# 'Connectivism' – a new paradigm for the mathematics anxiety challenge?

**Christopher M. Klinger**
University of South Australia
Adelaide, Australia
chris.klinger@unisa.edu.au

## Abstract

A major challenge for practitioners in adult mathematics education is to achieve effective learning outcomes in the face of prevailing negative attitudes in their students, often present as a consequence of unsatisfactory early mathematics learning experience and flowing from the well-established connection between adult innumeracy and mathematics anxiety. Whether in non-specialist mathematics teaching in diverse disciplines such as economics, nursing, and teacher education, or in adult numeracy teaching, the issues are essentially the same: traditional approaches to mathematics teaching, including constructivism, do not work for math-averse students. The need to find new ways to tackle old problems is further fuelled by the impact of the digital age, with mounting evidence that many aspects of accepted teaching and learning practices are being generally undermined by learners' exposure to technology. Consideration of some stereotypes of traditional methodologies in the context of behavioural and cognitive characteristics common among math-averse and math-anxious students motivates a re-framing of the practitioner's approach and outlines strategies for effective practice that find alignment with connectivist approaches to learning, which is a more flexible approach than the constructivism currently favoured as a positive way of presenting mathematics to learners.

## Introduction

The endemic adult innumeracy that is so deeply embedded in modern Western societies is inextricably linked with the spectrum of mathematics anxiety, negative mathematics attitudes and aversion to the learning of mathematics so often encountered by practitioners among adult learners. These aversive affective behaviours are typically founded on negative early mathematics learning experiences in the latter years of primary education, during the transition from instruction in concrete procedures to increasingly sophisticated and abstract concepts (Klinger, 2009). Pupils' difficulties during that period can be compounded, if not caused, by the 'guidance' they receive from teachers who may themselves be mathematically averse and even covertly innumerate in an integrative sense. Indeed, among undergraduate cohorts pre-service primary teachers who may be characterised by such negative attributes have been found to be disproportionately over-represented (ibid).

The challenge for teachers/practitioners in adult mathematics education, whether at the level of numeracy teaching or undergraduate non-specialist mathematics teaching (for instance, where mathematics is a service subject in non-mathematics disciplines such as economics, nursing, education, psychology, etc), is to find effective ways to break through the prevailing barriers of anxiety and disaffection so that their students may experience success in their mathematics learning, often for the first time, at a level that is at least sufficient for their immediate learning objectives. Ultimately, the greatest achievement will arise when students can overcome their anxiety and aversion to become independent learners with the capacity to extend their engagement with mathematics according to their inclination and intrinsic motivation rather than as a reaction to external drivers.

It will be argued here that, from adult numeracy teaching to undergraduate non-specialist mathematics teaching, the issues are essentially the same: ‘traditional’ approaches to mathematics teaching simply do not work for math-averse students. This will not surprise most practitioners but what, specifically, does not work? To respond to this, the basis of traditional pedagogies and their underlying epistemologies will be considered here in the context of behavioural and cognitive learning characteristics that typify math-averse and math-anxious students and which tend to undermine their learning goals. This will motivate a re-framing of the teacher/practitioner’s approach with the aim of outlining strategies for effective practice.

## Learner characteristics – the practitioner’s challenge

Students who enter university with the aim of becoming mathematicians, scientists, engineers and the like do so in the rather obvious expectation that their courses will encompass substantial mathematical content. But these comprise a relatively small fraction of all commencing undergraduates. Most students in other disciplines rightly have no such expectation yet a rather large proportion of them will, nevertheless, encounter components of a mathematical or statistical nature that they will need to master if they are to succeed in their studies. However, previous studies (Klinger, 2006-2008 inclusive) have shown that, regardless of students’ expectations as to content, mathematics anxiety and math-averse behaviours may be found, to varying extent, universally in all undergraduate student groups (but with greatest severity among student [pre-service] primary teachers) and particularly among pre-tertiary adult learners with aspirations of undertaking tertiary study.

The pervasiveness of mathematics anxiety in the community is well documented, with both practitioners and researchers reporting that perhaps a majority of adult learners exhibit at least some degree of anxiety when confronted with overtly mathematical tasks. This reality makes it apparent that many adult learners, and in particular those at the level of pre-tertiary adult education in numeracy and vocational mathematics, will bring to their mathematics/numeracy studies a strong affective load of negative preconceptions, both of mathematics and of their own capabilities. That is, they have only a vague concept of what mathematics is really all about, lack confidence in their own mathematical abilities, and often fail to appreciate the extent to which they actually and routinely engage in essentially mathematical thinking as they go about their daily activities. As a result, and notwithstanding that they may well display high-level competencies in other aspects of their lives (including study in subject areas other than mathematics), they display characteristics that will cause them to experience difficulty in their mathematics learning endeavours. Practitioners, then, are faced with the challenge of how best to respond.

