

Beyond questionnaires – Exploring adult education teachers’ mathematical beliefs with pictures and interviews

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Abstract

Because of the impact that mathematical beliefs have on an individual’s behaviour, they are generally well researched. However, little mathematical belief research has taken place in the field of adult education. This paper presents preliminary results from a study conducted in this field in Switzerland. It is based on Ernest’s (1989) description of mathematics as an instrumental, Platonist or problem solving construct. The analysis uses pictures drawn by the participants and interviews conducted with them as data. Using a categorising scheme developed by Rolka and Halverscheid (2011), the author argues that adults’ mathematical beliefs are complex and especially personal aspects are difficult to capture with said scheme. Particularly the analysis of visual data requires a more refined method of analysis.

Key words: adult education teachers, mathematical beliefs, qualitative methods, content analysis

Introduction

Beliefs and their influence on an individual’s actions have been researched for over a century. In the field of education belief research has gained momentum after the cognitive revolution, which led to more interest in teachers’ thinking and decision-making processes (Thompson, 1992). More specifically, beliefs relating to mathematics and the teaching of mathematics have been investigated in a number of contexts, and while in some areas concrete results have been produced, open questions remain. The “unevenness” of mathematical belief research not only applies to the geographic distribution and to particular thematic areas, as identified by Pehkonen (2004), but also to specific target groups. The field of adult education, more specifically adult basic education, seems to be particularly neglected. While Taylor (2002; 2003) or Dirkx and Sprugin (1992) discuss general beliefs of adult educators, only a limited number of studies on mathematical beliefs of adult educators could be identified¹⁷. The study presented in this paper, therefore, aims at contributing to this neglected field and describes the mathematical beliefs of five Swiss adult education teachers. In line with its exploratory nature, its overall approach is a qualitative one. On the basis of the work of Rolka and Halverscheid (2011) pictures created by the participants themselves are used in combination with interview data to explore these adult education teachers’ mathematical beliefs. In addition to interpreting these data, the usefulness of different data sources as well as the suitability of the employed methodology is assessed.

¹⁷ Among these few two contributions from previous ALM proceedings are worth mentioning: Henningsen and Wedege (2003) discussed the issue of values and mathematics, and Stone (2009) looks at how the institutions in which teachers work affect their beliefs and practice in the classroom.

Theoretical framework

The need for a generally accepted definition for beliefs has already been postulated more than twenty years ago, for example by Pajares (1992) and Thompson (1992). Yet even in the seminal work on mathematical beliefs by Leder, Pehkonen and Törner (2002), no generally agreed upon definition has been identified or proposed. Beliefs are considered “a messy construct” (Pajares, 1992) and the wide variety of definitions is occasionally considered one of the reasons for the lack of progress in this research field (Pajares, 1992). Very often terms such as conceptions, attitudes, values, judgements or personal theories – to name a few – are used synonymously. Some of them are broader than others and they stress cognitive and affective aspects to different extents (Pehkonen, 2004). As this study is based on a specific methodological approach presented by Rolka and Halverscheid (2011), it also follows these authors’ understanding of beliefs, namely that beliefs are considered to be a person’s world view (Rolka & Halverscheid, 2011, p. 521). It therefore adopts a very broad and inclusive approach, integrates cognitive as well as affective aspects and accommodates the view that beliefs entail both conscious and sub-conscious elements (for a more extensive discussion of these issues see Pajares, 1992; Furinghetti & Pehkonen, 2002; Pehkonen, 2004).

When it comes to mathematical world views, there is more agreement, particularly regarding the fact that there is not one right view of mathematics. Ernest (1989) describes three contrasting views of mathematics, namely (1) a problem-solving, (2) an instrumentalist and (3) a Platonist view¹⁸. Again they have also been used as a reference by Rolka and Halverscheid (2011) and are therefore also an integrated part of this study. These views can be summarised as follows:

- (1) a “view that mathematics is a useful but unrelated collection of facts, rules and skills” (Ernest, 1989, p. 21)
- (2) a “view of mathematics as a static but unified body of knowledge, consisting of interconnecting structures and truths” (Ernest, 1989, p. 21) and
- (3) “a dynamic, problem-driven view of mathematics as a continually expanding field of human inquiry” (Ernest, 1989, p. 21).

In addition to beliefs about mathematics, beliefs about the teaching and learning of mathematics are relevant for any person teaching mathematics. However, as they are not the focus of this paper, these areas of beliefs will not be discussed any further.

Methodological framework, procedure and participants

A broad variety of research instruments has been employed when researching beliefs, including large scale questionnaires, various types of interviews or the analysis of specific materials such as lesson plans or journals kept by teachers (see for example Leder & Forgasz, 2002; Speer, 2005; Forgasz & Leder, 2008). One key aspect of educational research is that the issues under investigation are often not easily accessible, something which also holds true for beliefs: people are not always conscious of their beliefs – a fact which presents specific methodological challenges. In her discussion of photo elicitation, Rose (2012) argues that “elicitation interviews with participant-generated visual materials are particularly helpful in exploring everyday, taken-for-granted things” (Rose, 2012, p. 306). This aspect combined with the experience that language can slow down the creative process, as it requires higher cognitive demands, provide important arguments for the use of drawings not only with children, but also with adults (Burton, 2010). This study therefore chose to adopt an approach used by

¹⁸ There are other authors who present very similar threefold perspectives, among them Dionne who called them the traditional, the formalist and the constructivist perspective (Dionne, 1984) and Törner and Grigutsch (1994) who presented the toolbox, system and process aspect of mathematics.

