Building Understanding and Fostering Empowerment Through Technological Interactions

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Abstract

This case is part of a teaching experiment that sought to provide insight into how the use of mathematics technology affects the internal mathematical representations possessed by adult developmental mathematics students. Sequences of dot patterns were examined and analyzed by the participant. He then used the algebraic features of a dynamic, interactive geometry software program to graph data describing the functional relationships present in those patterns. Insights about the effect of technological representations on adult students' thinking can be gained from observations of the subject's investigations of that data. Results show that appropriate use of technology can aid adult students in building an understanding of standard representations that is based upon their own thinking and their own choices.

Key words: mathematics technology, representations, empowerment

Introduction

Epper and Baker (2009) noted that although the use of technology may be critical to the success of adult mathematics' students, technological innovation has outpaced evaluation. Technology as it is used may only be fostering superficial knowledge (Caverly et al., 2000). The results of a recent teaching experiment show that adult learners, even those whose numeracy may appear to be considerably challenged, can use technological representations not only to work through misunderstandings, but also to build valid new internal representations of mathematics. Such representations may best be built if work occurs within the student's zone of potential construction (ZPC) which describes the potential students have to change their own understanding as a result of mathematical interactions (Norton & D'Ambrosio, 2008). The research was influenced by the work of Campbell (2003), who examined the use of dynamic tracking of students' computer interactions. It was also influenced by Falcade, Laborde, and

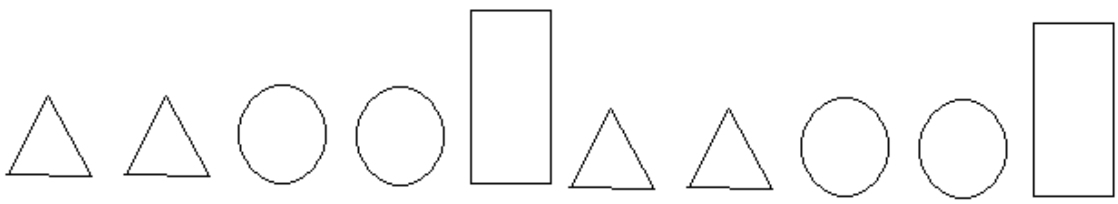
Marotti's (Falcade, Laborde, & Mariotti, 2007) examination of dynamic geometry software for the study of ideas related to functions. Inspired by Kaput's (1994) vision of technology's potential to increase all students' access to abstract mathematical ideas, the study sought to determine how dynamically connected representations might provide adult developmental mathematics students with greater access to standard representations. In this instance, the technological instruction that was presented allowed one subject, Marlon, to make his own choices as to the dynamic interactions he would undertake with standard representations of functions. The discussion of what Marlon experienced will be prefaced by a brief look at some aspects of the methodology used for this study.

Methodology

Subjects were recruited from Harrisville State University (HSU), a mid-sized university in the Southern United States serving a large developmental mathematics population consisting of adult learners as well as recent high school graduates who needed additional study in order to be prepared for college level mathematics work. Data collection occurred at HSU as well. The researcher conducted open recruitment, placing flyers around campus and entering the classrooms of HSU teachers in order to recruit students. Three subjects were selected, two of whom finished the teaching experiment. This article describes a case study of one of those subjects, Marlon, a 53-year-old African American male enrolled in his second developmental mathematics course. He had some experience with computer aided instruction, but not with the software used for the study, Geometer's Sketchpad 4.07S (Key Curriculum Press, 2006). An initial interview provided insight into his mathematical thinking and experience. This interview included the presentation of patterns of shapes and dots upon which a discussion of functions could be built. Figure 1 shows the two tasks used in the initial interview.

Looking at patterns

Study the pattern below and tell me everything you notice about it.



Looking at dot patterns

Study the pattern below and tell me everything you notice about it.

Figure 1. Tasks used in initial teaching experiment interview

The initial session was taped with two recordings, one view focused on the subject's work on paper and one view focused on the subject and interviewer. The later sessions in which the technology was introduced were taped with three recordings, adding a view of the computer screen to the other two views. Marlon participated in 7 total sessions. He explored "Looking at dot patterns" further, and was introduced to a follow up activity, "Another dot pattern", seen in figure 2.

