International Education Studies; Vol. 8, No. 4; 2015
ISSN 1913-9020 E-ISSN 1913-9039
Published by Canadian Center of Science and Education

The Experiment Check of the Efficiency of the Historical Component Formation in Teacher's Mathematical Methodological Culture

Mansur Fajzrahmanovich Gilmullin[1]

[1] Elabuga Institute (branch) of Kazan (Volga Region) Federal University, Tatarstan Republic, Russia

Correspondence: Mansur Fajzrahmanovich Gilmullin, Elabuga Institute (branch) of Kazan (Volga Region) Federal University, 89, Kazanskaya street, Elabuga, Tatarstan Republic 423604, Russia.

Received: November 12, 2014 Accepted: December 12, 2014 Online Published: March 30, 2015

doi:10.5539/ies.v8n4p218 URL: http://dx.doi.org/10.5539/ies.v8n4p218

Abstract

When teachers do not know the history of science well, when they misunderstand and underestimate its educational importance, it becomes a serious obstacle for the improvement of their methodological skills. This paper has a goal to describe author's method how to teach the history of mathematics. This method is aimed at training math teacher's profession-oriented qualities. These qualities form a historical component of teacher's culture of mathematics and methodology. Besides, the paper describes the techniques and results of experimental tests for the efficiency of this method. These materials can be useful for teachers who create a cultural and historical atmosphere on math lessons at school and for university teachers who use historical mathematical ideas in their work.

Keywords: history of mathematics, future math teachers, teacher's mathematical methodological culture, formation of historical component, math teacher's profession-oriented qualities

1. Introduction

The main purpose of modern math teacher consists in timely assistance in the integrated development of student's personality with the help of methodological means. The readiness to fulfill such a function is ensured by teacher's system of profession-oriented qualities which form the kernel of his professional culture. The whole complex of academic subjects provided by educational standards, including the history of mathematics, should help math students to obtain these qualities at a certain level.

The kernel of professional culture of a future math teacher is determined by values of profession and mathematical culture, general pedagogical and methodological fundamentals in the teaching of mathematics, the knowledge of basic mathematical objects of "elementary" mathematics and skills for using it. In this case, it is reasonable to use a more accurate term of teacher's "mathematical methodological culture". It means a specific kind of culture for such a professional. In order to identify profession-oriented qualities which could be trained during the course of the history of mathematics we use term "the historical component of mathematical methodological culture" (HCMC) for future math teachers.

The question about the history of mathematics as a support for math teachers' training is not new (Bobynin, 1886, 1913). Many models for teaching mathematics are based on psychologists' conclusions that school education should be built on a historical and genetic basis. The main formation stages of this method are connected with such names as J. Hadamard, J. Wallis, F. Klein, A. C. Clairaut, G. Leibniz, H. Poincare, H. Spenser, V. V. Bobynin, N. I. Lobachevskiy, D. D. Mordukhay-Boltovskiy and other mathematicians and educators (Beloborodova, 1999). T.S. Polyakova grounds in her research work the necessity to enlarge the content of historical and methodological training for math teachers. In her opinion, they should get a system of knowledge about the history of school mathematical education (Polyakova, 1997, 1998). A number of research papers include various models for profession-oriented historical, mathematical and methodological training of math teachers in high school (Beloborodova, 1999; Burova, 2000; Grattan-Guinness, 2004; Drobyshev, 2004, 2011; Karp & Schubring, 2014; Mordkovich, 1986; Safuanov, 2000; Tomilova, 1998, etc.). In spite of success, the reality is that the history of mathematics still has a very small place in teachers' training and does not correspond to its specificity. The main methodological problems are still not solved: what for, what exactly and at what level future math teacher should learn from a huge amount of information concerning the history of mathematical culture including mathematical education.

The author carried out a poll to reveal students' attitude to the course of mathematics history and a test to identify a real level of knowledge on the history of mathematics. The research shows that only 52% of students think that the history of mathematics is useful to develop the interest in mathematics. The majority of respondents (90%) do not see any connection between the history of mathematics and the problems future profession. Their study of mathematics history generally boils down to some names and separate facts.

A poll among math teachers showed that the majority of them (95%) mainly use the materials on mathematics history in extracurricular and school research work with students to develop their interest. In the main study time, certain historical facts are given to students occasionally. Many teachers do not see any reason to use these facts systematically. At the same time, 68% of teachers assess their knowledge on the history of mathematics as inadequate but do not link it to their professional goals and culture.

So, we face a question how to ascertain the main direction for the study of mathematics history in teachers' training university, and, consequently, to develop a methodological basis (elements of theory, content and methods) for its implementation. All above mentioned determine the topicality of research on the formation of historical component of math students' mathematical methodological culture.