Rolka and various colleagues who have used students' drawings to explore children's world views (Rolka & Bulmer, 2005; Rolka & Halverscheid, 2006; Halverscheid & Rolka, 2007). Their experiences are summarised in one article, in which they also present the classification scheme developed for the analysis of the drawings (Rolka & Halverscheid, 2011). The authors present two critical characteristics for each of Ernest's (1989) three views of mathematics and identify key questions and essential points for all of them (see Table 1 for details of this scheme).

Table 1. Characteristic features of Ernest's three world views (based on Rolka & Halverscheid, 2011, p. 528).

Perspective	Characteristics	Description
Instrumental	Non-coherent sequences (1a)	Are there several objects within the work which belong to a particular field of mathematics but do not show any relation with one another? Does the text consist of an enumeration or a classification of items rather than showing the parallels in-between? <i>Essential: the items instead of their characteristics are considered to be important, that is, the items are more important than their meaning in a wider context</i>
	Facile conception of usefulness/ application of mathematics in the course of life (1b)	Is there a slight evidence of the importance of mathematics and its application? Is the attention drawn to the fact that applications are useful rather than in which way? <i>Essential: the central motivation point for practicing mathematics is the convenience one can gain where the character of usefulness always comes to the fore</i>
Platonist	Display of mathematical coherence (2a)	Are there any references drawn between any mathematical items in the work? Does the text show a cohesive character within the implementations? <i>Essential: relations are identified but not necessarily self-drawn</i>
	Theory/history of mathematics (2b)	Is the development of mathematics referred to as a determined, somewhat stable construct of knowledge? Are scholars who once made mathematics crucial to the work? Is there an attempt to constitute a part of mathematics as methodical? <i>Essential: mathematics as a static entity predetermined by nature</i>
Problem-solving	Autonomous mathematical activities (3a)	Does the setting of tasks offer the occasion for using mathematics actively and self-dependently? Do certain actions enclose mathematical items as well and are not mentioned without any reference? <i>Essential: not only meta-mathematical explanations, but something inventive; an extract out of a mathematical process allowing not only to counterfeit, but also permitting independent thinking</i>
	The development of mathematics (3b)	Is the development of mathematics indicated by being a process? Does the description transcend the image of mathematics being a complete and static product? Is there somebody mentioned who actually produces mathematics? <i>Essential: dynamic of mathematics (through the author or somebody else) is described as a process</i>

As Rolka and Halverscheid (2011) have pointed out, using pictures as sole data sources entails highly subjective interpretations, which is why they have additionally asked their participants to write a text and, in some cases, also conducted interviews. Similarly, Rose refers to Collier who as one of the first to use photo elicitation already argued that the information entailed in an image can only be accessed through interviews (Collier, 1967 as cited by Rose, 2012, pp. 300-301). This study therefore follows the idea of data triangulation and also makes use of interviews conducted with the participants about the creation and content of their picture. The semi-standardised interviews allows enough flexibility with respect to the individual pictures, but also ensures that key issues are addressed in all interviews (see annex for a list of the questions used). The verbal data was analysed using the method of qualitative content analysis as described by Mayring (2010). At the heart of this approach is a set of categories (codes) which are defined and revised in the course of the work (feedback loop).

The codes used in this study are on one hand derived from the scheme by Rolka and Halverscheid (2011) as it is described in Table 1 (deductive codes), on the other hand they were developed from the available data (inductive codes). In addition to the six characteristics listed in the presented table above, the inductive codes consist of two large groups of codes, namely ‘Mathematics’ and ‘Other issues’. The category ‘Mathematics’ consists of two subcodes, that is ‘Characteristics of mathematics’ and ‘Mathematical terms and fields’ which were mainly used to specify particular aspects of the six characteristics presented in the table above. The category ‘Other issues’ identifies themes frequently mentioned in the interviews, but which cannot easily be integrated into the deductive codes. It encompasses a wide variety of themes, namely: ‘Personal issues’ (subdivided into ‘Emotions’ and ‘Personal experiences’), ‘Language’, ‘Definition of everyday mathematics’, ‘Education’ and ‘Nature’. The system of inductive codes was established in three steps, first on the basis of the pictures, secondly on the basis of the interviews and thirdly the two systems were integrated into one encompassing scheme. As the full interview material collected in this study goes beyond the issue of the created pictures, the presented code scheme will be altered once the rest of the interview material is analysed.

The data for this study were collected in the summer of 2012 in Switzerland. All participants are members of the Swiss network for everyday mathematics¹⁹ through which they were recruited. Ten days before the first interview they received a letter asking them to create a picture answering the question what mathematics is for them²⁰. Together with this task they received an A3-format piece of paper, which they had to use for the creation and presentation of their picture. They were asked to return the picture to the author no later than two days before the first meeting, as it was the basis for the first interview. A second interview focused on the participants’ biography and teaching. The average time between the two interviews was one month and on average they lasted 82 minutes (minimum 58 min., maximum 138 min.). Most interviews took place on the premises of the participants’ work place. They were conducted in Swiss German, recorded digitally and later transcribed in standard German. Only a small part of the interview data is included in the analysis on which this paper is based, namely the first section of the first interviews where the participants talk about the pictures.

