modeling in order to help the researchers develop expertise about cognition and problem-solving behavior proved to be a tool that could also be used to help *teachers* and *students* develop their own competencies. MEA research in classroom settings thus led to observable change among a) students, b) teachers and c) researchers. This pathway, unplanned in the early days of MEA research, led to the development of the multi-tier design experiment methodology as a means to investigate the parallel growth of expertise at all three levels in the same group of modeling activities [42, 54].

What is it specifically, though, about the nature of MEAs, their lineage as an educational research tool, or their evolution into curricular use that might attract the interest of the engineering education research community? And, how is the adaptation of the MEA approach by engineering educators re-shaping and expanding the construct? Several articles have distinguished MEAs from either problem-based learning or textbook word problems [46] or from the kinds of problem-solving typically experienced in the college engineering curriculum [25, 55]. While a taxonomy of other problem-based learning forms and their similarities and differences with MEAs is not the intent of this article, MEAs represent a promising direction for engineering curriculum largely because of their emphasis

The City Council of Sea Shell Island has asked your engineering firm to provide an analysis of their tidal power plant. Due to population and business expansion on Sea Shell Island, there is a need to obtain more energy from the power plant. In particular, the City Council is looking to increase energy production at the plant around 20%.

Background:
Tidal power plants generate electricity by trapping water from the rising tide behind a dam and then letting it out so that it turns one or more turbines. Currently, Sea Shell Island has a tidal power plant whose basin is 500 feet across and goes 1000 feet inland. Openings in the dam allow water to enter and leave the basin as the tide rises and falls. This passing of water through the dam generates energy that can be stored, processed and distributed. The amount of energy generated is directly proportional to the amount of work required to fill the basin. In the case of the Sea Shell Island plant, the energy produced each time the basin empties is 70% of the work required to fill it. The depth of the basin is 25 feet at the dam and gradually decreases to ground level at 1000 feet inland. The bottom of the basin follows the shape of a parabola. (See diagram.)

Task:
Write a feasibility study for the City Council that addresses the current specifications of the power plant and provides at least two alternative designs for achieving a 20% net gain in power output. Discuss the likely relative costs of the plans. Note that the City Engineer will read the study and present your findings to the Council. It is appropriate to provide detailed calculations along with relevant explanations for any solutions that you propose. Any charts and graphs you use can be computer generated and incorporated into the report.

The current specifications will include the volume of water that the basin holds and the maximum force on the dam at high-tide. It also includes the amount of energy produced each time the basin fills. Carefully consider the council's desire to increase the energy production by 20%. Discuss different construction options on the basin to achieve this result. Note that there is open space for another 200 feet inland, but beyond that there are buildings and roads. The community has expressed a preference for retaining as much open space as possible. You should consider options that require excavating the minimal amount of earth since the cost of the project will be directly proportional to the volume of earth that needs to be excavated.

Assumptions:
Changing the width of the dam is not practical since it already exists. Therefore all construction should be focused on changing the basin. The dam can be closed (thus not allowing any water to enter) so that construction can be accomplished. Currently the bottom of the basin is parabolic, but this is not necessarily required to be the case after excavation work is done.

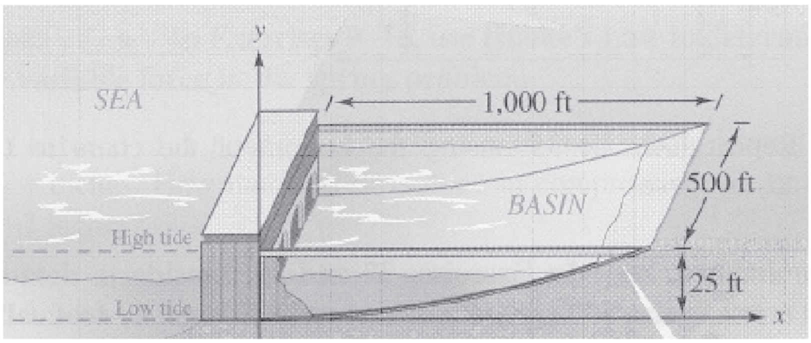

Figure 2. **Sea Shell Island.** ***Appropriate for Freshman Engineering or Freshman Calculus Students (Adapted from [83]).***