Another dot pattern

Figure 2. The additional representation of a functional relationship given to Marlon to explore

Marlon's technological explorations included graphing coordinate points, graphing data points representing "Another dot pattern", and making observations and predictions. He tried different options from the functions menu in order to graph a line passing through the coordinate points and tested ideas he was building about functions. He also followed instructions to create a dynamic representation that included a movable measured point, which was attached to the graph of function and generated a table of values. He was encouraged to create a table of values on paper representing the dot pattern, to graph those points, and to try to find the function that would pass through those points. The experiment, however, was not an intervention and he was allowed to make his own mistakes and discoveries so that the potential impact of the technology on his learning could be examined. Following is an examination of some of the events that occurred and the ideas that emerged as that process unfolded. Interview protocols for the semi-structured interviews may be found in Appendix A. Note that addition questions based on the subjects' thinking are part of those protocols.

Results

The evidence described below will show that Marlon possessed algebraic and graphical misconceptions that were interfering with his progress. It will also show that as he explored the representations, he was able to observe patterns, make sense of what he was seeing, and use technology became an aid for his reasoning. His use of technology appeared to eventually affect his internal representations, as evidenced by his statements and the coordinated gestures and mouse movements used to illustrate them, which were referred to as indicative movements. He appeared to gain a better understanding of standard representations and greater mathematical empowerment. Following is a closer look at some of these results, beginning with a description of a misconception that emerged about the representations of functions and coordinate points.

Function and coordinate point confusion

Marlon consistently confused the representations of coordinate points and the representations of functions. This was most commonly exhibited in his tendency to enter the function y = a + b in his attempt to graph a function which would pass through the point (a, b). His confusion was also manifested in his representation of a coordinate point as a sum. When I asked him to represent the point (0, 9) after its location was identified on a graph, he wrote 0+9. Following is part of that exchange.

Interviewer: What would the coordinates of that point be? [His cursor is at (0, 9)] If I were to ask you to put that point into a table . . . what would the table values be for that point?

Marlon: Okay . . . I would say um that x is going to be a negative 9. [He has given the x-intercept of the graph of the line rather than the coordinates of the point on that line which is being indicated. Interviewer gives him a clean sheet of paper]

Interviewer: Okay what would the table values for that point be?

Marlon: Especially when they're intersecting the x and the y axis.

. . .

Marlon: I would say that the x-axis [writes x on the paper] is equal - is a negative 9.

Interviewer: How do you know what the - if we're just looking at that place where the arrow is pointing right now, [referring to the position of the cursor] that point right there, . . . what would the table, . . . right at x and y

Marlon: Oh, that would be zero, that would be zero, nine, so that would be zero plus 9 [he writes 0+9]

Interviewer: How would you write it in a table? Write it as if it were in a table.

Marlon: In a table so um it'd be here , zero , plus 9 [he creates an x, y table and enters 0 in the x column and +9 in the y column] That'd be my table.

Figure 3 shows what Marlon wrote as he made these statements. When I asked him to write the same idea as if it were in a table, he said, "In a table . . . it'd be here, zero, plus nine" and entered 0 in the x column and +9 in the y column.

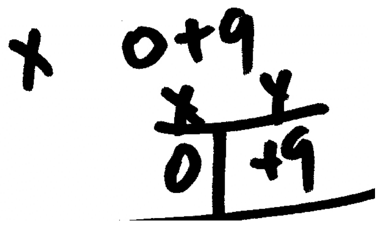


Figure 3. Marlon's representations of the point (0,9)

Technology as an aid to reasoning

In spite of his misconceptions, Marlon had exhibited ability to reason in his initial interview. As he learned to work with technology, it started to become a reasoning tool. Some of Marlon's interactions with the technology in session 4 seemed to indicate that he was beginning to reason logically based on the technological representations he was seeing. During session 4 he was asked to make predictions about the number of dots in the 20th, nth or xth pattern. Later he was introduced to the functions menu and encouraged to try the functions menu, using x as a variable and to try creating a function which would pass through the data points. Once he had graphed coordinate points for "Another dot pattern" up to (9, 10), he was asked to explain what was happening and where the next point would be located. He correctly located the point (10, 11) before graphing it and without directly referring to the x and y-axis until asked how he knew where it would be. Soon after this, he described the pattern he was seeing and gave the location of the next point, (11, 12), before graphing it. He described what he understood about the pattern of points, noting that "I'm going from the middle point all the way up and I'm also going to stay in my positives because I'm in my x-axis in the positive numbers and these are all positive numbers here that I have." As he said they were all positive numbers, he gestured along the paper copy showing the step numbers and numbers of dots which described "another dot pattern." He accurately predicted the location of the next point in the pattern, stating that "If I'm going to plot in the second one, the next one . . . if I plot my next one here . . . its 10, 11 so 11, 12, is actually going to, it should be here. And there it is." Later when I asked him to predict where the point (20, 21) would be, his indicative movements went along the x-axis to 21, and then to the general area where the point is actually located. He then made a precise prediction based on the locations of 20 and 21 on the x-axis and the y-axis. The use of technology allowed him to make and follow up on mathematical predictions and so aided him in his reasoning about mathematical patterns. With some facilitation in the form of guiding questions as described below, he was also able to use the technology to make progress in recognizing and clearing up some of his misconceptions.

