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Correcting Students' Misconceptions about Probability in an Introductory College Statistics Course

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Abstract

College students' misconceptions about probability are common and widespread. These misconceptions impede students' ability to make sound judgments in situations of uncertainty and master fundamental concepts of inferential statistics. In this paper the authors report the results of a study undertaken with the objective of correcting three common stochastic misconceptions within the framework of an introductory statistics course. Six instructors were recruited from an urban community college, along with their students. These instructors implemented a number of activities designed by one of the investigators and aimed at correcting the representativeness bias, the equiprobability misconception, and the outcome orientation bias. The purpose of the activities was to trigger cognitive conflict thereby leading students to bring out and correct their incomplete or erroneous concepts. Three of the instructors attended a workshop on misconceptions before implementing the activities; the other three implemented the activities without going through training. Instructors in the third (control) group did not implement any of the proposed activities and were not trained. An instrument designed by one of the investigators was used to measure the extent of students' misconceptions at the end of the semester in both treatment groups and the control group. The results show that trained instructors achieved significantly better outcomes than the control group in correcting two of the misconceptions: the representativeness bias and outcome orientation. By contrast, instructors who implemented the activities without being trained did not post better results than the control group in resolving any of the misconceptions. The results suggest that it is possible to improve students' conceptual understanding of probability and correct their misconceptions by targeting the misconceptions directly in an introductory college statistics course. They also suggest that training on misconceptions is critical in ensuring instructors' ability to successfully address students' erroneous concepts about probability.

Introduction

A substantial body of research demonstrates that students often hold non-standard concepts in mathematics and science (Garfield, 2001; Morton, 2008; Ozdemir & Clark, 2007; Vosniadu, 2004; Khazanov, 2008). These non-standard concepts are referred to in the literature by different terms: naive concepts, preconceptions, misperceptions, and misconceptions. The most common, and broadly accepted, term used in the literature is *misconception*, which is defined as "a student conception that produces a systematic pattern of errors" (Smith, diSessa, & Roschelle, 1993, p. 10). It is a well established fact that a large proportion of college students have misconceptions about probability (Antoine, 2000; Das, 2008; Giuliano, 2006; Hirsch & O'Donnell, 2001; Konold, 1989; delMas & Bart, 1989). These non-standard concepts could have developed as a result of informal encounters students have with uncertain events at home, at the workplace, or while playing games of chance. Some of them might have developed misconceptions at school while learning probability, statistics or other subjects. There are scores of misconceptions about probability described in the literature. The most common include the equiprobability bias, conjunction fallacy, outcome orientation, representativeness, availability bias, and various misconceptions of conditional probability (Shaughnessy,1992; Jones, Langrall, & Mooney, 2007).

There is also strong research evidence that misconceptions about probability do not disappear as a result of traditional instruction in probability comprised, for the most part, of formal definitions, rules, and procedures (delMas & Bart, 1989; Khazanov, 2005, 2008; Konold, 1995, Shaughnessy, 1977, 1981). Misconceptions can "peacefully coexist" with correct concepts and interfere with students' ability to apply these concepts consistently and with confidence (Khazanov & Gourgey, 2009; Ozdemir & Clark, 2007). Garfield (2001, 2007) points out that, although students may learn probability rules and procedures and may actually calculate correct answers on mathematical tests, these same students frequently misunderstand basic ideas and concepts and often ignore the rules when making their own judgment about uncertain events. Most researchers agree that instructional interventions designed specifically to eliminate students' misconceptions of probability are necessary if any tangible and stable improvements in students' concepts were to be expected (Khazanov & Gourgey, 2009; Khazanov, 2008; Konold, 1995; Hirsch & O'Donnell, 2001, Shaughnessy, 1981, 1992).

Need for study

Correcting students' misconceptions about probability has been broadly acknowledged as an important instructional goal for stochastic instruction. Shaughnessy, an influential scholar in the field of probability and statistics education, declared that one of the main goals of stochastic instruction should be to provide students with evidence of "how misconceptions of probability can lead to erroneous decisions" (Shaughnessy 1992, p. 482).

