Mathematics in vocational education: Revisiting a developmental research project, Analysis of one development research project about the integration of mathematics in vocational subjects in upper secondary education in Sweden

Lisbeth Lindberg
University of Gothenburg
Gothenburg, Sweden
<lisbeth.lindberg@ped.gu.se>

Barbro Grevholm
University of Adger
Adger, Norway
<barbro.grevholm@uia.no>

Abstract

In this article we describe and discuss the analyses of a developmental research project that took place in Sweden during 1998 to 2002. We carried out four different analyses in order to explore the learning outcomes from the project that could inform long term curriculum change and teacher collaboration in vocational education in Sweden. The analyses led to condensed descriptions of the project, including an exposition of its constraints and affordances as well as its developmental cycles. Also revealed was a comparison between the aims of the project and the actual outcomes. Based on the analyses and discussions, we suggest possible implications for practice and for future studies on vocational education.

Introduction

A reform of the upper secondary school in Sweden GY2011 will take place in 2011 and the preparatory discussions about the reform triggered the authors of this article to revisit and explore one developmental research project that was carried out during 1998-2002. We started this process of revisiting earlier data with current perspectives in 2007. The project KAM was initiated because of the former curriculum reform in 1994, Lpf94 (Utbildningsdepartementet, 1994). There were minor revisions of the reform during the time of the project (Skolverket, 2000). The questions that were asked focused on whether the project can inform us about how schools and teachers could improve mathematics learning experiences in vocational programmes and benefit from the results of the delayed analysis of the KAM-project.

The most critical issue in the discussion is whether pupils in vocational education benefit from studying the same first course in mathematics in upper secondary school as all other pupils. The argument for the first course to be the same for all students is that society is demanding more mathematics in almost all vocations and that all pupils should be prepared for that and be given the right to be able to directly build on to their upper secondary school education at a later stage in life. The arguments against are that pupils in vocational education need a different kind of mathematics course that is directly related, and seen by the students to be directly related, to their vocational studies and that it is the vocational interests that should guide the selection of topics and content.

The situation and problems behind the KAM-project

Over the years 1998 to 2002 a developmental research project, called the KAM-project, was carried out in an upper secondary school in a major city in Sweden. KAM stands for Karaktärsämnenas matematik, the mathematics of the vocational subjects. In this article we intend to describe, analyse and discuss this project in order to answer a number of research questions. We start by presenting the situation that gave rise to the project.

In 1994 a new curriculum, Lpf 94 (Utbildningsdepartementet 1994; Skolverket 2005, 2006), was selected for the Swedish upper secondary school. Under this curriculum, the organisation of upper secondary school meant, first of all, that pupils in the theoretical programmes and the vocational programmes were to study in the same school system and have the same study time of three years. Earlier, vocational education had taken place in separate schools, vocational schools, and followed a different curriculum over two years rather than three. Secondly, a compulsory mathematics course for all pupils, course A was introduced. Earlier pupils in vocational education did not study mathematics as a separate subject but were taught vocational calculations by the vocational teacher. Thirdly, the new curriculum Lpf 94 demanded collaboration between the teachers of mathematics and the main vocational subject in the programme, in order to ensure that the mathematics learnt was influenced by the needs of the pupils in the vocational part of the education. Additionally, it was the intention to ensure that the education would not prevent pupils who later wanted to continue to study at tertiary level from doing so.

All these demands were new to school leaders, teachers of mathematics and vocational subjects. The education of teachers of mathematics and vocational teachers were traditionally very different in nature and length and the teachers from the different areas had ways of working which were different and not well known to each other (Lindberg, 2009). Teacher education was not changed as a consequence of the new curriculum. Thus teachers were inquiring about how the new plans could be implemented and how to meet the new demands according to Lpf 94.

It was this demanding and confusing situation for teachers in vocational programmes that led some teacher educators to initiate what was going to grow to a developmental research study on the teaching of mathematics in those programmes. One of the authors of this article, Lindberg, took part as teacher educator and researcher in all the developmental cycles of KAM, while the other author, Grevholm, acted as researcher and scientific leader of the project between 1999 and 2002. Thus, being aware of our double roles in relation to KAM, we will carefully explain how we have tried to avoid subjectivity in the present analysis of the project.

In this article we will first present and discuss the new syllabus for mathematics, and after that discuss earlier research of relevance and related scientific literature. The research questions will be formulated, followed by the theoretical framework we intend to use for the analysis of the KAM-project. We then present and discuss the methods used, the data collection and the different stages of the analysis. Stage 1 is the analysis that resulted in the condensed description of the project. Stage 2 of the analysis consists of the findings of affordances and constraints in the project and stage 3 is an account of the developmental cycles we found in the study. Stage 4 is an analysis of the aims in the KAM-project in relation to the results of the project.

The vocational programmes in upper secondary school

The aim of the vocational programmes in upper secondary school is to provide an education that leads to the level where the pupil can have achieved acceptable vocational knowledge (Skolverket, 2006) that is to get their first job in a specific vocation. But the programme must also provide preparation that can be used as a basis for further studies later on. Most of the vocational programmes include practical components, where students are working in smaller groups (less than 15 per group) in a practical work situation, similar to an authentic work environment. These components include a main vocational subject taught by a vocational teacher. For example in the vehicle programme, pupils are working in a workshop similar to an authentic mechanical workshop, but most of the time without customers, time stress and the responsibilities for economic transactions.

The new syllabus of mathematics, especially course A

As mentioned, all pupils in the upper secondary school have to study the first course in mathematics, course A, as it is a core subject course and is included in all programmes. The content in the subject of mathematics is selected from a number of areas. Much of this content is covered in the mathematics courses of the compulsory school and the different parts are deepened and developed in the upper secondary school. Besides the content from the compulsory school new areas are introduced, deepened and gradually extended in the upper secondary school as the pupils take on more mathematics.

The directions of the studies are general so pupils should be able to master situations for themselves and for societal purposes. Emphasis is placed on mathematics providing knowledge to be used in the orientation of the students' programme of study, for example in a vocational programme. The exact wording is as follows:

> Both in everyday and vocational life, there is an increasing need to understand the meaning of and be able to communicate on issues with a mathematical content. … The power of mathematics as a tool for understanding and modelling reality becomes evident when the subject is applied to areas that are familiar to pupils. Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course. Knowledge of mathematics is a prerequisite for achieving many of the goals of the programme specific subjects. (Skolverket, 2000)

The written document indicates that there are many good opportunities to see the relevance of mathematics and the connections between mathematics and work. One can see that there is an emphasis on using mathematics within the study orientation and the vocational programmes. This means that the mathematics teacher could use content and applications from different vocations. The aims of the mathematics curriculum indicate a progression and extension from "to solve concrete problems in their immediate environment" to "solve problems that occur regularly in the home and society, which is needed as a foundation for further education" and further to "formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation" (Lpf94, 1994).

The curriculum is written in such a way that mathematics from the compulsory school is a base for applications and further development in course A. In algebra the goals in year 9 are "to be able to interpret and use simple formulae, to solve simple equations, as well as to be able to interpret and use graphs for functions describing real relationships and events" (Lpo94, 1994). This can be compared with the extension expressed in course A "to be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation", and "to be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems", and finally "to be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society". The pupils should "be accustomed, when solving problems, to using computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes" (Lpf, 1994). Thus the same kind of mathematical objects are treated but in somewhat different ways.

Here the study orientation is the same as the character of the specific programme the pupils are studying. It is obvious that it is intended that there is progression, extension and development over the years (See appendix 1). This intention does not necessarily reflect the development of knowledge in mathematics for each pupil.

The new Lpf94 curriculum was unique as it had a uniform post-16 educational system for the first time, and especially unique as the VET (Vocational Education and Training) was integrated in this system. The aim of the syllabus was a raised educational level in mathematics in Sweden for all citizens. The content of the course A was designed to strengthen the competence of the individual to function as a good citizen in a democratic society. At the same time there was a requirement that the course supported the students' understanding of the components of education that aim towards their future profession. National documents require the teachers to collaborate between the subjects and that the teaching of mathematics should demonstrate that the students can benefit from their mathematical knowledge when studying for the profession. The aim was to provide the student with opportunities to achieve the competence needed for citizenship and the labour market. The challenges for teachers were to make the connection between the content of the studies in mathematics and the reality of the post-school workplace visible. This requires more insight and knowledge for the teachers about different areas of content from mathematics and the vocational subjects that the pupils will meet within the vocational programme.