Principle for MEA Design	Comments on the Sea Shell Island MEA
Reality Principle (the "Personally Meaningful" Principle): Could this happen in "real life"? Will students be encouraged to make sense of the situation based on extensions of their own personal knowledge and experiences?	Undergraduates find the problem realistic and motivating.
Model Construction: Involves constructing, explaining, manipulating, predicting, or controlling a significant structural system? Does the task create the need for a model to be constructed (or modified, or extended, or refined?	Multiple models (discussed in the article) can be constructed, modified or refined.
Model Documentation: Will the response require students to explicitly reveal how they are thinking about the situation (givens, goals, possible solution paths)? What kind of systems (mathematical objects, relations, operations, patterns, regularities) are they thinking about?	This is a client-driven task and requires documentation of at least two solutions that meet requirements but entail different tradeoffs.
Self Evaluation: Does the statement of the problem strongly suggest the criteria that are appropriate for assessing the usefulness of alternative responses? Will students be able to judge for themselves when their responses are good enough? Will it be clear what purposes the results are intended to address? For whom? When?	Because of the initial parameters and constraints, self-assessment is straightforward: a cost per percentage increase in energy can be determined for every solution model that is tested.
Model Generalization: In "real life" situations, it is seldom worthwhile to develop a conceptual tool (such as a model) if the tool is only going to be used once. So, the model generalization principle says "Is the model not only powerful (for the specific situation and client at hand) but also sharable (with others) and re-useable (in other situations)?"	One interesting generalization pathway for this problem entails paramaterizing specific models (for example, the "extend the parabola further out" approach) as a function of how far the basin extends. Interestingly, few undergraduates at USAFA reach this level of generalizing, relying on continued trial-and-error.
Simple Prototype: Is the situation as simple as possible, while still creating the need for a significant model? Will the solution provide a useful prototype (or metaphor) for interpreting other structurally similar situations?	The situation has few complications; it can be remembered fairly easily and entails only a handful of defining structural features, yet requires model construction and the potential for generalization.

***Figure 3. Six Principles For Writing Effective Model-Eliciting Activities [25, 84] Activity Name:* Sea Shell Island.**

on *elicitation* and subsequent successive *alteration* and *generalization* of conceptual models. The important questions for both professors and researchers using MEAs focus on the models students use and how those models evolve. Perhaps the most important point of contrast with other problem-based learning forms is that MEAs focus less on notions such as ill-defined problems, problem-solving strategies, and rubrics and more on drawing out and changing the conceptual models that underlie strategies and on structuring problems to optimize that modeling process. Relating the experience of implementing MEAs across the freshman engineering class at Purdue, Diefes-Dux and colleagues [25] argued that while the approach bears superficial resemblance to common themes of freshman engineering curriculum, more substantively they represent a significant curriculum departure, and, perhaps most notably, that "the MEA framework fosters significant change in the way engineering faculty think about their teaching and students." (p. F1A-3).

Models and Modeling

By *model* in the acronym MEA we refer to the conceptual structures that an individual or a group uses to solve authentic or real-world problems. Lesh and Harel [46] provide a definition of models as:

> Conceptual systems that generally tend to be expressed using a variety of interacting representational media, which may involve written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors. Their purposes are to construct, describe or explain other system(s). (p. 158)

In earlier MEA literature, Schorr and Clark-Koellner [56] highlight the sense-making role of models. Their complementary description of a model is

