

## A Comparison of Pedagogical Practices and Beliefs in International and Domestic Mathematics Teaching Assistants

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### Abstract

*International and domestic mathematics teaching assistants (MTAs) are a critical part of mathematics education because they teach a substantial portion of low-level mathematics courses at research institutions. Even if there are several factors to build on MTAs' pedagogical practices, MTAs' beliefs significantly influence the MTAs' practices. The purpose of this study is to explore different beliefs and pedagogical practices between international and domestic MTAs. The findings reveal that there is consistency between the MTAs' beliefs and their pedagogical practices. In addition, the two groups adopt significantly different approaches of how to teach new concepts, definitions, and problem solving for students' conceptual understanding and how to interact with their students. These results contribute to the body of knowledge of MTAs and the adaptation of professional development programs of MTAs. In addition, faculty in mathematics has an opportunity to understand the differences in beliefs and pedagogical practices between IMTAs and DMTAs.*

**Keywords:** Domestic mathematics teaching assistants (DMTAs), international mathematics teaching assistants (IMTAs), beliefs, and pedagogical practices

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Since the late 1800s, mathematics teaching assistants (MTAs) have significantly influenced undergraduate education because of a wide variety of their roles in universities and their potential effect on education (McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001; Speer, Gutmann, & Murphy, 2005). Because of spending a number of times interacting with students through grading students' assignments and exams, providing tutoring services and office hours, and teaching classes (Hendrix, 1995; Speer, Gutmann & Murphy, 2005), MTAs' pedagogical practices directly influence their students' perspective on mathematics and achievement in mathematics education (Commander, Hart, & Singer, 2000; Speer, Gutmann, & Murphy, 2005). In the last two decades, the number of international mathematics teaching assistants (IMTAs) has been increased in mathematics departments in the U.S. because of the globalization of research institutions. Through the increased number of IMTAs, students in research universities have more opportunities to be taught by IMTAs than to be taught by domestic mathematics teaching assistants (DMTAs) (Twale, Shannon, & Moore, 1997). Because of MTAs' vital roles in mathematics departments, mathematics departments have provided and developed training programs such as short- or long-term orientations in order to improve their pedagogical practices and knowledge. Despite of these supports, the majority of MTAs have taught classes based on their personal knowledge derived

from their mathematics learning experiences as students because of their first priority goal, studying their fields, and their resistance to new methods of teaching (Chae, Lim, & Fisher, 2009; Baiocco & De Waters, 1998; McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001).

Even though researchers indicate that a wide range of factors influence teachers' practices, in particular, Speer (2005) and Thompson (1992) assert that there are significant relationships between teachers' beliefs and their teaching practices. In addition, educational experiences and philosophies influence the formation of MTAs' beliefs and teaching practices (McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001; Twale, Shannon, and Moore, 1997). Researchers have been interested in MTAs regarding MTAs' knowledge and beliefs, difficulties, aspects of their experience, curriculum development for MTAs; however, a limited number of studies have conducted MTAs' teaching practices. There is little literature that provides IMTAs' pedagogical knowledge, different practices, and beliefs in detail. In addition, there are few studies about comparisons between IMTAs and DMTAs among pedagogical knowledge, practices, experiences, and challenges. From this case study as a qualitative research project, faculty in mathematics departments has opportunities to understand different IMTAs' and DMTAs' pedagogical practices regarding their beliefs. Mathematics departments and developer of training programs are able to have insight into the appropriate support for IMTAs and DMTAs. The purposes of this study are to explore different beliefs and pedagogical practices between international and domestic MTAs and sought to answer the following two research questions: "What are the differences in beliefs and pedagogical practices between international and domestic MTAs?" and "How are MTAs' different teaching practices shaped by their beliefs?"