Using technology to reveal and clear up misconceptions

During the fifth session, he explored ideas he had been exploring previously, and was encouraged to test ideas he had been building about functions. As part of that recall and exploration, I asked him to remember the function he had found in a previous session that produced a graph that passed through his data points. He could not remember what that function was, and in his efforts to remember, his confusion over the representations for coordinate points and functions interfered. For example, he opened the function menu. He knew that the function had to pass through the point (1, 2) and so he had graphed f(x) = 1 + 2, as can be seen in Figure 4 below. Since the graph did not travel in a diagonal line through the points, he tried another function. He looked at the currently graphed point which was farthest to the top and right of the graph at that time, (8, 9), and graphed g(x) = 8 + 9. This graph turned out to be out of the viewing window, and he had to change the scale of the graph to see where it was. He did this himself with no facilitation after he had done some additional exploration which had produced an h(x) = 8 + 9 and the equation 1 +2 = 3. When he finally saw the graph he said "I'm getting a straight edge again here" and indicated 1+2 = 3 and then the graph of g(x) = 8 + 9 with his mouse. When I asked him where the graph of g(x) = 8 + 9 had come from, he at first replied "this last one I just put in" and indicated 1 + 2 = 3, then said "as you were" which was a phrase he commonly used when he realized something was wrong. He then counted to see that g(x) crossed the y-axis at 17. I asked him "Where might 17 come from?" This was genuinely puzzling to him, and he wondered aloud "How did I get 17 in there?" I asked him if there was anything on the screen that might give him 17. The situation at the time he responded is illustrated through the image presented in Figure 4, which is followed by his statements.

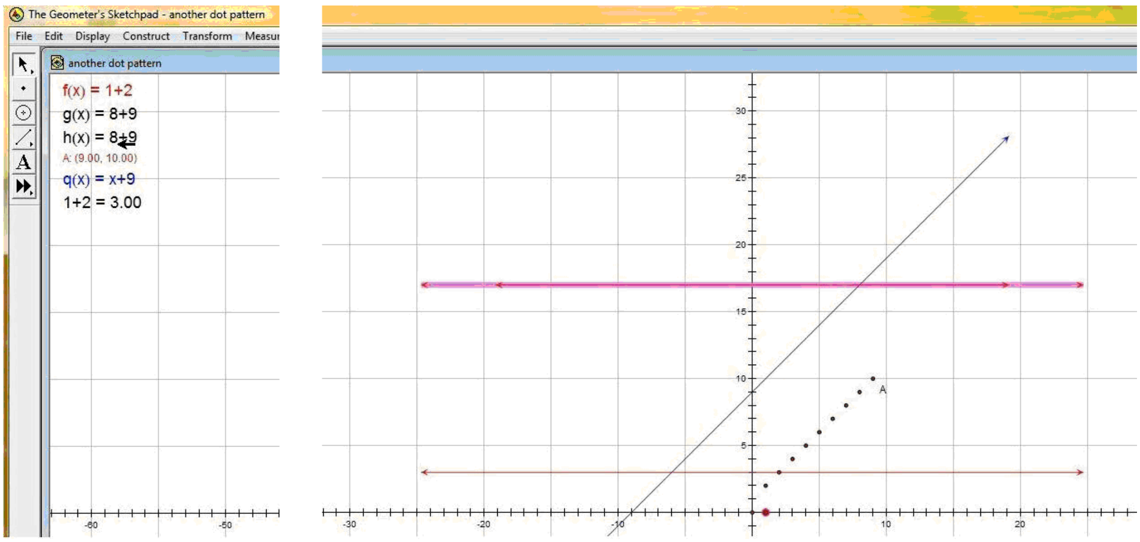


Figure 4. These portions of a screen shot show the situation at the time Marlon was asked to consider why the graph of h(x) = 8 + 9 might have been placed at y = 17.