> A way to describe, explain, construct or manipulate an experience, or a complex series of experiences. Models are organized around a situation or an experience. A person interprets a situation by mapping it into his or her own internal model, which helps him or her make sense of the situation. Once the situation has been mapped into the internal model, transformations, modifications, extensions, or revisions within the model can occur, which in turn provide the means by which the person can make predictions, descriptions, or explanations for use in the problem situation. (p. 192)

Modeling is the dynamic process of creating and manipulating these conceptual models in problem-solving. It can be described as an adaptive process, of creating solutions to previously unsolved problems. Modeling and its emphasis on the structure of ideas, connected knowledge forms and the adaptation of large ideas to new contexts is the focus of a diverse group of researchers, of which the MEA community is only one. Other education and learning science related fields with substantial literature on modeling include science education [57, 58], computer-supported collaborative learning [e.g., 59, 60], artificial intelligence and intelligent tutoring [e.g., 61], and interactive digital media [62–65]. Modeling in all of these areas involves crossing disciplinary boundaries and the kind of interpersonal communication stressed in ABET requirements. Complex and heterogeneous competencies are required to identify, organize and represent structure; connected reasoning ability is needed to create new knowledge structures; and the capacity to document, generalize and transfer solution paths is essential. For example, in all of the mathematics education MEAs referenced above, teams are required to make sense of multiple data sets by interpreting and differentiating them, prioritizing them, and connecting them to an overall task. Fairly sophisticated judgments about disparate data are required at the same time that more mundane skills in spreadsheet manipulation are crucial "point-of-entry" requirements for beginning the problem. Thompson and Yoon [66] elaborate on the nature of problem situations such as these or such as those implemented in engineering courses (e.g., the Sea Shell Island example in Figure 2), for which is it useful to consider *mathematical* models explicitly. A large share of research with MEAs has focused on how students *mathematize* elements of a situation, i.e., impose mathematical understandings or interpretations, and how they make judgments about what components of a situation are useful to consider with mathematical representations [67].

Systems Thinking

In broad strokes, when the situation can be interpreted as a *system* which possesses some mathematical components that affect the system and therefore the underlying problem, it lends itself to a modeling analysis. One central assumption of MEA research is the notion that mathematical thought functions as a set of systems, where mathematical ideas are connected to and embedded in contexts in which they are learned and used. The timeless example of youngsters able to compute batting averages or free throw percentages but unable to handle fractions in school is a simple illustration of a broader reality: individuals create mathematical understandings and models as they mathematize problems that are meaningful for them to solve. Meaning drives cognition. Many mathematical concepts, skills, and processes that students develop during real-world problem-solving are significantly less usable when taught in a manner unconnected to intrinsically motivating or meaningful contexts [67]. The process of making mathematical sense of more complex problems, including conjectures about how to proceed in a problem or whether to try one approach or another, is essentially a systems process—organizing, interpreting, connecting and manipulating different parts of the situation in order to make more sense of the whole and of possible solution paths. The assumption of the paramount importance of systems thinking is perhaps the most important intersection of MEA research with engineering education research.

Elicitation

Many of the debates in instructional reform, especially at the K12 level, have attempted to create a continuum from "direct instruction" to "constructivism", ostensibly with a skills to concept orientation

[68]. Modeling in general and the MEA framework in particular do not fall along either side or even on such a continuum. The premise of an MEA departs from imparting strategies and skills that add up to a hierarchically defined curriculum, and departs from expecting learners to "construct" strategies and skills; instead, it aims to leverage the models and conceptual systems that the learner already possesses. (If anything, the broader implementation of MEAs in engineering is likely to co-exist in balance with more didactic approaches, although the use of MEAs in college curriculum is too nascent for definitive conclusions.) Relative to other classes of problem-based learning, the focus of an MEA is on eliciting the model a student or group uses and revises to interpret or to make sense of a problem. *Eliciting* and *multi-cycle revision* of models rather than *constructing* or *imparting* is the foundational strategy of MEA design.