The research hypothesis is that the formation of historical component of students' mathematical methodological culture during the mathematics history course in teachers' training university will be effective if:

- Students are motivated to study the history of mathematics by the necessity to develop personal profession-oriented qualities connected with the usage of facts from mathematics history in future math teaching work;
- There are conditions and means for future teachers to use the potential of mathematics history in solving problems of research and education;
- Mechanisms and means for the formation of HCMC include training situations for professional development (TSPD), the series of training historical and methodological tasks (THMT), methods to solve them and also a dialogue between cultures in various forms, fragments of the regional, national and world history of mathematics and mathematical education.

2. Methods

Traditional education was aimed at the acquisition of knowledge and skills accumulated before which form the basis of profession. We think that the main result determining the today's guideline of education is the formation of professional's culture. In new conditions, a great importance belongs to the experience of cognition carried out, inter alia, by means of mathematics itself. This experience should be enough for self-education and cultural use of gained knowledge. Now the main goal of higher education is to help students to develop professional skills to required level while studying. The character of education changes from the subject to the professional one and from the fundamental to functional one (Gilmullin, 2009).

2.1 Teacher's Mathematic and Methodological Culture

We understand teacher's mathematical methodological culture as a specific kind of culture of a professional whose main activity is to teach mathematics at secondary school. We accept a model of professional's culture brought forward by Zhokhov (2007) as a starting point. Professional culture is thought to be an interpenetration and mutual supplement of the results of three processes:

Familiarization with knowledge from the relevant field. We designate the result of this process by "awareness" presented by a sum of information units considered necessary for this educational stage and "ability to use" knowledge at the level of professional means;

The improvement of operational fundamentals and professional means. The result is a skill to carry out activities necessary for one's profession or professional skills, though, for future teachers, it will be expressed in training;

It is reasonable to call the third process "dialogue" or rather "a dialogue of cultures" in a sense suggested by M.M. Bakhtin (Zhokhov, 2007). We designate the result of this process by "mutual understanding" or "readiness for the dialogue of cultures". In fact, they determine the interpenetration of meanings (saw, heard or read) and the belonging of different people to one type of culture.

The analysis of requirements to math teacher's training aimed at forming their professional culture makes it possible to identify the following constituents of mathematical methodological culture: content and knowledge; operation and activity; dialogue and reflection.

2.2 The Historical Component of Mathematical Methodological Culture (HCMC)

We modify professional's culture in accordance with the subject of research–a historical "section" of this culture.

The content-knowledge constituent is an informational basis for teacher's personal methodological system built on certain historical facts, the development regularities of mathematical culture or its separate content and methodological lines (numerical, algorithmic, algebraic, geometric, etc.) and the regularities of learning mathematics. The completeness criterion of this constituent is an ability to use knowledge on the history of mathematics at the level of educational means and, in future, at the level of teaching mathematics at school.

The main function of the operation-activity constituent of historical component is to promote the activity basis for student's professional culture. Its characteristic elements include the ability to learn professionally significant knowledge on the history of mathematics and to use them for solving professional problems. The skill in handling relevant methods of work forms its completeness criterion.

The dialogue-reflection constituent fulfils the coordinating and value-orientation functions. The completeness criteria of this constituent are dominant need for understanding the dialogue between different cultures and a positive tuning to use historical knowledge and experience in professional activity. The desire to join in the dialogue of cultures (a teacher and a student; methodological and content lines in teaching mathematics; mathematics as a science and a subject for study, etc.) and keep it running is one of its completeness indicators.

The content of HCMC constituents is formed by professional qualities sorted out to interconnected groups according to their professional direction (Table 1).

Table 1. The structural integrative model of historical component of mathematical methodological culture

The structure of historical component of mathematical methodological culture		
The dialogue-reflection constituent (1)	The operation-activity constituent (2)	The content-knowledge constituent (3)
Professional motives, emotions and evaluations	Professional skills	Professional knowledge
1) Connected with the dialogue between cultures 2) Reflecting, evaluating and developmental 3) Value-orientation 4) Predictive, translating	1) Goal-setting 2) Connected with source study, analysis and synthesis 3) Organizational and constructive 4) Content-genetic 5) Content-methodological 6) Motivating and developmental	1) Analytical, synthetic and object ones 2) Methodological 3) Connected with the history of motherland 4) Educational

The first constituent includes the following groups of qualities:

- Connected with the dialogue between cultures: desire to understand peoples actions, the importance of communication in profession; the ability to join in the dialogue between cultures, to create one's own objects of mathematical methodological culture and to assess them;
- Reflecting, evaluating and developmental: the ability to diagnose and assess the results of professional activity and to create conditions for students' self-development;
- Value-orientation: to identify the personal semantic and (or) methodological value of studied historical facts, to detect and understand their importance for educational problems;
- Predictive, translating: the desire and ability to predict the result of using detected means and methods in new conditions; the ability to transfer of acquired knowledge and actions into new situations and to construct them.
- The second constituent is defined as a whole set of the following professional skills:
- Goal-setting: to set goals for the use of historical materials in education, to analyze TSPB and THMT and to plan their use on math lessons;
- Connected with source study, analysis and synthesis: to work with sources; to analyze them; to adapt

mathematics history material to educational conditions; to study the experience of using historical material;