Earlier research and related literature on mathematics in vocational education

It seems to be unique for Sweden to offer vocational education as part of the upper secondary school system. Thus, we have not been able to find any international studies dealing with teaching mathematics in the kind of school programme we are investigating. We have reviewed methods used in studies on vocational education and found that they are of minor interest for this article.

The findings of this review were presented at a research conference and documented in Lindberg (2009). As the title, *To search for mathematics in the vocational teaching and learning- an overview of theories and methods,* indicates, the author searched for documents and research to find examples of theories and methods. The sources included databases such as MATHDI, proceedings from the Psychology of Mathematics (PME) and Adults Learning Mathematics (ALM) conferences. Most of the results presented came from research areas close to vocational education such as workplaces. The result shows that, over the years as technology became more accessible and easy to use, there was a trend towards using a multimodal approach when collecting data. It has, most often, not been obvious in the documentations what kind of theories the researchers were using, because theoretical underpinnings are rare.

The historical development of vocational education in Sweden was also investigated and documented in the paper *Historisk bakgrund till matematikens betydelse i yrkesprogrammen (Historical background to the importance of mathematics in vocational programmes)* (Lindberg, 2007). This paper is written in Swedish and gives an overview of the vocational education and training concerning mathematics up to the situation of today. Document analysis was carried out and based within a theoretical framework of hermeneutics (Gadamer, 1960).

In the ALM forum for researchers, practitioners and policy makers, Lindberg was one of the initiators of the topic group of mathematics education for the workplace. Over the years (2000-2004), when Lindberg was facilitator of the group, different aspects were discussed as concepts of workplace mathematics, data collection, differences between countries, lack of communication between mathematics educators and vocational educators (Lindberg, 2005).

The research questions

In revisiting the project from 1998 to 2002 we decided to try to answer the following questions:

- What are the characteristic features of the school based developmental research project KAM?
- What characterises the affordances and constraints that the involved participants in the project met during the work?
- What were the actual outcomes of the KAM-project compared to the aims?
- What are the conclusions and implications that can be drawn from this kind of developmental research project?

Theoretical considerations and framework

The implementation of a new curriculum and its syllabuses is a challenge for any school, school system and nation. The steps taken and signals given are not easy to see and interpret.

A development project, as described in this article, can illuminate and provide further insights to processes that can emerge out of changes and highlight outcomes that are both expected and unexpected. In order to answer the questions posed we have chosen to carry out several analyses based on complementary theoretical frameworks. Stage 1 and 4 in this process are based on a hermeneutic analysis of published documents. Stage 2 is based on the theoretical concepts of affordances and constraints and stage 3 is founded on theory of developmental research. We will in the following three subsections present the theoretical constructs and views that we used.

Hermeneutic analysis of documents

In a hermeneutic analysis the interpreter moves from parts of the text to smaller units, or vice versa, in trying to clarify meaning of the content. This movement between part and the whole, which leads to a deeper understanding, is called the hermeneutic spiral. The pre-understanding of the reader provides a certain holistic view as a starting point. The interpretation of different parts leads to a change of this holistic view and a new round of interpretation can take place (Ödman, 1979; Gadamer, 1960). There are three demands to fulfil: the system of interpretations should be coherent logically; the interpretations must have a connection with the object of interpretation; and the interpretation and its results must be disseminated to the reader in an appropriate way. From the presentation it should be explicit how the interpreter has reached the conclusions.

Theory of affordances and constraints

Using affordances and constraints to evaluate collaboration between teachers of different subjects in vocational education will give us one tool to analyse the project. Kennewell (2001) writes about use of the *concepts affordances and constraints* in didactical activity. According to Kennewell, affordances are the attributes that provide potential for action and the constraints are the conditions and relationships amongst attributes which provide structure and guidance to the course of actions. Affordances and constraints must be considered in relation to the abilities of the participants of the activity they support. Kennewell was studying use of ICT (Information and Communication Technology), but in our case this is replaced by integration of mathematics and vocational subject studies. In the KAM-project both pupils' abilities and teachers' abilities were studied. According to Kennewell (2001) the teacher's role is to orchestrate the supporting features in an attempt to make it possible for the learners to bridge the learning gap. Achieving learning effort on the side of the pupils is required to overcome the gap between their existing abilities and the intended abilities in the setting. Kennewell traces the supporting features of affordances to Gibson (1986) and of constraints to Greeno (1998). In the developmental project, KAM, the reports make use of similar expressions to describe what is going on. Thus we find it rewarding to try to use the *concepts affordances and constraints* in the analysis of the reported developmental work.

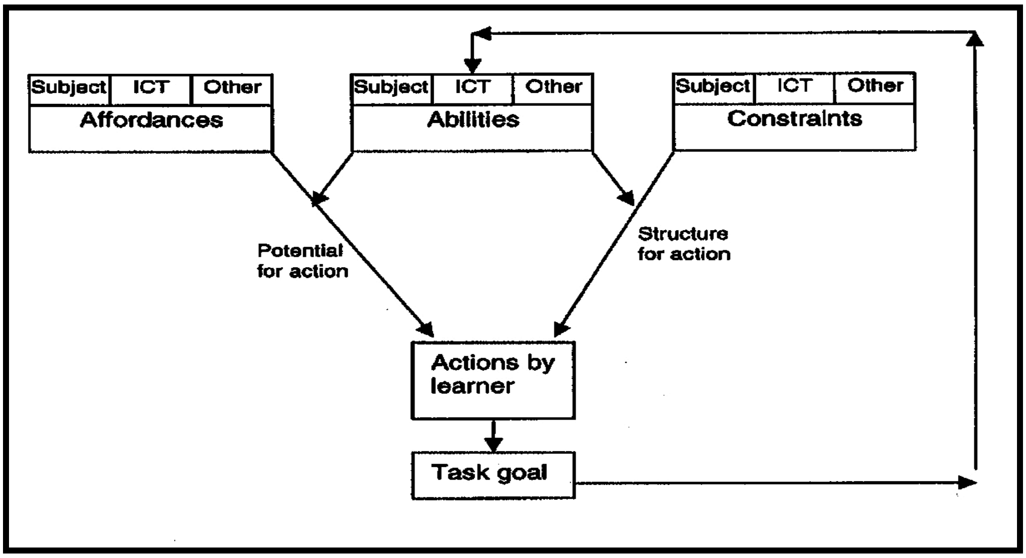

Figure 1. Kennewell's framework (Kennewell, 2001, p. 107)

As figure 1 shows, the teacher's intentions for the didactical activity are concerned with reflection and the development of abilities (Kennewell, 2001). Thus the framework used, incorporates task goals and outcomes according to the figure (, 2001).

Developmental research, the philosophy of the KAM-the project

An aim of the research project was to promote educational development, expressed as developing mathematics teaching and improving pupils' learning in mathematics by the integration of the studies in mathematics and the vocational subjects. Thus development of education is closely intertwined with the research project. Koeno Gravemeijer calls this developmental research and describes in some detail how he sees that (1994). He references Freudenthal (1988), who claimed that thought experiments are important in educational development. The developer envisions the teaching and learning process and, after implementing it, will try to find evidence to see if the expectations he had are confirmed or not. The feedback of practical experience into new thought experiments creates an iteration of development and research. Gravemeijer claims that this cyclic process is at the centre of Freudenthal's concept of developmental research (1994). Practice depends on a cyclic alternation of development and research and the cyclic process is more efficient when the cycle is shorter. The cyclic process can be seen as the learning process of the developer. A global theory, which Freudenthal, according to Gravemeijer, would prefer to call philosophy, guides the developmental work. This theory functions as the basis for the learning process by the developer and is nurtured by the alternation between thought experiment and practical experiment. That learning process can be interpreted as theory development.

Developmental research has some similarities with what is called design research. Eric Wittman is one researcher who sees mathematics education as design science (1998). He emphasizes that an important element in studying didactics of mathematics is building theory or theoretical frameworks related to the design of empirical investigations. Kelly (2003) writes about design research as a research dialect that attempts to support arguments constructed around the results of active innovation and intervention in classrooms. Di Sessa and Cobb

(2004) point out that in design research, theory must do real design work in generating, selecting and validating design alternatives. They also emphasise that development of theory should be one of the primary goals of design research.

