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ENHANCING MATHEMATICAL COMMUNICATION FOR VIRTUAL MATH TEAMS

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Abstract: The Math Forum is an online resource center for pre-algebra, algebra, geometry and pre-calculus. Its Virtual Math Teams (VMT) service provides an integrated web-based environment for small teams of people to discuss math and to work collaboratively on math problems or explore interesting mathematical micro-worlds together. The VMT Project studies the online math discourse that takes place during sessions of virtual math teams working on open-ended problem-solving tasks. In particular, it investigates methods of group cognition that are employed by teams in this setting. The VMT environment currently integrates social networking, synchronous text chat, a shared whiteboard for drawing, web browsers and an asynchronous wiki for exchanging findings within the larger community. A simple version of MathML is supported in the whiteboard, chat and wiki for displaying mathematical expressions. The VMT Project is currently integrating the dynamic mathematics application, GeoGebra, into its collaboration environment. This will create a multi-user version of GeoGebra, which can be used in concert with the chat, web browsers, curricular topics and wiki repository.

Key words: Dynamic mathematics, VMT, Math Forum, group cognition, GeoGebra

1. Introduction

The Virtual Math Teams (VMT) Project has conducted research since 2003 on how to support small teams of students around the world to collaborate in online discussions of stimulating mathematical topics. The project has developed an extensive web-based environment and logged about a thousand sessions of usage. Analysis of usage has resulted in over a hundred academic publications (see http://GerryStahl.net/vmt/pubs.html)—the most important of which are collected in *Group Cognition* (Stahl, 2006) and *Studying Virtual Math Teams* (Stahl, 2009)—and six doctoral dissertations (Çakir, 2009; Litz, 2007; Mühlpfordt, 2008; Sarmiento-Klapper, 2009; Wee, 2009; Zhou, 2010) (see summaries in Çakır, Zemel & Stahl, 2009; Sarmiento & Stahl, 2008).

The VMT environment—available at the Math Forum—currently includes a social-networking portal (http://vmt.mathforum.org/VMTLobby/), a Java application that integrates synchronous text chat with a shared whiteboard, social awareness indicators, and an asynchronous community wiki. We are currently porting the dynamic math GeoGebra system (http://www.geogebra.org) into the VMT environment. The integration of the open-source GeoGebra code will enable it to function in a multi-user, synchronous online environment. Integration into the VMT environment will support simultaneous text chat discussion of dynamic math diagrams, graphical referencing between chat and diagrams, scrollable history of chat and diagrams, and pasting of diagrams into the associated wiki.

The integration of GeoGebra into the VMT environment will provide significant mathematical content and functionality to enhance mathematical exploration and communication by virtual math teams. The integration includes the ability to support importing and exporting of GeoGebra dynamic worksheets; this will allow teachers and students to take advantage of available curricular materials. It

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will also provide a multi-user version of GeoGebra for the community of teachers and students currently using single-user versions of GeoGebra. The Math Forum plans to release the new system for worldwide usage, providing a convenient online venue for students to engage in synchronous collaborative learning within a rich environment for mathematical inquiry and knowledge-building interaction.

2. The Math Forum: An Online Service and Resource Center for School Math

The Math Forum manages a website (http://mathforum.org) with over a million pages of resources related to mathematics for middle-school and high-school students, primarily on algebra and geometry, mostly user generated (as a forerunner of the Web 2.0 philosophy). This site is well established; a leading online resource for improving math learning, teaching and communication since 1992, the Math Forum is now visited by several million different visitors a month. A community has grown up around this site, including teachers, mathematicians, researchers, students and parents—using the power of the Web to learn math and improve math education. The site offers a wealth of problems and puzzles, online mentoring, research, team problem solving, collaborations and professional development. Studies of site usage show that students have fun and learn a lot; that educators share ideas and acquire new skills; and that participants become increasingly engaged over time (Renninger & Shumar, 2002).

