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Peer-assisted learning in mathematics: An observational study of student success

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Cover Page Footnote

The authors are indebted to University of Minnesota Associate Professor David Arendale for his invaluable guidance throughout this study.

Peer-Assisted Learning in Mathematics: An observational study of student success

Dorothy Cheng and Matthew Walters

ABSTRACT

The Peer-Assisted Learning (PAL) program at the University of Minnesota has drawn from the best practices of Supplemental Instruction, Peer-Led Team Learning, Structured Learning Assistance, the Emerging Scholars Program, and other successful postsecondary peer cooperative learning models to establish guiding principles for structuring learning sessions. To estimate the impact attending weekly math PAL sessions has on students' chances of successful course completion, an observational study was conducted fall 2008 of 534 University of Minnesota students enrolled in two undergraduate math courses. Success was defined as passing the class with a C- or above, and failure as receiving a D+ or below, including withdrawals. In addition to PAL attendance, 16 other factors were considered in this analysis. Attending all PAL sessions during the semester corresponded with ten times higher odds of success than attending none. While further experimental studies are needed, these observations suggest that following these guiding principles result in effective peer cooperative learning sessions.

INTRODUCTION

Peer cooperative learning environments are characterised by positive interdependence, individual accountability, interpersonal communication, and cooperation (Johnson, Johnson, Holubee, and Roy, 1984). Such environments can be structured by clearly explaining educational objectives and activities to students, intentionally forming peer groups, and monitoring and evaluating group achievement (p. 25–26).

Supplemental Instruction (SI), Peer-Led Team Learning (PLTL), Structured Learning Assistance (SLA), and the Emerging Scholars Program (ESP) are examples of successful peer cooperative learning programs (Arendale, 2004). SI is well-known for its effectiveness based on success rates of attending students (Martin and Arendale, 1997; Lin and Woolston, 2008). Participation in PLTL workshops has been associated with increased persistence (Hewlett, 2004; Tien, Roth, and Kampmeier, 2002), lower rates of withdrawal, and a greater percentage of students earning grades of B- or above (Hockings, DeAngelis, and Frey, 2008). Attending SLA sessions has been shown to correlate with increased rates of success as defined by a final grade of C- or better (Doyle and Hooper, 1997; Doyle and Kowalczyk, 1999). The ESP has been associated with decreased rates of withdrawal and re-enrollment (Bonsangue and Drew, 1990; Bonsangue, 1992), failure (Fullilove and Treisman, 1990), and students earning a C or higher (Kosciuk, 1997).

The Peer Assisted Learning (PAL) program at the University of Minnesota has drawn from the best practices of these and other peer cooperative learning models to establish the following guiding principles for structuring learning sessions (Arendale and Ediger, 2007).

Facilitator and students share and model productive learning behaviors
PAL facilitators connect productive learning behaviors with course content by actively applying relevant study strategies during the peer cooperative learning session. For example, facilitators and students may share and model strategies for reading and outlining course materials, taking and reviewing lecture notes, preparing for exams, and improving vocabulary. They may look for "teachable moments" to introduce effective learning techniques during the session.

Facilitators share an increasing degree of responsibility and authority with students as the term progresses
Although the facilitator is always an authority in PAL sessions, an increasing degree of power and responsibility is shared with students as the term progresses. To foster such an environment, facilitators address questions by consistently referring students to the diverse sources of

knowledge available to them, such as textbooks, lecture notes, reference materials, and each other. In an ideal PAL session, outside observers cannot easily identify the facilitator.

Sessions are comprised of a blend of activities both planned by the facilitator and requested by students
Facilitators come well-prepared with activities, worksheets, and other materials necessary to conduct productive peer cooperative learning sessions. However, the actual agenda is developed collaboratively with students, as participants are also responsible for deciding how the session is conducted. For example, students may list topics they are having trouble with on the board. Topics difficult for many participants are selected and then supplement or replace components of the facilitator's planned agenda.

Sessions are formatted based on the nature of the academic discipline
While this study focuses on mathematics, PAL sessions at the University of Minnesota accompany classes in a variety of academic fields, including anthropology, history, chemistry, and geography. Facilitators thus select and customise activities based on the specific learning demands of the course and discipline. For example, some courses require problem-solving skills, while others challenge students to memorise information or synthesise concepts.