Twelve individuals volunteered to participate in the study after being informed about the project and the corresponding requirements²¹, eight of them were selected for the interviews. Out of this

¹⁹ The network is called “Netzwerk Alltagsmathematik” and is comprised of some 100 people from German speaking Switzerland who are interested in numeracy. See <http://www.netzwerk-alltagsmathematik.ch/> (last accessed June 17, 2013).

²⁰ The literal translation of the task is as follows: “Imagine you were an artist and have accepted the following contract work: What is mathematics? A personal view. Present your views in a pictorial, creative manner, working with materials and techniques of your choice (coloured pencils, watercolour, collage, etc.)”

²¹ The only requirement was that applicants were teaching adults and address mathematical topics in their classes. They were informed that they would need to invest a maximum of four hours of their time, during

group, five were selected in order to have as homogenous a group as possible. All of these five participants have attended one of the first two numeracy²² trainings for adult education teachers in Switzerland²³ and none of them studied mathematics at the tertiary level. Furthermore, all are currently working as adult education teachers, though to different extents, in different subjects and with different students. Three of them are teachers of German as a second language, one of them is a self-employed public relations worker (and teaching part time), the last is working as a course leader in the social affairs department of a large Swiss city. They have an average age of 50 years (between 43 and 57), three of them are male, two female. Their educational backgrounds are very diverse and many of them have been educated and trained in different jobs. Table 2 below presents a short overview of the five individuals labelled P1, P2, P3, P4 and P5 in the remaining text.

Table 2. Overview of the participants.

	P1	P2	P3	P4	P5
Sex	F	M	F	M	M
Age (in years)	57	53	52	43	45
Education	High school, commerce diploma, translator, music school, adult educator	Compulsory school, chemical technician, forestry manager, adult educator	High school, teacher training, speech therapy	High school, teacher training, public relations and communication specialist	Compulsory school, commercial apprenticeship, social work, adult educator
Work experience	Secretary, own company, translator, mother, adult education teacher	Chemical technician, craftsman in France and Spain, outdoor social worker, father, adult education teacher, case manager, trainer	Teacher, various small jobs, speech therapist, mother, adult education teacher	Radio journalist, self employed consultant, adult education teacher, trainer	Banking, IT coordination, social work, adult education teacher
Current position	German as a second language teacher with low qualified people	Course leader	German as a second language teacher, speech therapist, trainer	Self employed public relations consultant; trainer	German as a second language teacher, developing numeracy material
Time spent working on mathematics (P's own estimation)	0% *	10%	0% *	10%-15%	40%

* 0% indicates that these two people were not teaching specific numeracy classes at the time of the interviews. Most of these classes do not take place due to low enrolment rates, but both address related issues in their other classes.

which they would have to complete a small non-mathematical task and participate in two interviews in a location of their choice.

²² Without going into the details of conceptual discussions of mathematics and numeracy, the latter term is used in the context of this paper whenever the work situation of the informants is referred to, as they clearly see themselves as teachers of everyday mathematics (numeracy) and not mathematics in general. Numeracy is considered to be the appropriate translation for the German term everyday mathematics in this context.

²³ Specific numeracy training for adult education teachers is new in Switzerland. Currently the fourth training course aimed at trained adult educators with „no reservations towards mathematics“ is on-going (see also: <http://www.alice.ch/index.php?id=1330>, last accessed January 6, 2014). Some 45 individuals have attended the eight day course so far.

Findings

Visual data

Before analysing the pictures with the scheme presented in Table 1, they are first described in terms of the techniques used in their creation to provide a better understanding of their diversity as well as the shared characteristics. Four out of the five informants have chosen to present their view of mathematics in form of a collage: They put together various small bits and pieces and in one case combined these items with paintings and drawings (see figures 1 to 5 for reprints of the five pictures). Only one person (P5) presented an integral painting. Incidentally this was one of two who gave a title to their picture, namely “Chaos and Order” (the title of the picture of P4 roughly translates to “One Day Mathematics”, see figure 2). Looking more specifically at the items used for the pictures, the following types can be distinguished: (i) artefacts such as receipts, timetables, programmes, money, stamps, or a playing card (mainly used by P4); (ii) newspaper or magazine clippings (mainly used by P3) or pictures printed from the internet (mainly used by P1); and (iii) own writings and drawings, printed and cut out (mainly used by P2).

When asked why they chose this particular mode of creating the picture, the informants said it suited them or that it was the only way such as task could be approached. While all informants said to have thought about the task and how to approach it, before completing it, only one of them went about it systematically: P1 wrote down a list of words she associated with mathematics and then systematically looked for images which illustrated each term. The others worked more spontaneously, for example emptying his wallet (P2) or using those pictures from magazines that appealed to her (P3). P5 relied on a painting technique he was familiar with and liked. During the interview the informants had the chance to add to a copied version of their picture, however, none of them chose to do so and in spite of some of them identifying specific gaps when explicitly asked for them, they all were satisfied with their creation in the end and said that the picture represents them well as a numeracy teacher.

In the following paragraphs each of the characteristics presented in Table 1 will be discussed briefly in order to analyse the five pictures systematically. If necessary, references to interview statements will be included, and some specific methodological observations will be made.