## Literature Review

### Definitions and Categorizations of Beliefs

Even if researchers have studied definitions of beliefs for several decades, the definitions have been vague. Several researchers have been interested in beliefs because factors of making teachers' instructional decisions were not enough to explain the nature of teachers' instruction without teachers' beliefs. Because of raising concerns of beliefs, several researchers in mathematics education defined beliefs as personal philosophical conceptions, ideologies, worldviews and values that shape practice and orient knowledge (Aguirre & Speer, 1999; Ernest, 1989; Speer, 2005). Even though researchers have adequately defined beliefs, several researchers assert that there is not a certain definition of beliefs yet.

Researchers have suggested different categorizations of beliefs (Ernest, 1988, 1989; Kuhs & Ball, 1986; Lerman, 1990; Speer, 2005, 2008; Prawat, 1992). Each categorization has different characteristics of a variety of beliefs based on the content of beliefs. Currently, several researchers have proposed appropriated classifications of beliefs in mathematics education such as beliefs about teaching, learning, students learning, and mathematics (Cooney, 2003; Cooney et al., 1998; Cross, 2009; Ernest, 1989; Speer, 2005, 2008; Thompson, 1992). After researchers have suggested that beliefs significantly influence teachers' practices, they have shifted their focus to the relation between teachers' beliefs and their pedagogical practices. Speer (2005) and Thompson (1984) found that teachers' beliefs were consistent with their practices in classroom. On the other hand, Cohen (1990) and Thompson (1984) also found inconsistencies between teachers' beliefs and their practices. Even though there was a complex relationship between beliefs and practices, many researchers have suggested that studies of the relations between beliefs and practices are still valuable because there are no clear explanations for constructing and changing teachers' practices (Speer, 2005, 2008; Thompson, 1992).

### **Relationships Between Beliefs and Practices**

Researchers have been interested in the relationships between instructors' beliefs and their pedagogical practices because beliefs are vital elements for shaping and changing pedagogical practices (Speer, 2005, 2008; Pajares, 1992). Researchers found that there are consistencies and inconsistencies between instructors' beliefs and their pedagogical practices. Speer (2005) and Thompson (1985) found consistencies between instructors' beliefs and their pedagogical practices in class. In contrast, Thompson (1984) and Cohen (1990) also found inconsistencies between beliefs and practices. Because of consistencies and inconsistencies between instructors' beliefs and pedagogical practices, Thompson (1992) suggested complex relationships between them as "teachers' conceptions of teaching and learning mathematics are not related in a simple cause-and-effect way to their instructional practices" (p. 137).

Even though instructors' beliefs and pedagogical practices are often inconsistent, the studies of them are noteworthy because of other potential explanations for these findings and complex relationships between them (Speer, 2005, 2008; Thompson, 1992). In addition, without studying the relationships between instructors' beliefs and their pedagogical practices, researchers have not had clear explanations for particular findings in shaping pedagogical practices. Thus, the future focus of the studies would be the relationships between instructors' beliefs and pedagogical practices for professional development of instructors' pedagogical practices because of the complex relationships (Speer, 2008).

### **Mathematics Teaching Assistants**

There is limited literature of mathematics teaching assistants (MTAs) although research institutions have provided a number of mathematics graduate assistantships. In the past two decades, few researchers have done studies about the characteristics of mathematics teaching assistants (MTAs), MTAs' practices, challenges, and needs compared to the studies about K-12 teachers whereas the number of teaching assistants has been increased (Speer, Gutmann, & Murphy, 2005). After researchers have increased their concerns for quality mathematics education in universities, many researchers have found that MTAs' roles significantly influence the quality of mathematics education because of a high portion of MTAs' teaching hours. For example, MTAs teach 21% of mathematics and 17% of statistics undergraduate students at doctoral degree institutions (Lutzer, Rodi, Kirkman, & Maxwell, 2005). Commander, Hart, & Singer (2000) suggested that MTAs and undergraduate students show a cause and effect relation in regards to their education.