Khazanov & Gourgey (2009) surveyed statistics teachers and found that the majority of their respondents agreed that students' erroneous concepts and beliefs about probability need to be addressed in an introductory college statistics course. In another study, Khazanov (2005) found that the number of misconceptions held by students was significantly associated with, and predictive of, their achievement in an introductory statistics course.

Shaughnessy (2007) quoted examples from a number of studies in which misconceptions students have about probability impede the development of correct understandings of some important statistical concepts.

Teaching probability for conceptual understanding implies a major shift in emphasis from simply providing formulas, rules, and procedures for calculations to addressing students' erroneous intuitions and preconceptions (Garfield, 1995; Konold, 1995; Sharma, 2006; Khazanov & Gourgey, 2009). Khazanov (2005) pointed out that while misconceptions about probability have been extensively studied at different levels and for different age groups (see, for example, Fiscbein & Schnarch, 1997; Antoine, 2000) only a few studies reported attempts to correct misconceptions about probability in college instruction, and even fewer in introductory college statistics courses. The present study was designed to address the above deficiency.

Purpose of study

The main purpose of this study was to determine whether targeting three common misconceptions about probability – the equiprobability bias, representativeness, and outcome orientation–directly and systematically, by employing activities designed by the investigators in a college statistics course, would be instrumental in resolving the misconceptions.

The equiprobability bias involves attributing the same probability in a random experiment to different events regardless of their actual chances (Jones, Langrall, & Mooney, 2007; Lecoutre, 1992; Lecoutre & Rezrazi, 1998). Outcome orientation is characterized by treating the probability of an occurrence or non-occurrence of an event as an affirmation of certainty rather than a measure of likelihood (Jones, 2005; Konold, 1989, 1995). Representativeness bias implies estimating the likelihood of an event based on how well an outcome represents some aspect of the parent population (Hirsch & O'Donnell, 2001; Kahneman, Slovic, & Tversky, 1982; Shaughnessy, 1992). Another important purpose was to determine whether training instructors on misconceptions prior to implementing the activities would have an impact on student outcomes.

Review of the literature on the teaching and learning of probability

A number of investigators used computer modeling to teach about probability and address students' misconceptions. Krishnamachari (1988) explored how the use of computer simulations facilitated students' comprehension of basic concepts of probability. She used workbooks with problems based on computer simulations. The assessment showed that students understood the concepts of probability explored in the study. Konold (1989) used a computer modeling intervention in an attempt to influence students' misconceptions. The results were mixed; some students changed their interpretation while others persisted in their erroneous concepts. Garfield and delMas (1989) who used a computer program Coin Toss also obtained mixed results. While some students changed their ideas about variability after using their tutorial, others persisted in their misconceptions about sample size and variation. According to Snee (1993), computer simulations may not be helpful in changing misconceptions about probability in some students. Nevertheless, other researchers found that teaching with computers can facilitate conceptual understanding of probability by allowing students to explore and represent stochastic models, manipulate the parameters, vary assumptions, and analyze data (Jones et al, 2007).

Keeler and Steinhorst (2001) used an approach in which numerical information was presented in the form of frequencies rather than fractions, decimals or percent. They drew on research by Cosmides and Tooby (1996) who found that students have an affinity for counting things. Keeler and Steinhorst emphasize the use of frequencies within the constructivist framework to develop students' understanding of concepts such as independence and randomness. They also used simple probability density functions to allow students to focus on learning about probabilities associated with continuous random variables without getting bogged down in the mechanics of standard normal tables and z-scores. Keeler and Seinhorst did not report about any studies that measured the effectiveness of the above approach; they encouraged other researchers to investigate how students' thinking about probability changes as a result of this approach.

DelMas and Bart (1989) employed an instructional activity that required one group of students to evaluate predictions based on their intuitive understanding while the other group was not required to perform this evaluation. They found that the group required to evaluate predictions did significantly better on a mastery test than the other group. Garfield and Chance

(1999) demonstrated significant changes in student performance on items designed to assess their understanding of how sample size is related to the shape and variability of the sampling distribution. The activity asked students to select among a set of graphic displays the one that most likely represented a sampling distribution from a population, given a specified sample size. After making a prediction students used computer software to simulate the sampling distribution and compare the results to their predictions. While a significant number of students were found to overcome their misconceptions, subsequent studies revealed that for some of them the gains were short lived (delMas, Garfield, & Chance, 2002).