The process of expressing an existing model, especially one that is weak or uneven, by externalizing it and clarifying to others, then testing the model and collaboratively revising to higher order models, entails the development and use of critical competencies of interest both to engineering and mathematics educators. For example, verbally articulating models and clarifying connections between ideas, or elaborating on why a model might succeed or fail are all strategies documented to improve creative problem-solving both in mathematics learning [69] and in commercial engineering practice [70]. In both cases, elaborating on inadequate conceptual models drives other important learning behaviors, such as information search and retrieval and development of skills that are necessary to solve the scenario. If confirmed, the finding that elaborating on inadequate conceptual models can spur other important learning behavior challenges the amount of classroom time devoted formally to imparting information and to skill development, and suggests that professorial duties of the future may be oriented less to such tasks and more to insuring that students have succeeded in taking responsibility for them. While it may seem radical to propose less formal instruction in lower level information and skills in the future, in favor of increasingly expecting students to shoulder that responsibility themselves, it is unrealistic to think that current curriculum practices will remain adequate for building student knowledge bases as the knowledge that students need to acquire continues to expand. It is more likely that various STEM education communities will give up on expecting to *teach* everything that must be learned, and begin more fully to cultivate and nurture in college the life-long learning skills that future professionals will need to exercise throughout their careers.

Local Conceptual Development

One of the most important findings of MEA research in mathematics education goes to the heart of the Research Agenda Area on Engineering Learning Mechanisms [71]. That finding relates to what can be called "local conceptual development" by which a series of iterative models to solve an MEA can be documented to reflect Piagetian-like stages of concept development [46]. The finding is not interpreted to be confirmatory nor disconfirmatory of an overall stage-based theory of intellectual development such as that of King and Kitchener [72] and Perry [73]. Instead, it offers a fresh interpretation that developmental stages, rather than monolithic and inevitable changes in intellectual patterns that apply to all domains of cognition, can occur in stepwise and context-driven fashion. That is, developmental fluency, abstraction and complex reasoning ability may take place in variable contexts, and be deeper in some domains for which an individual possesses greater personal experience and meaning than in other domains. This interpretation appears supported by research by Louca et al [74] who argue that invariant stage transitions in intellectual development occur, but in finely grained and uneven contexts. It also sheds light on a swath of research on intellectual development and stages of complex reasoning, suggesting that exhibiting a developmental sophistication in one domain does not imply the same sophistication in all domains. This has important implications for research on spurring progress through stages of complex reasoning in undergraduate engineering education [75, 76]. MEAs can be seen as scenarios structured in such a way as to stimulate local *conceptual development* relative to the context of the scenario and to the contexts to which a team's final model can be re-used or generalized. Mathematics education researchers have found that as youngsters iteratively develop, express and test models in solving a scenario, they produce new approaches and cognitive structures that are often far more sophisticated than what might be taught in a classroom [46]. They also found that students who traditionally underperformed in more traditional mathematics curriculum settings were very successful performers in team modeling sessions. If confirmed by further research, this is a profound finding. Like other STEM disciplines, the engineering community needs to find ways to expand its welcome and accessibility to those who are underrepresented in the profession's demographics by addressing the social and mentoring climate for learning [77, 78]. Frameworks such as MEAs may prove highly strategic for expanding the reach and effectiveness of undergraduate program with a more cognitively attuned curriculum practices.