Despite of universities and mathematics departments offering several training programs to support MTAs to teach class, MTAs believed that the training programs were inadequate to prepare them to teach class (Moore, 1996). In addition, the first priority of MTAs is to earn their degrees and find jobs based on their excellent research. Because of these circumstances of MTAs, they often have less motivation to improve their pedagogical practices for mathematics education. Even if MTAs complete a training program for their teaching, MTAs often uses teaching practices based on their mathematics learning experience as students because of institutional constraints and each MTA's resistance to new methods of teaching (Chae, Lim, & Fisher, 2009).

Along with the globalization of universities in U.S., the number of IMTAs has been increasing in mathematics and science departments (Twale, Shannon, & Moore, 1997). Because of a high ratio of MTAs' population, undergraduate students at research institutions have more opportunities to be taught by IMTAs than be taught by domestic MTAs. IMTAs have had a variety of teaching challenges because of cultural differences, lack of understanding and knowledge of American college contexts, and poor English proficiency (Chae, Lim, & Fisher, 2009; Luo, Grady, & Bellows,

2000; Tang & Sandell, 2000). For example, IMTAs realize that their initial expectations for students are significantly higher compared to the actual level of students' mathematical competency at the beginning of teaching (Chae, Lim, & Fisher, 2009). After the initial teaching experience, IMTAs have adapted their teaching practices for the students' level of mathematical competency. Despite IMTAs' efforts for teaching, the instructional changes were based on their prior pedagogical knowledge and learning experiences as students (Chae, Lim, & Fisher, 2009). For supporting IMTAs' teaching, universities and departments have developed training programs. Several studies reported that general training programs for IMTAs have not significantly changed IMTAs' teaching (Etkina, 2000; McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001). Recent researchers have suggested that a number of training programs for IMTAs' pedagogical practices involve not only acquisition of information about American educational contexts but also the communication issues such as fluency in spoken English for non-native speakers (Tang & Sandell, 2000).

### **Theoretical Framework**

Employing a post-positivism perspective, this study verified regular patterns of differences in beliefs and practices between IMTAs and DMTAs because of different cultures and experiences (McGivney-Burelle, DeFranco, Vinsonhaler, & Santucci, 2001; Twale, Shannon, and Moore, 1997). Even though certain different patterns between IMTAs and DMTAs would be discovered, the findings do not consistently not true because there is no accurate equipment for measuring MTAs' beliefs and there is inconsistency between MTAs' beliefs and their pedagogical practices (Cohen, 1990; Thompson, 1984). However, from a post-positivist perspective, the findings of this study are valuable because those contribute to the body of knowledge of MTAs and professional development programs. In addition, the results help readers to understand differences in beliefs and pedagogical practices between IMTAs and DMTAs.

### **Method**

This study is a case study as a qualitative research project. It compared six international and six domestic MTAs regarding their beliefs and pedagogical practices at a coeducational public research university with about 30,000 students. After receiving IRB Approval, the participants were selected by four criteria: MTAs were all Ph.D. students in the department, MTAs were classified by domestic versus non-domestic MTAs, MTAs taught their own classes, and MTAs' classes were low-level courses in the department according to criterion sampling (Creswell, 2007). The five IMTAs taught precalculus classes and one IMTA taught a business calculus. On the other hand, the four DMTAs taught four precalculus and two taught business calculus classes. Through triangulation, I employed three different data sources: an observation for one class period, a questionnaire, and a semi-structured interview with 12 MTAs. After one class period observation, I collected data from the questionnaires and then interviews with a digital voice recorder for later transcripts following a semi-structured interview protocol based on MTAs' beliefs and pedagogical practices. The total time of questionnaire and interview was approximately one hour. Twenty eight closed-ended questions on the questionnaire consisted of the MTAs' background information, pedagogical practices, and beliefs. The interview was semi-structured with 12 open-ended questions consisting of six questions about the MTAs' pedagogical practices and six questions about their beliefs. To improve the validity for this study, the MTAs checked their transcriptions and findings from the data. After removing the participants' identifiable information, several researchers in educational research repeatedly reviewed the data, which consisted of the expanded field notes, transcripts of interviews, critical themes, categories, and code books.