Marlon: [he put the cursor at h(x) = 8+9 then moved it down toward the bottom of the list between q(x) =x+9 and 1+2=3] Ummmm, [He moved the cursor back up the list and then to 1 + 2 = 3] I don't see, I mean, that would give me 17. How'd I get that one there? [Cursor near 1+2=3 and q(x) = x + 9] And this is the, this is the same -similar to the one that I gave down here [cursor at (0, 3)] through 3. Okay [cursor to 1 +2 =3, cursor up to h(x) = 8 + 9]. Oh, not unless these added together.

By looking at the multiple representations presented by the technology, he realized that in the functional notation f(x) = a + b, those two numbers were in fact added together to determine where the graph would be located, and were not representative of the two numbers describing a coordinate point, which was the way he had been trying to use them to graph the function. The location of the graph of h(x) = 8 + 9 in a different place than he expected it to be revealed the misconception and examining the different representations present helped clear up the confusion. Even though the representation 1 + 2 = 3 was not the functional representation which matched the graph which passed through (0, 3), the presence of that representation may have been important to his building an understanding that "these added together." Though he appeared to be learning, it was not always clear what changes were taking place in his internal representations.

Internal representations

One of the goals of the study was to determine the effect of technological representations on the subject's internal representations of mathematics, which are those mathematical forms that exist within the subject's mind (Goldin, 2003). As the study progressed, and it became clear that my time with Marlon was growing short, I decided to give him a more dynamic representation in the last session which might build on what he had been experiencing and affect his internal representations in some way. The activity described below presented him with a prescribed set of technological instructions, since this would introduce a new feature of the technology, and I wanted the technological steps to be clear. Even though the steps were described, he still needed some help in interpreting those instructions, for example confusing the selection arrow tool with the point tool (perhaps because it points to things). A copy of the activity as provided to Marlon is found in Appendix B.

Marlon's explorations to that point had led to his speculation that functions of the form f (x) = x + b cross the x-axis at y = b. The activity called for him to put a sliding point on the graph of f(x) = x + 9, create an electronic table of values to track where the point was located, and eventually animate the point. Once the representation was created, he was free to study it and make observations. Marlon's language about what he was seeing seemed to change from the language he had previously been using. During the study he had at first focused on the y-intercept of the graph. He then noticed some other locations where the graph intercepted other parts of the xy plane. After seeing the dynamic representation, he said:

> When it was sliding up and down, it was actually giving me these different locations *[cursor to table and then graph]* where it was crossing over the line. Every single one was giving . . . in this case here x and y *[cursor from line to table]* . . . . So all of these here are actually on this particular connected.

His mention of "different locations where it was crossing over" seemed to connect to his previous explorations. The rest of his meaning was unclear. When I asked him what he was trying to describe, he said "The whole line itself." It seemed that his understanding of the graph had moved to a new level. The sliding point and its accompanying table seemed to have helped him to consider the idea of the entire line – an idea conceptually beyond the multiple points he had noticed previously. This is supported by his statement that:

> The whole line comes from actually . . . connecting all the different points in a straight line, connecting every last one . . . because they (were) plotted and they all . . . intersected each other.

His internal representation seemed to have gone beyond a focus on the y-intercept and "intercepts" of other lines which the graph of $f(x)$ crossed. He now seemed to consider the entire line as a collection of connected points. The phrase "every last one" in particular seems to indicate that his thinking may have been broadened and gone beyond the idea of "crossing over." The following section the use of technology in understanding and using standard representations.

Technology as an aid in the use of standard representations

In addition to building his internal representations, technology was also an aid in the use of standard representations in Marlon's work on an occasion when he was trying to find a function, which would pass through the data points he had graphed for "Another dot pattern." He had tried $r(x) = x + 2$, and I asked him to think about what he had done.

Marlon: Okay so now what I did especially is I provided x plus 2, that's what gave me this (he traced that graph).

Interviewer: Okay

Marlon: And again, x, this is my x axle (indicating the x-axis) I mean axis and plus two gave me right here (indicated (0, 2)) Okay. So if I want to do this here I can actually say x + 1 (indicated (0, 1)) so if I go to . . . the graph a new function (he opened that menu) I can say x plus 1 (he entered it).

He had not chosen plot new function. Once the function was plotted he said:

Marlon: There she go: x + 1 and the reason why I got it is because here again is my x-axis (traced the x-axis) and plus 1 is right here {he went up to (0,1) from (0,0)] and it intersected through those points that I graphed last week (he traced along the graph).