Non-coherent sequences (1a)

While collages can be presented as integral pictures, as it has been done by P3 and P4, they can also be seen as illustrative of presenting non-coherent sequences, the first of the classification criteria described by Rolka and Halverscheid (2011). A prototypical example of such a presentation is the picture created by P1, which consists of eight individual elements, laid out systematically:

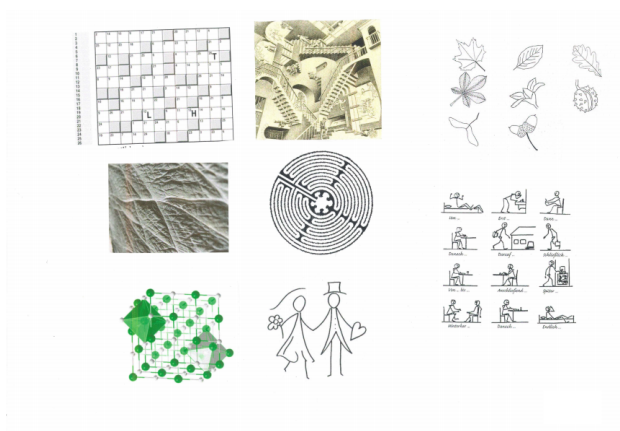


Figure 1. Picture created by P1, illustrating non-coherent sequences.

Similarly when talking about their pictures, the participants in some cases moved from one element to the next (particularly P1), while others talked in more coherent and general perspectives about mathematics. However, it is worth noting, that there are hardly any “objects [...] which belong to a particular field of mathematics” (Rolka & Halverscheid, 2011, p. 528) depicted in the pictures. Again the creation by P1 is a good example: None of the eight pictures can be identified as an object belonging to a field of mathematics. The pictures by P2 and P3 contain some elements which can easily be allocated to fields of mathematics (for example equations belonging to the field of algebra), but they clearly do not constitute large shares of their pictures. The majority of the specific elements show mathematics in a wider and applied context, such as shopping or time keeping. In the remarks made by P1 it becomes clear, that her elements are actually representing characteristics of mathematics rather than objects of mathematics – each item stands for a specific aspect of mathematics, such as regularity, symmetry or confusion. And even though they all seem unconnected and unrelated, P1 stresses that “They [the eight small pictures] are related to me, to my experience of mathematics. [...] Amongst themselves they are not directly – they represent the same in different ways. They represent mathematics.” (P1).

In short, while many of the pictures present unrelated objects, the stories the informants tell convey a different message, namely that the specific elements do belong together and can be seen as different sides of the concept mathematics. Furthermore they indicate that the wider context is eminently important – an aspect that is highlighted in the next point.

Facile conception of usefulness/application of mathematics in the course of life (1b)

When it comes to the usefulness and application of mathematics it is interesting to note that applications are dominant in most of the pictures and that in the interviews many informants stress how present mathematics is in everyday life. The picture created by P4 is an illustration of the presence of mathematics in everyday life, as it consists mainly of real objects such as money, a stamp or a lottery ticket. The creator of the picture also stresses, that one cannot choose to do mathematics respectively numeracy, one has to do it: “It is part of our everyday life, well, yes. But also it is smashed into your face. One has to do it. [...] It is not simply, just flowing along somehow.” (P4)



EINTAGSMATHEMATIK

Donnerstag, 28. Juni 2012
06.00 – 12.00 Uhr

Figure 2. Picture created by P4, consisting mainly of everyday objects.

This focus on mathematical aspects of everyday life is not surprising, considering the participants' background as numeracy teachers. Their pictures present rich illustrations of the many contexts in which mathematics is found: in games, timetables or receipts, measuring time or money. These applications also illustrate one characteristic highlighted by many, namely that mathematics is frequently hidden and needs to be identified first. It is therefore interesting to note that in their talks the participants often refer to basic activities such as counting, categorising or organising when speaking of mathematics, rather than identifying specific operations such as adding, multiplying or calculating percentages which stand behind these applications. Furthermore, they all very clearly divide mathematics into two areas – applied mathematics, which is what they know and do, and abstract mathematics, which is beyond them. This duality is very often referred to in the context of their own educational experiences:

To a certain extent, math²⁴ was playful for me. I understood it well, I liked doing equations. [...] But, when it was too abstract or so, at some point maths became a problem zone [pointing to the same word she had glued on her collage]. (P3)

Or: *“Being a linguist, I’ve quite a high affinity for math. I had that at school as well. I was left behind when it was not connected to everyday life.”* (P4)

There is a strong sense of the relevance of applications in everyday life and at the same time a division of mathematics into an applied and an abstract or theoretical part, an issue which will be discussed in the next section.

One last point which needs to be mentioned in this context is an illustrative example of the difficulty of interpreting pictures, as it has also been pointed out by Rolka and Halverscheid (2011). P3 has drawn a series of artefacts on her picture and commented:

At some point it occurred to me that actually all inventions that we have such as a car, a plane, a submarine and here the mouse and the computer – all of them have to do with math. [...] Our modern cities would not be possible without math. (P3)

²⁴ As Swiss German is predominantly an oral language in which the term mathematics does not exist, the English equivalent term math is used in all quotations to reflect this aspect.

Both the mentioned images as well as this statement can easily be interpreted as illustrations of mathematics being useful. But when asked about the usefulness of mathematics, P3 answered:

Pfffft, that is a question I have never asked myself. I think that, well categorising and sorting, those are excellent skills that one needs to have in order to do math. I think. And at the same time they are also cognitive functions. (P3)

The interviewee does not provide a direct answer; also she never used the word useful in her answers herself. So while her picture clearly could be interpreted as showing the usefulness of mathematics, the informant does not stress that aspect herself at all.