*Figure 1* (below) illustrates this challenge: the math-averse adult learner is surrounded, overwhelmed, by the task at hand of acquiring specific skills, dealing with math content, and developing effective learning strategies. The needed insights have to be facilitated from outside but they are blocked by the barriers of anxiety, negative beliefs and stereotypes. Therein lies the challenge – effective teaching strategies for these students go well beyond the ‘usual’ scope of mathematics teaching. Before specific content and techniques can even begin to be addressed, it is necessary to uncover the nature, extent, and source of this outer barrier. The wall has to be breached somewhere. Many of these students will state at the outset that they ‘hate maths’, either because they are ‘no good at it and never have been’ or because of negative early mathematics learning experiences. They tend to be quite entrenched in their views and may even be hostile and resentful at being confronted with the material since they feel they are being forced to do something they ‘know’ they’re ‘no good at’. Such students may appeal for help and yet, paradoxically, not expect to change their perceptions of mathematics and of their own ability and so the first task for the teacher/practitioner is to adopt an approach that allows them to admit the possibility that their negative views, no matter how valid they may once have been, need not be absolute and insurmountable.

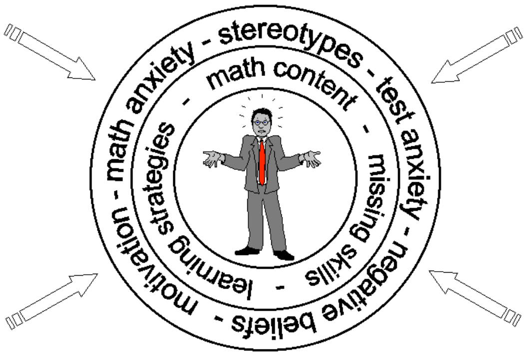


Figure 1

In this, both the students’ and the practitioner’s perceptions of their respective roles, the interaction between them, and the effect of these on student attitudes are particularly critical factors for successful outcomes and should, therefore, be examined carefully with the overarching goal being to educe transformative behaviours from an informative process that includes recognition of the following learner characteristics (Klinger, 2004):

- Confusion
- Lack of confidence
- Negative perceptions
- Lack of strategies
- Narrow focus
- Assessment-driven motivation
- Little or no appreciation of the concept of mathematics as language

When students are unclear about what is required of them, they tend to over- or under-estimate the difficulty of subject material and display little insight about the extent and relevance of their prior knowledge (mathematical and otherwise) and pertinent skills. Commonly, confusion dominates their behaviour: they do not know what it is that they ‘don’t know’, have difficulty in organizing notes and other materials and make little use of text books or other resources. Repeated lack of success impacts on confidence, in which they are lacking, and manifests in low self-efficacy beliefs, often accompanied by strong negative emotions of embarrassment, self-deprecation, and helplessness (reported also by Karabenick and Knapp, 1988). Thus low expectations are common, as are lack of persistence and little interest in attempting to acquire deeper understanding (as opposed to ‘quick fix’ outcomes). Negative perceptions can also contribute to low self-esteem and may include their perceptions of self, particularly in terms of cognitive competence, extending to their instructor(s), topic organization, and reactions of their peers and family.

Although they may be very competent problem solvers in other contexts, confusion and negative perceptions and emotions are too strong an influence in this context, resulting in an obvious lack of strategies with which to negotiate or cope with their difficulties. The capacity to generalise is limited and there is little facility to synthesise knowledge connections to aid their understanding and this is often aggravated by a lack of attention to creative thinking and problem-solving skills (Kessell, 1997), which may stem from a narrow focus and strong tendency to see mathematics as ‘different’ from other intellectual activities. Consequently, motivation is likely to be assessment-driven rather than intrinsic – that is, these students pursue the ‘extrinsic goals’ described by Ryan and Pintrich (1997), with all the attendant consequences and implications. This may also extend from low self-efficacy beliefs.

In particular, while most will readily acknowledge that mathematics is loaded with specialised terminology, much of it involving common words whose definitions are much narrower than their everyday usage, and while they accept the use of specialised language to describe mathematical concepts, they fail to recognise the converse: that mathematics language describes specialised concepts. That is, there is little or no appreciation of the concept of mathematics as language.