The Math Forum offers a number of online services, including the following. Most of these services were developed with research funding and volunteer support; some of the established services now charge a nominal fee to defray part of their operating costs:

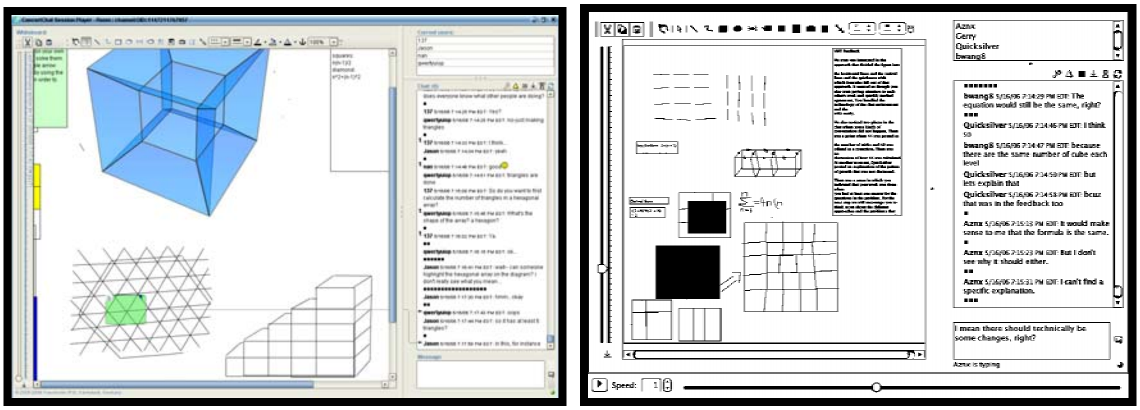
- *The Problem of the Week (PoW)*. This popular service posts a different problem every other week during the school year in a number of categories, such as math fundamentals, pre-algebra, algebra or geometry. Challenging non-standard math problems can be answered online or offline. Students can submit their solution strategies and receive feedback from mentors on how to improve their presentations. The best solution descriptions are posted on the Math Forum site.
- *Ask Dr. Math*. Students and others receive mathematics advice from professionals and expert volunteers.
- *Math Tools*. Visitors to the site explore the world of interactive tools for understanding math concepts and communicate with teachers using them in their classrooms, discussing and rating the tools.
- *Teacher2Teacher*. Classroom teachers and educators from around the world work together to address the challenges of teaching and learning math.
- *Other*. Math Forum staff also provide online mentoring and teacher professional development, lead face-to-face workshops and work with teachers in their math classrooms, under contracts with school districts.
- *Virtual Math Teams (VMT)*. The VMT service builds on the highly successful Problem-of-the-Week service. Students who once worked by themselves on PoW problems can now work on more open-ended problems with a group of peers. This can be organized in a variety of ways and can bring many advantages, as discussed in the following sections.

3. The VMT Project: A New Form of Math Education

The Virtual Math Teams Project explores the potential of the Internet to link learners with sources of knowledge around the world, including other learners, information on the Web and stimulating digital or computational resources. It offers opportunities for engrossing mathematical discussions that are rarely found in most schools (Boaler, 2008; Lockhart, 2009). The traditional classroom that relies on one teacher, one textbook and one set of exercises to engage and train a room full of individual students over a long period of time can now be supplemented through small-group experiences of VMT chats, incorporating a variety of adaptable and personalizable interactions (Scher, 2002).

While a service like PoW or VMT may initially be used as a minor diversion within a classical school experience, it has the potential to become more. It can open new vistas for some students, providing a different view of what mathematics is about. By bringing learners together, it can challenge participants to understand other people's perspectives and to explain and defend their own ideas, stimulating important comprehension, collaboration and reflection skills (Sfard, 2008; Stahl, 2008).

As the VMT library grows in the future, it can guide groups of students into exciting realms of math that are outside traditional high school curriculum, but are accessible to people with basic skills (see Figures 1 and 2). Such areas include: patterns, combinatorics, symbolic logic, probability, statistics, finite math, number theory, infinity, group theory, matrices, non-Euclidean geometries. Many math puzzles and games also build mathematical thinking and stimulate interest in exploring mathematical worlds (Livingston, 1999).



Figures 1 and 2. Images of actual student online collaborative work on patterns, showing the importance of shared visualizations tied to the math discourse. In Figure 1, a student points from his chat message to a smallest hexagon pattern composed of 6 triangles. In Figure 2, the VMT Replayer displays a chat exploring the composition of different 3-D pyramids. (Note the VCR interface of the Replayer at the bottom of Figure 2).