Hirsh & O'Donnell (2001) used the instrument they developed for identifying the representativeness bias to determine the effectiveness of a number of approaches aimed at correcting this misconception. In the instructional intervention they carried out, three methods of triggering and resolving cognitive conflict were employed: direct instruction, individual activities, and small group activities. The effectiveness of the three treatment models was compared to a control group that received instruction not specifically designed to trigger cognitive conflict. Although the researchers documented slightly better outcomes in sections were the cognitive conflict approach was utilized, the improvement did not reach the level of statistical significance.

Fast (2001) successfully used analogies and anchors to facilitate conceptual reconstruction of common misconceptions about probability in high school students. He found that his method led to stable outcomes. Students tested half a year after the treatment still retained the correct concepts and a significant proportion of them did not show evidence of misconceptions.

Lecoutre investigated the various opportunities for resolving the equiprobability bias. This bias involves attributing the same probability to different events in a random experiment regardless of the chances in favor or against them. According to Lecoutre:

> There exists an intra-subject vicariance of different cognitive models in various structurally isomorphic situations. (Lecoutre, 1992, p. 7)

The specific activation of a particular model was found to be linked to the "surface features" of the situation; the chance context of a purely random situation evokes in most subjects an implicit model which is not adequate: random events are thought to be "equiprobable by nature." Lecoutre succeeded in activating an appropriate combinatorial model by masking the random aspect of the situation in spite of the fact that the chance model is highly resistant to change. Nevertheless such activation was found to be superficial. The transfer of an appropriate model to an isomorphic random situation was less frequent than had been expected. Lecoutre surmised that the little transfer occurred because the subjects were unable to construct an abstract representation of the situation. In a follow-up study (Lecoutre & Rezrazi, 1998) it was found that in situations when subject succeed in constructing an adequate abstract representation for themselves (with situations of cognitive conflict, learning situations with feedback, anchoring, etc.) the result of such cognitive activity was, not infrequently, the abandonment of an inappropriate "chance model" and a more stable transfer to isomorphic situations.

Khazanov (2005) developed instructional materials aimed at addressing three misconceptions: representativeness, outcome orientation, and the equiprobability bias. He also developed a test instrument aimed at identifying the above misconceptions. In his 2005 study, Khazanov trained some instructors and then had them use the instructional materials in a statistics classroom at their own discretion. Instructors could also use their own materials in addition to those suggested by Khazanov. The results were inconclusive. While some instructors managed to achieve a significant reduction in certain misconceptions, the majority did not report any major changes. It was not clear whether these outcomes could be attributed to the tenacity of misconceptions, instructor inability to use the materials properly, or simply to the insufficient number of activities used to address the misconceptions. The Khazanov 2005 study was broad in scope and exploratory in nature. The study recommended that a more structured approach be utilized to determine whether the instructional materials used in the study were helpful in correcting some common misconceptions about probability. The present study was designed to implement the approach recommended by Khazanov.

Method

The experimental phase of the study was conducted at an urban community college with a diverse student population situated in a large metropolitan area in the Eastern United States. One of the principal missions of that college is to provide access to higher education for non-traditional adult learners who seek professional and economic advancement and personal fulfillment. According to the college's office of institutional research, the median age of students is 22 years old.

The introductory college statistics course offered in this college satisfies the mathematics requirements for students majoring in such diverse fields as Accounting, Business, Liberal Arts, and Mental Health. The prerequisite for this course is basic algebra. A custom edition of a popular textbook was used that included all the topics represented in the syllabus. The topics in this course are measures of central tendency, measures of dispersion, graphs, the binomial and normal distributions, sampling distributions of statistics, t-test, chi-square, correlation and regression.

In particular, the probability segment of the course includes fundamentals of probability, addition and multiplication rules, and conditional probability.