This study was conducted to find patterns and finally identify salient themes by inductive analysis. First of all, the data were analyzed based on four classifications of beliefs: beliefs about

teaching, student learning, students, and calculus. In addition, there were seven categories of pedagogical practices: teaching organization, explanations of concepts and definitions, question forms, responses to students' answers, methods to encourage students to participate in class, methods of summary, and teaching materials. I read and analyzed multiple data in order to find tentative codes through combining and reducing codes about each group's patterns regarding beliefs and pedagogical practices based on the strategy of Miles & Huberman (1994). Using the derived codes, I constructed initial categories with the labels or codes. Through combining and refining the categories, I finally found critical themes, which aided in noting significant differences in beliefs and pedagogical practices between two groups.

## **Results**

From the inductive analysis, one of the results was that the twelve MTAs were consistent in their beliefs and pedagogical practices. The other results revealed three patterns of beliefs and two patterns of pedagogical practices among the IMTAs and DMTAs. The three different beliefs were beliefs about teachings, beliefs about student learning and about students, and beliefs about calculus. In addition, the three beliefs were arranged in hierarchical organizations. From the expanded field notes and questionnaires, there were two different overarching patterns of pedagogical practices: (1) New concepts, definitions, and problem solving and (2) interactions with students.

### **Beliefs About Teaching**

This section describes three middle-level topics: important aspects of teaching mathematics, efficient pedagogical practices, and instructional goals and roles under beliefs about teaching.

**Important Aspects of Teaching Mathematics.** Even if all mathematics teaching assistants considered motivation as a critical factor in teaching mathematics, they had several different views about teaching mathematics. The international MTAs believed that instructors' knowledge of mathematics and pedagogy and preparation were important factors for teaching mathematics. In addition, the IMTAs emphasized recognition of students' mathematics abilities because they had experienced the gap between their initially high expectations for students and their students' mathematical competency. The domestic MTAs, on the other hand, considered instructors' strategies such as problem solving by repetition and visual explanations as important factors for teaching mathematics.

There were significantly different views about how to improve students' motivation. The IMTAs believed that they could develop motivation for students through asking questions and challenging problems, while the DMTAs thought clear explanations regarding why mathematics ideas were valuable would help students improve their motivation. Here are two interview quotations illustrating the IMTAs' and DMTAs' views about improving students' motivation:

Paul (an IMTA): For pre-calculus, you need to motivate students to understand subjects and should help them. And many of them you should give them challenging questions.

David (A DMTA): I think the most important is trying to explain...um...why these processes are important. And exactly how they evolved more because I can teach the processes and I can actually teach you how to just use technology to figure it out. But if you don't know why it's important, then you uh...you probably won't remember it.

The interview with Paul revealed that he believed challenging questions encouraged students to be interested in the lessons. On the other hand, David believed that explanations of the reasons why students learn mathematics were significant for motivation.

**Efficient Pedagogical Practices.** The IMTAs believed that the most efficient practice is clear explanations of concepts by their own methods because students can solve problems if the students understand concepts. Here are two IMTAs' interview responses about explanations of concepts and definitions:

Daniel: I believe that if they really understand concepts, they can do any kind of problems. I also give them ... using a calculator all day. I spend a lot of time making sure they understand what is happening.

Jason: My idea is that I do not just solve problems but also give some definitions and something which is in my own ways.

Daniel and Jason believed that explanations of concepts and definitions are more important than problem solving.

By contrast, the DMTAs stressed doing problems by repetition for students to learn the procedures. In addition, many homework assignments and quizzes were provided for students in order to develop their pattern recognition. One of the DMTAs described her view about efficient pedagogical practices:

Kelly: It's mostly repetition. You just got to keep practicing, keep doing examples, keep doing homework problems. And...hopefully by then, they'll...and there's a lot to memorize, of course. And the only way you're going to remember it all is if you just keep using it and doing it and using it and doing it.