Ultimately, whole curricula within mathematics could be structured in terms of sequences of VMT topics with associated learning resources (Boaler, 2008; Cobb, Yackel & McClain, 2000; Lockhart, 2009; Moss & Beatty, 2006). Students could form teams to explore these sequences, just as they now explore levels within video-game environments. A Problem-Based Learning (PBL) approach could cover both the breadth and depth of mathematical fields, just as PBL curricula currently provide students at numerous medical schools with their academic training in face-to-face collaborative teams (Barrows, 1994; Koschmann, Glenn & Conlee, 1997). In varying degrees, students could pursue their own interests, learning styles, social modes and timing. Assessments of student progress could be built in to the computational environment, supplementing and supporting teacher or mentor judgments. The collaborative, small-group VMT approach would be very different from previous automated tutoring systems that isolated individual learners, because VMT is built around the bringing together of groups of students to interact with one another. Students can work with peers in other schools, even from other countries and cultures.

4. Promoting Knowledge Building through Math Discourse

For most non-mathematicians, arithmetic provides their paradigm of math. Learning math, they assume, involves memorizing facts like multiplication tables and procedures like long division. But for mathematicians, math is a matter of defining new concepts and arguing about relations among them. Math is a centuries-long discourse, with a shared vocabulary, ways of symbolically representing ideas and procedures for defending claims. It is a discourse and a set of shared practices. Learning to talk about math objects, to appreciate arguments about them and to adopt the practices of mathematical reasoning constitute an education in math.

To mathematicians since Euclid, math represents the paradigm of creative intellectual activity. Its methods set the standard throughout Western civilization for rigorous thought, problem solving, and argumentation. We teach geometry to instill in students a sense of deductive reasoning. Yet, too many people end up saying that they “hate math” and that “math is boring” or that they are “not good at math” (Boaler, 2008; Lockhart, 2009). They have somehow missed the true experience of math cognition—and this may limit their lifelong interest in science, engineering and technology.

According to a recent “cognitive history” of the origin of deduction in Greek mathematics (Netz, 1999), the primordial math experience in 5th and 4th Century BC was based on the confluence of labeled geometric diagrams (shared visualizations) and a language of written mathematics (asynchronous collaborative discourse), which supported the rapid evolution of math cognition in a small community of math discourse around the Mediterranean that profoundly extended mathematics and Western thinking. The vision behind VMT is to foster communities of math discourse in online communities around the world. We want to leverage the potential of networked computers and dynamic math applications to catalyze groups of people exploring math and experiencing the intellectual excitement that Euclid’s colleagues felt—leveraging emerging 21st Century media of shared math visualization and collaborative math discourse.

Classical training in school math—through drill in facts and procedures—is like learning Latin by memorizing vocabulary lists and conjugation tables: one can pass a test in the subject, but would have a hard time actually conversing with anyone in the language. To understand and appreciate the culture of mathematics, one has to live it and converse with others in it. Math learners have to understand and respond appropriately to mathematical statements by others and be able to critically review and constructively contribute to their proposals. The VMT Project creates worlds and communities in which math can be lived and spoken.

The learning sciences have transformed our vision of education in the future (Sawyer, 2006; Stahl, Koschmann & Suthers, 2006). New theories of mathematical cognition (Bransford, Brown & Cocking, 1999; Brown & Campione, 1994; Greeno & Goldman, 1998; Hall & Stevens, 1995; Lakatos, 1976; Lemke, 1993; Livingston, 1999) and math education, in particular, stress collaborative knowledge building (Bereiter, 2002; Scardamalia & Bereiter, 1996; Schwarz, 1997), problem-based learning, dialogicality (Wegerif, 2007), argumentation (Andriessen, Baker & Suthers, 2003), accountable talk (Michaels, O’Connor & Resnick, 2008), group cognition (Stahl, 2006), and engagement in math discourse.

These approaches place the focus on problem solving, problem posing, exploration of alternative strategies, inter-animation of perspectives, verbal articulation, argumentation, deductive reasoning, and heuristics as features of significant math discourse (Powell, Francisco & Maher, 2003). By articulating thinking and learning in text, they make cognition public and visible. This calls for a reorientation to facilitate dialogical student practices as well as requiring content and resources to guide and support the student discourses. Teachers and students must learn to adopt, appreciate and take advantage of the visible nature of collaborative learning. The emphasis on text-based collaborative learning can be well supported by computers with appropriate computer-supported collaborative learning (CSCL) software.