Students learn to evaluate their own learning
Students participating in PAL sessions learn and apply techniques to self-monitor their comprehension of course topics. Proactive and continuous self-evaluation is important if students are to optimise their learning behaviors in anticipation of upcoming tests, papers, and other graded coursework. For example, students may compose and answer questions they suspect will be on the next exam. They may spend a few minutes identifying and sharing course topics they are having difficulty with or identifying concepts essential to finding the solution to a math problem. Facilitators also benefit from these activities, as this feedback helps them plan appropriate and effective sessions.

Multicultural sensitivity characterises all aspects of sessions, including the selection of activities, choice of words, and social environment

It is critical in increasingly diverse institutions of higher education that facilitators and students actively promote a respectful and safe learning environment. This requires careful consideration of participants' personal identities and how they may be influenced by characteristics such as gender, culture, and socio-economic status. For example, facilitators may take into account not only the identities of immigrant students in their sessions, but also reflect on their own cultural biases and assumptions affecting the session. They may apply concepts such as critical discourse analysis or ideological models of literacy to meet this challenge of increasing diversity (Couchman, 2008).

Students are actively engaged with both the course material and each other

PAL facilitators encourage student involvement by planning engaging sessions. They introduce a variety of peer cooperative learning activities, allot time for these activities, intentionally form small peer groups, monitor and consult groups as they work, encourage discussion, and reconvene as a large group when necessary.

Educational theory guides learning

Effective learning theories should provide the basis for session planning and facilitation. For example, facilitators may encourage students to assume ownership of course content by challenging them not only to memorise information, but also comprehend, apply, analyse, synthesise, and evaluate this knowledge (Bloom et al..). This may be accomplished with carefully constructed discussion questions prompting students to activate and develop these higher order thinking skills.

Previous Evaluation

At the time of this writing, one public evaluation of PAL program operations had been conducted. Ediger (2008) coded the results of open-ended questionnaires to find that PAL facilitators felt their students experienced increased analytical skills, confidence and positive risk-taking behaviors, effectiveness working in small groups, comfort

with other students, and academic autonomy. Ediger followed this with a quantitative study of students enrolled in *General Chemistry*, an undergraduate chemistry class and common prerequisite for advanced coursework, over a three year period (2005–2007). Each year, a portion of the students were required to attend exclusive PAL sessions due to their enrollment in an animal science cohort program. To simulate an experimental design, cohort members were paired with classmates not in the program on the basis of composite college admissions test scores (administered by ACT, Inc.), course instructor, gender, and year. For the first year, Ediger found that significantly fewer participants failed or withdrew from the course than non-participants. The second year, she found that significantly more participants earned A's.

However, the implementation of PAL in math has not been publicly evaluated. This study aims to follow up on Ediger's evaluation by determining to what degree, if any, attending math PAL sessions increased students' chances of successful courses completion.

METHOD

A quantitative observational study was conducted fall 2008 of 534 University of Minnesota students enrolled in one of two undergraduate math courses: *College Algebra and Probability* or *Precalculus I*. To complement these courses, the PAL program organised weekly peer cooperative learning sessions following the nine guiding principles listed above. Sessions lasted 50 minutes and were held once per week for 13 weeks in normal classrooms. Attendance was not mandatory, but sessions were added to students' course schedules upon registration to avoid scheduling conflicts and limit enrollment in each session to approximately 25 students. In addition to these sessions, students were expected to attend 50-minute lectures held three times per week and 50-minute recitations conducted once per week by graduate teaching assistants.

Each session was facilitated by a model student who had previously completed the course with exemplary marks. Facilitators attended one lecture meeting per week and

reviewed all required coursework. Because time invested in the form of preliminary and ongoing training is critical to the administration of a successful peer cooperative learning program (Arendale, 2000), facilitators were also trained before and during the semester in peer cooperative learning strategies, effective study skills, and classroom management techniques in accordance with the guiding principles listed above.

The original data set contained 816 students, but 12 were not considered because they opted to take the class on a pass or fail basis and incomplete data resulted in the omission of an additional 270. No students were enrolled in both math courses. A total of 16 potential predictors of student success were considered: PAL session attendance, course, course section, instructor, teaching assistant, session facilitator, gender, ethnicity, college admissions test scores (administered by ACT, Inc. and separated into reading, science, math, and English sections), high school class rank, academic level based on credits earned (e.g., sophomore or second-semester junior), major (as declared or undeclared), and college within the University (e.g., College of Biological Science or College of Information Technology). Because the majority of participants were first semester freshmen with no previous college coursework, prior college grade point average (GPA) was not considered in the analysis.

