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(MIS)CONCEPTIONS ABOUT GEOMETRIC SHAPES IN PRE-SERVICE PRIMARY TEACHERS

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Abstract: During the last reform, Slovakia has reduced geometry in curriculum in primary mathematics education. Pre-school education schools devote more time to geometry in curriculum, but this is not mandatory for all the children. In the primary schools, teachers devote very little lessons time to this problem. These lessons are mainly focused on technology of drawing rather than on creating the right conceptions, which results in below the average scores on international test results in Slovak students. One possible solutions is to improve the level of geometrical knowledge of primary education teachers and pre-service primary education teachers.

This paper is focused on the knowledge of plane geometric figures and their properties. We found out that pre-service teachers in primary education have many misconceptions that started when these students were learning the subject matter during their lower level education. In order to be able to more effectively design the curriculum, didactics and methods for future math teachers, we decided to test pre-service primary education teachers in planar shapes during their first year of university. Some results are presented in the article.

Key words: conceptions, geometric shapes, misconceptions, pre-service primary teachers

1. Introduction

Math curriculum for primary education in Slovakia is defined by National Education Program, also known as "NEP", (National Institute for Education, 2009). It is called Mathematics and Work with Information. It provides objectives for primary education and minimal mathematical standards which math students must acquire. Geometry represents a very small part of curriculum in Slovakia. Slovak students in TIMSS 2007 and 2011 study, scored lower on average on geometric shapes test questions than their international counterparts in EU and OECD (Galádová et al., 2013). This may be caused by recent National Education Program for primary education which has significantly reduced geometric curriculum. Scholtzová (2013) analyses geometry education in primary schooling in Slovakia and she mentions that students comes to the next stage of education with a limited range of geometric knowledge.

Reduction of geometry in curriculum (as well as the number of hours) in Slovakia has changed teachers attitude towards teaching geometry in primary education. Teachers usually dedicate very little time to geometry in primary education and therefore they may do it very broadly. We assume that this situation will significantly improve if and only if there will be a change of attitude about geometry in teachers and future teachers. Scholtzová (2013) also notes few things about math (geometry). "Education of future teachers for primary schools must incorporate maximum of determining factors from legislative and educational reality." Therefore, we consider superior geometric teacher training for primary education as one of the starting points for addressing this situation.

Nowadays, mathematical preparation of teachers for primary education in Slovakia is done at six universities. Every university has its own approach towards mathematical education. Some faculties prefer teaching didactic approaches in math education, while others emphasize professional mathematical components. In general, we have seen a drop in the level of geometrical knowledge in pre-service primary education teachers (for example, in studies of Žilková, 2013; Gunčaga, Kopáčová and

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Paksu, 2013). These students are the ones who have completed secondary education and should be able to think at some level of abstraction. We observed that many students lack the knowledge of plane geometric shapes and their properties. Thorough analysis has shown that often these misconceptions in students about elementary planar shapes are acquired during lower levels of their education. It is possible to correct these misconceptions only if they are precisely identified. For this reason, we decided figure out which problems students have in recognizing geometric figures and their properties.

2. Theoretical Basis

The knowledge of geometry in pre-service primary education teachers is also described by several foreign research studies. Marchis (2012) notes insufficient knowledge of geometry. She also identified four important reasons why students fail to define the basic geometrical shapes:

- “they can’t recognize the geometrical shape;
- they don’t know the correct properties of the shapes;
- they know the properties of the shapes, but they repeat some properties in the definition;
- they know the properties of the shapes, but they miss some properties from the definition” (Marchis 2012).

Knight (2006) in her dissertation describes interesting and disconcerting conclusions. The author statistically verified low level of geometrical knowledge in pre-service primary education teachers at the beginning and also at the end of their studies. Fujita and Jones (2006) describe more specific problems with definitions and classification of rectangles in primary education students in Scotland. Çontay and Paksu (2012) do the same for Turkey. All of these studies show that one of the causes of poor performance may be misconceptions about planar shapes in students. There are many reasons for these misconceptions, but usually the main one is the lack of personal experience and manipulation with geometric figures.