We see the KAM-project as developmental research and the cyclic alternation process of development and research is crucial and we intend to illustrate parts of this process. What Freudenthal calls the philosophy of the work, the global theory behind the study, is in our case based on two important theoretical constructions, namely: integration of mathematics and vocational subjects; and a co-learning community of teachers. The integration of the subjects is both theoretical and practical. The theoretical part is the analyses of syllabuses and work material. The practical part is the performances in the school-settings. The co-learning community (COL) or professional learning community (PLC) is as Hord (1997, p. 1) describes it ”a powerful staff-development approach and a potent strategy for school change and improvement”.

Methods and methodology

The KAM-project, which was an empirical case study, had the character of developmental research project and has been reported to The National Agency for Education (Skolverket) in three parts. The project can be seen as a mainly qualitative case study and it ended in year 2002. We decided to explore the project again in an interpretative mode and learn more from it from a research perspective. Because of our involvement in the empirical study during 1998-2002 and risk for subjectivity we decided to use only the printed material from the project as sources in this part of the research. The documents available are the three KAM-reports (KAM1:1, 1999; KAM 1:2, 1999; KAM 2, 2001; KAM 3, 2002), the DUGA-report, printed working material, stored working notes and published conference papers presented about the project during the years. See table 1.

Report	**Date of publication**	**Number of pages**	**Target group**
DUGA	August 1996	35	National Agency for Education Gothenburg University
DUGA, attachment	August 1996	28	National Agency for Education Gothenburg University
KAM 1 part 1	January 1999	11	National Agency for Education
KAM 1 part 2	June 1999	42	National Agency for Education
KAM 2	September 2001	31	National Agency for Education
KAM 2 attachment	September 2001	78	National Agency for Education
KAM 3	November 2002	41	National Agency for Education

Table 1 Written documents used in the data collection

We started in 2007 by collecting all the available printed material and reading it. In order to be able to answer the research questions in the article we decided to divide the analysis in four parts or stages.

- Stage 1 is the analysis that resulted in the condensed description of the project. This step is needed both to make explicit our understanding of the project and for the reader to be able to follow the further presentation.

- Stage 2 of the analysis consists of the findings of affordances and constraints in the project. In the project reports challenges, problems and obstacles of different kinds became visible and the concepts of affordances and constraints seemed to be a helpful tool to use for this part.
- Stage 3 is an account of the developmental cycles we found in the study and the analysis of the developmental cycles of the project that we found by investigating the documents.
- Stage 4 is an analysis of the aims in the KAM-project in relation to the results of the project. Here we want to compare the actual outcome of the project as it was reported with the aims that were set.

For the condensed description in stage 1 of the KAM-project we have relied on the written reports from the different parts of the project. We have tried not to rely on our memories of how the work progressed but used the written reports from the different parts of the project as our source material. The analysis process started by reading and re-reading the reports. The interpretation of the text and the understanding of the project were developed in a hermeneutic spiral, where the relation between the parts and the holistic picture was used (Gadamer, 1960). The first purpose was to listen to the story told and to get a description of the work. The first reading activates one's memory but not in detail as the intention is to gain an overview. The second reading starts the reflection on the different parts of the project. The readers must return to the different project reports to check if the description still fits into their overall image of the project. Much time was spent on reading, rereading, checking and discussing. In the time when not focusing directly on the texts, reflection starts and questions are formulated for the next return to the text. The mode of reading thus changes. The answers are not easy to find at once, so the reading starts all over again with a new focus. The process goes in circles or as in a spiral. In the condensed description we decided to include the purpose and aim of the parts of KAM, the processes and data collection made, and the results.

For the analysis in stage 3 we use the theory of developmental research also presented earlier. In stage 4 again, we interpret the report texts in a hermeneutic way in order to reach understanding of presented aims and outcomes both in detail and from a more holistic perspective.

Methodological discussion and justification of methods used in this article

In order to interpret and create understanding of such a large and long-lasting project as KAM there are no self-evident methods to use. We had to struggle before we made decisions about use of theories. Our discussions will be made at least partly visible here.

The theory of legitimate peripheral participation in a community of practice (Lave & Wenger, 1991) could be considered to be used in this study. The discussion led us to realise that there was no existing community of practice with stable conditions for the teachers to enter into in the kind of school we are investigating. For both the mathematics and the vocational teachers the new situation means that they have to face new demands and there are no established practices for them to enter. The concept of apprenticeship might be used as an instrument for the analysis, but the project we are analysing is not organised as an apprenticeship model so this would not be adequate in our study. In reading the reports we found that the concept constraints had been used spontaneously and it was thus natural to use the theory of affordances and constraints, particularly as there were strong design aspects to the project. The reports from KAM talked about problems, challenges and opportunities, which seems to align well with Kennewell's theoretical views of affordances and constraints (Kennewell, 2001).

From the overview of the whole of the KAM-project it was clear that the project is a developmental research study. This was of course not obvious at the beginning of the project when the development parts dominated and the practical aims were in the major focus. Later the alternation between developmental work and theoretical influence became more explicit. We find it productive to interpret the process in the parts of the project in terms of developmental cycles, where empirical parts nurture the research influence of the project.

It is rare to find such delayed analyses of a developmental research project as the one we have made in this article. In the light of discussions about the usefulness and justifiability of educational research (Burkhard & Schoenfelt, 2003) we find this kind of investigation helpful and worthwhile. Especially as the structure for vocational education in upper secondary school is going to change again in 2011, this article can inform teachers and policy makers about possible actions in order to improve the mathematics in vocational education.

Participants in the different parts of the KAM-project

The KAM-project took place in an upper secondary school with vocational programmes. Involved in the project were university lecturers, a scientific leader, the project leader who was one of the mathematics teachers, other mathematics teachers, teachers in Swedish language, vehicular mechanics teachers and transport teachers. Furthermore, classes from the Vehicle programme and occasionally student teachers took part. All the teachers from the upper secondary school were the normal teachers in the school for the pupils in those programmes. The teachers who took part were practitioners that worked with the same groups of pupils. See table 2.

Project Part	**KAM 1**	**KAM 2**	**KAM 3**
Year	98-99	99-01	01-02
Mathematics teachers	3-5	3	3
Vocational teachers	2-5	5+3	3
Other teachers	2	1+1	1
Student teachers	2		
Pupils	2 classes (30 pupils)	1 class (14 pupils)	120 pupils
Researchers	2	2	2

Table 2 Number of participants in KAM 1-3 parts

Results from the analysis in step 1, the condensed descriptions of the project parts

In this section we describe in a condensed way the different parts of the KAM-project. In each case we include the purpose, aims, carrying out, data collection and results. Our main sources are the KAM project reports, i.e. the KAM 1-report (two reports, KAM 1:1 and KAM 1:2), the KAM 2- and KAM 3-reports.

Attention has been paid to research ethical aspects during the entire project and the following quote from the report of KAM 3 (p. 8) has among other things been a guiding star in the work:

> The project has been carried out and been accounted for in a fair manner, i.e. the scientific demands and accepted ethical norms are respected. The researchers also observe an ethically acceptable behaviour in their relations with the surrounding world. This includes for instance to avoid making public statements in the name of science regarding issues for which one lacks scientific competence. The equality between women and men is a moral value to protect. Equality in the scientific community is to be viewed as an ethical issue when it comes to research. The research activity should be documented in an open way to enable outside examiners to follow the entire process (Forsman, 1997).

The theoretical founding idea for the three parts of the KAM-project was collaboration between teachers in order to be able to integrate the course mathematics A and vocational subjects. The students were supposed to experience relevant knowledge as one unit to get a holistic impression of the knowledge for their vocation.