- Organizational and constructive: to build models and fragments of lessons using detected historical facts; to organize the training research for students;
- Content-genetic: the ability to make tables of significant stages of mathematics development; to formulate questions about the origin of notions;
- Content-methodological: to look through new literature on mathematics history and methodology regularly; to study the origin of content and methodological lines in the school course of mathematics;
- Motivating and developmental: to be aware of the necessity to use historical facts in teaching mathematics as an incentive for professional activity; to reexamine the knowledge on the history of mathematics.
- The content characteristic of the third constituent is defined as knowledge on the history of mathematics:
- object (analytical and synthetic) knowledge: to identify and know the characteristics of mathematical objects: the origin, personalities and dates; difficulties in understanding and use of an object, etc.;
- Methodological: to know the origin and use of general mathematical methods for cognition and training;
- Connected with the history of motherland: to know and use in cognition the history of development of national and regional mathematics and education;
- Educational: to know the origin of mathematical education, the origin of subjects included in the school course of mathematics, and the origin and history of content and methodological lines.

2.3 The Completeness Levels of Historical Component of Mathematical Methodological Culture

The formation of HCMC is the process of filling student's personal experience of training and professional activity with separate but interconnected qualities our groups of qualities from its named constituents. The formation of HCMC is controlled by the methodological system of mathematics history teaching. The indication intensity of criteria for each constituent is the reason to discern "the levels of completeness" of future teacher's HCMC: initial, middle and high. They are determined by student's reproductive, reproductive-productive and creative activity respectively. The following table shows the schematic view of these levels (Table 2).

Table 2. The completeness levels of historical component of mathematical methodological culture

Levels	The constituents of historical component of mathematical methodological culture		
	The dialogue-reflection constituent	The operation-activity constituent	The content-knowledge constituent
Initial (reproductive)	There is no any interest in historical knowledge and ways to use it in education. The evaluation of one's own knowledge and skills is not developed.	The knowledge on the history of mathematics is not used in training and teaching or is used haphazardly. There are no skills of working with teaching material.	The knowledge on the history of mathematics is superficial, disjointed and is not systematized.
Middle (reproductive-productive)	Students want to study the history of mathematics and to use it at school. Their self-appraisal is adequate. There is no readiness to transfer knowledge and actions.	The style of learning is based on copying the experience of other teachers that is not necessary for an individual system of teaching. There are elements of information adaptation to use it in training or future professional activity.	The knowledge on the history of mathematics includes separate facts or groups of facts. There is systematization on different bases and spontaneous links with professional goals.
High (creative)	There is a stable motivated need for forming personal	The techniques how to use historical material in teaching are	The professional knowledge on the history of mathematics is mastered to a level of

	profession-oriented qualities. The ability of control and self-control is strongly pronounced. The fragments of experience of transferring acquired knowledge and actions on the history of mathematics into new conditions.	mastered. Students search for new forms and methods for work on the basis of mathematics history. Students acquire a habit to search for facts of mathematics history, to process them creatively and include them in their personal professional experience.	personal instruments for mathematical cognition and means for teaching mathematics at school. Knowledge is complete and systematized by separate content-methodological lines or mathematical theories.

2.4 Training Situations for Professional Development (TSPD)

The methodological system for teaching the history of mathematics described in this paper uses a system of training situations as one of the main means to form the elements of mathematical methodological culture of future teachers. Training situation is a certain combination of conditions which can appear while training. They can be spontaneous or created by a teacher to reach the goals of education using relevant means. The second alternative is preferred. Usually, this is a whole set of problems to be solved. We use training situations interesting from the viewpoint of HCMC formation. We call them "training situations for professional development" (TSPD). They require solving actions and thus promote the formation of student's profession-oriented qualities.

A "training historical and methodological task" (THMT) is the organizing kernel of TSPD. THMT materializes TSPD in a form of concrete task with historical and mathematical content and contains a system of knowledge revealing the whole range of contradictions in this TSPD. If solved successfully, these tasks help to achieve the intended educational results.

Here is an example of TSPD.

TSPD-1. (Type: search mobilization; historical analysis and synthesis; prediction). In everyday life, man sometimes faces the names of large numbers. Future math teachers will have to answer students' questions, such as "What is "trillion" and why is it called so?" How to explain this? Is there any regularity in number names?

THMT-1. Italian traveler Marco Polo (1254-1323) is believed to be the first who used term "millione"–"a large thousand". Nicolas Chuquet (1445-1500), a French mathematician and doctor, introduced such terms as billion, trillion and so on up to nonillion in his treatise "The Science of Numbers in Three Parts" (1484).