In short, when looking at the aspects constituting the instrumental view of mathematics, it can be said that visually a few of the pictures imply the presentation of non-coherent sequences, but that this view is modified by their verbal explanations. Furthermore, the application of mathematics is dominant, in both the pictures, and the interviews. There is also a particularly strong notion of a division of mathematics into an applied and an abstract part.

Display of mathematical coherence (2a)

This characteristic is one that is most difficult to analyse in the five pictures. As stated above, there are very few mathematical items are presented, therefore it is difficult to identify references between them. Particularly interesting in this context are the two pictures by P3 and P4. At first glance they depict a coherent image, but when looking closer both consist of various individual elements. And while many of these elements could be sorted or grouped (for example visual presentations, formulas or measurement tools in the image included below), this is not done by the creators of the image.



Figure 3. Picture created by P3, presenting a mix of technologies used.

When examining the statements made during the interviews, all of the pictures can be considered to contain at least some cohesive elements, as the interviewees identify references between the elements. Since coherence was visually not always present, the participants were specifically asked for it and only P2 says that he is not sure that the elements on his picture have any connections, except to him as a person and his thinking: *"They are really the thought clouds that I associate with it [mathematics]. In the end it should illustrate a little bit the ambivalent attitude that I have towards math."* (P2)

And while this sentiment of the creating person itself as the connecting aspect of the picture is partially reflected by others, two people said that all shown elements “*have something to do with math [...] it meets on the meta level of math*” (P3). This and other statements imply that the participants do see mathematics as a unified (and unifying) body comprised of different parts or levels: “*The depth is reflected in the colour blue, but also in the sea and the waves [...] also from the easy to the difficult, always this, from the profound to the superficial, from the simple to the complex.*” (P3)

Or:

For me, math as a concept is not really interesting. Until the point, when I reach some limit and then I have to go one [level] deeper and say, okay, what does it look like more abstractly, what are the structures that are behind it. (P4)

These statements again reflect the profound duality of mathematics that has been identified before, namely the two parts of applied mathematics – or in terms of the participants those aspects of mathematics they are able to do – and the abstract, conceptual side of mathematics.

Theory/history of mathematics (2b)

This characteristic is clearly absent in all pictures – there are no references to any mathematical scholars or historic events. Similarly in the oral discussion, history of mathematics or its development are very rarely touched upon and are definitely not a reoccurring theme. In some statements development is implied, for example: “*I believe that in math we are currently at this point, most likely also, because time and again we were curious.*” (P3)

This statement implies not only a history of mathematics, but also leaves other possibilities for the future. At the same time it shows clearly that this development is not determined and stable. Similarly the following statement: “*At some point there was a decision for the decimal system.*” (P4)

This observation additionally implies that there are actors who actively influence the development of mathematics by taking specific decisions. Mathematics is therefore clearly not seen as being “*a static entity predetermined by nature*” (Rolka & Halverscheid, 2011, p. 528). However, it is worth noting, that though largely absent in the pictures, nature is a key issue in the interviews. It is mentioned by four participants and all identify a close relationship between mathematics and nature, namely that there is a lot of regularity and symmetry in nature and that this can be described with mathematics. However, it is clearly people who “*put patterns over nature. [...] It [picture of a daily routine] stands for the human who has made this division of something that is determined by the sun*” (P1).²⁵ But this active use of mathematics is the core theme of the problem solving view of mathematics, which is described after a short paragraph summarising the key elements of the Platonist perspective.

When it comes to assessing the aspects constituting the Platonist perspective it can be said that virtually no aspect of this view is represented in the participants’ pictures: Coherence in their pictures is strongly linked to the participants themselves as the pivotal point of the presented elements, rather than being identified between the items in the work. Scholars or historic issues are not depicted at all and the only element, which is implicitly present, is a methodical part of mathematics, namely

²⁵ Interestingly P5 does not refer to nature itself, he refers to the environment in cities, namely people and public transport and he assigns mathematics a very similar role in this context, namely that it describes and explains patterns in our environment, in this case the man-made: “If you look around, if you walk around the city, you get a feeling that people walk around chaotically. And if you have the correct algorithm, you find an order in it, how it all works and so on. Or also a train, how the entire thing works. It looks very chaotic, but it is all planned meticulously.” (P5)

everything that is not applied mathematics. However, that aspect remains very vague and needs to be explored further.

Autonomous mathematical activities (3a)

The focus of this aspect lies on the independent thinking by the person using mathematics. The picture by P2 is an example which – upon closer inspection – is very illustrative of this process: It consists of creative adaptations of word problems (sentence in the centre at the bottom), an own presentation of the number Pi (little square in the upper middle) as well as other changed and adapted quotations or principles (the line around spider web is a free adaptation of the Haiku, a Japanese form of poetry where the precise number of phonetic units is decisive):

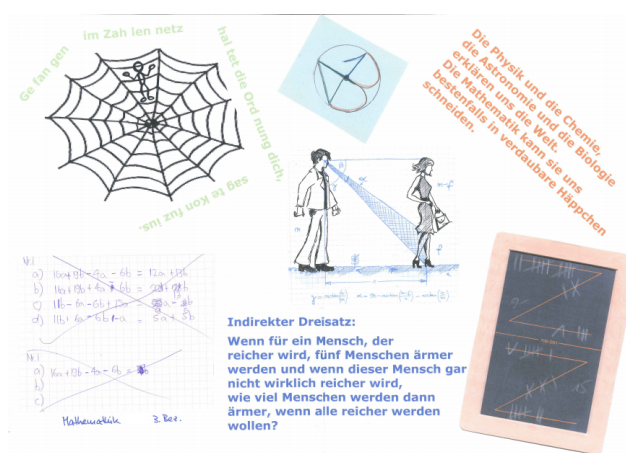


Figure 4. Picture created by P2, representing some autonomous mathematical activities.