Report	Theoretical framework/ tacit or communicated	Methods /Data collection	Number of pages	Authors
KAM 1:1 KAM 1:2		Tests Analyses of syllabi, materials, text-books, etc	11 42	Kilborn & Maerker
KAM 2	Situated cognition and learning, ethnography	Study visits, Classroom Observations, Field notes, Interviews, Recordings, Tests, Questionnaires	31	Grevholm, Lindberg, & Maerker
KAM 2 attachment		Summary of test results, examples of planning- and teaching material, examples of pre-tests, teaching analyses, classroom observations, reflections, recordings and diaries	78	Grevholm, Lindberg, & Maerker
KAM 3	Action research and ethnography	Renewed analyses, of materials, text-books, etc Tests, Interviews, Observations, Field notes, Questionnaires, Surveys with quantitative and qualitative data	41	Grevholm, Lindberg, & Maerker

Table 3 Overview of printed reports from KAM

We now present the short accounts of the three parts of KAM following the structure sketched in table 3.

KAM 1

Purpose

According to the reports from KAM 1 (KAM 1:1, 1999, KAM 1: 2, 1999), one purpose was to work out mathematical models, where the students could apply and develop the mathematics, via co-operation between mathematics teachers and vocational teachers in a systematic way. Those models should be more appropriate and better suited to the students' preknowledge and abilities. Another purpose was to analyse what content in mathematics that is required to be used as a tool for students in vocational programmes where only mathematics course A is compulsory. One more aim was to analyse how mathematics teacher education could better prepare teachers to adapt to the reality of the schools, especially for vocational programmes.

Aim

In KAM 1, one aim was to make the pupils feel that mathematics is meaningful by starting with problems directly related to the vocational courses. In the long run the aim was to see mathematics more intertwined with the vocational subject. Another aim was to help the pupils to see the relevance of mathematics to manage to solve tasks in the vocational subject that they had previously had difficulty mastering. One more aim was to start from the pupils' level and to let them work in their own pace. In achieving those aims the pupils would get a better chance to achieve a pass grade in mathematics.

Carrying out and data collection

The project had scheduled time for meetings and individual work for the project members. In the beginning the vocational teachers informed the mathematics teachers about the content and how they taught this to the pupils. Then the mathematics teachers analysed this content and compared it to the content in mathematics, course A. After that they came back and discussed this with the vocational teachers to find out if the vocational teachers thought that the ideas of changing the explanation and procedures could work for the pupils. From the work of gear ratio and changes in revolutions for example there was a need to work with fractions, proportionality and estimations via mental calculations (KAM 1:1, 1999, p. 6). The teachers trialled the teaching material with some groups of pupils. Much of this work was done in order for the teachers to use the same words/concepts when teaching the pupils. The focus group during KAM 1 was mainly the pupils. Textbooks, teaching material as tools and manuals were part of the data collection. These data were used for the analyses in the project. Both the tests used and the results for testing the students' standard were collected.

Results

The project produced material (KAM 1:1, 1999, p. 5) for in service training for the teachers in a school. Thus the vocational teachers would be able to show the pupils the relevance of using mathematics in the vocational subjects and vice versa. This material was later used as teaching material in one class. Another class was used as a control group. The pupils' results from the tests showed an improvement (on short term basis) when using the mathematical tool to understand the element gear ratio and their motivation increased (ibid, p. 18).

KAM 2

Purpose

In KAM 2 (KAM 2, 2001) one purpose was to continue in the same way as in KAM 1 to build a coherent platform out of content and teaching/learning in some vocational courses as Vehicle, basic course, Pneumatics and Electricity year 1 and the core subjects Mathematics, course A and Swedish, course A and B. Another purpose was to have in-service training for the teachers in the programme enabling the pupils to feel this coherence and to become more motivated to study parts in their programmes where mathematics was involved.

Aim

KAM 2 continued from KAM 1 to analyse mathematical models and teaching methods in one upper secondary school in vocational programmes regarding the mathematics that is present in course A, using a process oriented approach. The aim was to use cooperation between teachers and together develop the teaching content and new approaches to improve the interest and success of the pupils.

Carrying out and data collection

Part 2 of the KAM-project was an extension of part 1 of the KAM-project and mainly aimed at the teachers carrying out a competence development programme through working together as a team to plan the programme for the pupils. When analysing the Vehicle electricity course many of the goals of mathematics, course A, became very visible. For example, "to have deepened and extended their understanding of numbers to cover real numbers written in different forms" and "with and without technical aids, be able to apply with judgement their knowledge of different forms of numerical calculations", and "to be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems" (Lpf94, 1994).

Through the classroom observations questions were raised about the teaching both in relation to the subject and the didactics and these were discussed in sessions with the teachers. To facilitate the subsequent discussions the teachers used tape recorders to report about and reflect on the classroom activities. The teachers also maintained a written diary periodically documenting the work during the project. During the development of the KAM-project the need for evaluation arose, both of the pupils and of the teachers. The pupils' evaluation was carried out in cooperation with the teacher in Swedish. The teacher evaluation took place through portfolios. Based on these, the teachers received feedback about their own teaching which otherwise could be hard to obtain. Qualitative evaluation completed involved interviews of teachers and students. There were even visits and studies out in the motor work shops.

Results

The collaboration between teachers of the core subjects and the vocational subjects improved during the course of the project. The teachers were interested in taking part in the planning of each others' subjects. The results show that if the subjects are taught with mutual support then the pupils will be more successful in their studies of the theoretical parts in all of the involved items. But it is important that the pupils understand the coherence in the content and in the organization of the education for the results to be a success. The teachers involved in the project noticed that it brought a better instruction and improved learning, but that the process was slow (KAM 2, p. 23). A great deal of the time was used for discussions, and the communication process was further developed from the KAM-project part 1 as the mutual language developed. Some of the teachers expressed pride in the work that had been done in the project in their evaluation of this project (ibid., p. 23). The work with discussions and carrying out change, demands proceeding with caution as the project takes place at the same time as all the other activities that teachers work with on a daily base. At the same time, it is important to be purposeful and to have the goal visible during the work even when the process takes a long time (ibid, p. 27).

KAM 3

Purpose

The purpose of the work in KAM 3 was to extend the work of KAM 1 and KAM 2 in all the six classes in grade 1 at the school thus scaling up the project.

Aims

From the results from KAM 1 and KAM 2, one aim was to study the strengths and weaknesses of the intervention when up-scaling. Another aim was to widen the coordination with core courses and mathematics. A further aim was to revise the earlier produced teaching material and to further investigate teaching methods and teachers' competences.

Carrying out and data collection

More and more the project was carried out as an action research project (Rönnerman, 2010) with a flavour of ethnography (Cohen & Manion, 1994; Bessot & Ridgeway, 2000) where the focus was the activities related to the subjects visible in the teaching/learning situation. The lessons were implemented as developed in KAM 2. In the course Vehicular Electricity Course A, some of the aims for Mathematics Course A were the same, i.e. number sense, to interpret and handle formulas and they were carried out there. In Vehicular Mechanics Basic Course simple solving of equations was illustrated with the concept of torque. In Transport Vehicle

Basic Course the aim was to work with the concept of velocity and units for length and time were in focus (KAM 3, pp. 10-11).

The lessons were followed up in seminars where teachers representing the different subjects took part. This was partly for revision of the material from the lessons but also as planned in-service training / professional learning for the teachers where they learnt more about the didactics of the different subjects. Observations in classrooms and workshops were documented as field notes and annotated after discussions with the respective teacher (ibid., p. 12). Qualitative analysis was used here and where the pupils were asked about their attitude towards the subject (ibid., pp. 20-22). The scientific leader interviewed the teachers and the principals of the school during the project (ibid, pp. 28-36). The same analysis method was used for the teachers' evaluation where they wrote in an open ended form about their views of the project and the result was summarized (ibid, p. 23). Quantitative analysis was used where the aim was to test the students' knowledge before and after a specific unit of work (ibid, pp. 13-20).

Results

The increase in scale of the project occurred in fewer classes than planned mainly because the school reorganised and the school leaders became less supportive. The survey showed that the students improved in mathematics, but even more in the vocational subject, when they were exposed to the KAM-model (ibid, p. 19). The interviews with the teachers indicated that the project required support from a solid organisation and support from the head of the school (ibid, p. 23). The lack of support in carrying out the project could be viewed as a weakness. The strength was the teachers' beliefs and engagement in the project (ibid, p. 23).

Results from stage 2 of the analysis - Affordances and constraints in the project

Using Kennewell's model we can now analyse the project. In the table we have listed affordances and constraints mentioned in the reports.