The theoretical basis of the research studies mentioned above was van Hiele model of geometric thinking. The authors were the Dutch educators - van Hiele married couple. Their model consists of two parts. **Levels of thinking in geometry** and their characteristics is the first part**. Phases of learning** is the second part, which is devoted to teachers - how to effectively organize the teaching of geometry. Different authors have different numbering systems and different names for each levels of thinking. The van Hiele married couple originally divided learning stages the following way: (a) the recognition level, (b) the analysis level, (c) the order (or simple deduction) level, (d) the deduction level, and (e) the rigor level (see figure 1). Usiskin (1982) used this categorization too and he added characteristics of individual levels by using Hoffer´s study (1979, 1981 in Usiskin, 1982): recognition, analysis, order, deduction, rigor. In this paper we will (in accordance with studies of Marchis, 2012; Mason, 1998; Musser, Burger and Peterson, 2001) number, name and describe levels as follows:

- *Level 0 Visualization* - learners recognize shapes only according to their appearance, holistically, they often use prototypical example in order to identify them. Learners are unable recognize shapes in non-standard positions and sizes.
- *Level 1 Analysis* - students recognize and name geometric shapes based on their characteristics. Location and size does not cause any problems, but they do not see some of the common characteristics of different shapes. They do not distinguish between the characteristics that are necessary and sufficient to describe the shapes.
- *Level 2 Abstraction* - students understand the relationship among different sets of shapes and they can define inclusive relationships among them, they understand logical implications. They can formulate a very simple definition or justify their decisions.

- *Level 3 Deduction* - students are able to understand the principles of construction in geometry, they understand how to make proofs in geometry and they can autonomously formulate geometric proofs.
- *Level 4 Rigor* - learners understand Euclidean geometry and principles of non-Euclidean geometry models, they can use deductive reasoning to compare mathematical systems.

We can see based on the results described above that pre-service primary education teachers have difficulties formulating definitions of elementary geometric shapes. They know only some properties, they are unaware of some and they can not choose the ones that are necessary for the definition of the given shape.

For this reason, we decided to **identify problems** only on van Hiele´s levels of visualization (level 0) and analysis (level 1).

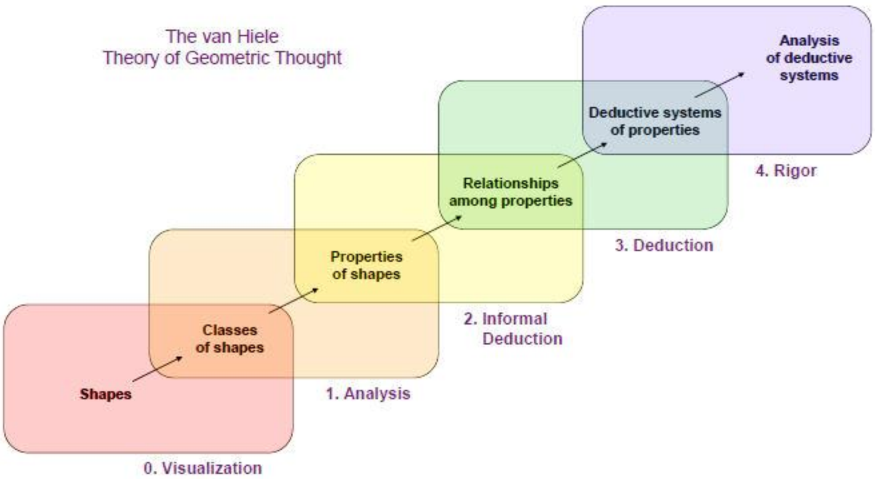

Figure 1. *The van Hiele's Model of Geometric Thinking according to Rynhart (2012)*

3. Research Design

The sample included 95 students in their first year of undergraduate studies in the field of teacher training for primary education at two Slovak universities. Data collection was conducted in 2014.

The research tool was a knowledge test, which consisted of 10 complex tasks aimed at identification and elementary properties of geometric shapes (triangle, rectangle, square, parallelogram, rhombus and deltoid). The tasks were designed to be on the first two van Hiele levels. According to the structure in Figure 1, the test was focused on shapes, classes of shapes and properties of shapes. A picture was included with every task which made the test easier. (Many studies have confirmed that tasks with pictures are easier for students than the tasks without pictures.) Test questions were focused on topics that are listed in table1.

Table1. *Focus of test tasks*

Task	Focus of the task
U_1	Identification of a triangle in the picture.
U_2	Elementary properties of triangles (sides and their length, internal angles and their size).
U_3	Identification of a rectangle in the picture.
U_4	Elementary properties of rectangles (sides and their length, internal angles and their size, diagonals and their properties).
U_5	Identification of a square in the picture in vertical-horizontal position.
U_6	Identification of a square in the picture in non-standard position.