Students learn math best if they are actively involved in discussing math. Explaining their thinking to each other, making their ideas visible, expressing math concepts, teaching peers and contributing proposals are important ways for students to develop deep understanding and real expertise. There are few opportunities for such student-initiated activities in most teacher-led classrooms. The VMT chat room provides a place for students to build knowledge about math issues together through intensive, engaging discussions. Their entire discourse and graphical representations are persistent and visible for them to reflect on and share.

5. Research in Designing an Online Chat Community

The VMT Project is an effort to explore some of the opportunities and issues posed above. In order to understand the experience of people and groups collaborating online in the VMT service, the researchers in the project look in detail at the interactions as captured in computer logs. In particular, the project is studying groups of three to six middle- or high-school students discussing mathematics in chat rooms.

The VMT Project was designed to foster, capture and analyze instances of "group cognition" (Stahl, 2006). The project is set up so that every aspect of the communication can be automatically captured when student groups are active in the online community, so that the researchers have access to everything that enters into the communication and is shared by the participants. All interaction takes place online, so that it is unnecessary to videotape and transcribe. Each message is logged with the name of the user submitting it and the time of its submission. Similarly, each item placed in the shared whiteboard is tagged with the name of its creator and its creation or modification time. The chat is persistent and the history of the whiteboard can also be scrolled by participants, and later by researchers.

Although many things happen "behind the scenes" during chat sessions—such as the production of the messages, including possible repairs and retractions of message text before a message is sent, or things that the participants do but do not mention in the chat—the researcher sees everything that the participants *share* and all see. While the behavior of a participant may be influenced on an individual basis—such as by interactions with people outside of the chat or by the effects of various social and cultural influences—the researchers can generally infer and understand these influences to the same extent as the other participants (who often do not know each other outside of the chats). These "external" factors (including the participants' age, gender, ethnicity, culture) only play a role in the group interaction to the extent that they are somehow brought into the discourse or "made relevant" in the chat. In cases where they play a role in the group, then, they are also available to the researchers.

In particular, the sequentiality of the chat messages and of the actions in the whiteboard is maintained so that researchers can analyze the phenomena that take place at the group level of interaction among participants. The other way in which the group interaction may be influenced from outside of activities recorded in the chat room is through general background knowledge shared by the participants, such as classroom culture, pop culture or linguistic practices. If the participants meet on the Internet and do not all come from the same school and do not share any history from outside of the VMT chats, then researchers are likely to share with the participants most of the background understanding that the participants themselves share.

This is not to say that the researchers have the same experience as the participants, but their resources for understanding the chat are quite similar to the resources that the participants had for understanding and creating the chat, despite the dramatic differences between the participant and researcher perspectives. Participants experience the chat in real time as it unfolds on their screen. They are oriented toward formulating their messages to introduce into the chat with effective timing. Researchers are engaged in analyzing and recreating what happened, rather than participating directly in it. They are oriented toward understanding why the messages were introduced when and how they were.

The VMT Project wants to understand how groups construct their shared experience of collaborating online. While answers to many questions in human-computer interaction have been formulated largely in terms of individual psychology, questions of collaborative experience require consideration of the group as the unit of analysis. Naturally, groups include individuals as contributors and interpreters of content, but the group interactions have structures and elements of their own that call for different analytic approaches. In particular, the solving of math problems in the chat environment gets accomplished collaboratively, interactionally. That is, the cognitive work is done by the group.

We call this accomplishment *group cognition*—a form of distributed cognition that may involve advanced levels of cognition like mathematical problem solving and that is visible in the group discourse, where it takes place. It is possible to conduct informative analyses of chats at the group unit of analysis, without asking about the individuals—e.g., their motivations, internal reflections, unexpressed feelings, intelligence, skills, etc.—beyond their participation in the group interaction. Of course, there are also intriguing questions about the interplay between group cognition and individual cognition, but we generally do not consider those in the project.