	Affordances	**Constraints**
1	New curriculum and new syllabus	Pupils' prerequisite knowledge
2	School based	Different teacher cultures –different didactical contract/ teacher role
3	Teacher education	Didactical competence
4	School management	New school leader
5	Schedule/Timetable	Schedule/timetable
6	Routines	Organisation
7	Teacher learning	Substitution of teachers
8	Money	Lack of money or resources
9	Time	Lack of time or simultaneous time

Table 4 Affordances and Constraints

We will start by giving examples of affordances, i.e., potential or possibilities for action. The affordance is that the curriculum (Lpf94, 1994) gives every pupil opportunities "to solve mathematical problems of importance for vocational and daily life" (Lpf94, 1994), and "Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course." (Lpf94, 1994). In the teaching situation there should be cooperation between the mathematics and vocational teachers to give unity or wholeness for the pupils.

In chapter 1.2, Common tasks for the non-compulsory school system, it says: “Developments in working life mean inter alia that traditional boundaries between vocational areas need to be revised and that demands are imposed on our awareness of not only our own but also the competence of others. This in its turn imposes demands on the school/s working structure and organisations” (Lpf94, 2006, p.6) Further in the section Knowledge and Learning it says “The pupil’s acquisition of knowledge is dependent on developing the ability to see interconnections.” And in the section Development of the Individual School: “the school shall attempt to arrive at flexible solutions for its organisation, range of courses and working structures. Co-operation with the compulsory school, and universities and university colleges shall be developed as shall the co-operation between non-compulsory schools.” (Lpf94, 2006, p. 7). The affordances in this paragraph are related to the curriculum but as the implementation of the curriculum was the aim of the project they also became affordances in the project.

There is one specific chapter of the curriculum stating the responsibilities of the school head: “The school head is responsible for the school results and thus has, within certain limits, special responsibility for ensuring that: Teachers and other personal receive opportunities for the development of competence required for them to be able to carry out their tasks professionally” (Lpf94, 2006, p. 18) In reports this came to be an example of the constraints in this project. “Half of the teachers who were scheduled to teach in the courses involved in the project had not taken part in the in service courses for the project” (KAM3, 2002, p. 24).

The constraints arose mostly during the daily work of the project. Critical aspects were the tensions between the project leader and the school management, when the project leader asked for pedagogically motivated changes, which were uncomfortable for the school leaders (ibid., p. 35). Further organisational changes decided by the school management became obstacles to achieving the project aims and this also created tensions and frustration.

One constraint, i.e. the hindrances and problems for the projects, as well as being an affordance is the new curriculum. The pupils were not aware that they had to study more mathematics when going to a vocational programme. This could show as lack in motivation. In the former programme the mathematics had only been calculations that were relevant to do in the workshop or directly related to the vocational subject and this was taught by the vocational teacher. Now the pupils had to be taught by a mathematics teacher and this could be a problem because of the students’ expectations (KAM 3, 2002, p.32-33).

Many of the students also had poor prerequisites in mathematics as documented by tests (KAM 1:2, 1999, pp. 8-14) at the beginning of the project part KAM 1. The teachers came from different cultures and had different didactical contracts which gave rise to different teacher roles (KAM 1:2, 1999, p. 20). The teachers had different didactical competence as they had taken different teacher education programmes with different content (KAM 3, 2002, p. 27). During the project there were exchanges among teachers some of whom were not aware of the project and its purpose (KAM 3, 2002, p. 24. p. 28).

The school leaders did not support the organisation of the project by supporting changes in schedule/timetable when needed. “These changes of the leaders gave rise to many changes of the organisations at the school.” (KAM 3, 2002, p. 26) The school management was not part of the project and did not feel involved. The project took more time than expected. Here is one statement about that:”The project is so enormously big, so many involved. There is a KAM-project meeting every week.” (ibid., p. 31).

To summarize this section it is fair to say that the constraints related to the teachers were expected and taken in consideration, but the reactions and restrictions from the school leaders had not been predicted. Among the affordances at the school were that all the teachers, but one, who were participating in the project, were qualified teachers with many years of practice in the school system in their specific subjects. One teacher had many years of teaching and he obtained his teacher qualification during the project (KAM 2, attachment 3, p. 11). He joined the project during his studies. The school had access to computers, tape recorders and on the premises there were well equipped workshops for car mechanics. In the workshops and garages there was much material to use to demonstrate and build models that could be used in mathematics education. The school had recently been involved in an ICT project to develop the ICT competence among the teachers and the school had a well equipped computer room (KAM 1:1, p.10). There was also one teacher who was responsible for this room who taught in the computer subject (ibid., p. 1; KAM 1:2, p.5).

From the table it is obvious that some features can be seen as both an affordance and a constraint. For example, the new curriculum is the affordance that initiates and inspires the project as such. But the curriculum also represents some limiting structures and creates obstacles. When pupils' prerequisite knowledge is too far from what is expected it becomes difficult for the teachers to reach the goals of the curriculum. Thus affordances and constraints can in some cases be seen as two sides of the same coin.

Results from the analysis in stage 3. The developmental cycles in the KAM-project

This section shows how the KAM-project became visible as a developmental research project. The iteration of empirical and practical development work intertwined with theoretical discussions and reflections is illustrated below.

A pre-project of KAM, called DUGA (1996), was initiated and driven by teacher educators together with student teachers. Teacher educators felt the need to find out how the integration of mathematics and vocational subjects could be organised with the help of the mathematical content in course A and what the needs were for mathematics in vocational subjects. The opportunities for integration of subjects and to teach the new content in course A to pupils in vocational programmes were to be considered and given form in practice. There was a wish to document what kind of mathematics was needed in the vocational subjects. Another aim was to construct new teaching materials and suggest relevant applications in the vocational programmes. The planned development was inspired by the theoretical conditions given by the new syllabus in Lpf94. The demands for changes can be seen as theory based prescriptions in the form of the new curriculum, which had to be taken into practice by both teacher education staff and in teachers' daily teaching in school. The DUGA-project was not fully completed but left a number of questions to be answered by the teacher educators. Some of these questions (KAM 1.1, 1999, p. 2) seem to have triggered the first part of the KAM-project.

In KAM 1 the developmental research character of the project becomes explicit. The aim was to study whether the pupils would succeed better in the studies when the content of mathematics was integrated into the vocational subjects and vice versa. In order to answer that question some systematic investigations were completed and documented. Another part of the aim was to construct developmental material to be used both for competence development of the teachers and for teaching in one class to gain experience, while another class was used as a control group. In the construction process, theoretical insights were used and inspired the development component. The empirical and practical knowledge from teachers combined with the theoretical knowledge that teacher educators brought from research in mathematics education. In the developing learning community (Jaworski, 2002) the shared knowledge of all parts contributed to the construction of teaching material. That learning process was a learning situation for both the teachers in school and the teacher educators.

In this part of the KAM-project the major focus was the improvement of the students' learning of mathematics with the teachers of mathematics and vocational subjects collaborating to achieve this through the creation of learning opportunities.

It seemed as if the theoretical goals expressed in the new curriculum were accepted by the teachers and they were prepared to strive to reach the goals with the tools that were suggested in the plan that is integration of the subjects. In the official booklet about programme goals (1997) it is stressed that the applications in mathematics should be locally adapted to the pupils' study direction/ programme and to the pre-knowledge, interest and needs of the pupils (Skolverket, 1997, p. 107). This met the requirements of the Lpf94 to adapt the mathematics to the students' study programme and for the local work plans to respect mathematical knowledge in the vocational subjects and ensure that the applications in mathematics arise from the vocation thus making mathematics meaningful for the students (Skolverket, 1997). Thus we implicitly see theoretical foundations inspiring the development work in this first developmental cycle of the project. In the report from KAM 1 there is a list of references but these are not mentioned specifically in the text as such. Rather, we interpret it to be a list of theoretical sources that were used in the project. Most of the references are non-scientific works, such as popular scientific papers or books, or books used in mathematics teacher education. One doctoral thesis is mentioned.