Six instructors teaching in aggregate 9 sections of the course were recruited in the spring semester of 2007, along with their students. Three of these instructors were randomly selected to attend the workshop on misconceptions that was designed and conducted by Leonid Khazanov. The other three instructors did not partake in training. All participating instructors were provided with the instructional materials and a list of common misconceptions with explanations and examples. All instructors had advanced degrees (doctoral or masters) in statistics, mathematics, or mathematics education obtained from accredited institutions. It must be noted, that the control group used in this study was drawn from sections of Introduction to Statistics offered at the college in 2005. This control group was used in the Khazanov 2005 study. Although the control group was not drawn from the same population as the treatment group, it is a valid comparison group for the following reasons:

- Both treatment and control sections were drawn from the same statistics course "Introduction to Statistics" offered at the same urban community college
- In both cohorts students taking the course majored in the same areas
- Prerequisite requirements for enrolling students in the course did not change from 2005 to 2007
- The same syllabus for the course was used for both cohorts
- The same textbook was adopted by the department
- The same test instrument was used to identify the misconceptions
- Four out of nine instructors from the control group were in the treatment groups in the present study (by random selection, two of them were trained and the other two untrained); the remaining five instructors in the control group were similar to the instructors in the treatment group in terms of education and teaching experience

The workshop

The workshop was designed to inform statistics teachers about common misconceptions of probability and equip them with effective strategies for resolving the misconceptions. The workshop was conducted by one of the investigators in June, 2006 in two 1.5 hour sessions. Three instructors randomly selected from a list of six participating instructors attended the workshop which covered the following topics:

- What are misconceptions of probability; overview of prevalent misconceptions
- Why is it important to address misconceptions in the first statistics course
- Methods of assessing students' misconceptions; the PRQ
- Methods of addressing misconceptions; the Instructional Materials
- Implementation. The pitfalls and how to avoid them. Case studies

This workshop is described in detail by Khazanov (2008). The workshop was attended by all three of the selected participants. They were actively involved in the training and shared with one another their experiences in dealing with students' naïve intuitions and erroneous concepts. Each participant received a copy of the Instructional Materials and guidelines on how to use them in class.

Instructional Materials

The Instructional Materials (IM) were prepared by one of the investigators (L. Khazanov). The misconceptions addressed in IM are representativeness, equiprobability bias and outcome orientation. The materials are comprised of hands-on activities and problems for discussions. Some elements of the materials were designed by the investigator; others were borrowed from published sources and modified to varying degrees (Shaughnessy, 1977; Lecoutre, 1998; Fast, 2001; Konold, 1995). The proposed intervention included two hands-on activities and seven topics for discussions. The activities and problems were chosen so that they address all three misconceptions.

The theoretical underpinning for the instructional intervention based on the IM is the theory of 'cognitive dissonance' or 'cognitive conflict. The theory holds that people prefer their multiple cognitions to be consistent with one another. When their cognitions are inconsistent or 'dissonant' people feel uneasy and are motivated to make them consistent (Bernstein, et al., 1994). In addition to suggested activities and questions for discussions, the instructional materials contain guidelines for their implementation in the classroom. For each activity and question there is an indication of the misconception(s) treated, links to important concepts in the statistics course, suggested placement in the course, estimated time and possible organizational formats. Below is an example of activities utilized in the instructional intervention[1].

Tossing a coin

- Misconceptions treated**:** representativeness
- Links to important concepts**:** sample space, independence, binomial probabilities
- Placement in the course: introduction to probability
- Time required: approximately 20 minutes if some work is done at home
- Materials**:** fair coins

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[1] Reproduced with the permission of the author from Khazanov, 2008