The VMT Project is studying how small groups of students do mathematics collaboratively in online chat environments. We are particularly interested in the *methods* that the chat members must develop to conduct their interactions in an environment that presents new affordances for interaction. “Member methods” (Garfinkel, 1967) are interactional patterns that participants in a community adopt to structure and give meaning to their activities. A paradigmatic example of member methods is the set of conventions used by speakers in face-to-face conversation to take turns talking (Sacks, Schegloff & Jefferson, 1974). The use of such methods is generally taken-for-granted by the community and provides the social order, meaning and accountability of their activities. Taken together, these member methods define a group culture, a shared set of ways for people interacting to make sense together of their common world. The methods adopted by VMT participants are subtly responsive to the chat medium, the pedagogical setting, the social atmosphere and the intellectual resources that are available to them. These methods help define the nature of the collaborative experience for the small groups that develop and adopt them. Through the use of these methods, the groups construct their collaborative experience. The chat takes on a flow of interrelated ideas for the group, analogous to an individual’s stream of consciousness. The referential structure of this flow provides a basis for the group’s experience of intersubjectivity and of a shared world.

As designers of educational chat environments, we are particularly interested in how small groups of students construct their interactions in chat media that have different technical features. How do the students learn about the meanings that designers embedded in the environment and how do they negotiate the methods that they adopt to turn technological possibilities into practical means for mediating their interactions? Ultimately, how can we design with students the technologies, pedagogies and communities that will result in desirable collaborative experiences for them? Our response to the question of how cognitive tools mediate collaborative communities is to point to the methods that interactive small groups within the community spontaneously co-construct to carry out their activities using the tools.

The VMT Project pioneered the study of online collaborative math discourse—both its nature and modes of computer support for it. The studies in (Stahl, 2009) present some of the most important of the publications related to the project. They include a number of dissertation-level case studies of interactions in the VMT environment by middle-school, high-school and junior-college students, which analyze: how math problem solving can be effectively conducted collaboratively among students who have never met face-to-face; how the structure of text chat interaction differs from spoken conversation; how the media of graphical diagrams, textual narratives, and symbolic representations can be intimately interwoven to build deep math understanding; how deictic referencing is important to establishing shared understanding; how students co-construct a joint problem space; how collaborative meaning making and knowledge building are accomplished in detail; how online math discourse can be supported by a software environment that integrates synchronous and asynchronous media with specialized math tools; and how a methodology based on interaction analysis can be used for a science of group cognition (Stahl, 2010a; 2010b).

6. VMT: A Multi-User Platform for Synchronous and Asynchronous Math Discourse

In our design-based research at the VMT Project, we started by conducting chats in a variety of commercially available environments, including AOL Instant Messenger, Babylon, WebCT and Blackboard. Based on these early investigations, we concluded that we needed to include a shared whiteboard for drawing geometric figures and for persistently displaying notes. We also found a need to minimize “chat confusion” by supporting explicit referencing of response threads. We decided to adopt and adapt ConcertChat, a research chat environment with special referencing tools (Mühlpfordt & Wessner, 2005). By collaborating with the software developers at Fraunhofer IPSI in Germany, our educational researchers have been able to successively try out versions of the environment with groups of students and to gradually modify the environment in response to what we find by analyzing the chat logs.

The ConcertChat environment—which is now available in Open Source—integrates text chat with a shared whiteboard. A unique feature of ConcertChat is its support for graphical referencing. It allows for three forms of referencing from the text chat:

- A chat message can point to one or more earlier textual postings with a bold connecting line. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the selected chat posting above.
- While someone types a new chat message, they can select and point to a rectangular area in the whiteboard. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the area of the whiteboard.
- While someone types a new chat message, they can select and point to a graphical object in the whiteboard. When that message appears in the chat as the last posting or as a selected posting, a bold line appears connecting the text to the area of the whiteboard.

This referencing is just one form of integration of media in the VMT environment. The overall technological integration of the VMT Lobby (or portal), chat room/shared whiteboard, and wiki should be understood theoretically as a pedagogical integration of learning at the individual, small-group and community levels. The VMT Lobby provides a portal for the *individual user* to browse the people and topics of the community and to select a room for group work. The chat rooms are basically meeting and work places for the *small groups* as they engage in synchronous collaborative learning. The wiki, on the other hand, primarily provides an asynchronous *community space* in which the work of all groups is coordinated, commented upon and perhaps summarized.

Figure 3. The VMT Lobby, with social networking features on the left and a list of chat rooms on the right, organized by math subject and problem topics.