So while the issue of autonomous mathematical activities is not always easily accessible in the pictures, the explanations the participants have given clearly illustrate its importance at different levels: *“It [mathematics] makes independent, from other people. It empowers. It enables you to participate in society without depending on other people.”* (P1)

While this person stresses a fundamental idea of independence facilitated by mathematical abilities, another makes the link between specific mathematical items and individual activities: *“Anything that is related to measuring. [...] Measuring time, measuring weight, ehm presenting things in graphs, translating it into curves.”* (P3)

Another stresses the relevance for everyday life:

There are larger themes such as shopping, where a lot of mathematical things come into play. Dates, prices, weight, being able to compare. The rule of three always comes into play relatively quickly. And this is the hidden [math], that needs to be carved out. (P5)

For many of the participants the setting of the task or identifying the mathematical aspects in a given situation is the starting point for autonomous mathematical activities – people need to solve a problem and mathematics is considered to be helpful:

And if a mathematician is looking at what a person, who understands nothing of math, is doing, he could probably very easily formalise that and present it with numbers or transfer it into graphs or so. And the person himself only wanted to have a larger harvest or make a dress or a shoe or something. (P3)

It is interesting, that in the context of these participants, it is not always easy to distinguish between the instrumental and problem-solving perspective, as the idea of application and usefulness is very closely related to or even a prerequisite for independent thinking.

The development of mathematics (3b)

This issue has already been touched upon when discussing the aspect of history of mathematics, where it was shown that if history is talked about by the participants it is understood as a process. And while no names of people producing mathematics are mentioned, there are references which imply that such people exist:

And I think math is something to be developed. Even though one can define laws and the like, they are defined on the basis of our own boundedness. That means, because we are limited, what we define here can also only be a form of boundedness. [...] But it doesn't really reflect anything complete, because we are not complete, so it can't be complete either. (P5)

Many of the participants mention the possibility of being creative with the application of mathematics, for example cheating when keeping scores in games, but are at the same time aware of the fact that there are clear limits as to what one can do:

I can't, suddenly, well, I can claim something absurd in math, but then to prove it is a lot harder than for example in other areas. [...] In math that is quite difficult. There is relatively little room for manoeuvre. And I don't really know whether that has already been exhausted. (P2)

Out of the three perspectives of viewing mathematics, the perspective of problem solving is undoubtedly the most present one in the analysed data. The pictures very clearly present views of mathematics implying a strong personal engagement, particularly in specific applications of mathematics. Furthermore, the statements indicate that mathematics is dynamic rather than complete and static – even though the participants recognise that these developments are clearly beyond their own capabilities.

Summarising and very broadly speaking, it can be said that on the basis of the dominant characteristics in their pictures, the participants' views of mathematics are clearly mixed. It is equally clear, that the least relevant view is the Platonist, as neither mathematical coherence nor its theory or history feature prominently in the presented pictures. The analysis has also shown that it is indeed very difficult to work with pictures only, when trying to understand and describe their creators' view of mathematics. The picture created by P5 is a good example of this fact that visual data on its own can be very hard to interpret. Without the accompanying interview this picture could not have been classified. However, once formulated, its key message comes across very clearly:

For me, the circle represents absolute order [...] Order also means to be able to orient yourself in the chaos and if one takes the relation to math, math to me is like a language with which you can create order in a chaos. So if from the outside something looks chaotic and you then look closer, with the help of math you can explain certain things or identify new dimensions within and a new understanding." (P5)



Figure 5. Picture created by P5, representing an integrated presentation of mathematics.

In order to complete this sketchy analysis based on the pictures, some select issues which only surface in the interviews will briefly be presented in the next section. This will help to get a better understanding of the complex views that the interviewed adult education teachers have.

Verbal data

While the analysis of the pictures has particularly highlighted the relevance of application for the creators of the pictures, and in some instances also their autonomous mathematical activities, they not only raised a number of questions with respect to the interpretation of specific elements, but also left out a number of issues which only emerged from the analysis of the interviews. As a first element of this section the issue of language, which does not arise at all from the pictures will be discussed, afterwards a number of issues, which are only marginally or implicitly present in the pictures will be presented.

Language

One issue which was addressed by all interviewees at least once, but cannot be identified in the pictures is language. Mathematics was seen as complementary to language in mastering everyday life: *“And numeracy is more like [...] let's say decisions and organising or I don't know exactly what. Either way, it is not only language, I have to deal with something else as well.”* (P2) And: *“It [mathematics] is a help, like language. Language helps mastering your life and communication, sure, and math also helps mastering your life, doesn't it?”* (P1)

As this statement already indicates, the participants identified similarities between language and mathematics, not only that both have an instrumental function for mastering everyday life, but also at a more fundamental level: *“For me math is like a language where one can create order in a chaos. [...] And math is the same for me, it is like a language for me to also explain something, phenomena.”* (P5)

More specifically, two interviewees mentioned that mathematics is like the grammar in language which explains how things work and by contrast, *“numeracy is in this sense the expression of what I do in daily life. Like I can speak without ever wasting any thought on grammar or linguistic structures or on metalinguistic condition.”* (P3) Other points mentioned include differences between mathematics and language, for example the fact that language has developed more organically, without seemingly arbitrary decisions such as the naming of numbers (P4), or difficulties of non-native speakers with word problems (P2).