- Organizational format: instructor may choose to do his/her own demonstrations or have students work individually or in small groups
- Procedure: this activity begins with the teacher asking students to compare the probability of obtaining the following 3 sequences of heads and tails in tossing a coin four times: TTTT, THHT, and THTT. Say this to your students: When you flip a coin 4 times and record the outcomes, the following sequences may occur, among other sequences: THHT, TTTT, and THTT. Compare the probabilities of these sequences. You may state your position in words, for example, the probability of TTTT is less than the probability of THTT, or the probability of TTTT is equal to the probability of THTT. Alternately, you may estimate and compare the probabilities numerically.
- Then the experiment (tossing a coin four times) is performed a large number of times and the results are recorded. It may be a good idea to ask the students to perform the experiment at home. Ask each student to do it 30 times at home and record the results. Make sure your students understand that they need to toss the coin 120 times to obtain 30 foursomes. This way you will have at least 600 sequences. Ask a group of diligent students to compile the data and compute the experimental probabilities for the above three sequences. The experimental probabilities for the aforementioned sequences are calculated using the relative frequency model. After that the students compare their predictions with the experimental results (some might observe discrepancies). The next step involves building a theoretical model for this experiment. Students should be asked (and if necessary helped) to list all possible sequences of length four and compute the probability of each sequence using the multiplication principle. Finally the theoretical probabilities are to be compared with the calculated relative frequencies and guesses. It is likely that students who hold the misconceptions of representativeness would predict a greater probability for THHT, than for TTTT. Both, the result of the experiment and the theoretical model are expected to contribute to the resolution of the misconception.
- This activity can be performed in a variety of organizational formats depending on instructor preferences. One possible format includes a teacher demonstration followed by students working in small groups. As was mentioned earlier, you may ask the students do some work at home. If you prefer the small group format, I can provide suggestions as to how students might be grouped to maximize their learning. In addition to addressing the misconceptions of representativeness, this activity can be extended to address the equiprobability bias; students may be guided to observe that different random events may have different probabilities (for example, the probability of getting exactly 2 heads without regard to order is greater than the probability of getting exactly 0 heads).
- Enrichment: have students explore various outcomes when a coin is flipped 6 times or 4 coins are tossed simultaneously (same denomination or different denominations). Students may be given a link to the website http://shazam.econ.ubc.ca/flip/index.html
- This website allows students to instantly view the results of tossing a large number of coins. They can make a variety of observations. For example, break down the long sequence into foursomes and count the number of sequences HTTH and HHHH.

Test Instrument

The test instrument entitled The Probability Reasoning Questionnaire (PRQ) was developed by one of the authors (L. Khazanov) and is discussed in detail in Khazanov (2005). The instrument has 16 items. Each test item is comprised of two multiple-choice parts: the principal question and justification. This approach has several advantages over the one-part items. A number of studies report that correct conceptual understanding is significantly overstated if no justification for the chosen answer is required (Konold, 1995; Hirsch & O'Donnell, 2001; Rubel, 2002). Konold (1995) described how a correct answer to a question that involved comparison of probabilities could be obtained using an erroneous concept. On the other hand, answers to both parts that are consistent with a particular misconception serve as strong evidence that the misconception is indeed part of the students' conceptualization of certain random processes and events.

The instrument was designed to test students for three highly prevalent misconceptions: representativeness, equiprobability and outcome orientation. To increase the utility of the instrument, an effort was made to create distracters consistent with at least two misconceptions on each item. The careful selection of items and creation of suitable distracters achieved this objective. When creating distracters, priority was given to common explanations consistent with one of the three misconceptions subject to investigation and treatment. These common erroneous explanations were extracted from the literature (Shaughnessy, 1992; Lecoutre, 1992, 1998; Konold, 1995, Konold, et al., 1993), as well as the investigator's own teaching experience. Distracters that reflect some other common misconceptions and skill deficiencies have also been used, where appropriate. About half of the items on this instrument are original. The other half are similar to those proposed by Konold (1990), Garfeild (1998), and Hirsch and O'Donnell (2001). However, the majority of the borrowed items were modified. The modification normally included adding or replacing distracters, rephrasing the problem or editing it.

The validity of the instrument was confirmed by three experts who have advanced degrees in mathematics or statistics. The experts were asked to pass judgment on how well the questions reflect the construct domain, and whether they are appropriate at the introductory college level. They were also asked to identify correct responses, as well as responses consistent with the specific misconceptions. This method known as "backward translation" is recommended by Sundre (2003). Items about which the experts disagreed were revised until a consensus was achieved. The experts also established the cutoff scores and estimated the time required to complete the test. The consensus was that a student would be considered to have a specific misconception if he/she scored two or more misconception points for that misconception on the test. The instrument was then subject to a known-group validation procedure, another accepted validation method recommended in the literature (Hirsch & O'Donnell, 2001). An agreement coefficient (Berk, 1984; Subcoviak, 1988; Garfield, 1998) was calculated for each misconception to establish the reliability of the test.