Solution versus Process Orientation

While problem-based learning is often associated with so-called inquiry, hands-on and constructivist approaches [79, 80], MEAs are not bound to any single set of knowledge forms, strategies, or procedures. That is for a very simple reason—MEAs, in simulating real-world *experiences*, are designed to simulate real-world *problem solving* where solutions are usually more important than how one gets to a solution. This flies in the face of conventional education reform orthodoxy, for example, where "process" is ostensibly stressed more highly than "product." And it may appear to fly in the face of the original intent of MEAs to understand the processes occurring as problem-solving competencies develop, or the reported experiences of those implementing MEAs in the engineering curriculum [25]. There is an important and instructive irony, though, that arises when a solution becomes the deliverable rather than acquisition or mastery of a procedural or conceptual competency that might be required for that solution. MEAs take a solution-orientation in order to engage the powerful and comprehensive system processes that authentic human adaptive effort entails. In order to get to powerful processes, MEAs are need-based—they require a product that the modelers can evaluate while they are building the product. To the degree that they are well-designed, MEAs are realistic and connect ideas, intuitions, skills, values and preferences—elements of human experience that are intertwined with cognition and problem solving [81, 82]. They may require or touch on larger ideas that are more advanced than anything the student has seen in a textbook at the same time that they require certain types of lower level procedural skills that require clever application. Real world problem-solving through use of carefully designed tools such as MEAs will not only challenge the granularity of common engineering curricula, but how curricula are sequenced. Our suggestion is not that big ideas in mathematics and engineering and situations that embed those ideas should not be taught directly [67]—to the contrary, next generation learning environments are more likely to succeed if they invoke multiple approaches that have been considered mutually exclusive. Different pedagogical frameworks each possess some validity, in the sense that in different contexts and for different purposes they may be more productive than at other times. This important issue is discussed below as a central challenge to a current MEA research collaborative.

A Sample MEA: Sea Shell Island

MEAs simulate authentic problem contexts in order to reach the goals described above, such as systems thinking, elicitation, local conceptual development and iterative modeling skills. As simulations they require a careful design process. A sample MEA used at the Air Force Academy, *Sea Shell Island*, appears in Figure 2 as noted. Students are placed in the position of being in an engineering consultancy that has been asked by its client, the City Council for Sea Shell Island, to design means to increase the power supply generated by a dam. It is one of a growing collection of MEAs in use with undergraduates in engineering and mathematics courses. This problem, adapted from a textbook-based project problem [83] elicits a variety of interesting student models from the initial brainstorming process through the types of solutions that are submitted as final documents. Models that students test include squaring out and extending the bottom of the basin; diagonalizing the parabolic bottom; extending the basin x feet with a diagonal to extend from the new outer limit of the basin to intersect the parabola y feet below the water level, or creating an extension with a purely diagonal or parabolic arc to it. Students have proposed use of reverse turbines as a single solution or in combination with other approaches. Some of the big ideas with the MEA that we expect students to use, form or clarify are that work and energy are roughly proportional; that filling a basin takes as much work as emptying it (though it is the latter that is more commonly taught); that gravity as a force contributing to work should be optimized so that work (roughly proportional to energy) can be optimized—meaning, eventually, that square bottom solutions that put water as low as possible are not promising. Another area of testing solutions involves calculating the force of stationary water at the base of the dam versus calculating work when the water is moving through the dam. Of course, the modelers also have to compute the cost, and to attend to the community desire to aesthetics and to retaining as much open space as possible.

MEA researchers in mathematics education have distilled a set of six design principles [25, 84], subsequently adapted for engineering education [49] that appear in the left column of Figure 3, with comments on the relationship of each principle to the Sea Shell Island scenario. Each principle plays a role in eliciting from teams successively sophisticated (or re-usable) models for resolving a scenario for a client. The discussion above deals with the first two principles—the "reality principle," that the scenario must be realistic and meaningful and the "model construction" principle, that is, it must require structuring a model [66]. The "model documentation" principle for MEAs is a design feature that goes to the heart of eliciting models—MEAs require explicit articulation of solution paths, conjectures, and ways that each individual thinks about the problem. The "self-evaluation" principle is an essential assumption for propelling the cycles of "express-test-revise" that are at the core of modeling activities. The MEA design must permit the modelers to evaluate and test solution paths iteratively and to make judgments about whether the paths are likely to be productive. A fifth principle, "model generalization" entails designing MEAs so that they permit both particularized solutions and then more generalizable models that might be sharable or usable on a class of similar problems. This is related to the sixth, "simple prototype" principle: is the situation as straightforward as possible yet still require a significant model, a model that might be usable for structurally similar situations?