Note that he used the cursor to indicate the location of the previous attempt and that once he graphed the new attempt he used the cursor to indicate and trace along key aspects of the new attempt. It may be that such indicative movements serve to help the learners solidify in their minds the knowledge they are building. His statement about "why I got it" shows that he is experiencing some mathematical empowerment. Following is a further discussion of technology's use for mathematical empowerment.

Empowerment through the use of technology

Marlon used the technology for his own explorations of ideas he was having about what was happening and worked to understand the functions he was creating. In this way technology empowered him mathematically. During session 4, I asked him to enter the variable x into the functions he was graphing. He graphed f(x) = x + 9 and g(x) = x - 9 and noticed that they crossed the y-axis at (0, 9) and (0, -9) respectively. He was building some understanding that functions of the form f(x) = x + b cross the y-axis at (0, b). He also observed that the two graphs were parallel. When I questioned him further, he was able to deduce and demonstrate that f(x) =

x + 1 went through his graphed points. In session 5, he had reasoned his way back to that representation again after forgetting it.

Later in session 5, after some discussion, I asked him to try something that he hadn't done before. He entered v(x) = x – 9 and then said "Could I add more?" and I told him he could try that. The software automatically entered parentheses to what he entered to give the function v(x) = (x – 9) + 6. After giving the graph and equation matching colors, he moved the equation close to the graph, studied it and said: "Now how did I get that one?" I turned the question back over to him. During the exchange he moved the cursor from the algebraic representation to just below the y-intercept briefly, back to the algebra and then to a blank spot in the second quadrant.

Interviewer: Think about what's happening with that one. Why is it going the way it's going?

Marlon: Okay. x negative 9 - so - x negative 9 here - x negative 9 (cursor at algebra) — I'm coming across here to a 3, (cursor at (0,-3)) Ah! What's happening (cursor at algebra until the word "giving" when it moves to (0, -3)) is that it's subtracting (cursor moves back and forth along the algebra until "negative 3" when he moves it back to (0, -3)) the negative 9 from the 6 and it's giving me a negative 3 and that's the reason why it's intersecting here (indicates (0,-3)) because . . . its subtracting the -9 from a positive 6 which gives me actually negative 3 and again it is diagonal.

His cry of "Ah!" seemed to indicate confidence, as did his clear explanation. Although there was most likely much he still did not understand, he understood a correct mathematical idea which made sense to him and was built on his own demonstrated prior knowledge. He had chosen the exploration himself, he had put his plan into action using the technology, and he had drawn a correct conclusion about the connection between the algebraic and graphical representations. Such an exploration may have been difficult for him to do without the use of technology, and in this way he was empowered by his use of technology. Technology helped him see his misconceptions, helped him gain insight into and build greater understanding of standard representations, strengthened his internal representations, and provided him with mathematical empowerment.

Internal representations, which were being built by Marlon, seemed to be based upon his own thinking. This can be seen in the earlier stages of his explorations when his understanding was weaker. Marlon's choice to try f(x) = 1 + 2 to pass through the point (1, 2) was based on his conception of a coordinate point as a movement to one side and then an upward motion symbolized by addition. This conception was evident from his mouse movements and written representations of coordinate points. He moved the mouse to the right and then up to show the locations of coordinate points. When asked to write down a representation of the point (0, 9), he wrote 0 + 9, and said, "It'd be here, zero, plus nine." In session 4, as he considered how to find a function to pass through his coordinate points, including the point (1, 2), he noted that he should have said 1 + 2 and moved the cursor from (0,0) to (1,0) to (1,2). His choice to try f(x) = 1 + 2 as a function which might pass through the data points for the pattern of dots found in "Another dot pattern" was based upon his idiosyncratic thinking about coordinate points.

Later in the study after he had begun to build some understanding, I asked him to create a function of his choice. He had noticed that functions of the form f(x) = x + b passed through the y-axis at (0, b). He had already been working with g(x) = x – 9. When I asked him to create a function of his own, he decided to "add more" and created v(x) = (x – 9) + 6. He then observed that the function passed through the y-axis at -3 and was able to deduce correctly that the -9 and the +6 were combined to give the information as to where the graph crossed the y-axis. He was building upon his own knowledge to understand a new function that he had created. Such episodes and others carry implications for adult students.

Implications

What can adult mathematics students learn?