In the report from KAM 2 the theoretical foundations were more explicit. The report included a section that discussed the theoretical background and referred to relevant research reports. Situated knowledge and learning was discussed and mathematics in vocational education was seen as activity (KAM 2, 2001, p. 9). The teacher's role as guide and builder of bridges was emphasised as was the importance of the knowledge of the teacher when it comes to motivate the pupils for studies that is to make the studies meaningful for them (ibid., p. 10; Pehkonen, 2001). These theoretical views influenced the work of the project and became visible, for example, in the process-oriented work-forms that were used (ibid., p. 17). Among the experiences mentioned as an important component was giving the teachers opportunities at several occasions to discuss changes and let their thoughts mature and develop over time. As a growing community of practice the teachers engaged in competence development, where they discussed the working material that had been developed and visited the motor workshops (ibid., p. 20-21; Jaworski, 2002).

In the next cycle of the KAM-project, reported in KAM 3 (2002), the main aim was to upscale the project, that is to involve all six classes in the school in the kind of work that earlier had been carried out in smaller proportions. In this process the importance of documenting the results was recognised and thus multimodal data collection was carried out. Both quantitative and qualitative methods were used in order to analyse observations, test results, questionnaires and interviews. The report indicates that the methods were similar to the ones used in action research with an ethnographic view (ibid., p. 7). The activities in the respective subjects were in focus in the study. The choice of methods was guided by a wish to report the outcomes of the project from many perspectives. Up-scaling of smaller case studies is not well documented in the research but had been requested by the teachers (Adler et al, 2005). Not much guidance about how the up-scaling can be carried out can be found in the literature. In the case of KAM 3 a number of complicating factors evolved and created obstacles for the final phase of the project. The practical outcomes that were documented in this up-scaling process can inform theories in mathematics education research about some of the possible affordances and constraints that influence the up-scaling process. The conclusion is still that it is possible to carry out local school development but it demands that both school leaders and teachers have a strong will to create change and a conviction that they work for the best of the pupils (ibid., p. 36). Developmental projects of this kind have to be allowed to take time, demand careful planning and have funding for some resources.

It was not until we had an opportunity to revisit the project and study the reports and documents again from a more long term perspective, that it became clear to us that the KAM-project must be seen as a developmental research study. While we were closely involved in the KAM- project and when the data and analysis was not complete it was difficult to see the character of the project. Thus we find it rewarding to be able to study all the documents and describe and analyse what was actually going on, now from a post-project reflective research perspective. The KAM-project over time evolved to become a school project that was more and more research based and it is clear that the reports developed to become more systematic, structured and linked to earlier documented research.

Results from the analysis in stage 4, Results of the KAM-project related to the aims

Stage 4 is an analysis of the results of the KAM-project in relation to the aims. The KAM-project was, as mentioned before, completed in three parts: KAM 1, KAM 2 and KAM 3. The overarching aim was to teach mathematics in ways that could enhance the pupils' mathematical knowledge in vocational programmes. Pupils' grades were expected to improve with the result of an improvement in the pass rate. The purpose was to make stronger connections between mathematics and vocational subjects to convince pupils of the relevance of mathematics.

Aims and results of the first part, KAM 1

According to the reports from KAM 1 (KAM 1:1, p. 1) the aims were

1. "to increase and deepen the pupils´ mathematical knowledge in the vocational programmes in the upper secondary school in a national perspective" and
2. "to analyse the real need of mathematics in different vocational subjects". One important part in the project was
3. "to bring about a systematic co-operation between the vocational teachers and the core subjects teachers".

The aim number 1 was to help pupils to get higher grades in mathematics and better understanding of the role of mathematics in the vocation. The tool to get there as stated in number 2 was to analyse the vocational subjects and compare with the content of mathematics, Course A. This process was pursued in collaboration between vocational teachers and mathematics teachers as stated in aim number 3. An implicit aim number 4 was also to come up with a model for in-service training for teachers in a school. One model was worked out to give strategies and methods to work together and some teaching material was produced. The teachers tested out teaching material in some groups of pupils. Most of this work was done in order to educate the teachers to use the same words/concepts when teaching the pupils. Gradually this cooperation became a routine that worked. Some of the aims as number 3 and 4 are very wide-ranging and it is hard to see if they have been achieved.

Concerning aim number 2, some analysis of the need for mathematics in vocational subjects was carried out during this phase but there was no clear outcome from this other than that the mathematics in course A was used in different ways in the vocational subjects.

For aim 1, some tests showed progress for pupils but the tested group was very small and it is hard to see whether pupils really increased and deepened their knowledge in this short period of time. Additionally the simple written tests evaluated only limited aspects of the mathematical learning process. There is no convincing evidence of pupils' lasting, improved knowledge. Some evidence of fruitful cooperation between vocational and mathematics teachers exists. The intention to develop a course to be used in teacher education was not fulfilled in this part.

Aims and results of the second part, KAM 2

In KAM 2 (2001) the aim (1.) was to get a common platform for three vocational courses and mathematics, course A, and in some cases Swedish, course A and B. This meant that all teachers were expected to work via integration in the subjects involving mathematics teachers, vocational teachers and one Swedish teacher. The cooperation was intended to occur in the vehicle programme, year 1, in one school. The aim was to work with a process orientation. Another aim

(2.) was to develop competences for the teachers in this work. The pupils were intended to perceive their studies as something united and holistic and to feel more motivated to study mathematics in their programme.

In KAM 2 the intention was to continue in the same way as in KAM 1, i.e. to analyse mathematical models and teaching methods in one upper secondary school in vocational programmes regarding the mathematics that was present in course A using a process oriented approach. The aim partly differed from that of the initial plan because the project received less funding than was requested. In order to evaluate the results the questions below were posed in the report of KAM 2 (p. 11-13). We try to answer the questions in connection to this text.

- *What change was there in the pupils′ understanding of the vocational subjects if the mathematics teaching was integrated in the content of the core subject?*

  The report claims a small enhancement but there is no significant evidence for that. It is pointed out that it is necessary for the pupils to realise the connection between the subjects. The analyses of the project indicates that there is no significant evidence for pupils' improved understanding due to the fact that the time that elapsed in the project was very short and the number of participants was small.

- *What were the changes for the teachers when the mathematics teaching was integrated with the content of the core subject?*

  Obstacles for the cooperation between vocational and mathematics teachers were observed. The teachers declared that they gained better insight in the content of the different subjects and were positive to the working procedures. This outcome was demonstrated only in teachers' written and verbal reports and not in any actions observed in the classrooms. The teachers were offered opportunities to exercise collaboration. The fact that teachers declared the positive outcome of the project in terms of gaining better insight in the different subjects is very challenging. That this was not observed is not because it might not exist, but there was not enough recourses in the project to follow up with more classroom observations.

- *What was the effect of tutoring the teachers in connection to the activities taking place in the classroom for the developmental change of teachers?*

  The teachers were able to reflect more about the effect of integration. Their reflections were documented by using portable recorders. This recording was more efficient and convincing than for them to write down the reflections.

- *How did the teachers and the leaders in the school consider the project and the work with integration between the subjects?*

  Teachers who were involved in the project from the start said that the project resulted in a better teaching and learning situation, but that the process was slow. They claimed that they were proud of what they achieved. The leaders in the school supported the project but there were changes in the organisation. The new leaders were not able to understand the process and how the development sometimes created conflicts in the daily work. This result is very important as it points out the school leaders' role as part of development in the schools. When there is a change of school leaders it becomes crucial that they commit themselves to what has been decided and to follow through.

- *How could a portfolio be used as a method of evaluation in subject integrated work? In what way could the Swedish subject support the pupils?*

  When the pupils were using the portfolios the teachers received spontaneous feedback from the pupils' writing. The students' writing indicated small improvements. This was a very good help for the teachers to see how the pupils could make the transfer from one subject to another. The whole idea of the cooperation and integration was to help the pupils to see the connections between the subjects and as a result get a better understanding.

What was the result of this part of the project compared to the aim?

The report showed that co-operation and collaboration worked to a greater extent than what was normally the case in schools. Teachers shared their knowledge and competence within and between the groups and felt that they achieved wider competence. The result was better shared understanding of the connection between core subjects and the vocational subjects. The teachers also produced new integrated teaching material. One aim was to develop competences for the teachers in this work so that the pupils experience their studies as something united and holistic and for them to feel more motivated to study mathematics in their programme. There is evidence that the pupils felt more motivated as they asked for more KAM-mathematics. They experienced what they called KAM-mathematics as something different and more useful.