U_7	Elementary properties of squares (sides and their length, internal angles and their size, diagonals and their properties).
U_8	Identification of a parallelogram in the picture in non-standard positions.
U_9	Elementary properties of a rhombus (sides and their length, internal angles and their size, diagonals and their properties).
U_10	Elementary properties a deltoid (sides and their length, internal angles and their size, diagonals and their properties).

We scored tasks very strictly. This means that if a student solved the entire task correctly (all the subtasks) he or she got a point. If he or she did not solve all the subtasks correctly, then he or she did not get a point. Reason for the strict scoring was mainly due to easy test questions. All the tasks were according to revised Bloom's taxonomy (Anderson and Krathwohl, 2001, in Scarborough) assigned to the first two dimensions of knowledge (facts and concept), see Table 2.

Table 2. *Specification of test tasks according to cognitive processes*

THE KNOWLEDGE DIMENSION		THE COGNITIVE PROCESS DIMENSION				
		remember	**understand**	**apply**	**analyze**	**create**
factual knowledge	knowledge of terminology		U_5, U_3		U_4	
	knowledge of specific details and elements		U_1	U_6, U_8		
conceptual knowledge	knowledges of classifications and categories				U_7, U_2	U_9
	knowledges of principles and generalizations					U_10
	knowledges of theories, models and structures					

3. Results and discussion

4. 1 Reliability

The main factors that affect the reliability of the test is the number of items and their difficulty. The test had 10 tasks and it was focused on the first two van Hiele levels. Therefore, we can assume only a low reliability of the test. We used Cronbach's a and we calculated that its value of reliability is 0.51. Table 3 shows how the coefficient would change after elimination of individual tasks. We can see from the table that the reliability of the test increased to 0,541 after elimination of the task U_4 , which means that this task decreases the reliability of the test. There is no other task which reduces the reliability of the test.

Table 3. *Reliability of the test after removal of tasks*

Task	**Cronbach's □ after removal of task**
U_1	0,511
U_2	0,454
U_3	0,511
U_4	0,541
U_5	0,496
U_6	0,48
U_7	0,433

U_8	0,504
U_9	0,451
U_10	0,431

4. 2 Difficulty level of tasks

The Table 4 shows success rate of each test task. We compared the performance of the test tasks using Cochran Q - test. The attainable value of the test statistics $Q = 114.46$, $df = 9$, $p < 0.001$ means that **tasks are statistically significant with different levels of difficulties.**

Table 4. *Success rate in task solving*

	Sum	Percent 0´s	Percent 1´s
U_1	89	6,3	93,7
U_2	79	16,8	83,2
U_3	93	2,1	97,9
U_4	80	15,8	84,2
U_5	88	7,4	92,6
U_6	81	14,7	85,3
U_7	65	31,6	68,4
U_8	74	22,1	77,9
U_9	64	32,6	67,4
U_10	50	47,4	52,6

U_10 is the most difficult task. It is statistically more difficult than any other task. The task was focused on identification and properties of kites. Slovak students do not normally see these types of the problems. These type of quadrilaterals are seen very rarely in math work books. Also, other studies have shown (eg. Žilková, 2013) that conceptions of Slovak students about the concept of kite are incomparable with other geometric shapes because these knowledge levels are completely different.

U_3 (identification of a rectangle), U_5 (identification of a square) and U_1 (identification of a triangle) were the easiest tasks. These tasks do not statistically differ from one another, but in comparison with the other tasks they are significantly less challenging. This means that task on van Hiele level zero were tasks U_6 (identification of a square) and the task U_8 (identification of a parallelogram) were more difficult for our students. These tasks showed shapes in non-standard positions (not vertical-horizontal positions). This fact is confirmed by Bloom´s classification of tasks (Table 2).

4. 3 Evaluation of students according to universities and in general

We wanted to figure out whether they are statistically significant differences among students of different universities. Table 5 shows descriptive statistics according to universities and in general.