There are important implications of this study that might be shared with adult developmental mathematics students. This study reinforces the observation that some of the challenges adult learners face is their own lack of self-efficacy, lack of study skills, and learning experiences which may be procedurally rather than conceptually based (Epper & Baker, 2009; Wadsworth, Husman, Duggan, & Pennington, 2007). Marlon was able to make and check his own conjectures through the use of technology. Adult learners can consider that some of those investigations would have been difficult for him to engage in without technology. They can think how this type of empowering investigation may help them to make valid mathematical learning choices. Such experiences may help build their self-efficacy. Student centered investigations may also help adult developmental mathematics students to understand the learning process better in general and thus enhance their learning skills. In addition, they may see that being able to create and test their own conjectures can help them learn conceptually and see the advantage of learning conceptually. In the current study, for example, the subjects sometimes appeared to remember and discuss more clearly their own investigations.

Adult learners may be encouraged by the progress Marlon made over the course of the study. He began with great confusion about what he was seeing, and an inability to properly represent coordinate points, representing them as a sum of the x coordinate plus the y coordinate. He did not understand what the functional notation the software required meant and saw it as just another way to graph coordinate points, entering f(x) = a + b in an attempt to graph a line passing through the point (a, b). During the study he was able to discover that this was a misconception and that such functions create a horizontal line crossing the y-axis at y = a + b. By the end of the study, he had learned that functions of the form f(x) = x + b create a diagonal line passing through the point (0, b), and that such functions pass through other points on the plane. He learned to associate the step numbers for the table of values representing “Another dot pattern” with the numbers along the x-axis. He also seemed to begin to understand that a function is associated with points along the entire line, representing “the whole line itself.”

What can teachers of adult mathematics students learn?

Teachers of adult development students can note that misconceptions were revealed through and affected the use of technology. A better understanding of their students’ misconceptions will allow adult developmental mathematics educators to recognize students’ zones of potential construction (ZPCs) (Norton & D’Ambrosio, 2008). Understanding this will help them to better understand how best to implement the use of technology (Norton & D'Ambrosio, 2008). The findings of this study imply that technology has the potential to help build learning within adult learners’ ZPCs.

The results of this study also imply that teachers of adult learners who are incorporating technology would benefit from having multiple forms of assessment they can draw upon. They can consider their students’ indicative technological movements and verbal descriptions of what they are doing. They can also examine their students’ use of both paper and technological representations in order to more clearly assess their understanding. Validity in making inferences about student understanding should include adequate relevant evidence (NCTM, 1995). Allowing students to represent mathematics in different media and asking students to explain those representations may allow teachers to make more valid inferences as to their students’ learning.

Teachers should also take note that Marlon seemed to be able to build more enduring representations when he had some choice over the avenue of exploration and was building on the choices he had made. Teachers can choose tasks in which students have control over their avenues of exploration and which are situated with those students' ZPCs. In considering the challenges which adult learners may have in remembering technological procedures, teachers may also wish to provide clear facilitating tools to accompany the use of technology so that the dynamic representations and the empowerment they provide can be activated by the student for learning as easily as possible.

Conclusion

Coming from a setting in which he was failing at his attempts to remember algorithms, Marlon's investigation of $v(x) = (x - 9) + 6$ built upon his understanding of functions of the form $f(x) = x + b$ was an example of the kind of mathematical empowerment technology can provide adult students. Technology has the potential to prevent mathematics from being the "insurmountable barrier" it was recently described as being by Bryk and Treisman (2010), who noted that difficulties in mathematics was "ending . . . aspirations for higher education" (p. 19). Adult mathematics students need not be victims of traditional teaching methods which have been passed down as an artifact, but which have not been effective in meeting their learning needs (Tate, 1995). The results of this study seem to indicate that a wise use of technology, which encourages individual exploration and knowledge building with the student's ZPC, can change adult education for the better.

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Appendix A: Interview Protocols

Interview Protocol: Initial Interview

Project: the Effect of Technological Representations of Developmental Mathematics Students' Understanding of Functions

Obtaining of informed consent: Before anything else, I need to obtain your signature on this document. It officially lets you know about the study and what your rights and obligations are. *(Point to first paragraph).* You can read here that the purpose of this project is to find out how using computer software to study mathematics affects how a person thinks about mathematics. Look over the rest of the document and let me know if you have any questions. *(Answer questions as needed, obtain signatures.)*

Obtaining personal information: *Ask subject to fill out the subject information sheet.*

Further description: This first interview will help me get to know you a little bit and find out what your experiences with math have been. I'll be selecting four people to continue on and participate in more interviews where we'll look at some math computer software and I can see how they think. No one will have to pass any kind of a test to be selected to continue. I'll be picking people to continue based on how they will fit in with what I want to learn and what I'm doing to do to learn it. Do you have any questions about the project?