It is common within teaching and learning literature to associate these characteristics with shallow, or surface, learning styles and their presence is frequently associated with avoidance behaviour such as that commonly observed in the math-averse. An important distinction should be made, however: while students may exhibit such characteristics in association with their mathematics learning, it should not be assumed that they more generally typify their learning style in other learning situations – I contend that in many instances the mathematics learning style is anything but intrinsic and is instead directed by a student’s anxiety and prior mathematics learning experiences. In many respects, then, the above characteristics are reactive, manifesting in self-defeating behaviours that undermine the learning situation unless the teacher/practitioner can affect successful intervention.

Since the proximal cause is likely to be strongly related to past learning situations, forms of instruction that derive from a deficit model of remediation, echoing negative early encounters in the mathematics classroom, will be necessarily ineffective and serve to validate the student’s poor perceptions. Rather, there is a need to establish an entirely different framework whereby students have a genuine opportunity to experience the epiphany they need to shed their history and construct new understandings. This is a matter of good practice (a value-laden term) for which ‘it is vital to understand the epistemological basis that underlies the teaching of numeracy in the adult classroom.’ (Swain, n.d.)

## Epistemology and pedagogy in perspective

### Behaviourism to social constructivism

‘Skill and drill’ teaching is the archetype of *behaviourism* in mathematics education. In this framework, the ‘ideal’ learning environment is one that maintains a focus on procedures and outcomes arranged hierarchically so that mastery of basic skills serves as a scaffold to more advanced activities in a linear and cumulative progression. Here, mathematical knowledge is external, absolute, and transmitted didactically with emphasis on learning as the correct application of appropriate algorithms to obtain correct answers, conditioned and reinforced positively by 'rewards' of success and concomitant approval and negatively by failure and disapproval (even to extremes of physical and psychological punishment, as commonly reported by sufferers of more severe mathematics anxiety). The practice of correct application is undertaken by studying worked examples and emulating the procedures in situations that are similar to the examples. While such practice might be termed 'problem solving', it is not thus in a modern sense since it lacks genuine creativity; rather, competence lies in the ability to find an appropriate mapping from a known example to a given scenario.

Orton (2004, p29) observes that, '…much of the teaching of mathematics has traditionally consisted of the teacher demonstrating a method, process, routine or algorithm to be used in particular circumstances, followed by the class attempting to solve routine questions using the set procedure. … Exposition by the teacher followed by practise of skills and techniques is a feature which most people remember when they think of how they learned mathematics'. He goes on to explain that while the objective is to establish strong stimulus-response bonds (if, indeed, such a construct is valid) teachers know well that these are frustratingly short-lived, with subsequent examination after several months interval revealing that 'most pupils demonstrate only that they cannot respond correctly'. Orton uses the example of the addition of fractions, the algorithm of which is taught, re-taught, and practiced throughout early schooling only to be forgotten repeatedly (*ibid*). This is not true understanding.

While the 'use it or lose it' impermanence of such mathematics learning is problematic in itself, this aspect is vastly more so because it admits no structured, regularised or intrinsic opportunities to promote understanding, which is redundant in the face of the imperative to elicit correct behavioural responses. Whatever learning takes place is discrete and disconnected, with each topic essentially partitioned from the body of discipline knowledge as a whole and considered often in isolation from, and generally without appeal to, the often considerable knowledge base that students may hold as a result of their learning in other disciplines and life experience.

There is a long tradition of teachers, from primary to tertiary levels of education, adopting an essentially behaviourist approach in their mathematics teaching when they would reject such methods in other curriculum areas. Those who are least prepared for the mathematics classroom are likely to have a limited procedural and rules-based view of mathematics. This accords with their own recollections of primary- and secondary-school mathematics classes and perhaps their everyday experiences as functionally numerate adults, where practical concerns often dictate a pragmatic reliance on procedures and algorithms (to the extent that these are remembered) that can be implemented mechanically and reliably to satisfy an immediate need. But it is not just inexpert teachers that employ this mode of instruction. At the other end of the spectrum, teachers with highly-developed mathematical ability may adopt a similar approach (Golding, 1990) as a result of their unconscious competence and lack of awareness of the actual complexity of their expertise, which may lead them to pursue perceived efficiencies in the transmission of 'obvious' knowledge. Moreover, it 'takes less time to state a well-established method than it does to guide students to its discovery… And, of course, the prevailing emphasis on skills tests influences many teachers toward the short-term goal of teaching rote procedures' (Golding, 1990 p46).

The foregoing is not to reject in its entirety all aspects of behaviourism; it is the means that are questioned rather than the ends. Clearly, the use of mathematics in the 'real world' – such as for vocational purposes – most often demands a focus on outcomes (the 'right answer') that can only be satisfied by the functional fluency that can only be found as a result of practiced ease.