Ethnicity, instructor, PAL facilitator, course, course section, gender, teaching assistant, year in school, major, and college were treated as categorical variables. Reference cell parameterisation was used to code these data using the value appearing most frequently as the reference parameter. The names of instructors, PAL facilitators, and teaching assistants were all replaced with random, unique integers to protect their anonymity. The coding process resulted in a total of 104 potential predictors of student success.

A dichotomous response of success or failure was modeled. Success was defined as earning a final grade of C- or higher in the course. Students who failed the course, withdrew, or received a D+ and lower were considered to have been unsuccessful. For the analysis, the response was coded as 1 or 0, respectively.

The probability of success based on the predictors listed earlier was estimated by fitting a logistic regression model. Logistic regression is recommended over linear regression when modeling dichotomous responses and allows the researcher to estimate probabilities of the response occurring (Lemeshow and Hosmer, 2004). Preconditions associated with ordinary least squares regression, such as homoscedasticity and normality, are not important given the binary nature of the response. The logistic regression equation takes the following form.

$$\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$$

(1)

The estimated probability of the response occurring (*p*) divided by the probability of it not occurring (*1-p*) is called the odds. Knowing the odds of success can of course lead us to the probability of success.

$$\frac{\text{odds}}{1 + \text{odds}} = p$$

(2)

The logarithm of the odds, also called the *logit*, is assumed to follow a linear model composed of variables ($X_1, X_2, \cdots, X_k$) and coefficients ($\beta_0, \beta_2, \cdots, \beta_k$), as seen above. The odds can be rewritten as follows.

$$\frac{p}{1-p} = e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k}$$

(3)

The odds ratio refers to the odds for one student (*a*) divided by the odds for another (*b*). In other words, if the odds ratio of student *a* to student *b* is greater than one, student *a* has a greater predicted probability of success than student *b*, and vice versa. Furthermore, if students *a* and *b* have the same values for all variables except X_i, and X_i is one unit greater for student *a* than for student *b*, then their odds ratio can be simplified as follows.

$$\frac{\text{odds for student } a}{\text{odds for student } b} = \frac{e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\cdots+\beta_i(X_i+1)+\cdots+\beta_k X_k}}{e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\cdots+\beta_i X_i+\cdots+\beta_k X_k}} = e^{\beta_i}$$

(4)

This means that by applying the exponential function to any coefficient, we can see the impact on the odds of a one unit increase in its corresponding variable.

Because of the large number of predictors, the lasso method was used to select the coefficients with the best fit (Tibshirani, 1996). Lasso is a form of LARS (Least Angle Regression) that constrains the sum of absolute values of the coefficients (*s*) and eliminates predictors if their non-zero coefficients reach 0 during the iterative selection process (Tibshirani, 2003). As the lasso progressed, ten-fold cross validation with maximum log-likelihood estimation was used to find the set of coefficients maximising the probability of observing our data set.

RESULTS

Figure 1 is the coefficient map produced by the lasso procedure as *s* was increased. This is a graphical representation of how new coefficients were added to the model based on their correlation with the response. Figure 2 shows the cross validation scores as the lasso progressed. The maximum log-likelihood estimate occurred at *s=0.0136*. This best fit included 21 variables, listed in table 1. For categorical predictors, the reference parameter follows in parentheses.

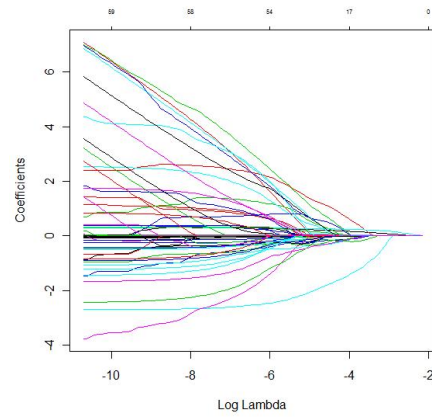Figure 1: Lasso coefficient map
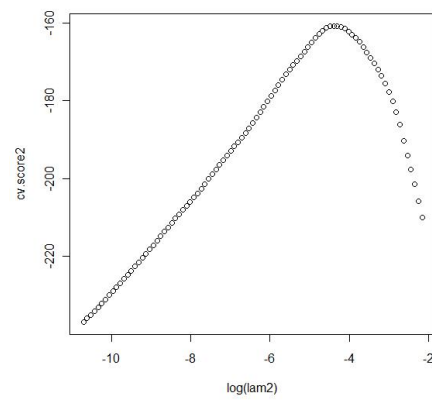
Figure 2: Cross validation scores