The *VMT Lobby* provides a social networking portal for students to log into the system (see Figure 3). It includes tools for defining and viewing personal profiles. In general, students in a VMT group have no knowledge about each other except for what is revealed in the chat interaction; with the functionality available in the VMT Lobby, they can define their own profiles and view profiles of each other, as well as send messages to individuals or groups in their communities. Communities are defined for various VMT constituencies, such as participants in a given Spring Fest or in a given course. There is also support for defining buddies, listing favorite chat rooms, etc. In addition, there is an interface for searching and browsing available chat rooms, usually listed for a given community. This provides access to chat rooms on different topics. Students may be told by their teachers to find certain rooms, may be invited by buddies, may search for rooms on interesting topics or may create new rooms and invite peers to join them.

A typical *VMT chat room* consists of the text chat interface on the right and a shared whiteboard on the left. The history of the whiteboard state can be scrolled through, much like that of the chat, but unlike the chat it usually retains inscriptions in the visible board as long as they are relevant. VMT chat rooms have a tabbed interface, with multiple workspaces—and users can add additional spaces as needed (see Figure 4). One kind of workspace is the shared *Workspace*, supporting graphics and text boxes. Another is a similar shared whiteboard, intended for preparing a *Summary* of the group's work for posting to a special wiki page associated with this chat room. A third tab may display the *Topic* for the room, stored on a wiki page by an instructor. A *Wiki* tab displays a page of the VMT wiki; a special page is created for each room, linked to other pages on the Topic, math Subject or Community. A *Browser* tab provides a simple multi-user web browser that can support the graphical referencing tool from the chat and a history scrollbar. The final tab displays wiki pages containing the VMT *Help* manual and associated information.

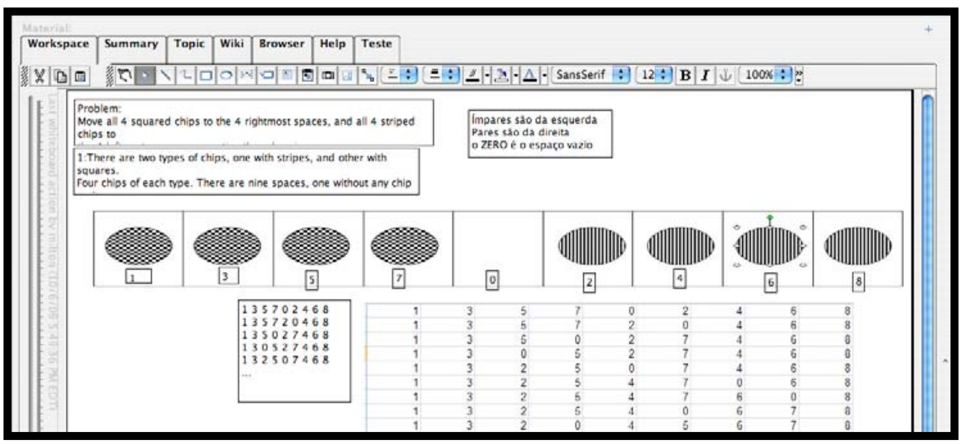


Figure 4. A team including students in Newark and in Brazil collaborated on the chips combinatorics problem. They dragged circles representing game chips according to specific rules. They could then scroll the history of the "Workspace" shared whiteboard to *animate* the sequence of moves taken. Note that the students have added a new tab named "Teste" in addition to the tabs defined for this curriculum topic.

The *VMT wiki* acts as a digital library repository for summaries of work posted by teams. If there is a course that involves multiple chats by several teams, a wiki home page can be constructed for the course. The home page would then point to pages describing the course and each assignment. Group assignments are all posted to linked wiki pages. The course wiki includes index pages that bring together the student assignments in various combinations and allow the instructor to post feedback that is visible to all. The student groups can also rate and provide feedback to each other's previous reports.

The VMT wiki can be used flexibly to structure mini-repositories. For instance, a wiki page for the VMT Spring Fest 2007, which involved probability problems, provided a knowledge-building space, analogous to Wikipedia (see Figure 5). That is, anyone in the community could add information to this catalog of knowledge about K-12 probability as well as browsing the space. The space was seeded with a number of different probability problems and several strategies for solving such problems. During the Fest, student groups were to each initially select a problem and try to solve it with one of the strategies. Then they would post a summary of their solution path on the wiki page linked to from the home page for that problem and that strategy. Subsequent work would involve trying the same strategy on other problems or other strategies on the same problem, followed by comparing the results posted by other groups. The idea was that this kind of knowledge-building repository could persist and evolve through use in the future.