Questions and Prompts:

1) Tell some things you remember about school when you were growing up.
2) Describe a *(another)* good experience you remember having in school.
3) *(If not already stated)* What helped you learn?
4) *(If not already stated)* What made learning harder?
5) Talk to me about your most memorable teacher.
    a. How did they teach?
    b. How did you feel about that class?
6) *(If not already mentioned)* Describe your most memorable math teacher.
    a. *(If not already stated)* How did they teach?
    b. *(If not already stated)* How did you feel about that class?
7) What about your other math classes?
8) What other things do you remember about your math classes?
9) Did your teachers ever use any kind of technology in teaching mathematics?
    a. That includes calculators, computers, and the internet . . .
    b. *(If so)* What did they use?
    c. How did they use it?
    d. How did you feel about using ________________ to study mathematics?
    e. Do you think ___________________ helped you learn?
10) *(Give them a blank sheet of paper and a pencil)* Show me and tell me about some mathematics you remember. It can be anything that you remember.
    a. Why did you pick this to share?
    b. *Additional probing questions might focus on the mathematics they share, but not for the purpose of measuring achievement, e.g.*
        i. Can you tell me anything else about ___________________?
        ii. *NOT* What's the answer to __________________________?
11) I'm going to give you something to look at now, and I want you to just tell me everything you can about it. Don't worry about what the right answer is, or what math you're supposed to use or anything like that. Just look at this, read the directions, and follow them. One of the other things I want you to do is talk out loud about what you are doing as you work,

talking continually, as if you were thinking out loud and I were listening in on your thoughts. *(Give them "Looking at patterns).*

a. *Probing questions for this and number 12 might include:*
    i. How will the pattern continue? Can you draw the next shapes in the pattern?
    ii. What will the 20th shape in the pattern be?
    iii. Can you tell what the 35th shape in the pattern will be (*without drawing it?)* What about the 41st? How do you know?

12) *(If this seems not to challenge them, you may continue with one or more of the following)* Here is another idea for you to think about. *(Give them "Looking at dot patterns" and/or "Soda cans").* Read the directions and follow them. Talk out loud about what you are doing as you work, talking continually, as if you were thinking out loud.
    a. What will the next pattern be?
    b. Can you tell me how many dots the 10th pattern will have? How?

13) Thank you for participating. Is there anything else you'd like to say?

Teaching Experiment Interview Protocol: Exploring Patterns

Recalling the last session and examining the data:

1) What do you remember about this dot pattern that you saw last time?
    a. *Follow with probing questions, such as those listed below, to elicit student thinking*
        i. Why did you think that?
        ii. Is there anything else you've noticed about the pattern?
        iii. How do you know what the number of dots in the next pattern will be?
            1. Is there a way you can write down this information?
        iv. Do you know what a table of values is?
            1. *If they do:* Show me what you know about tables by creating a table of values showing what you know about the step numbers and numbers of dots.
                a. Tell me about your table.
            2. *If they don't or are unsure: then go ahead and give them page 1 of the exploration, so they can see the blank table.*
                a. Does this look familiar? What do the columns tell us?
                b. Enter what you know about the step numbers and number of dots.

2) Look at the information in your table and see if you notice any patterns. Record your thoughts and talk about them as you write.
    a. *Subject may also compare back to the dot pattern representation.*
    b. What is the number of dots in the _____ the pattern? (*Several steps beyond the data they have).*
    c. Can you give me a rule for how many dots would be in any pattern?
        i. *If they can, show them another pattern until you find one for which they can't. Finding the rule will be the overall question this teaching experiment seeks to answer.*

Introduction to the software

1) We are going to use mathematical software to study this question. The name of the software is the Geometer's Sketchpad. *(Open the software)* Have you ever heard of it?
    a. *(If so)* Open a new file and show me what you know about the Geometer's Sketchpad.
    b. *(If not)* We're going to explore it. Open a new file. Make it as large as you can.

i. What do you notice?
ii. Hold your cursor over the buttons at the left.
1. What do you notice?
2. What are the names of the buttons?
3. What happens when you hold the buttons down?
iii. Try some of these buttons and tell me what you notice.