Table 1: Predictors and coefficients at maximum log-likelihood estimate

Predictor	Coefficient	Odds Ratio

	(β)	(e^{β})
Total attendance	0.190	1.21
Black (white)	-0.151	0.860
Hispanic (white)	-1.64	0.194
Instructor 5 (instructor 4)	-0.182	0.834
PAL facilitator 5 (PAL facilitator 6)	0.905	2.47
PAL facilitator 7 (PAL facilitator 6)	0.419	1.52
PAL facilitator 13 (PAL facilitator 6)	0.107	1.11
PAL facilitator 15 (PAL facilitator 6)	-0.247	0.781
Section 5 (section 14)	0.0360	1.04
Section 7 (section 14)	0.00430	1.00
Section 13 (section 14)	0.0175	1.02
Section 15 (section 14)	-0.0152	0.985
Precalculus I (College Algebra and Probability)	-0.148	0.863
ACT math	0.248	1.28
ACT English	0.00848	1.01
ACT science	0.0224	1.02
High school rank	0.0128	1.01
Teaching assistant 1 (teaching assistant 3)	0.349	1.42
Teaching assistant 12 (teaching assistant 3)	-0.00273	0.997
Teaching assistant 13 (teaching assistant 3)	0.0771	1.08
Undeclared (declared)	-0.218	0.804

The third column of table 1 contains the results of applying the exponential function to the coefficients, transforming them into more easily interpretable odds ratios. For example, 1.21 is the odds ratio for PAL attendance, meaning that this model reports an increase in a student's odds of success by 21 percent for each PAL session attended. Consider for example a student with an odds of 3, or a 75 percent probability of success. Attending one additional session would increase the student's odds to 3.63 (3×1.21) and thus probability of success to 78.4 ($3.63 \div [1 + 3.63]$) percent. A total of 13 sessions were held, so the analysis suggests that all other variables being equal, a student with perfect attendance had an 11.9 (1.21^{13}) times higher odds of success than a student who attended none. Thus, if the

student in the previous example had a baseline odds of 3 without attending any PAL sessions, perfect attendance would increase that student's odds to 35.7 and probability to 97.2 percent.

To further illustrate this example, figure 3 depicts the non-linear increase of this student's probability of success as total session attendance increases. The logarithmic shape of the plot illustrates that the positive correlation of attendance with probability of success appears highest early on. For example, this model predicts an approximately 9 percent increase in probability of success after attending just three sessions. However, after completing ten sessions, attending three more will correlate only with an estimated 2 percent increase in probability of success.

Figure 3: Probability of success versus total attendance for a student with an estimated 75% probability of success before attending any sessions
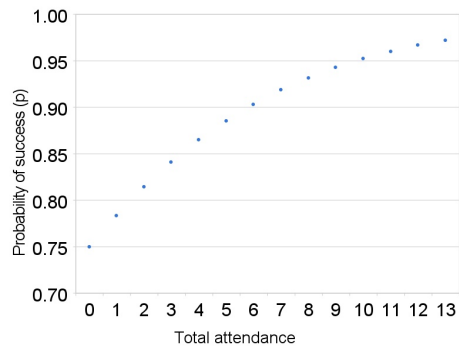
Furthermore, the logistic curve is S-shaped, so students with low baseline probabilities of success will not benefit from large increases in estimated odds after attending the first few sessions. For example, figure 4 depicts the odds increase for a student with an estimated 0.1 odds (9 percent probability) of success without attending any sessions. In this case, attending just three sessions would only result in a 6 percent increase in probability of success, while each session attended after ten correlates with a 14 percent increase. Furthermore, even if this student attends all PAL

sessions, this model still predicts a probability of success only marginally greater than 50 percent.