School

There is only one element on the picture by P2 which can be clearly attributed to a school context, namely a piece of an exercise book with several crossed out attempts of solving a problem. Contrary to this single artefact, school was mentioned several times in the interviews: On one hand in the context already mentioned when discussing the pictures, namely the experience that all of the participants at some point of their educational career reached a point where mathematics became too abstract. On the other hand, several references were made to the way mathematics is taught and experienced at school in general. In their role as adult educators the interviewees are often confronted with students who had very negative experiences with school in general and particularly mathematics classes. They are very aware of this aspect and clearly state that in adult education (or in their classes) other principles apply to the teaching of mathematics:

But it was stressful at some point and I can imagine that my participants, that they are simply stressed. They get somewhere where they can't continue, it just doesn't go any further with their imagination. [...] And that as a course leader you are always aware of that, that each individual has his or her limits. I notice precisely if a person simply can't, that doesn't work and then I don't insist, because I don't want them to panic. [...] As adult educator I can do that. I have the liberty of simply stopping. (P1)

Personal experiences

The interviewees' own educational experiences with respect to mathematics teachers and classes are an important aspect that is not directly reflected in the pictures, but is constitutive of how they see mathematics. In addition to their specific school experiences which did not leave any of them "*a typically traumatised person*" as P5 said, many positive memories connected to mathematical activities were mentioned, for example solving puzzles, mental arithmetic when shopping, playing cards or being particularly good in a specific field of mathematics area such as solving equations or probability. Again, some of the participants make a direct link from their experiences to their students' by stating that one of their goals in their courses is that their participants are also able to experience mathematics as something fun, that they are able to enjoy doing it – like they themselves did at some point.

Affective issues

Personal experiences are closely related to emotions which can be equally difficult to express in images. Even seemingly explicit presentations, such as the stick man wedding couple in the picture by P1, can stand for something very different, as the interviewee explains:

Here [pointing to the picture] I actually googled stick man, this represents a stick man. [...] It is probably not the right image, [...] but it is a symbol for dumbing down. And for the two dimensional presentation of something that is three dimensional and has so many sides. Well, and if you just want to flatten something on to a plain and reduce it to some lines which do not conform to reality [...] a simplified presentation [...] And it came to my mind that also with math – we are sometimes dumbed down by the mindless calculations and by how we learn math in school. (P1)

Overall it is interesting to note, that the participants view their own experiences with mathematics generally positively and those of others, mainly their students, predominantly negatively.

Characteristics of mathematics

One last aspect, which is represented indirectly in many pictures, particularly in the picture by P1 who argues that each of the small images stands for one characteristic or aspect of mathematics, is its nature. When asked to describe the nature of mathematics the use of adjectives is predominant and while the participants were not directly asked to do so, many of them used numerous adjectives when talking about mathematics, for example: mathematics is useful, explaining, precise, organising, abstract but also hidden, playful, reliable or contradictory. The one feature, which was named most often and in each interview at least once, can be summarised with the adjective fundamental. It includes statements like "*I think, yes, maths is like always included*" (P3) or "*It is always and everywhere*" (P4). This is one aspect which seems central to the belief of the interviewed teachers, but which cannot easily be reconciled with any of the three perspectives described in the used scheme.

Overall, a number of additional aspects of mathematics, such as its close relationship with language, how it is experienced in school and other everyday situations as well as the connected affective issues gain more clarity in a verbal exchange than in visual representations. Furthermore, it

seems that they are better suited to provide insight into dynamic aspects of beliefs, namely their development or how they influence teaching practices. The combined results of the visual and verbal descriptions will briefly be discussed in the next paragraph before some concluding remarks are presented.

Discussion

Many characteristic features of mathematical beliefs and other issues prominently discussed in mathematics education can be recognised in the beliefs of the five adult education teachers presented in this study. Among them the relevance of affective issues (Evans, 2000), the invisibility of mathematics (Wedegge, 2010) and findings described in many other studies such as the early formation of beliefs, or their influence on an individual's behaviour. One aspect which is completely absent is the issue of gender: Lim (1999) has found that mathematics is generally perceived to be a male domain and Pehkonen (1994) has identified gender differences as one area of belief research which is well documented – however, it has not been an issue in the data used for this paper. In the next paragraphs, two aspects of the findings will be discussed, namely how the described beliefs fit into Ernest's categories and methodological issues.

Instrumental, Platonist or problem solving views?

On one hand, it seems easy to allocate specific views of mathematics to the participants. Both their pictures as well as the interviews illustrate the relevance of the application of mathematics in the course of life – one aspect of the instrumental view – as well as the importance of autonomous mathematical activities – one key aspect of the problem solving view. Both these aspects can be explained with the context in which they work and their identity: They see themselves as numeracy teachers and the issue of autonomous applications in real life, in specific contexts, is a key aspect of numeracy. On the other hand, there are also some issues, which seem to be unique to these five individuals, among them the relevance of language when discussing mathematics. Potential explanations for this aspect are on one hand the informant's background – all of them also work as German as a second language teacher – on the other hand their training: In their education as numeracy teachers, comparing language and mathematics was a prominent theme (oral information by the respective course leader).