THE MEA-PHASE III CCLI COLLABORATIVE

How MEA-libraries might be expanded and then more systematically tested in engineering education curriculum is a large question for those seeking to move classroom experience to a heavier concentration of scenarios and systems-thinking. With substantial attention both to scenario design and to cognitive growth, the MEA approach provides a testable pathway to bringing future engineers into proximity with authentic problems of the kind that they are likely to face, in a manner that appears appropriate for pre-professional developmental pathways. The approach has a growing qualitative and theoretical literature that documents their success in spurring development of complex reasoning (e.g., [45, 46, 58]), and it has won a place at the table of engineering education reform discourse. The undergraduate engineering curriculum experiences at Purdue, the Illinois Institute of Technology, the Air Force Academy and elsewhere have served as important precursors to new efforts to build foundations for testing and implementing MEAs. For the approach to become a candidate for more expansive implementation in the engineering education curriculum, it will need a more sophisticated evaluative research literature, one that can elaborate more fully on when, how and why the approach succeeds with engineering students. Developing that literature, in turn, requires stable testbeds whose implementation involves strategic development challenges. The most salient for the nascent engineering MEA community is to optimize the blend of MEAs in existing engineering curricula and then to formulate and test assessment tools that can assure viable progress both of the programs implementing the MEAs and of the students who are learning through modeling.

A consortium of six colleges and universities, through a series of collaborative grants from Phase III of NSF's Course, Curriculum and Laboratory Improvement (CCLI) Program, has begun a systematic effort to address these challenges. This Phase III CCLI Collaborative seeks to test different combinations of MEA use at a set of diverse institutions, in industrial engineering (Pittsburgh) [1], environmental engineering (Air Force Academy) [31], chemical engineering (Colorado School of Mines) [34], mechanical engineering (California State University at San Luis Obispo) [35], electrical and computer engineering (University of Minnesota) [33] in addition to introductory courses (Purdue) [32]. Blending MEAs into existing engineering curricula will involve a stepwise process of mapping existing and new scenarios against course objectives, and testing the optimal time to situate a scenario in a course.

In the CCLI grants, researchers will vary the number of MEAs in a given course, and experiment with whether the key ideas on which they rely appear before or after the MEA episodes in the course material. Literature on existing programs that blend problem-based learning, lectures and other pedagogies for undergraduate curriculum generally report the kind of balance that the programs implement, but not the experimental or decision-making process for arriving at a particular blend [85–87]. The use of MEAs in multiple institutions in varied contexts should permit variation and experimentation in arriving at optimal mixes. If the finding is confirmed that significant evolution of complex reasoning can occur at a local concept level through iterative modeling, then both mathematics and engineering education researchers and the broader fields that they represent will inevitably need to re-think curriculum priorities. MEAs and other problem-based learning forms challenge the granularity of instruction; MEAs also challenge traditional notions of what might be considered the granularity of cognition, and they suggest greater emphasis on developing engineers by exposing them more frequently to systems of ideas or to models. Promising approaches, however, even with substantial evidence of success, do not speak to the intrinsic value of competing approaches that may reflect different theories of learning. In fact, approaches that rely both on rehearsed skills through repeated exercises and the kinds of local conceptual stage development that MEAs promote may prove to be an important hybrid or blend for effective learning and transfer within the undergraduate curriculum. While an approach to modeling seems at odds with lecture-based instruction and traditional textbook exercises, variations both approaches might reasonably be hypothesized to blend into an effective portfolio of pedagogies, depending on research findings [88]. One of the primary conclusions of modeling research is that a particular problem-solving strategy is not inherently better or worse than others, but rather more appropriate for a particular context. The MEA-Phase III CCLI Collaborative will be able to test this finding as it transfers to a conjecture about pedagogical strategies, namely that modeling compares favorably to more didactic forms (such as lectures) in some contexts, not so in others. It should, in short, contribute to a science of blended pedagogies.