Tasks: 1. Describe mathematical objects these terms are used for. Describe their origin. Find out what the latest term means. If possible, continue this row of terms. 2. Analyze how these terms are used today in comparison with Chucuet's times. Do they include such terms as "octillion" and "milliard" and what do they signify? 3. What names large numbers had in different nations at different times? Compare historical information about this issue connected with Slavs and Hindus. 4. In what modern field of knowledge man faces the necessity to name large or small numbers? What terms he uses for this? Recollect astronomy, information science and other fields. Explain what "nanotechnology" is from the viewpoint of the general principle.

The salvation of this TSPD takes part in forming the next group of qualities inherent in HCMC: knowledge about the origin and practical application of mathematical objects which can be found in training texts; knowledge about content-methodological lines of school mathematical course; the ability to find in known sources the well-grounded answers to the questions about knowledge objects; the ability to use knowledge about the history of mathematics to identify links between objects; skills in investigating the etymology of each mathematical term; work with etymological dictionaries and so on.

For the integrated quantitative assessment of THMT, a term of "THMT potential" is introduced. The results of task solution made by a student characterize some formed quality which enters one or another group of HCMC qualities. Then the THMT potential is set by a vector (K, AS, D) where each coordinate takes its value from a relevant sequenced collection: K = {K-1, K-2, K-3, K-4}, AS = {A-1, A-2, A-3, S-1, S-2, S-3}, D={D-1, D-2, D-3, D-4}. Here the collection of values for each component of this vector corresponds to the quantity of its characteristics in Table 1. Letter K and AS signify knowledge, abilities and skills accepted in the standards of student's progress in training course. That is why they are the first components of the vector though they correspond to the second and the third columns of Table 1. Letter D signifies the third characteristic of the potential – the dialogic one (the first column of the same table). If THMT takes part in forming one of the four qualities of the content-knowledge group of HCMC (the third column of Table 1), parameter K gets value H (high). If the task is not aimed at this, the value of this parameter is signified by letter A (absent). We assess the capabilities of task to form this quality by an ordinal scale {initial–I, middle–M, high–H}. For example, the potential of the THMT-1 is set by collections K = {H,A,H,H}, AS = {H,H,H,H,H,H}, D = {H,H,A,H}. THMT solved by a certain student can be used for the integrated assessment of some qualities from three constituents of HCMC formed in this student.

3. Findings

3.1 Experimental Methods

The effectiveness of developed methods was tested experimentally on the basis of the Physics and Mathematics Faculty of Yelabuga Institute of Kazan Federal University (former Yelabuga State Teachers' Training University) in 2004-2009. The experiment had three stages: statement, search and formation. All the stages are logically interconnected but each of them has its own goals, objectives, means and methods. In general, the experiment was to test the hypothesis about the conditions for effective HCMC formation of a future teacher. During the experiment, a group of students underwent a purposeful influence with the help of a new methodological system.

While training teachers in the history of mathematics, it is necessary to assess the characteristics of separate students or groups of them, i.e. how close they are to the goal of training. In traditional teaching of mathematics history, they assess knowledge on the origin and development of mathematics. In experimental teaching of mathematics history, other goals are set and other means are used. The objects should be assessed by the indicators adequate to the goals of training and research (Novikov, 2004; Afanasyev, 2007). The general theory of measuring describes various methods for transforming data obtained by various scales and methods for making decisions on them. The majority of them can be used in teaching the history of mathematics. First of all, it is necessary to ascertain the content aspects of measuring for such training: how to measure, what to measure and what the criteria of effectiveness are. The usual form of evaluating the state of an object is testing (Tomilova, 1998, 2001). Mostly, the number of correct answers is the result of measurements. They are shown in a ratio scale. Then goes aggregated (joint) information about the group: the number of members with one or another level of knowledge (for example, low, middle or high). The measurement can look like a test or a poll.

3.2 Diagnostic Techniques for the Completeness of Historical Component

Now we will describe the method of statistical calculations performed during the experiment. Students in experimental and control groups were compared by the indexes of HCMC completeness at the beginning and at the end of the experiment in general and by separate constituents.

An important requirement for the experiment is the relatively equal initial level of tested quality that students display in experimental and control groups. The Pearson's chi-square criterion (χ^2) and the Wilcoxon-Mann-Whitney criterion (WMW) were used to evaluate the statistical significance of equal or different levels of competence in student groups by different methods.