*Cognitivism* arose largely in response to behaviourism. Whereas the latter ignored processes internal to the learner because they were not observable, the former argued that intentional action deriving from a learner's mental states contributed to behavioural and learning outcomes as the learner seeks to adapt to the learning environment. From a cognitive perspective, knowledge may be transmitted between individuals but is stored as internal mental constructs or representations. For mathematics education, the influence of cognitivism manifests most directly in an emphasis on learning by problem solving as a recursive process of assimilation and accommodation whereby a problem is interpreted by assigning it to existing internal representations or *schema* (Bartlett, 1932). This approach has been shown to yield superior learning outcomes for more experienced learners, for whom worked examples become increasingly redundant (Kalyuga, Chandler, Tuovinen, and Sweller, 2001). It must be stressed, however, that in traditional approaches to mathematics and numeracy teaching, the cognitivist approach augments rather than supplants behaviourist practices, which persist as the dominant mode of instruction when new (and, particularly, fundamental or foundational) procedures are introduced.

*Social cognitivism*, largely credited to educational psychologist Albert Bandura, fuses elements of behaviourism and cognitivism with social aspects of learning (Bandura, 1986). The theory recognizes that learning is at least as much a social activity as it is behavioural and cognitive and emphasizes the importance of observational learning, by which behavioural and/or cognitive changes are effected by the learner's comparative observations of others and of self, a process incorporated in the concept of self-efficacy beliefs (Bandura, 1997), which have been found to play a prominent role in the learning activities of math-averse students (Klinger, 2004-2009) through a complex interplay of motivational, behavioural, cognitive, and affective factors.

For at least three decades, *constructivism* has been the darling of many in the education community. Alenezi (2008) claims that mathematics and science instruction is 'increasingly grounded in constructivist theories of learning' (p17). Essentially, the so-called 'math wars' – the debate between traditional versus reform mathematics education – derives from philosophical differences between behavioural and cognitive approaches and those of constructivism. The central tenet of constructivism is that knowledge cannot be transmitted but is a construct of the mind as a consequence of experiential learning. *Information* may be transmitted but its transformation to knowledge is an internal process affected by learners discovering relationships between new information and their inner knowledge representations and constructions of reality.

That is, the learner is not a passive vessel to be filled (the behaviourist's standpoint) but has an active role in building understanding so as to make sense of the world. Learning results from an ongoing process of hypothesizing, rule-creation and reflection, with new information being evaluated in the context of existing rules that are themselves subject to revision or rejection if found incapable of accommodating the data or otherwise discovered to lack internal consistency. The teacher is dispossessed of the role of didactic authority to become instead an information conduit and facilitator of the learning process, charged with finding ways to present information that is relevant to the student's learning needs in a manner that makes it meaningful. This is to be achieved by providing students with opportunities to discover, explore and apply ideas that will satisfy learning objectives.

*Social constructivism* adds the further proposition that there can be no sensible definition of knowledge that ignores its social context. That is, knowledge must necessarily be grounded in the social values, standards, mores, language and culture by which the learner acquires an understanding of the world. It is also to be developed socially in the classroom via open communication between instructor and student and by peer interactions in common exploration activities. In this sense then, while knowledge is in one sense individual and internal, learning is a social activity and social interaction extends the location of knowledge via communicated and shared understandings.

While constructivism dominates current pedagogy, I suggest that there are profound flaws in the context of mathematics and numeracy education. First, the principal required curriculum outcome is identical to that of behaviourists and cognitivists – a demonstrated ability to *perform* by applying the appropriate procedures to a given situation, in a manner that is consistent with the formalism, to arrive at a correct result that may be communicated to and verified by others (that is, the formulation, solution, and answer must be presented according to agreed conventions). Second, there are implicit assumptions that self-directed learners have 'sufficient prior knowledge and skills (particularly basic literacy, numeracy and study skills) to engage effectively and productively for generating new learning' (Rowe, 2006 p101). Elementary or foundational formalism and conventions, in particular, are not reasonably accessible to exploration and discovery; they need to be learned in essentially the same way as vocabulary and rules of grammar – no matter how they arrive at their ideas, students must know *what* to write and *how* to write it in order to reliably record and communicate them.

Last (and far from least), the word 'basic' is often applied to the everyday arithmetic of addition, subtraction, multiplication, division, fractions, decimals, and percentages. There is a tendency for those who 'can do' to use the word in a dismissive or belittling fashion with those who 'can't do' – "You should be able to do *that* [at least]… it's just basic arithmetic". Math-averse learners have heard statements like this throughout their mathematics learning history and in the work-force, often accompanied by expressions (both verbal and non-verbal) of disparagement, derision, frustration, or anger and, sometimes physical chastisement. While concepts of number, number representation, and arithmetic operations are certainly *fundamental,* there is nothing *basic* (in the sense of 'simple' or 'obvious') about them – it has taken humankind millennia to invent/discover these concepts and to formalize them in terms of definitions, rules, procedures, and algorithms. One might wonder how many of those with greater facility in the use of the procedures of 'basic' arithmetic can actually sensibly talk about their use without resorting to parroting rules once learned. It is neither reasonable nor sensible to expect students at any level to *actually* discover 'basic' mathematical concepts and corresponding procedures by pursuing a literal constructivist agenda.