Two other dominant aspects of the described mathematical beliefs include firstly the fact that the participants see mathematics as being divided into what they know and are able to do and the rest – an aspect that is not reflected in either Rolka's and Halverscheid's (2011) or Ernest's (1989) original presentation of the three views. Again, this can partially be explained by the teachers' identity as well as their experiences: they see themselves as numeracy teachers and not mathematicians therefore needing only specific knowledge in the former field. Furthermore, during their education they all have experienced that there are issues in mathematics that they do not understand. One could argue that this division of mathematics into what they understand and are able to do, that is above all numeracy, and the rest, namely “abstract mathematics”, is not only based on their experiences but also allows them to see themselves as competent numeracy teachers. The second aspect is that the participants see mathematics as something fundamental and universal and in this function they provide it with a trait that goes beyond the strict instrumentality of specific procedures, namely that mathematics per se explains and organises the world. One could argue that this characteristic is constructivist rather than instrumental, however the notion of usefulness is still strongly linked to this function of explaining

and organising the world, therefore the relevance of the instrumental view for these adult education teachers is justified.

Methodological issues for future research

Overall, it can be said that using pictures to explore adult education teachers' beliefs has worked very well. Many of the participants commented positively upon the task of creating the picture and were engaged in the research process. Furthermore, data collected from the pictures and interviews have proven to be somewhat complementary – two positive aspects of visual methodologies as they are also identified by Rose (2012). The complementarity of issues raised has also been reflected in the elaboration of the codes, where it can be seen that certain codes predominantly occur in the visual respectively verbal data (for example time and money respectively language). However, looking at the diversity and richness of the pictures presented and the classification reached according to the three views of mathematics, a sense of inadequacy remains. The benefits of working with visual data seem to vanish if one of the main results is a categorisation that hardly goes beyond what could also be attained by using questionnaires. The obtained results therefore not only underline the relevance of the question raised by Halverscheid and Rolka (2007) whether the identified categories describe the works extensively, but point to the more fundamental question of whether these categories which focus on one aspect of beliefs are adequate to analyse the richness of visual data. Particularly if beliefs are understood in the broad sense of world views taking into account affective as well as cognitive aspects, a more encompassing analysis which allows capturing these diverse components of beliefs is needed.

Kress and van Leeuwen (2006) argue that verbal and visual communication both have their specific possibilities and limitations in constructing meaning and in the shift towards a new visual literacy the corresponding abilities of reading visual communication are essential. They present a system of categories which can be used to critically analyse images and which could be a starting point for a more fundamental and inclusive analysis of the pictures presented here. For example, the layout of images: While Rolka and Halverscheid (2011) reduce this aspect to the issue of connectedness (point 1a in Table 1), Kress and van Leeuwen (2001) suggest various types of narrative and conceptual representations of different types of layout – an aspect which could enrich the presented analysis. Moreover, as the created images were discussed extensively, a multimodal perspective taking the specificities of and interaction between the verbal and visual data into account as described by the same authors (Kress & van Leeuwen, 2001) could be even better suited for a more in depth analysis of the presented data. Such an approach could most likely help to address some of the challenges encountered when using the scheme developed by Rolka and Halverscheid (2011), namely that adults' perspectives of mathematics seem to be somewhat richer (for example including less mathematical objects, but more characteristics of mathematics) and are therefore sometimes more difficult to capture with the provided characteristics only. Or the specific difficulty, which these authors also encountered and which became even more prominent in the analysis of pictures created by adults, namely that of separating and identifying the extent of each of the three perspectives in mixed world views. As mentioned before, it has proven to be particularly challenging to differentiate between the aspect of application of mathematics in everyday life and autonomous mathematical activities. Once again, the question that Rolka and her colleague (Halverscheid & Rolka, 2007) ask might need to be reformulated, namely that using visual data is not best suited to estimate the extent of each of the three perspectives, but rather how they are related.

Furthermore, it would be interesting to explore, how the perspectives relate to the division of mathematics into what the participants can do and the rest, as it has been described above. As Rolka and Halverscheid (2011) have observed an overwhelming dominance of the instrumentalist view

amongst younger students, a move towards more autonomous mathematical activities might also be explained with increasing life experience. Another emerging question in this context is what factors facilitate a change from an instrumental use to an autonomous use of mathematics.

In short: the performed analysis has proven a valuable starting point for exploring adult education teachers' beliefs, but other more suitable analytic methods are needed in order to find answers to some of the questions raised or formulate alternative questions.

Conclusion

When taking stock of the experience of using pictures to explore mathematical beliefs of adult education teachers, the overall assessment is a positive one. Furthermore, combining visual and verbal data (data triangulation) has proven to be beneficial as the two sets not only confirmed the centrality of particular themes, therefore validating each other; they also complemented each other, as some issues only emerged in one of the two sets, indicating that data triangulation also helped to improve the quality as the two sets enriched each other thematically. These benefits gained from data triangulation could be exploited even more, if triangulation was also applied to the methods of analysis. More specifically, if a particular visual method of analysis was used or if content analysis was adapted to reflect the specificities of both the visual and verbal data. Each of them has their characteristics and unique qualities that need to be respected and taken into consideration in the analytic process.

Annex: Interview questions

List of questions used for talking about the creation and content of the picture. While the first question was always the same, the order of the following questions was adapted depending on how the interview developed.

1. You received the written task and what happened then?
2. Why have you decided to use this mode of presentation?
3. How is mathematics presented in your picture?
4. What is the connection between the elements of your picture?
5. Are there aspects of mathematics that you wanted to present but could not do so?
6. Is there anything you would like to add to what we have said?

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