Figure 4: Probability of success versus total attendance for a student with an estimated 9.1% probability of success before attending any sessions
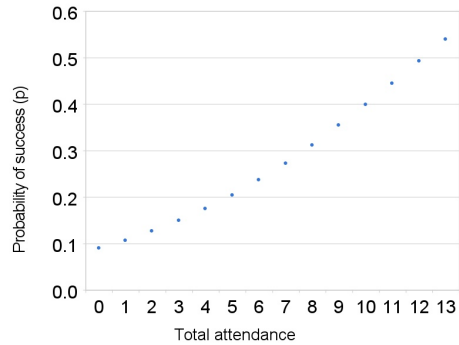
To examine the fit of this model, a cutoff probability (p) of 80 percent was used. In other words, we predicted students with a probability of success greater than 80 percent would succeed. With this cutoff value, 84.1 percent of the model's predictions were correct, and only 5.4 percent of 424 students predicted to succeed actually failed. Table 2 contains a summary of these predictions.

Table 2: Predicted versus actual success, *p=0.80*

Predicted	Actual	
	Success	Failure
Success	401	23
Failure	62	48

DISCUSSION

PAL principles have been drawn from the best practices of successful peer cooperative learning programs, including Supplemental Instruction, Peer-Led Team Learning, Structured Learning Assistance, and the Emerging Scholars Program. A correlation between odds of success and PAL session attendance is expected considering the numerous

studies documenting the effectiveness of these models, as well as Ediger's (2008) evaluation of PAL in chemistry courses. These observations suggest that following PAL guiding principles in math may result in effective peer cooperative learning session planning and facilitator training.

Limitations

A number of limitations were present in this study. For example, due to numerous missing values, only 65.4 percent of the 816 students enrolled in the two courses could be included in the analysis. The absence of some data (such as gender and ethnicity) was inevitable due to student privacy concerns, but a more complete set would have contributed to the analysis.

Decisions regarding the categorisation of factors may also have impacted the usefulness of these findings for at least two reasons. First, because of the large number of majors represented, students were categorised based only on whether or not they had declared their major. Not only did this reduction eliminate the possibility of recognising group trends based on discipline of study, but some overlap with academic level may have occurred because juniors and seniors have already declared their majors. A greater sample size or filtered set representing a smaller but manageable number of majors would have allowed us to better consider the effect of this predictor on student success. Second, the decision to treat a C- or higher as success precluded any further attempts to determine how each individual student may have defined success. While most students must attain a C- or higher in all major coursework at the University of Minnesota, students seeking admission to competitive majors, planning to attend graduate school, or taking the course solely for credit may have had very different definitions of a successful final grade.

Controlling for selection bias is also a central issue in the design of observational studies due to the inability to control fully which participants have access to which treatments (Rosenbaum, 2002). In this study for example, it is unclear whether we measured the effect of PAL on student success or the effect of some other unmeasured predictor highly correlated with both PAL attendance and success.

Finally, this research was conducted during one semester at a research intensive university, so the results cannot be assumed to apply to students at others types of institutions. Additional studies validating the effectiveness of PAL in other academic environments are necessary.

Further Research

This study sought to determine to what degree attending PAL sessions increased students' chances of completing a course successfully. However, perhaps a more important question suggested implicitly above, concerns how PAL increases students' chances of realising their own unique definitions of success. For example, determining what grade defines success for each individual student based on his or her major or anticipated field of study may prove a more insightful analysis. This data could be approximated by taking into account the individual requirements for admission to desired majors or simply surveying the students directly. Perhaps some students even come to PAL solely for its social aspect, in which case a response based on final course grade may not always be entirely appropriate.

An experimental design should also be used to suggest more convincingly that attending PAL sessions caused an increase in students' odds of success. Such an experiment would of course be no small undertaking given the sheer number of variables to control. Permitting one group of students to avail themselves of services unavailable to another also raises important ethical and democratic concerns, especially at a state-funded land grant institution such as the University of Minnesota.

CONCLUSION

The PAL (Peer Assisted Learning) program at the University of Minnesota has drawn from the best practices of Supplemental Instruction, Peer-Led Team Learning, the Emerging Scholars Program, and Structured Learning Assistance to establish a set of guiding principles for session planning and facilitation. In this observational study of

students enrolled in two math courses offered fall 2008 at the University of Minnesota, a logistic regression model was fit using the lasso technique to estimate students' odds of success based on 16 predictors, many of which were categorical. While the resulting model suggests PAL session attendance correlated impressively with increased odds of success, limitations suggest not only that an experimental design be used in future studies to lend greater credibility to results, but also that the way each individual student defines success be investigated further to produce a richer and more nuanced investigation.

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