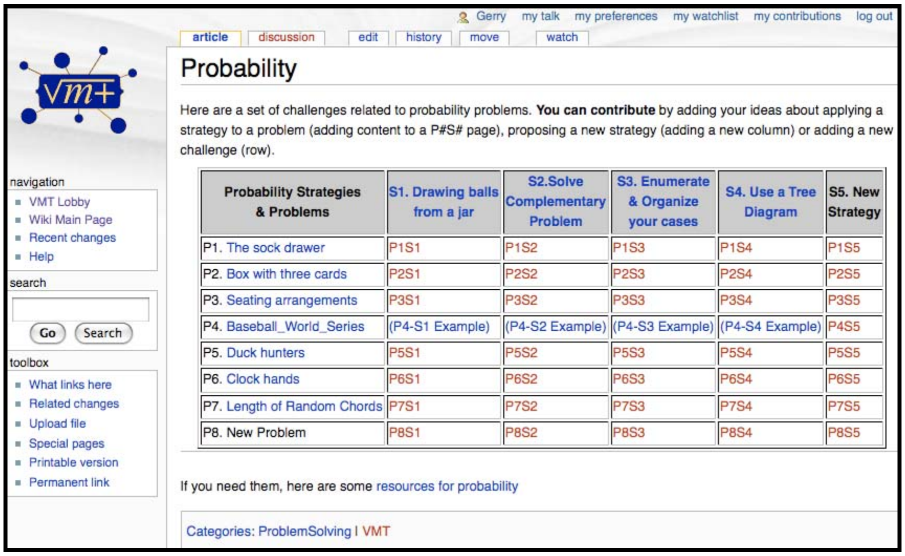


Figure 5. A VMT wiki page associated with chat rooms exploring proability topics.

The VMT environment has come a long way from the simple AOL Instant Messaging system to the current lobby/chat/tabbed-spaces/wiki multiple-interaction space. In part, this increased complexity parallels the shift from simple math exercises to open-ended explorations of math worlds, from one-shot meetings to multiple-session Fests, from problem-solving tasks to knowledge-building efforts. Along with the considerable gain in functionality come substantial increases in complexity and the potential for confusion. This has been countered by trying to extend and supplement the integration approaches of ConcertChat. The graphical referencing and the history scrollbars have been extended to the multiple tabs. New social awareness notices have been added to track which tab each group member is viewing or referencing.

The VMT collaboration environment has been tuned to the needs of high-school math students. There are specifically math-oriented functions—like a partial implementation of MathML for displaying equations (see http://vmt.mathforum.org/VMTLobby/VMTHelp/mathequations.html) and the whiteboard's stock of Euclidean shapes. In addition, there are tools for integrating the multiple work spaces—like the graphical referencing from chat, the creation of wiki pages corresponding to each chat room and the automatic posting of summary text to the proper wiki page.

Integration across modules has been important. Logins and passwords have been unified across the Lobby, chat rooms and wiki, so that logging into one automatically logs into the others. People registered in one module show up in the profiles and messaging system, by their selected community. When a new chat room is created, it is categorized by a community (e.g., a school), subject (e.g., combinatorics), a topic (e.g., Week 3's assignment) and a group (e.g., Team D). A new wiki page is generated for posting the summary from this room. The MediaWiki functionality of categories automatically associates this new page with aggregation pages for the community, subject, topic and group.

7. GeoGebra: Dynamic Math Support for Group Cognition

Our next major enhancement to the VMT environment is to port the single-user GeoGebra application into VMT as a multi-user component of the tabbed chat room. This will allow groups of users to co-develop and co-explore a GeoGebra geometric construction. They will be able to chat about the drawing and reference parts of it from their chat postings. There will be a history slider, so users can scroll back and forth, watching the changes take place in the drawing for convenient review and reflection.