Kelly believed that her students can understand lessons through rote problem solving. Thus, the IMTAs emphasized instructors' clear explanations more than the DMTAs did. By contrast, the DMTAs believed that providing many problems is more critical for efficient pedagogical practices than the IMTAs believed.

**Instructional Goals and Roles.** Even though the international and domestic MTAs had similar top instructional goals--teaching students and students doing well on exams--their next prior goals were different. The IMTAs' next instructional goal was motivating their students as helpers and not as instructors. To help their students, the IMTAs believed that instructors need to make materials easier, share knowledge, and prepare students for exams. An IMTA in this study explained his role:

Paul: I think that my role is to present the material as best I can and help them as much as they want. I don't think I can... I mean I try to motivate them. So I want to help them achieve whatever they want to achieve.

Paul emphasized his instructional role is a guide or helper.

By contrast, the DMTAs thought that their roles are teaching lessons, interacting with students, and grading exams as primary instructors in order to provide best opportunities to learn materials and prepare their students for next level classes. Here is an interview quotation about DMTAs' views about their roles:

Alley: I view myself just like a teacher. Do answer questions, I feel like I understand and I feel well enough, they answer any questions they have. That is what a teacher is.

Alley believed that her roles are the same as other professors or instructors.

Table 1

*MTAs' Beliefs About Teaching*

Beliefs About Teaching	IMTAs	DMTAs
Important Aspects of Teaching Mathematics	Instructors' knowledge of mathematics and pedagogy and preparation  Recognition of students' mathematics abilities	Instructors' pedagogical practices such as problem solving by repetition and visual explanations
Efficient Teaching Practices	Clear explanation of concepts by their own methods	Providing time to work problems on students' own  Doing problems by repetition
Instructional Goal & Roles	As helpers, motivating their students to get involved in mathematics	As primary instructors, providing best opportunities for their students to learn materials and to prepare their students for next level classes

**Beliefs About Student Learning and About Students**

This study revealed several common beliefs about student learning and about students between the IMTAs and DMTAs. The majority of both groups believed that all students cannot learn mathematics the same way because of the students' negative attitudes, which resulted in students missing classes with a variety of excuses and students' minimal efforts. In addition, the important abilities students need to have to learn mathematics are curiosity, logical thinking, patience, diligence, dedication, and paying attention in class. However, the DMTAs were more considerate towards pattern recognition as an important ability than the IMTAs were. Two DMTAs' interviews revealed they believed some students cannot solve similar problems because of a lack of pattern recognition.

Jane: A lot of times students see a problem they should know how to do, but there is something slightly different about it. They'll think that they don't know how to do it when really they do, but they just get stuck on this one little part that's different. What I think that is really important is learning to look fast that and view what the question actually is and recognizing that they really do know how to do it.

David: I give it to them...the exact same problem with a little bit different numbers and maybe a little bit different words. Like, instead of using...like, if we're doing...talking with probability...and they know how to do it...with, like, cards...blind cards, and if I switch it to...like, sandwiches or something like Fritos...and they, uh...they just...are stumped. And they don't...they don't see the connection between these problems. Even though they are just different words...and they're different numbers.

Jane and David believed that the pattern recognition is a vital ability to learn mathematics. If students recognize the pattern, they can solve similar problems even if words or numbers are changed.

The DMTAs wanted their students to realize mathematics is practical and to have the same respect for the DMTAs as other professors. According to the questionnaire, 80% of the DMTAs

responded that they often used real world problems. Two DMTAs' interviews explained that students need to realize that mathematics is useful and valuable:

Jane: I want them to see that it is valuable whether they are interested in it or not. And even if they are going into writing, recognizing that even though math is not going to be foundational to what they do, it's foundational to the way that most of the world work. I don't like the attitude of math is pointless. Math is useless. And that's what I don't want them to have. They don't have to love it but I want them to see that it is useful.