Elementary mathematics instruction texts appear to have changed considerably in the period since constructivism came to the fore of educationalists' thinking. Modern texts tend to be colourful and inviting, with many diagrams. They contain numerous 'real world' scenarios to accompany and supplement more formal aspects of the text; these are usually provided to motivate students' engagement by providing social grounding for the presented concepts and worked examples. In addition, practical exercises presented as 'experiments' identify opportunities for learners to verify various aspects by quasi-independent enquiry.

It takes little more than a cursory examination to discover that these apparent changes are largely superficial. The core material (definitions, rules, procedures, algorithms and 'drill' exercises) continues to be present and largely indistinguishable from that of texts compiled during pre-constructivist eras. That is, constructivist principles appear to have been applied as a veneer that mostly serves to augment cognitivist problem-solving activity. Ultimately, and fundamentally, little has really changed; teachers must still be able to demonstrate, via skills tests and league tables, that they are engaged in practices that satisfy benchmarked standards and learners must demonstrate, through standardized assessments, that they have acquired sufficient mastery of the curriculum.

There are other problems, which constructivist thinking appears to mask. Figure 2 shows a 'gifted' Grade 5 student's work, taken from Davis and Maher (1990). In a typically constructivist approach, the student, Ling, has been given a problem to solve (stated in the figure) and a range of manipulable materials, including Pattern Blocks, which she used to explore and correctly solve the problem. Her teacher had then asked if she could write her solution and she first recorded the diagram.

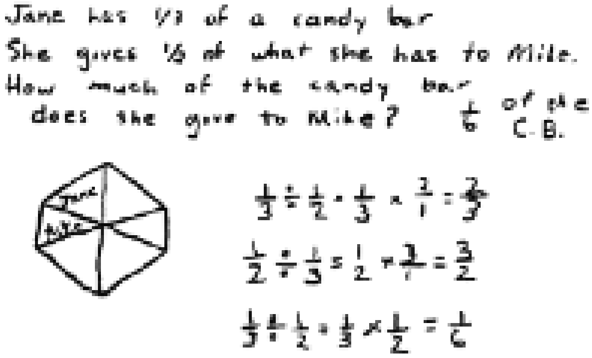


Figure 2 The Candy Bar Problem (Davis and Maher, 1990, p.75)

That Ling started her diagram by drawing a hexagon indicates that she *knows* the correct answer – she’s ‘done the maths’. The teacher asked if she could do it in figures and this is where her troubles began; she responded by writing the first line in the figure, 1/3 ÷ 1/2 = 1/3 × 2/1 = 2/3. Davis and Maher write:

> ‘Ling is clearly a good student; she has learned the "invert and multiply" rule correctly. But she has produced an incorrect answer! How come? Because she has not called upon the correct algorithmic solution procedure. There is no reliable way to go from a problem statement to a solution procedure unless you get a correct representation of the problem.’ (Davis and Maher, 1990 p75)

The first attempt at writing the problem and solution being not in accord with her experience, Ling wrote the following two lines with some frustration. Eventually she was apparently satisfied that she had finally written something that produces the answer she ‘knows’ is correct. But of course, no doubt out of desperation, she has claimed that division and multiplication are the same thing in this instance.

Now, this example is interesting not because of the student’s approach or, directly, because of the constructivist pedagogy. Rather, it is interesting here because of the authors’ observations, which focus on the student not having called upon ‘the correct algorithmic procedure’ (*ibid*) instead of identifying that this example is not one of problems with mathematics but, rather, problems for Ling with the language of mathematics – a different thing entirely. Thus Davis and Maher entirely misses the real point! Ling’s mistake was not in calling upon the wrong *procedure* but in her failure to correctly *translate* the English expression, ‘half of what she has’ (i.e. one half of one third), into the corresponding mathematics expression. This is not a procedural error nor is it a retrieval error (as in faulty memory recall of a rule), it is a *language* error. No-one with a fluent grasp of mathematics as *language* could make such an error without seeing the mistake immediately or on reflection – which is something that Ling never did, apparently.