EXTENSIONS OF THE MEA CONSTRUCT

The implementation of MEAs in undergraduate engineering has prompted researchers and developers to explore several important ways to expand the utility of the construct. A full treatment of these are beyond the scope of this article, but a brief discussion of five of them, involving student reflection tools, technology, misconceptions, ethics, and advanced curriculum, may help to highlight areas of research opportunity.

Student Self-Assessment Through Reflection Tools

Within the broad discussion on assessment challenges associated with evolving curricula and pedagogies, one relatively recent development in MEA research merits note. It has focused on assessing metacognitive understandings that students develop through modeling processes. This line of research has entailed the use of devices called *Reflection Tools* to elicit from students their evolving conception of how team problem-solving unfolds, the individual roles that they play in modeling teams, and how and under what conditions particular problem-solving strategies might be productive [89]. Reflection Tools furnish a means for learners to develop, document, and self-assess characteristics or capabilities that are important components of a productive problem-solving personality. They are designed to help make explicit the range of group and personal dynamics students have experienced in a modeling or problem-solving sequence, with those dynamics tied to the specific properties of the problem the group is attempting to solve. A central purpose of the tools is to evoke from learners structured observations about their interactions in MEAs, and, like MEAs, they are designed for use by learners, professors, and researchers. Four examples of RTs appear in [89]; these focus on (a) changing roles of productive individuals in a modeling activity; (b) changing functions of a group over a modeling activity, (c) values, attitudes, or feelings which contribute to high levels of engagement, and (d) problem-solving strategies that are productive at different stages of modeling. Reflection Tools form a potential direction for building a suite of student-based self-assessments that stress the kinds of competencies and processes that are difficult to probe with more traditional instruments.

Reliance on Learning Technologies

The origin of MEAs as research tools for understanding mathematical cognition may help to explain the relatively low profile for technology in MEA research in the 1990s. The analysis of conceptual models does not inherently require extensive technological mediation. Yet both relatively simple tools such as basic spreadsheet or MATLAB functions, and more sophisticated tools (e.g., decision-making support software, laboratory equipment or statistical visualization systems) alter problem-solving, and the pervasive use of these tools underscores the need to understand the cognitive models students use in deciding how or when to rely on them. The horizon promises more areas for expansion of the MEA construct. Web 2.0 and advanced distributed conferencing tools create fundamentally different possibilities for MEA team interactions. These include opportunities to advance the vision reported in the literature for international student problem-solving teams [90]. The emergence of artificially intelligent systems is a related avenue of expansion. Avatars that can supply information, coach teams and organize progress are likely to expand the scope of implementation of MEAs, simply by providing some content-rich and anthropomorphically-credible accompaniment to groups [91, 92]. Among efforts to expand the technological profile of MEAs are a project supported both by NSF's Human and Social Dynamics Program and the Chinese National Natural Science Foundation explores the use of such accompaniments in MEAs [93]. The Phase III Collaborative, primarily through [35] will develop a series of MEAs that rely heavily on laboratory instrumentation.