The experimental group contained students (N=24) who study the course "mathematics and informatics". The control group contained students (M=20) who study the course "informatics and mathematics". The contingent of students in these two groups has no any statistical significance at the beginning of the experiment. This fact is confirmed by the test assessment of the initial completeness level of HCMC and its separate constituents. The test mostly included tasks assessed only by one qualitative indicator from K-1–K-4 (the content-knowledge constituent). At first, these tasks (Part 1) were assessed only by the fact of correctness and were used to put data in the ratio scale as a number of correctly solved tasks. The second part of the test consisted of THMT. After that both parts were assessed in an ordinal scale. So, matching characteristics of the two groups was detected by two methods.

I. The test method of unambiguous assessments by ratio scale

It is carried out by a well-known Wilcoxon's statistical W-criterion.

II. The diagnostic method of integral assessments by ordinal scale

This method uses information about how both parts of the test were fulfilled. The assessments have a three-valued (L=3) ordinal scale {initial, middle, high}. First of all it is necessary to distribute the students of experimental and control groups by these categories. The difficulty is that the task has many criterion and the assessments are integrated. This scale does not afford summing marks. Besides, it is forbidden to compare levels from different groups of qualities.

At first, the assessment is performed by each of HCMC constituents (K, AS and D) for each student separately. For example, let us consider an assessment by the K constituent (professional knowledge). Each task is assessed by set {K-1, K-2, K-3, K-4}. Each coordinate has values H (high), M (middle) or I (initial) if it is assessed for this task. The assessment is absent only if this parameter is not taken into account. The aggregated assessment of each THMT task is found by each student with the help of vector (a_1, a_2, a_3), where nonnegative whole numbers a_1, a_2, a_3 signify the number of marks I, M and H respectively.

If a_1 =0, condition $a_2>a_3$ is checked. If it is met, the aggregated assessment is M, otherwise H.

If $a_1>0$ и $a_1> a_2+ a_3$ the assessment is I, otherwise M.

According to these rules, every student gets a single aggregated assessment for each constituent and a general assessment for all components. Then these data is used to characterize groups by the number of students distributed by three levels of HCMC completeness. Then these data is processed using the Pearson's chi-square criterion (χ^2).

Suppose the number of members assessed by the completeness level of measured indicators is set for the experimental and control groups of vector (n_1, n_2, n_3) and (m_1, m_2, m_3) when the order is: "initial", "middle", "high".

The empirical value (χ^2) [chi-square] is calculated using formula

$$\chi^2_{'exp} = NM\sum_{i=1}^{L}\frac{\left(\frac{n_i}{N}-\frac{m_i}{M}\right)^2}{n_i+m_i}.$$

Then this value is compared with the table value of (χ^2) [chi-square] when significant level is 0.05 $\chi^2_{0.05} = 5.99$ and a conclusion is made about the reliability of coincidence in compared samples.

The final test is processed using the same diagnostic methods. The test consists of two parts. The first part includes tasks of two levels. The first level of tasks checks the closed factual knowledge with the possibility to choose the only one or several correct answers. The second level contains creative tasks requiring the ability to analyze and make conclusions. These are open tasks with a direct input of answers and tasks to ascertain correspondence or order. All tasks are assessed unambiguously and processed using the first method. The second part consists of training tasks on the history of mathematics. They have different levels of difficulty and different potentials. The results of this part of the test are processed by the second method.

3.3 Data Processing

1) Measuring the initial condition of the groups

Here are the calculations of the initial measurement test. In this case, we used one THMT with potential: K = {H,H,A,H}, AS = {H,H,A,H,M,H}, D={H,A,A,H}. After the test is complete, the data is processed and the results are put in "The Record of Tests" for each group (Table 3). It contains the results obtained by each student. In the first part of the test, it is possible to write down only the number of correctly solved tasks. Letters H, M and I are put in each position and characterize the expert assessments of completeness level for each group of qualities.

Table 3. The record of tests

	Experimental group									Control group						
	1		2		3		…	24		1		2		…	20	
	P-1	P-2	P-1	P-2	P-1	P-2		P-1	P-2	P-1	P-2	P-1	P-2		P-1	P-2
P-1	8		6		4			5		9		7			6	
K-1		H		M		I			H	H	M		M			I
2		M		I		I			M		M		I			I
3																
4		M		M		I			M		I		I			I
A-1		M		M		I			M		M		M			I
2		M		M		I			M		H		M			I
3																
S-1		I		I		I			M		M		M			I
2																
3		M		I		I			M		M		M			I
D-1		M		I		I			M		M		M			I
2																
3																
4		M		I		I			M		M					I

I. Now let us process data obtained during the test using such method as “The Test Assessment of Unambiguous Assessments by Ratio Scale”. The working hypotheses are: H_0 (zero)–the completeness levels of HCMC in control and experimental groups are equal; H_1 (alternative)–the levels are statistically different. The required samples are found as numbers of correctly solved tasks in groups. The calculation data are given in Table 4. Conclusion: the characteristics of compared samples coincide with significance level 0.05. It means that there is no reason to reject the zero hypothesis.