This simple example is profound in its illustration of a very common feature of students’ mathematics work (even those operating at advanced levels). They write to create, largely by mimicry when working at the elementary level, the *appearance* of mathematics rather than writing to express meaning using mathematical language. Advanced-level students will frequently write mathematics by manipulating symbols according to the rules that they have acquired rather than, again, writing to express meaning so that the written mathematics tells a 'story'. That the teaching of mathematics and numeracy generally focuses on the doing of mathematics, without explicitly attending to the language of mathematics and students' fluency therein, is no particular fault of any of the 'isms' discussed thus far as these epistemological and philosophical standpoints are equally deficient in this regard.

### Connectivism

The latest contender in educational theory has been termed *connectivism*, a 'learning theory for a digital age' advanced by George Siemens (2005) in response to an awareness that technology is increasingly undermining many aspects of accepted teaching and learning and that prevailing learning theories are inadequate in the present era. In fact, the term, 'connectivism, was not coined by Siemens and has been used by others in a different sense – for instance De Geest, Watson and Prestage (2002, p22) describe connectivism as a desire to 'view mathematics as a connected, holistic way of working rather than as separate topics'. Siemens' connectivity, on the other hand, is considerably different, being motivated by the exponential growth of knowledge (in the sense of a hypothetical audit of what is known, which is a debatable concept in its own right and beyond the scope of the present work) and the observations that, in an increasingly technological and networked world in the 'digital age', 'know-how and know-what is being supplemented with know-where' (p4) and the 'capacity to know more is more critical than what is currently known' (p7).

In proposing connectivism, Siemens borrows, somewhat speculatively (even extravagantly), concepts from the science of complexity, including chaos theory, networking, and self-organization. A significant flaw, though, is the absence from his 'model' of the essential criterion for self-organization, which is the presence of randomness (self-referential noise) as a driver. One might speculate that cognition in a living organism is sufficiently 'noisy' to provide the missing component. Still, despite this omission, there is considerable appeal in many aspects of the proposal. Self-organization and emergent higher-order phenomena from self-referential complex systems is a universal principle (Klinger, 2005) that is strongly reflected in the behaviour of networks and random graphs. The complex and self-referential nature of the human mind is, perhaps, obvious, and in simplistic terms the human brain is certainly a network of interconnected neurons with memory and other cognitive activities being associated with changes to neural connectivity.

However, rather than further pursuing what is, essentially thus far, just an analogy, it will be useful to focus attention on one specific aspect of connectivism that provides a good illustration for the purpose of reframing adult numeracy practice (and, indeed, fundamental mathematics education more generally). This is encapsulated in Siemens' statement that connectivism 'posits that knowledge is distributed across networks and the act of learning is largely one of forming a diverse network of connections and recognizing attendant patterns' (Siemens, 2008 p10).

## Reframing practice – a connectivist approach

What I am suggesting here, as a conceptual discussion based on many years experience of working with adult learners with mathematics anxiety and math-averse behaviours in pre-tertiary and tertiary settings, is that the particular value of the connectivism paradigm in mathematics and numeracy teaching lies in exploiting the properties of network connectivity in complex systems. By actively pursuing opportunities for students to forge links that promote an understanding of mathematics as *language*, they may establish connections that permit mappings between mathematical concepts and their various skills and understandings of the world. That is, mathematics language is to be understood in terms of things and language that the learner already knows (through appeal to common-sense and intuition by metaphor and analogy).

This view posits that the connectivity attained by forming links between mathematical know-how, language and other skills from the student's existing knowledge base serves to build understanding so that dependence on mathematical rules becomes redundant (in that, as a consequence of increasing fluency, they come to be seen as obvious consequences of the mathematics language rather than algorithmic procedures to be applied mechanistically). As the internal and self-referential (reflective) knowledge network grows by the formation of new connections that incorporate more and more nodes of both congruent and disparate knowledge and experience, it undergoes periods of self-organizing criticality whereby there are cognitive phase transitions (to borrow physics terminology) that spontaneously yield flashes of emergent deeper understanding (epiphanies or 'ah-ha' moments). Increasingly, the learner is empowered to undertake self-directed learning according to need or inclination.

It is to be emphasized that the foregoing is entirely speculative. Nevertheless, in terms of pedagogical practice, there is substantial merit in the approach of considering mathematics first and foremost as language and focussing on ways and means to develop students' fluency therein while utilising their existing skills and knowledge-base as leverage. For the teacher/practitioner working with math-averse and mathematically anxious adult learners, this will most often demand a reframing of the learning situation away from traditional practices towards techniques that explain and demonstrate how the context and methods of mathematics are revealed through its application as *language*, mapping these onto concepts and language with which the student is otherwise familiar and confident. Of paramount importance is the need to first expose and debunk the preconceptions that underpin negative self-efficacy beliefs, recognizing that these stem predominantly from prior unsatisfactory mathematics learning experiences that were likely grounded in pedagogy derived from behaviourist principles.