Administration of Test

The test instrument was administered in class at the end of the spring semester of 2007 in the nine participating sections of the Introduction to Statistics course. Students were given 40 minutes to complete the test. The test was administered on different days of the week for different sections. To maintain the integrity of the study, the instructors were directed to collect all question sheets after the administration of the test and not to discuss it with colleagues or students. In such a way, the independence of the outcomes was assured.

Results

A summary of results is presented in table 1.

Table 1: Percentages of different student groups by categories of probability misconceptions

Probability Misconceptions	CGNA (263)	TFA (106)	UTFA (65)
Representativeness	67% (176)	54% (57)	68% (44)
Equiprobability	83% (218)	80% (85)	80% (52)
Outcome orientation	37% (97)	27% (29)	34% (22)

CGNA=Control Group of College Students (No Activities); **TFA**=Trained Faculty Implemented Activities; **UTFA**= Untrained Faculty Implemented Activities

The first column of table 1 contains data based on a large sample of the college population who took introduction to statistics in an earlier semester. Instructors teaching the students in this sample did not receive any training and did not implement any targeted activities aimed at correcting the misconceptions. The second column contains data aggregated from the five sections whose instructors attended the workshop before implementing the activities in the classroom. The third column includes the data from the four sections where the instructors implemented the activities, but had not attended the workshop. The three categorical variables in this study are specified in the header of table 1. The observed values of these variables for each specific misconception are recorded in the respective columns. For analyzing the data, z - test for comparing two sample proportions from the population was used. The results show that trained instructors achieved significantly better outcomes than the control group in correcting the representativeness bias (p-value 0.0089) and outcome orientation (p-value 0.0404). There was no significant difference between the treatment and control groups on the equiprobability bias (p-value 0.7299). Notably, this study did not find any significant evidence that implementing the activities without training was instrumental in resolving any of the misconceptions that students might have.

Figure 1 illustrates the data presented in Table 1. Each triple of bars corresponds to a specific treatment category. The height of the bars reflects the prevalence of the misconceptions after the intervention.

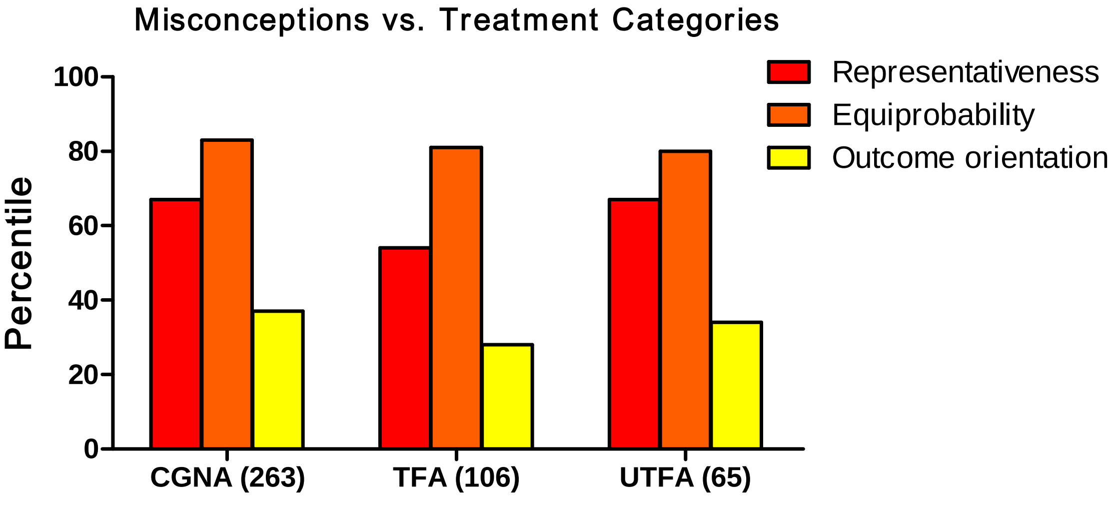

Figure 1: Misconceptions vs. Treatment Categories

Discussion

In this section we will discuss a number of points in our findings that warrant explanation. First, why did the instructors who had been trained prior to implementing the activities overall achieved better results than those who had not been trained? Second, why was there no significant improvement in the resolution of the equiprobability bias in any of the treatment groups? And third, why a large proportion of students in the treatment sections still did not have their misconceptions corrected?