The report also concluded (KAM 2, 2001, p. 27) that it is possible to achieve the aims and goals of the curriculum for mathematics in the vocational programmes. This demands that pupils have a pass result from the compulsory school. The integration of subjects facilitated pupils' opportunities to achieve the goals. During the project much time and effort was spent on communicating the project at conferences and meetings to raise the awareness of the needs in this area as well as to share findings. This was not articulated as an aim in the project, but it grew naturally to discuss and value the relevance for this kind of project. It was of course expected of the main funding body 'Skolverket' that dissemination from the project would take place.

Aims and results of the third part, KAM 3

One of the aims in this part was to build further on the experiences from KAM 1 and KAM 2. The ideas and material produced earlier were to be tested in all classes in grade 1 (year 10 of school but grade 1 in the Vocational Programme) at this school. The second aim was to use the same model in grade 2 (year 11 of school). This part included the making of an inventory of the material, and documenting the teaching methods and teacher competences in a similar way to the approach in KAM 2. The first aim was to scale up the developmental work and the second aim was to implement the work model again.

The first aim was partially achieved and the second one was just executed to a low degree. The challenges and problems which arose during KAM 3 are presented and discussed in the report. Among them were:

- a reorganisation of the school,
- several new school leaders,
- changed timetable and class- and group-structure, and
- new teachers not informed about the earlier parts of the project.

Still the teachers had faith in the project and fought to continue it. The processes and ideas from the earlier work in KAM 1 and 2 could be implemented in a larger group of pupils but not in all classes in year 1 as was the aim. The work planned in year 2 could not be carried out.

Implications of the results in KAM 3

Cooperation between teachers of mathematics and vocational subjects promotes pupils' development in vocational education. Motivation was raised and better learning outcomes in both mathematics and the vocational subjects were achieved according to the teachers' evaluations. Cooperation demanded from teachers a widened competence and a willingness on the part of a teacher to both enter into the work of colleagues and allow them to enter into the teacher's own work. The reward was greater work satisfaction and a stronger feeling of being able to support pupils' learning. There was a demand for new evaluation methods and flexibility. Teachers needed to be open to a great variety in methods, tools and models of explanation. An important condition was support from the school management. Long term and sustainable planning is also important if such change is to become part of the culture of the school.

The KAM-project was disseminated at different meetings and conferences. (See appendix 2).

Discussion and conclusions

In revisiting the project in which we participated between 1998 and 2002 we have tried to answer the following questions:

- What are the characteristic features of the school based developmental research project KAM?
- What characterises the affordances and constraints that the involved participants in such a project meet during the work?
- What were the actual outcomes of the KAM-project compared to the aims?
- What conclusions and implications can be drawn from this kind of developmental research project?

One evident characteristic feature of the KAM-project is that the project started as a school based development project initiated by teacher educators, grew into a more teacher-driven project and became more and more a developmental research project, where theory and practice were interwoven in a fruitful iteration process in the different components of the project. The teachers and researchers collaborated in all aspects of the project. In the beginning, the theoretical driving force was the new curriculum and the integration of vocational education programmes into the general upper secondary school system. In later stages, when more refined questions about integration of mathematics and vocational subjects were studied, mathematics education research provided the theory that inspired the project. The character of the reports developed from more descriptive narratives (without scientific references) in the early reports into papers and reports which followed the structure of a research paper, establishing theory and referencing literature.

Another feature is that unexpected affordances and constraints became noticeable as the project progressed. The established goals and aims of the project were not all realistic. When teachers are expected to do work that they never have done before (like for example developing textbook material) the completion of the task and hence the aim may be unattainable. But when teachers are expected to develop and reflect upon their daily teaching in a community of practice with other teachers the outcome is valuable and important and the knowledge thus gained can be used by them. The work in the community of teachers of mathematics and vocational subjects has shown that it is possible to integrate the teaching in those subjects. It also shows that if such integration is practised, it improves the outcomes for the students and increases the efficiency of the work and its value for the teachers. The small scale case of integration was convincing for the teachers and researchers but in trying to up scale of the intervention to all classes in a school there were too many unexpected obstacles and factors which impeded the project.

The project also shows that the decision from a historical point of view to combine the more theoretical programmes and the vocational programmes in upper secondary school in one school in a form with the same length and structure (Lindberg, 2009) can create a basis for different groups of teachers to collaborate and share each others' knowledge. In this combined school organisation different kinds of knowledge (sometimes called theoretical and practical) are visibly given the same value. In fact the vocational subjects are valued more highly than the theoretical as they may cost much more to teach, are taught in smaller groups (15 instead of 30 pupils) and have a visible impact on the students' future vocations. The integration of mathematics and vocational teaching would be more difficult to implement if the programmes were more separate in the school organisation. The groups of teachers confirmed that the mathematics content in course A is useful and needed for the learning of the vocational subject. Together they convinced the pupils that they were guiding them in the same direction, towards learning for adult life, both in their vocation and for full participation in a democratic society. It must be seen as both appropriate and possible to develop theoretical and vocational paths of education to become equivalent in length and structure as in Lpf94.

The parts of the project that dealt with identifying pupils' learning show that most of the pupils lack the expected pre-knowledge from the compulsory school. For example the teaching material produced as appropriate for the learning of fractions was about content which was part of teaching in year five and six in compulsory school. It seems clear that many of the pupils in the vocational programmes have received a pass in mathematics from compulsory school implying that they have attained knowledge from the curriculum but which does not equate to the knowledge they carry with them to upper secondary school. The fact that pupils do not acquire the expected knowledge in compulsory school then leads to the creation of problems in upper secondary school. The mathematics course A is blamed for the failure of the pupils, but in fact the failure happened already early in compulsory school. The wise thing to do is not to reduce the mathematics in course A, but to focus on the compulsory school ensuring that the pupils will be offered adequate learning opportunities so they enter upper secondary school with relevant pre-requisites. Pupils need appropriate mathematics for their vocational education and later for professional life.

The most influential affordances and constraints for the work in the KAM-project were the opportunities that were created by the new curriculum and school organisation, the support from school management and leaders, the organisation of time schedule and other resources for teachers, and the competence of teachers. It is obvious that the new curriculum meant that teachers were faced with new expectations and thus were ready to take action. Other studies indicate the upper secondary mathematics teachers in many cases tend to be loyal to the educational system (Kleve, 2007; Hundeland, 2010). The KAM case also convinces us that the full support of the school management is absolutely crucial for the success of a project. When during KAM, part 3, the organisation was changed in ways that did not support the project, this effectively prevented some of the work on the project, and hence aims of the project could not be met.

The competence of the teachers is central for the success of a development project and in this case the professional learning that was offered inside the project was well received and seen as meaningful learning for the teachers. Their experience in the project was that they could implement in the classroom what was learnt directly through their work on the KAM project. They had influence on the content of the teaching programme and had control of what was offered in the teacher seminars. The ownership of the project work is thus important.

Integration between mathematics and the vocational subjects does not occur naturally. There is a need for focussed professional learning opportunities for teachers involved in both the mathematics Course A and the vocational teachers. The KAM-project convinces us that teachers are willing and eager to be part of such competence development related to their daily teaching and they are able to design collaboratively challenging interventions and involve their pupils in a constructive manner.

Thus, one overarching theme of the project was the collaboration. The models for collaboration can differ depending on the purpose. Some models have been described in the report "Villkor och vägar för grundläggande yrkesutbildning" (2001), ("Conditions and paths for basic vocational education"). Collaboration does not necessarily occur automatically. But in periods of change as in the implementation of a new curriculum, an important way of working is to involve teachers in discussing the consequences of this new curriculum for education. Accordingly, the teachers can perceive their profession not to be a one-person profession but a profession where people work together daily. In the KAM-project another underlying motive was to encourage the pupils to act cooperatively. Some collaboration also took place in occasional activities in the form of project work and thematic work. These ideas have emanated from the cooperation which was established between the teachers from the separate subjects. This was an essential part of the KAM project. Rather than emanating just from one subject didactical point of view, the planned curriculum in the particular vocational programmes in KAM was designed cooperatively.