Every new mathematics learning activity should be approached from a language perspective, first identifying a common base of understanding with which students can connect so that concepts can be discussed in natural language before proceeding to translate them into the formalism of symbolic mathematics language. This methodology can, and should, be openly explained so that students understand explicitly that they are engaged in learning a language so that if their initial efforts are frustrated they will recognize that many of their difficulties are, in reality, language difficulties and that these will abate as the mathematics language becomes less unfamiliar. Fortunately, students may be reassured, the vocabulary and grammar of mathematics is small compared to natural languages and very literal. There must be particular emphasis that any mathematics that the student reads or writes *must* make sense – it must 'say something'; that is, it must always be possible to translate freely in either direction: mathematics language to natural language and the converse.

As with the learning of natural languages, students must be guided to cultivate an 'ear' (or eye in this case) for dissonance between what is understood and that which is written or read and to develop the ability to self-correct. Whenever meaning is obscured by ambiguity or lack of semantic clarity, or whenever an attempted procedure fails despite apparent correct application, both the instructor and the learner should be alert to inappropriate language construction or interpretation. This may prompt the need to unpack whatever is being attempted to examine, test, and remedy underlying language weaknesses or misunderstanding. Because the mathematics will be shown increasingly to 'make sense' and be something other than obscure procedures and rules, this attention to language is essential as a first step to reducing confusion and anxiety and to broaden students' focus.

While maintaining this concentration on language, the introduction of new concepts should be effected by referral to prior concepts that are properly understood (that is, the student has gained fluency at that level) and, wherever possible, by seeking to identify analogous or parallel ideas in non-mathematical every-day domains. The aim is to establish, wherever possible, connections between what students already know and that which they seek to learn. Often, skills and procedures that appear alien and intractable in a mathematical context may be shown to be intrinsic (metaphorically if not literally) to many ‘ordinary’ adult activities and situations – seeking to embed new mathematical material by forming connections with students’ existing knowledge network creates familiarity by association, adds leverage to the learning task, and brings learners closer to achieving the cognitive phase transition that transforms information into knowledge and understanding. With each advance, students gain confidence, overcoming their negative perceptions to discover intrinsic motivation for actively pursuing their further learning as an end in itself, rather than simply to satisfy assessment requirements. The outer barrier depicted in Figure 1 is thus penetrated. The math-aversion and anxiety may never disappear entirely but future encounters with mathematics can at least be directed by informed, adult decisions rather than dominated by attitudes and emotions imposed by an unfortunate history.

## Conclusion

The needs and characteristics of math-averse and mathematically anxious adult learners present teachers/practitioners with a significant challenge for which the traditional ‘isms’ – behaviourism, cognitivism, constructivism – underpinning conventional pedagogy in mathematics and numeracy teaching are in not only in deficit but can be seen to be directly associated with aversive affective behaviours that typically stem from students’ prior experiences of mathematics schooling. A review of the learner characteristics and their association with surface-learning styles, together with the recognition that these stem from past mathematics learning experiences, provides both insight into the nature of the challenge and a context from which to consider the epistemological foundations of traditional mathematics teaching and, more particularly, to give consideration to the utility of the latest ‘ism’, connectivism, and its value as a means to effect behavioural and cognitive transformations.

Aspects of connectivism resonate with techniques and approaches known from professional practice to be broadly successful in alleviating mathematics anxiety and achieving effective learning outcomes. The concept of invoking the properties of network connectivity in complex systems (of which the human brain is an archetypal example) as a means to explain learning provides a potential theoretical framework to support the advocated reframed adult numeracy practice: specifically, the pursuit of opportunities to approach the learning of mathematics as language and to form multiple links to connect new information with students’ existing knowledge networks.

The paradigm of connectivism, at least along the lines indicated here, needs considerably more research to become established as significantly more than a rhetorical device. Nonetheless, there is obvious intuitive appeal in the insights suggested by this new ‘ism’ that surely warrants further investigation.

## References

Alenezi, D.F. (2008). *A study of learning mathematics related to some cognitive factors and to attitudes*. Centre for Science Education, Faculty of Education, PhD Thesis, University of Glasgow. Retrieved on 7 May, 2009 from <http://theses.gla.ac.uk/333/01/2008_Aleneziphd.pdf>

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Eaglewood Cliffs, NJ: Prentice Hall.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: W.H. Freeman & Co.

Bartlett, F. C. (1932). *Remembering: a study in experimental and social psychology*. 1977, London, Cambridge University Press.

DECS. (2004). Profiling high numeracy achievement. *Australian Primary Mathematics Classroom* , *9 (4)*, 35-38.