Table 4. The calculation of empirical value for the WMW criterion of the initial state of experimental and control groups

No. of a EG member	Experimental group: the number of tasks solved correctly by a EG member (N=24)		The number of CG members who solved correctly the largest number of tasks	No. of a CG member	Control group: the number of tasks solved correctly by a CG member (M=20)	
	x_i	Rank	a_i		y_i	Rank
1	8	38.5	2	1	9	41.5
2	6	21.5	6	2	7	33.5
3	4	9.5	15	3	9	41.5
4	7	33.5	3	4	8	38.5
5	6	21.5	6	5	6	21.5
6	9	41.5	0	6	6	21.5
7	6	21.5	9	7	6	21.5
8	4	9.5	15	8	7	33.5
9	4	9.5	15	9	3	3

10	3	3	18	10	4	9.5
11	7	33.5	3	11	4	9.5
12	4	9.5	15	12	6	21.5
13	3	3	18	13	7	33.5
14	6	21.5	9	14	6	21.5
15	3	3	18	15	3	3
16	6	21.5	9	16	4	9.5
17	10	44	0	17	6	21.5
18	7	33.5	3	18	4	9.5
19	9	41.5	0	19	6	21.5
20	7	33.5	3	20	6	21.5
21	7	33.5	3			
22	6	21.5	6			
23	6	21.5	9			
24	6	21.5	9			
Sum		552	167			438
Average	6				5.85	
T_x=552, U_{exp}=228, $U_{cr}(0.05)$=169, $U_{exp}>U_{cr}$				W_{ex}=1.72, $W_{0.05}$=1.96, $W_{ex}< W_{0.05}$		

II. Let us carry out the initial comparison of groups by the diagnostic method of integrated assessments taking into account all tasks. Table 3 contains data not of all students, thus we will explain how calculations, for instance, for the first student from the experimental group were carried out: by the number of correctly solved tasks of the first part of the test 8=M; by the second part K=(H, M, M)=M, AS=(M, M, I, M)=M, D=(M, M)=M; and the final aggregated assessment is (M, M, M, M)=M.

Table 5. The distribution of completeness levels for HCMC (The frequency by the second method at the beginning of the experiment)

Group	Control		Experimental	
Assessments	Number of students	Percentage	Number of students	Percentage
Students	20	100%	24	100%
Initial	8	40.00%	10	41.67%
Middle	12	60.00%	13	54.16%
High	0	0.00%	1	4.17%

The vectors of experimental and control groups: (n_1, n_2, n_3)=(10,13,1) and (m_1, m_2, m_3)=(8,12,0).

$$\chi^2_{\exp} = 24 \cdot 20 \cdot \left(\frac{\left(\frac{10}{24} - \frac{8}{20}\right)^2}{10+8} + \frac{\left(\frac{13}{24} - \frac{12}{20}\right)^2}{13+12} + \frac{\left(\frac{1}{24} - \frac{0}{20}\right)^2}{1+0} \right) = 0{,}96 .$$

The table value for χ^2 if the significance level is 0.05 $\chi^2_{0.05} = 5.99$. Since 0.96 < 5.99, we conclude that the coincidence of compared samples is reliable, i.e. the zero hypothesis is true.

2) Changes in the final states of groups.

The calculations are carried out by the same methods to check hypotheses H_0 and H_1.

Table 6. The calculation of empirical value for the WMW criterion of the final state of experimental and control groups

No. of a EG member	Experimental group: the number of tasks solved correctly by a EG member (N=24)		The number of CG members who solved correctly the largest number of tasks	No. of a CG member	Control group: the number of tasks solved correctly by a CG member (M=20)	
	x_i	Rank	a_i		y_i	Rank
1	6	37.5	1	1	6	37.5
2	4	15.5	6	2	5	28.5
3	3	5.5	16	3	7	41.5
4	5	28.5	2	4	5	28.5
5	6	37.5	1	5	4	15.5
6	9	43.5	0	6	4	15.5
7	4	15.5	6	7	4	15.5
8	4	15.5	6	8	5	28.5
9	5	28.5	2	9	2	1.5
10	3	5.5	16	10	3	5.5
11	5	28.5	2	11	5	28.5
12	6	37.5	1	12	4	15.5
13	3	5.5	16	13	5	28.5
14	4	15.5	6	14	4	15.5
15	3	5.5	16	15	2	1.5
16	5	28.5	2	16	4	15.5
17	9	43.5	0	17	4	15.5
18	5	28.5	2	18	3	5.5
19	7	41.5	0	19	4	15.5
20	6	37.5	1	20	4	15.5
21	5	28.5	2			
22	4	15.5	6			
23	6	37.5	1			
24	5	28.5	2			
Sum		615	113			375
Average	5.08				4.2	
T_x=615, U_{exp}=165, $U_{cr}(0.05)$=169, $U_{exp}<U_{cr}$			W_{ex}=2.99, $W_{0.05}$=1.96, $W_{ex}>W_{0.05}$			

Conclusion: the reliability of differences between the characteristics of compared samples is 95% by the Wilcoxon's W-criterion (the levels are statistically different; the alternative hypothesis H_1 is confirmed) and Mann-Whitney's U-criterion.