A possible explanation of better outcomes for students taught by trained instructors is that instructors who belonged to the trained cohort gave more thought to the misconceptions and had a better understanding of their nature and, hence, how to address them in instruction. This might have affected how they led and summarized students' discussions and activities and addressed students' misconceptions when confronted with them. Another possibility is that trained instructors were more alert to misconceptions when implementing other activities in the course; this could have reinforced the impact of the activities and affected students' long term outcomes. Notably, the test instrument was administered at the end of the semester. Hence, the outcomes might have reflected the aggregate effect of teaching the course while being "misconception vigilant" (Khazanov, 2008), rather than just the net effect of the activities from the Instructional Materials. Although, as far as we know, none of the instructors targeted the misconceptions directly in any other activities beyond those recommended by the investigators, they might have kept an eye on misconceptions better than untrained instructors. We have anecdotal evidence to corroborate this speculation. One instructor shared with us that the idea of being "misconception vigilant" emphasized in the workshop had a deep and lasting effect on his teaching.

Our findings did not show a significant response to the attempt to correct the equiprobability bias. (Notably, this bias has the highest prevalence of the three considered in this study). A possible explanation is that there were not enough activities to address this bias which appears to be more difficult to correct than other biases. To learn their way out of the equiprobability bias, students need, in addition to understanding chance events in general, to develop the ability to quantify random events and compare probabilities numerically (Khazanov, 2005). Based on our findings, it appears that in order to resolve the equiprobability bias students would need more exposure to the unequally likely probability models. (These models are not sufficiently represented in many popular textbooks). Additionally, students must become proficient in the fundamentals of combinatorics.

It must be noted, that in spite of the best effort put forth by our participants to correct their students' misconceptions, a large proportion of students still retained some of them at the end of the term. When we come to think about the factors that detracted from the effectiveness of the intervention, time constraint appears to be a major factor. An introductory statistics course is replete with topics that need to be covered. Because of the time constraint, we had to limit the number of activities and topics for discussions included in the intervention.

Another important factor that surfaced in the course of carrying out the intervention was that some instructors were unable to trigger cognitive conflict in all students; some students could not see the discrepancy between their concept and the evidence presented. Triggering cognitive conflict is a powerful approach to resolving misconceptions. Its effectiveness has been confirmed in a large number of studies. However, it does not necessarily work for all students. It is possible that for some students, gradual knowledge refinement and reorganization would be a more productive approach to correcting their probability misconceptions (Roschelle, 1995). This speculation needs to be tested in future studies. The investigators have already begun to revise the IM to incorporate this approach.

And finally, a plausible reason why so many students still had misconceptions in spite of the intervention is that people's naïve concepts about probability are difficult to change. A deep insight into this issue is offered by Steven Pinker in his book "How Mind Works" (1999). Pinker pointed out that human brain has undergone extensive growth about 100,000 years ago. At that time our brain has evolved modules instrumental in the processing of important survival information. Some probability misconceptions are perfectly consistent with the kind of probability experiences our ancestors had and which we ourselves still experience in everyday life. For example, the belief in Gamblers Fallacy, which is a manifestation of the representativeness bias, may be explained by the experiences created by observations such as 'a sunny day is more likely to follow a series of rainy days' or 'it is more likely not to see a wild animal after several of them passed by'. Change is a norm in nature, not an exception. Thus believing in change had some survival value and is therefore deeply ingrained in human psyche. To overcome the misconceptions, students sometimes have to go against their natural instincts, and this proves to be difficult.

In conclusion, we would like to map out a few directions for future research in the area of probability misconceptions. We believe that it is important to conduct a study designed with the objective of getting a significant improvement in the resolution of the equiprobability bias, which proved to be particularly resistant to change. Another possible direction is to conduct a qualitative study that will include classroom observations and interviews with instructors and students in order to further our understanding of factors that affect students' conceptual understanding of probability. It may also be useful to develop a test instrument to identify other misconceptions, especially those effecting comprehension of inferential statistics.

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