De Geest, E., Watson, A., & Prestage, S. (2002). Building a holistic view of mathematical thinking - data evaluation of improving attainment in mathematics project. In S. Pope (Ed.), *Proceedings of the British Society for Research into Learning Mathematics 22(3) November 2002*. Retrieved 7 May 2009 from <http://www.bsrlm.org.uk/IPs/ip22-3/BSRLM-IP-22-3-4.pdf>.

Kalyuga, S., Chandler, P., Tuovinen, J., & Sweller, J. (2001). When problem solving is superior to studying worked examples. *Journal of Educational Psychology*, *93*, 579-588.

Karabenick, S.A. & Knapp, J.R. (1988). Help seeking and the need for academic assistance. *Journal of Educational Psychology*, *80(3)*, 406-408.

Kessell, S. R. (1997). Improving science students’ problem solving skills. *Proceedings Western Australian Institute for Educational Research Forum 1997.* Retrieved 4 May 2009 from <http://www.waier.org.au/forums/1997/kessell.html>.

Klinger, C. M. (2004). Study skills and the maths-anxious: reflecting on effective academic support in challenging times. In K. Dellar-Evans, & P. Zeegers (Ed.), *Language and Academic Skills in Higher Education, Vol 6; Refereed proceedings of the 2003 Biannual Language and Academic Skills Conference* (pp. 161-171). Adelaide: Flinders University Press.

Klinger, C.M. (2005). *Process physics: bootstrapping reality from the limitations of logic.* School of Chemistry, Physics and Earth Sciences, Faculty of Science and Engineering, PhD Thesis, Flinders University. (Available online from the Australian Digital Theses Program at <http://catalogue.flinders.edu.au/local/adt/public/adt-SFU20080430.132508/index.html>.)

Klinger, C. M. (2006). Challenging negative attitudes, low self-efficacy beliefs, and math-anxiety in pre-tertiary adult learners. In M. Horne, & B. Marr (Ed.), *Connecting voices in adult mathematics and numeracy: practitioners, researchers and learners. Proceedings of the Adults Learning Mathematics (ALM) 12th Annual International Conference, Melbourne, July, 2005* (pp. 164-171). Melbourne: ALM.

Klinger, C. M. (2007). On mathematics attitudes, self-efficacy beliefs, and math-anxiety in commencing undergraduate students. In V. Seabright (Ed.), *Crossing Borders - Research, Reflection and Practice in Adults Learning Mathematics. Proceedings of the 13th International Conference on Adults Learning Mathematics, Belfast, July 2006* (pp. 88-97). Belfast: ALM.

Klinger, C. M. (2008). Experience, the difference: maths attitudes and beliefs in commencing undergraduate students and pre-tertiary adult learners. In J. O'Donoghue (Ed.), *Proceedings of the 14th International Conference on Adults Learning Mathematics, Limerick, Ireland, July 2007* (pp. 197-206). Limerick: ALM.

Klinger, C.M. (2009). Passing it on: linking adult innumeracy to mathematics attitudes, low self-efficacy beliefs, and math-anxiety in student primary teachers. In K. Safford-Ramus (Ed.), *Proceedings of the 15th International Conference of Adults Learning Mathematics, Philadelphia, July2008* (pp. 123-132). New Jersey: ALM.

Orton, A. (2004). *Learning mathematics: issues, theory and classroom practice*. London: Continuum International Publishing Group.

Rowe, K. (2006). Effective teaching practices for students with and without learning difficulties: Issues and implications surrounding key findings and recommendations from the National Inquiry into the Teaching of Literacy. *Australian Journal of Learning Disabilities, 11(3)*, 99-115.

Ryan, A.M. & Pintrich, P.R. (1997). “Should I Ask for Help?” The role of motivation and attitudes in adolescents’ help seeking in math class. *Journal of Educational Psychology*, *89(2)*, 329-341.

Siemens, G. (2005). Connectivism: a learning theory for the digital age. *International Journal of Instructional Technology and Distance Learning, 2(1)*, 3-10. Retrieved on 12 May 2009 from < http://www.itdl.org/Journal/Jan_05/Jan_05.pdf>.

Siemens, G. (2008). *Learning and knowing in networks: Changing roles for educators and designers*. University of Georgia IT Forum. Retrieved 7 May 2009 from <http://it.coe.uga.edu/itforum/Paper105/Siemens.pdf>.

Swain, J. (n.d.). What is good practice in adult numeracy teaching? *Reflect Online, Issue 2, NRDC*. Retrieved on 9 May 2009 from <http://www.nrdc.org.uk/content.asp?CategoryID=592&ArticleID=487>