Let us carry out the final comparison of the groups using the diagnostic method of integrated assessments taking into account all the tasks.

The vectors for experimental and control groups: (n_1, n_2, n_3)=(3,17,4) and (m_1, m_2, m_3)=(8,12,0).

Table 7. The distribution of completeness levels for HCMC (The frequency by the second method at the end of the experiment)

Group	Control		Experimental	
Assessments	Number of students	Percentage	Number of students	Percentage
Students	20	100%	24	100%
Initial	8	40.00%	3	12.50%
Middle	12	60.00%	17	70.83%
High	0	0.00%	4	16.67%

$$\chi^2_{\exp} = 24 \cdot 20 \cdot \left(\frac{\left(\frac{3}{24} - \frac{8}{20}\right)^2}{3+8} + \frac{\left(\frac{17}{24} - \frac{12}{20}\right)^2}{17+12} + \frac{\left(\frac{4}{24} - \frac{0}{20}\right)^2}{4+0} \right) = 6{,}87$$

The table value for χ^2 if the significance level is 0.05 $\chi^2_{0.05} = 5.99$. Since 6.87>5.99, we conclude that the differences between compared samples are reliable, i.e. the alternative hypothesis is true.

All the methods of assessment give the same conclusion that the initial state of experimental and control groups coincides and the final state of them is different. We can decide that the changes are caused by the experimental method of training.

4. Discussion

The research allowed finding the mechanisms and means to influence effectively the formation of students' HCMC qualities during the course of mathematics history in teachers' training university. From the viewpoint of the problem set forth by this paper, the most significant of them are: the methodological system; the dialogue of cultures as a condition and means for communication; types of TSPD and series of THMT supporting the formation of HCMC. The hypothesis about the effectiveness of HCMC formation conditions in the developed methodological system of teaching the history of mathematics is confirmed.

The mathematics history teaching methods worked out by the author for the students of teachers' training universities has a practical significance. Author's tutorial "The History of Mathematics" for students was published and approved (Gilmullin, 2009). During the experiment, the most effective special training means, forms and methods were found and tested: the training methodological tasks; the historical analysis of mathematics teaching materials; the local history of mathematics; the regional and national component of mathematics history; historical identification; synoptic tables; historical methodological lines; tests, compositions, projects, elective courses on the history of mathematics; the historiography of tasks; the museum of mathematics history and so on.

5. Conclusion

The course of the history of mathematics in teachers' training university can be used to form the HCMC of future teachers. It is possible to make them carry out some profession-oriented actions using the facts of mathematics history (to set the goals for math lessons; to choose the methods and means of teaching with a glance to historical experience; to find the motivating and developing elements in teaching mathematics, etc.).

This research can be developed in introducing the worked out methodological system for the history of mathematics into the practice of teachers' training universities and centers of professional advancement for mathematics teachers. It is possible to use similar methodological system and their components (particularly, TSPD and THMT) for teaching other subjects. Besides, the theory of math teacher's professional culture and the role of mathematics history in its improvement needs further development. It seems possible to continue the research of uninterrupted teaching of mathematics history at university and school. It should form and develop the mathematical culture for professional goals.

At present, all university courses for the professional training of future math teachers should be connected with new Federal State Educational Standards (*The Federal* …, 2011). A teacher of mathematics is required to be ready to use a cultural-historical approach to teaching mathematics at school. Now the school mathematical course includes an additional methodological section: “Mathematics in its Historical Development” (*The Exemplary* …, 2011).

Acknowledgements

Thanks to A. L. Zhokhov for the help in researching.

References

Afanasyev, V. V. (2007). *The Probability Theory*. Moscow: VLADOS. Retrieved from http://bookgdz.ru/teoriya-veroyatnsti/10561-afanasev-v-v-chitat-teoriya-veroyatnostej-onlajn

Beloborodova, S. V. (1999). On the Historical Genetic Approach. *Mathematics at School, 6*, 7-10.

Beloborodova, S. V. (1999). *The Professional Educational Line of the Mathematics History Course for the Teachers of Mathematics* (Ph.D. thesis, Moscow City Teachers’ Training University, Moscow, Russia). Retrieved from http://www.dissercat.com/content/professionalno-pedagogicheskaya-napravlennost-istoriko-matematicheskoi-podgotovki-uchitelei

Bobynin, V. V. (1886). *The Philosophical, Scientific and Educational Significance of the History of Mathematics*. Moscow: Published by the editorial board of journal “Physical and Mathematical Sciences in their Present and Past”.