Repairing Misconceptions and Developing Ethical Frameworks

The investigation of optimal blending of MEAs will advantage two other crucial research topics that are a particular focus of the Phase III CCLI Collaborative, *misconceptions* and *ethical* *frameworks*. In both of these areas, traditional pedagogies appear inadequate for reaching long-term educational goals. The original Force-Concept Inventory research, for example, produced the well-known and troubling finding that deep misconceptions about fundamental ideas in physics resist correction by formal instruction [94]. Miller and Olds confirmed the persistence of misconceptions about thermal dynamics among college seniors in spite of successful completion of completed courses in fluid mechanics and thermodynamics [95]. The MEA framework is designed to expose and to help learners refine inadequate conceptual models. Researchers in the MEA-Phase III CCLI Collaborative have conjectured that identifying misconceptions through elicitation activities, followed by testing and revising underlying models, is more likely to repair misconceptions than an approach that primarily relies on lectures and testing. This conjecture has sufficient face validity for testing; an evidentiary base in support of MEA frameworks to address misconceptions will eventually entail comparisons of students who participated in MEAs designed to expose and repair misconceptions with peers who did not. Data such as that reported by Miller and Olds [95] (e.g., over 40% consistently cannot distinguish between the rate and amount of heat transfer between two bodies at different temperatures and approximately 50% cannot distinguish between the quantity and quality of energy as described by the second law of thermodynamics; and nearly 30% cannot logically distinguish between temperature and energy in simple engineering systems and processes) will provide an important test and baseline for the approach.

A related area of expansion of the MEA construct involves the new ABET Criteria, specifically relative to understanding of professional and ethical responsibilities, and, going a step further, students' ability to recognize and resolve ethical dilemmas similar to those that they might see in practice. Eliciting and nurturing existing ethical models from students may develop into an important component of future pedagogies of ethics. Most literature reporting frameworks for ethics education [96–98] recommend the use of case studies. Wareham et al [99] go further and relate the introduction of ethics instruction by placing students in simulations in which they are unaware that ethical issues will arise; the simulations are designed to elicit ethical frameworks and to provide a basis for reflective discussion of those frameworks. The starting point for the instruction was to elicit the models that the students possessed. This, coupled with the evidence of local stage development, suggests that building opportunities for eliciting, elaborating on and revising ethical models in MEA scenarios may prove to be a promising strategy to reach the ABET criteria. Again, though, the body of research necessary to form conclusions about optimal approaches does not yet exist. The ethics rubrics developed at Pittsburgh [100], variants of the King and Kitchener Reflective Judgment metrics, will likely be an important platform for efforts to establish baselines for assessing the efficacy of an MEA approach to ethical framework development.

Advanced Courses and MEAs

A fifth area of expansion of the MEA construct entails moving beyond the introductory engineering curriculum to more advanced courses. Upper level engineering curricula typically feature substantial capstone projects that require extensive integration of content from multiple courses. These differ from the MEA approach, which stresses 45–90 minute activities, focus on models more than on solutions, and emphasize the elicit—test—revise iterations of model building. Mathematical MEAs in middle school and high school settings leverage sizable mathematical understandings, intuitions, and tacit knowledge that students possess but that are not invoked in formal instruction. By the time students have moved past calculus, the nature of the MEA design necessarily changes. Shuman has coined the term "model-integrating activities" to refer to MEAs that are formulated for upper level students in such a way as to force connections between material from prior courses [101]. The hypothesis that repeated use of shorter, integrative modeling activities a) produces more integrative modelers and b) is worth the opportunity cost is an important research question for upper level curriculum reform.

CONCLUSION

While still in its early stages, MEA research and subsequent classroom implementation, including that reviewed in this prospectus-oriented article, suggests the possibility that prioritizing large ideas and creating scenarios by which students develop and refine models that bring those ideas to bear in varied circumstances may engender the kinds of transformative shifts in undergraduate experience that have been called for by the National Colloquies on Engineering Education Research [102]. Navigating the testing, improvement and expansion of such possible directions entails concerted theoretical and empirical research in varied contexts. MEAs are at a sort of "mezzanine" level, whereby the construct has an initial literature base in both mathematics and engineering education. The construct is a candidate for contributing to the engineering education community's ambitious goals for the future. Efforts such as the CCLI Phase III Collaborative comprise crucial and high stakes *in situ* research opportunities to determine conditions by which such candidate approaches can be expected to succeed in educating future engineers.

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