Table 5. *Descriptive groups statistics and in general*

Group	N	Average	Median	Modus	Min.	Max.	Standard deviations	Skewness	Kurtosis
G_1	36	7,97	8	9	4	10	1,44	-0,55	0,05
G_2	59	8,07	9	9	4	10	1,73	-0,83	-0,18
Sum	95	8,03	8	9	4	10	1,62	-0,74	-0,13

The total sum of points in either of the groups is not distributed normally (the hypothesis was verified by Shapiro-Wilks test (G_2: $p < 0.001$, G_1: $p = 0.016$)). Therefore, we verified the hypothesis of the same success rate G_1 and G_2 group by non-parametric Mann-Whitney test. The value of the tested statistics Mann-Whitney test, $U = 977.0$ and p-value is 0.514, which means that the **success rate of G_1 and G_2 is not significantly different.** Therefore, we can evaluate the results as a whole. We will try to find the potential implication in relationship among students' answers to the tasks. We use statistical analysis of implications for this purpose.

4. 4 Statistical Implicative Analysis

Statistical implicative analysis was originally designed for research purposes in mathematics. It allows us to examine the quasi-implications among variables, also called association rules. The data were processed using program called CHIC. CHIC allowed us to present the results in the form of a similarity tree, tree hierarchy and implication graph which visualized potential relationships among variables.

Figure 2 shows oriented hierarchical tree, which shows existing implication relations among students´ responses to different tasks. The rate of cohesion is shown above the arrows in Figure 2.

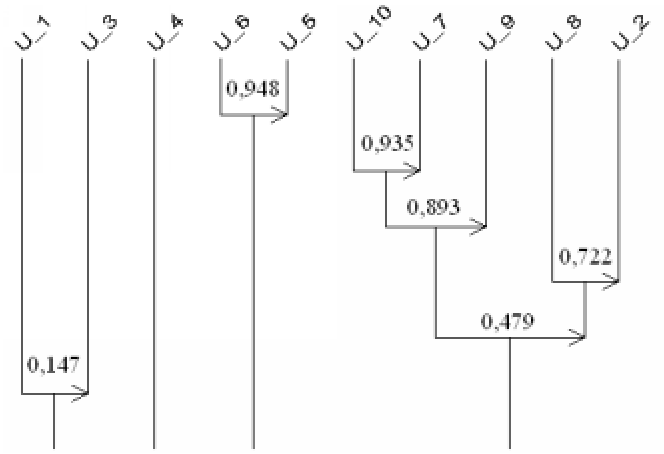

Figure 2. *Hierarchy tree*

It is clear looking at the graph that there is an implicative relationship between responses to tasks U_6 and U_5. Both tasks are focused on identifying the square and both belong to the van Hiele levels 0 (visualization). We can characterize relationship as follows: if the student was able to correctly identify a square in a non-standard position, task U_6, he also knew how to correctly identify a square in a standard position, task U_5. We can see that task U_6 was more difficult for students than task U_5. This is also confirmed by success rate in solving of these two tasks. Task U_6 had a success rate of 85.3% and the task U_5 had a success rate of 92.6%. The graph also confirms U_4 has no informative value and therefore it can be removed from the test.

Tasks U_10, U_7 and U_9 demonstrate another relationship among the students´ responses. All these tasks required knowledge about the properties of plane shapes and also formulation of the questions verified knowledge of mathematical thinking (understanding of quantifiers and negation). The arrows show that if a student knew the right answer to U_10 (success rate of 52.6%) and then he or she also correctly answered U_7 (success rate of 68.4%). U_10 was focused on properties of deltoids and U_7 was focused on properties of squares. The results show that tasks about properties of squares were easier for students (square as a special case of a deltoid). Of these tasks U_10, U_7 and U_9, U_10 was the hardest. It required knowledge of circles and quadrilaterals and it was generally focused on deltoids (we stated the reason for the low success rate in the part called difficulty of the tasks). The

oriented hierarchical graph showed the implication rule (U_10 $\Rightarrow$ U_7) $\Rightarrow$ U_9 which illustrates the relationship among the responses to the tasks about the shapes and their properties (deltoid $\Rightarrow$ square) $\Rightarrow$ rhombus. The difference in success rate in tasks which ask about squares (68.4%) and rhombuses(67.4%) is minimal. The final rule, which can be seen in oriented graph, but also in implicative graph (Figure 3) perfectly illustrates the existing inclusive relationship among these three planar shapes square -> rhombus -> deltoid. Also the organization and position of theses tasks on implicative graph in Figure 3 confirms the relationship above.