Bobynin, V. V. (1913). The Goals, Forms and Means to Introduce Historical Elements into the Course of Mathematics in Secondary School. *The Proceeding of the 1st All-Russian Congress of Mathematics Teachers*. Saint-Petersburg: The Sever Publishing House.

Burova, N. S. (2000). *The Course of Mathematics History as a Factor of Humanization and Humanitarization of Mathematical Education in Teachers’ Training University* (Ph.D. thesis, Novosibirsk State Teachers’ Training University, Novosibirsk, Russia). Retrieved from http://www.dissercat.com/content/kurs-istorii-matematiki-kak-faktor-gumanizatsii-i-gumanitarizatsii-matematicheskogo-obrazova

Drobyshev, Yu. A. (2004). *The Historical Aspect in Teacher’s Methodological Training*. Kaluga: The KSTTU Publishing House.

Drobyshev, Yu. A. (2011). *The Multi-Level Teaching of the History of Mathematics for Future Mathematics Teachers* (PhD thesis, Moscow City Teachers’ Training University, Moscow, Russia). Retrieved from http://www.dissercat.com/content/mnogourovnevaya-istoriko-matematicheskaya-podgotovka-budushchego-uchitelya-matematiki

Gilmullin, M. Ph. (2009). *The Historical Component of the Mathematical Methodological Culture of Students Taking the Course of Mathematics History at Teachers’ Training University* (Ph.D. thesis, Ushinsky State Teachers’ Training University, Yaroslavl, Russia). Retrieved from http://www.dissercat.com/content/formirovanie-istoricheskogo-komponenta-matematiko-metodicheskoi-kultury-studentov-pri-obuche

Gilmullin, M. Ph. (2009). *The History of Mathematics*. Yelabuga: The YSTTU Publishing House. Retrieved from http://www.egpu.ru/lib/elib/Data/Content/129936576714671565/Gilmullin_M.F._Istorija_matematiki._Uchebnoe_posobie._(2009).pdf

Grattan-Guinness, I. (2004). The mathematics of the past: Distinguishing its history from our heritage. *Historia Mathematica, 31*, 163-185. http://dx.doi.org/10.1016/S0315-0860(03)00032-6

Karp, A., & Schubring, G. (2014). *Handbook on the History of Mathematics Education*. New York: Springer Science+Business Media. http://dx.doi.org/10.1007/978-1-4614-9155-2

Mordkovich, A. G. (1986). *The Professional Training of Mathematics Teachers at University* (PhD thesis, The Moscow City Teachers’ Training University, Moscow, Russia). Retrieved from http://www.dissercat.com/content/professionalno-pedagogicheskaya-napravlennost-spetsialnoi-podgotovki-uchitelya-matematiki-v-

Novikov, D. A. (2004). *Statistical Methods in Educational Research (Standard Cases)*. Moscow: MZ-Press.

Polyakova, T. S. (1997). *The Historical Methodological Training of Mathematics Teachers: The Methodological Mechanism*. Rostov-on-Don: The RSTTU Publishing House.

Polyakova, T. S. (1998). *The Historical Methodological Training of Mathematics Teachers at University* (PhD thesis, Rostov State Teachers’ Training University, Rostov-on-Don, Russia). Retrieved from http://www.dissercat.com/content/istoriko-metodicheskaya-podgotovka-uchitelei-matematiki-v-pedagogicheskom-universitete

Safuanov, I. S. (2000). *The Genetic Approach to Mathematical Subjects in Teachers’ Training Universities* (PhD thesis, Moscow State Teachers' Training University, Moscow, Russia). Retrieved from http://www.dissercat.com/content/geneticheskii-podkhod-k-obucheniyu-matematicheskim-distsiplinam-v-vysshei-pedagogicheskoi-sh

The Exemplary Curricula. Mathematics. 5-9 Forms. (2011). Moscow: Prosvescheniye.

The Federal State Educational Standard for Basic General Education. (2011). Moscow: Prosvescheniye. Retrieved from http://standart.edu.ru/catalog.aspx?CatalogId=2588

Tomilova, A. Y. (1998). *The Method to Choose the Content for the Course of Mathematics History and to Implement it in Teachers' Training Universities* (Ph.D. thesis, Pomor State University, Arkhangelsk, Russia). Retrieved from http://www.dissercat.com/content/metodika-otbora-soderzhaniya-kursa-istorii-matematiki-i-ego-realizatsii-v-pedagogicheskom-vu

Tomilova, A. Y. (2001). *Tests on the History of Mathematics: the Methodological Aid*. Arkhangelsk: Lomonosov State University.

Zhokhov, A. L. (2007). *The World-View: Formation, Development and Upbringing through Education and Culture*. Yaroslavl: The Yaroslavl Branch of the Institute of Management.