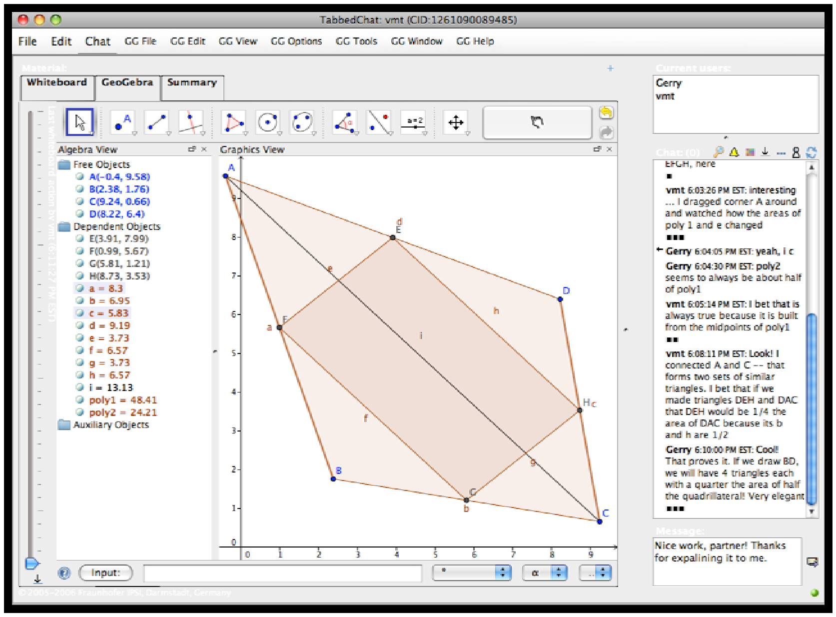


Figure 6. A GeoGebra construction created and discussed collaboratively in a prototype of the VMT 2.0 learning environment. (Not real interaction data.)

The project is porting GeoGebra—a comprehensive and well established application for dynamic math exploration—to the VMT learning environment described above. It will make the application fully multi-user. It will integrate the application in a tab of the environment (see Figure 6). GeoGebra is a particularly appropriate dynamic math application for this project because its source code is freely available as open source, there is a development community to support on-going development, the lead developer and the founder are consulting with us, the application supports a wide range of math from Euclidean construction to calculus and 3-D, GeoGebra has won international prizes, and it has been translated into about 50 languages.

Like all other dynamic math applications, GeoGebra now exists only as a single-user application. While users can send their static constructions to each other, display screen images, or awkwardly include a view of the GeoGebra application within other environments (Blackboard, Moodle, Elluminate, etc.), only one person can dynamically manipulate the construction. Our port will convert GeoGebra to a client-server architecture, allowing multiple distributed users to manipulate constructions simultaneously and to all observe everyone's actions in real time. Every action in the GeoGebra tab will be immediately broadcast by the server to all collaborating clients.

In addition, incorporation of GeoGebra in the VMT environment framework allows users to engage in text chat while manipulating the construction. Importantly, users can graphically point from a chat posting to an area of the construction that they want to index—an important support for math discourse that is unique to VMT (or its now-defunct basis, ConcertChat). They can also scroll back and forth through the history of the GeoGebra construction, animating its evolution—a powerful way to explore many mathematical relationships (see Figure 4 above). In addition, a complete record of the collaborative construction is available to the participants, their teachers and project researchers, allowing them to analyze and reflect upon the complete interaction, including the construction actions synchronized with the chat.

The VMT version of GeoGebra will be compatible with the standard version. Thus, constructions can be imported and exported seamlessly between the two versions. This will facilitate use of legacy

GeoGebra curriculum within the collaborative VMT environment. Images of GeoGebra co-constructions can be created and pasted by users into the VMT wiki or into Word documents. Logs of the corresponding chats can also be saved as spreadsheet files and pasted into documents.

The integration of GeoGebra will significantly enhance the mathematical domain-orientation of the VMT system. On the other hand, for the GeoGebra community, it will make available for the first time truly multi-user dynamic geometry support within a rich collaborative environment. With the flexible system of tabbed components, a curriculum designer, instructor or even a student can define topics for rooms with just GeoGebra and chat or with a more complicated mix of additional browsers and support components.

For researchers of math learning, the enhanced environment will provide a laboratory for hosting virtual math teams engaged in GeoGebra-based tasks. The entire interactions of these teams will be logged in detail. The logs cannot only be generated in a variety of convenient formats, but the team interactions can actually be replayed from the logs like digital videos for careful study. With these tools, researchers can explore the group cognition of small teams accomplishing creative problem solving involving geometric constructions that are shared, visible and dynamic.

We hope the enhanced VMT environment will provide an attractive and effective platform for collaborative mathematical discourse and will appeal to students, teachers and researchers.

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