What were the actual outcomes from the KAM-project compared to the set aims and goals? From the analysis in stage 4 the most important conclusion is that there is convincing evidence that it is possible to integrate the teaching of mathematics with the teaching of vocational subjects. When teachers collaborate in such ways it is more demanding for them but they perceive it as more fruitful and meaningful. There is no clear indication that the learning outcome of pupils improved, mainly because the studied groups were too small and there was no opportunity to follow up the development over time. The short term tests indicate some improvement, but this might have been the case even if no intervention had taken place. More clear is the evidence that the pupils' motivation has improved and that they subjectively experience what they call "KAM-mathematics" as more meaningful and relevant. They are also able to see the coherence of the studies in mathematics and vocational subjects. As research has shown, pupils' beliefs about mathematics influence their learning opportunities (Pehkonen, 2003). This is an important finding.

The collaborative production of teaching materials was most important as a tool to trigger the teachers to open for discussion about their actual teaching. Teachers' reflections about their teaching and the written documentation created opportunities for deeper investigations of how the subjects could be better connected and seen by pupils as a united whole rather than isolated parts. There is very little research on mathematics in vocational education and more studies are needed and necessary for the development of the teaching and learning, which is perceived by pupils as a meaningful experience. The perspectives of pupils, their results and experiences from instruction and learning must be included in such studies.

Implications from the KAM-project

What are the conclusions and implications that can be drawn from this kind of developmental research project for future school based projects? A project of this kind needs support in two ways. One is support and encouragement from the leadership within the schools to facilitate the project and ensure local issues such as timetable do not impede the project. The school management should be involved in the project. This includes scheduling time for meetings. The other is finance to enable time for teachers, researchers and project participants to carry out the work. It is not possible to run such a project within normal school settings without the extra resources. Even with extra resources the demand on teachers is that they devote more time and effort than in the normal case to their work. The project indicates that teachers are willing to do so in order to get a more rewarding working situation and see increased motivated in pupils.

Skolverket (The National Agency for Education) funded the KAM-project but as far as we know they have not promoted the knowledge and results obtained by the project. Even if the participants in the project, as has been shown, have disseminated the project widely, it seems as if most Swedish teachers in vocational programmes know little about the findings. In the preparatory discussions for the new curriculum during 2008-2010, there has been no mention of the outcome as far as we know. Thus changes in the mathematics courses will be taken without any reference to what teachers learnt about opportunities to integrate mathematics and vocational subjects. One of the political suggestions, though, seems to be in line with the conclusions from KAM. Mathematics in compulsory school will be given one more teaching hour per week in years 7-9. Hopefully this can lead to a situation where the pass grades from compulsory school have real meaning demonstrating pupils' acquired knowledge.

The KAM-project also shows that, in order to gain deeper knowledge about what is actually going on in classrooms, there is a need to involve teachers in the research studies as equal partners offering them real opportunities to influence the project work. For that kind of study a developmental research design would be fruitful, where teachers and researchers collaborate closely, and let theory and practice inform each other in the cycles of the work. Future changes of curriculum could be based on this kind of developmental research studies. It is also a suitable way for carrying out analysis of the curriculum and creates a more long-term discussion about the content of the curriculum.

This project can also have an impact on how to organize teacher education when students are doing the practical studies or practice. There students can meet and discuss the relations between their different subjects and what impact it has on pupils' understanding if the students know more about the bridges and connections between different subjects. There could be specific tasks for the students to carry out in the practice period that focus on collaboration and integration for better learning opportunities for the pupils. This approach could also help teacher educators to be more involved in the school development.

Because this project pointed out the importance of the role of the school leader, it may be appropriate to research how they can support the implementation of the mathematics courses in the new curriculum GY2011.

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Appendix 1

Syllabus Mathematics Course A, for upper secondary school, from Lpf94

Below are the goals that pupils should have attained on completion of the course A. Pupils should:

- be able to formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation
- have deepened and extended their understanding of numbers to cover real numbers written in different forms
- be able to apply with judgement their knowledge of different forms of numerical calculations linked to everyday life and their study orientation, with and without technical aids.
- have an advanced knowledge of geometric concepts and be able to apply these to everyday situations and in the different subjects of their study orientation
- be sufficiently familiar with basic geometrical propositions and reasoning so that they understand and are able to use concepts and different ways of thinking in order to solve problems
- be able to interpret, critically examine and with discrimination illustrate statistical data, as well as be able to interpret and use common co-ordinates
- be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation
- be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems
- be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society
- be accustomed when solving problems to use computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes
- be familiar with how mathematics affects our culture in terms of, for example, architecture, design, music or the arts, as well as how mathematical models can describe processes and forms in nature. (Skolverket, 2005-11-02) (http: www.skolverket.se/skolfs?id=637)

Development of Mathematics from the compulsory school to course A

In table 1 below different aspects of mathematics, as it is written in the syllabus of mathematics, can be followed from school year five via school year 9 to course A.

Goals from the Syllabus	Goals that pupils should have attained by the end of the fifth year in school	Goals that pupils should have attained by the end of the ninth year in school	Goals that pupils should have attained on completion of the course A
A. Aims	Pupils should have acquired the basic knowledge in mathematics needed to be able to describe and manage situations, and	Pupils should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as solve problems that occur regularly	Pupils should: be able to formulate, analyse and solve mathematical problems of importance for everyday life and their

Goals from the Syllabus	Goals that pupils should have attained by the end of the fifth year in school	Goals that pupils should have attained by the end of the ninth year in school	Goals that pupils should have attained on completion of the course A
	also solve concrete problems in their immediate environment.	in the home and society, which is needed as a foundation for further education.	chosen study orientation be accustomed when solving problems to use computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes.
B. Numbers and operations on numbers	Within this framework, pupils should – have a basic understanding of numbers, covering natural numbers and simple numbers in fractions and decimal form, – understand and be able to use addition, subtraction, multiplication and division, as well as be able to discover numerical patterns and determine unknown numbers in simple formulae.	Within this framework, pupils should – have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form.	have deepened and extended their understanding of numbers to cover real numbers written in different forms.
C. Calculations in different modes	– be able to calculate in natural numbers – in their head, and by using written calculation methods and pocket calculators.	– have good skills in and be able to make estimates and calculations of natural numbers, numbers in decimal form, as well as percentages and proportions in their head, with the help of written calculation methods and technical aids.	with and without technical aids, be able to apply with judgement their knowledge of different forms of numerical calculations linked to everyday life and their study orientation.
D. Measuring, Geometry	– have a basic spatial understanding and be able to recognise and describe some of the important properties of geometrical figures and shapes, – be able to compare, estimate and measure length, area, volume, angles, quantities and time, as well as be able to use drawings and maps.	– be able to use methods, measuring systems and instruments to compare, estimate and determine length, area, volume, angles, quantities, points in time and time differences, – be able to reproduce and describe important properties of some common geometrical objects, as well as be able to interpret and use drawings and maps.	be sufficiently familiar with basic geometrical propositions and reasoning so that they understand and are able to use concepts and different ways of thinking in order to solve problems.

Goals from the Syllabus	Goals that pupils should have attained by the end of the fifth year in school	Goals that pupils should have attained by the end of the ninth year in school	Goals that pupils should have attained on completion of the course A
E. Handling of data, Statistics	– be able to read off and interpret data in tables and diagrams, as well as be able to use some measures of location	– be able to interpret, compile, analyse, and evaluate data in tables and diagrams, - be able to use the concept of probability in simple random situations.	be able to interpret, critically examine and with discrimination illustrate statistical data, as well as be able to interpret and use common co-ordinates.
F. Algebra		– be able to interpret and use simple formulae, solve simple equations, as well as be able to interpret and use graphs for functions describing real relationships and events.	be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society.

Table 5 Goals from the syllabus in mathematics year 5, year 9 and course A.

Appendix 2

The table below indicates some of the dissemination points of the KAM-project

Place	Year	Event	Mode
Manchester	1999	Section of ALM 6	Oral presentation
Boston	2000	ALM 7	Published paper
Tokyo	2000	ICME 9	Oral presentation
Gothenburg	2000	Mathematics biannual conference	Paper
Östersund	2003	Swedish Mathematical Society	Oral Presentation
Kungälv	2004	ALM 11	Topic Group
Melbourne	2005	ALM 12	Published Paper

Table 6 Dissemination at conferences of KAM