c. Open a new file. From the menu at the top, select *Graph, define coordinate system.*
i. Does this remind you of anything?
ii. *(If so)* What is it? What can you tell me about it?
iii. *(If not, unsure, or more scaffolding is needed regarding the xy plane)* Use the point tool to put a point somewhere in the coordinate system. Use the measure menu, and measure the coordinates of the point. Use the selection arrow tool and move the point around. Talk about what you see. What are the coordinates telling you?
1. *(If needed to elicit further observations).* Use the graph menu and choose snap points if you want the point to always land at an intersection. Look at the coordinates.
a. What do you notice as you move the point around? Try this for a minute and as you do so, talk about what you see.
2. Put the point at a place where[1]
a. Both coordinates are the same
b. The right is twice as much as the left
c. The right is half as much as the left
d. The right equals the negative of the left or the left equals the negative of the right

d. Using the *selection arrow tool*, select the point at the middle of the crossed axes (called the origin) and move it to see how the coordinate grid moves around.
i. What's happening to the point you put on the grid?
ii. What about its coordinates?

e. Using the *selection arrow tool*, select the other point you see, the one at 1 on the x-axis and move it to see how the grid changes.
i. Talk about what you see.

f. Using graph, plot points, enter some coordinates of your choice and see where those points lie. See if you can predict where they will be.
i. Move these points around. How do they behave differently from the other points you have put on the graph?

Using the software to look at their data:

1) How can you use these tools to explore your data?
a. What data do you have?
b. What value in your data could be the left coordinate of a point? What could be the right coordinate to go with that point? Why?
2) *If they have graphed their points:* What pattern do your points form?
a. (*If they think the points are in a straight line).* How could you find out for certain?
3) What if you had another step in the sequence of dot patterns, whose data is not already listed in your table? Where do you think the point representing that data would fall?

[1] As suggested in Key Curriculum Press (2002)

Additional questions may be asked, based on the student's thinking.

Teaching Experiment Interview Protocol: Exploring Functions

Project: the Effect of Technological Representations on Developmental Mathematics Students' Understanding of Functions

Questions and prompts:

Recalling the last session and starting algebraic notation

1) Begin with GSP in its opening configuration
    a. Show me what you remember from last time. Talk about what you see
        i. What is the problem we're trying to solve?
        ii. What do you know about that problem?
        iii. What have we done so far to explore that problem with GSP?
        iv. What does the table tell us?
        v. What does the graph tell us?
        vi. If I asked you now how many dots were in a certain step number, how would you find out?
        vii. If we had a rule, how would we describe it?
            1. *Look at a simpler pattern for which a rule can be found and help the subject to see how to use algebraic notation to describe it.*
                a. How is the number of dots related to the step number?
                b. Can you write that in a sentence?
                c. How can we write that in a simpler way?
                    i. Use boxes to represent the step number if scaffolding to the use of variables is needed.

Using the software to explore algebraic notation

2) Go to graph, new function. In the rectangle, you can enter expressions, or rules.
    a. Enter the rule we found for our pattern
    b. Hit okay.
    c. Hit plot function.
    d. What do you notice?
    e. Why does the graph look the way it does?
        i. Explore some different expressions. You may change the colors to match using the display, color menu.
    f. What patterns do you notice?

Additional questions may be asked, based on the student's thinking.

Appendix B: Technological Activity Used in Marlon's Last Session

Geometer's Sketchpad

Finding more points on the graph of a function

1) Use the point tool to attach a point to the graph. Make sure you see the line of the graph turn blue. This will mean the point is attached to the line. Use the selection arrow tool to test and make sure the point is attached to the line. You should be able to graph the point and slide it up and down the line. It should not leave the line.
2) With the sliding point selected, measure its coordinates. What are they? Slide the point some more. What do you notice?
3) Go to Edit, preferences, and change "other's" precision to units or tenths. Hit okay. What happened to the coordinates?
4) GSP can help you keep track of the different coordinates in a table. Select the coordinates (not the point itself, just its coordinates). Go to graph, tabulate. After the table appears, go back to the graph menu and select add table data. Click on the second bullet so the program will add several entries to the table. You can change the 10 to a higher number if you want more entries in your table. When the point moves data will be added to the table.
5) Move the point and watch what happens to the table.
6) What patterns do you notice?

P.S. Here's another way to move the point!

1) Select the point.
2) Go to edit, action buttons, animation. A button will appear which says "animate point."
3) Click on the button and see what happens.
4) If you want to change how the point is moving, select display, show motion controller.
5) Click on animate point and the motion controller will also activate. In addition to play, stop, and pause, you will see a button which allows you to change the direction the point is moving in and a place to slow the point down or speed it up.