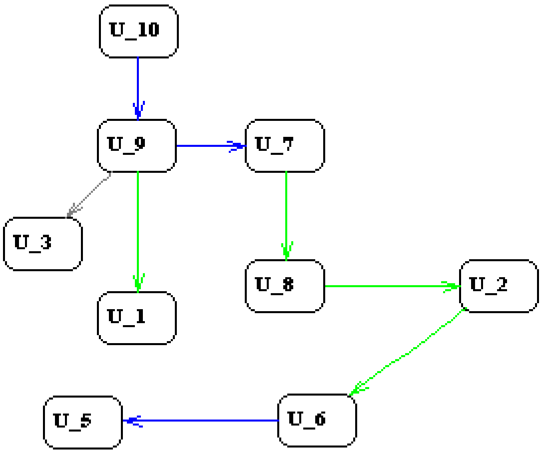

Figure 3. *Implication graph with implicative intensities 95, 85, 75, 65*

There were test tasks on the lowest van Hiele level (identification of shape in the picture, U_1, U_3, U_5, U_6, U_8) and there were tasks on the first van Hiele level (elementary properties of shapes U_2, U_4, U_7, U_9, U_10). This breakdown also shows oriented hierarchical graph (Figure 2) in which the majority of the harder tasks are positioned in group on the right side. Looking at the implicative graph it is clear that if a student has a higher level of thinking (can solve more demanding tasks) then he is more likely to solve easier task as well (lower-level of geometrical thinking). Position of tasks U_10, U_9 and U_7 on the implicative graph is also confirmed by success rate in solving these tasks. It is interesting that none of the selected relationships is in red color. This means that relationships are not significant, which would proof strong and stable students conceptions among these shapes.

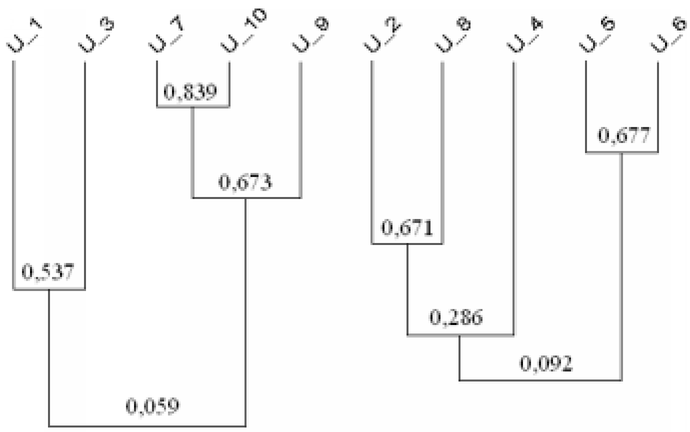

Fig. 4 *Similarity tree*

Similarity tree (similarity tree, fig. 4) created classed based on similar students responses to different tasks. For example, the class (U_7, U_10) U_9) represents, that U_7 and are U_10 similar and that this class is similar to U_9 from similarity point of view. Or tasks U_5 and U_6 created class (according to similarity of responses), which was aimed to identify squares, which means that students' answers to these two tasks were similar (implicative relationship between U_6 $\Rightarrow$ U_5 is true).

Given the fact that this was a task only on the first level we think that the success rate was very low. If there were harder tasks included in the test, for example tasks which require higher level of geometric thinking, we assume that the students would have had serious problems solving these tasks.

3. Conclusion

Despite the fact that we used only tasks focus on determining the level of geometric thinking on van Hiele level zero (visualization) and one (analysis) and also difficulty levels were evaluated by revised Bloom's taxonomy and the task were focused only on factual and conceptual knowledge, success rate of solving these tasks was lower than expected, but definitely lower than we should expect from pre-service primary education teachers.

The test results showed that the first van Hiele level of geometric thinking causes many problems to a lot of our pre-service primary education teachers. Students were most successful in solving rectangle problems followed by triangle and then square problems. As expected, students were less successful in parallelogram tasks. Not surprisingly, the students had the least knowledge about deltoids, because our educators spend very little time teaching this shape. We believe that it is insufficient that pre-service teachers can identify planar shapes only in standard position and they do not have an accurate understanding of the properties of the shapes or they are uncertain in terminology.

Our research has produced interesting but not surprising results even when compared with other countries. It was shown that students do not have solid and stable conceptions even on the lowest cognitive levels about elementary geometrical shapes and their properties or even if they have some they are often wrong. We see our research as a pilot one, we intend to continue tests and especially improve the training pre-service primary education teachers and as well as regular teachers (in lifelong learning).

We believe that teachers who create conceptions for younger students should not have any misconceptions and they should be on higher van Hiele level of geometric thinking than their pupils are. Therefore teachers should be at least on level two or three.

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