Brian: I want them to...um...realize and be able to acknowledge...um...that...all this stuff that they were doing is for a purpose. And I want them to realize that...it's something that they can...and will probably have to use it some point if they end up going into...um...whether it is business or accounting or...or anything in general.

Since Jane and Brian have practical views for mathematics, they want their students to have the same views for mathematics.

On the issue of respect, the DMTAs had a desire for their students to respect them like any other professors or instructors.

Brian: I, for the most part, view myself as the...or maybe conduct myself as the...the primary instructor. Like, not as a T.A. You know, I teach every day.

Even though DMTAs are mathematics teaching assistants, they want their students to treat them with respect because the MTAs are the primary instructors.

On the other hand, the IMTAs considered the students' respect less than the DMTAs because they accepted the students' attitudes as a cultural difference. Daniel explained his view of students' respect for him:

Daniel: Sometimes I feel they don't really respect their teachers. It is cultural differences compared to India or Korea. American students are not really careful about teachers. It does not motivate you to be like a real teacher. You would not care that sometimes the students are not respectful.

Table 2

*Beliefs About Student Learning and About Students*

Different Beliefs About Student Learning and About Students Between the IMTAs and DMTAs
The DMTAs emphasized students' pattern recognition as an important ability for learning mathematics more than the IMTAs did.
The DMTAs emphasized that students realize the value of mathematics.
The DMTAs considered students' respect for MTAs more than the IMTAs did.

**Beliefs About Calculus.** Both groups believed that calculus was a foundation for mathematics; however, they had different perspectives about who needs to learn calculus. The IMTAs believed that calculus is for all undergraduate students because it is a foundation for students' learning and understanding in other fields. The following is the interview responses from the two IMTAs regarding their views of calculus:

Paul: Calculus is one of the most basic math courses. Basic knowledge of calculus is required for all kinds of students because you should see students in different departments trying to take calculus courses in math department.

Daniel: I think calculus is a very important subject, very foundational subject. We all should at least learn to be calculus II level. I think it is very important.

The IMTAs believed that all undergraduate students need to learn calculus because calculus is the foundation for all majors.



By contrast, the DMTAs believed that calculus is a tool for students majoring in science and not every student. The two DMTAs described their views of calculus:

Jane: I think that it is very essential especially for any students going into any kind of engineering or physics or any kind of applied science at all. So I think it's very useful, necessary, and fundamental.

Alley: I think it's essential for mathematician [laughs] but not necessarily to everybody in the university.

Jane and Alley thought that only some students need to learn calculus. Thus, the DMTAs believed that calculus is a tool for only science majors; however, the IMTAs' view of calculus is it is a tool for all majors.

Table 3

*MTAs' Beliefs About Calculus*

Beliefs About	IMTAs	DMTAs
Calculus	Foundation for all majors  Calculus is for all undergraduate students	Foundation for only science majors  Calculus is a tool for students majoring in science, not every student

### **Pedagogical Practices Influenced by Beliefs**

As the literature states, there are consistencies and inconsistencies between beliefs and pedagogical practices of instructors, and in this study the MTAs' beliefs significantly influenced their pedagogical practices. Because of different patterns of beliefs between the two groups, the results revealed two classifications of each groups' pedagogical practices: (1) New concepts, definitions, and problem solving and (2) interaction with students.

**New concepts, Definitions and Problem Solving.** From the expanded field notes and interviews, the IMTAs spent more time preparing for their class than the DMTAs because of the IMTAs' beliefs about teaching. One of the IMTAs in this study described how he prepared his class.

Sam: My idea is that I do not just solve problems but I also give some definitions and something which is my own ways of solving problems. This is not designed for this course. For this reason, basically what I do is that I open the book and I look over the booklet to see what it mentions there. And sometimes I look up topics and definitions on Google because some definitions are easier than what is in the book.. And once I am done with sorting things and definitions I will then look at problems and try to find the easiest way to solve these problems. Not the way I used to but I try to find the easiest way. As you know, mathematics problems can be solved in many ways. So I prepare for every lesson. When I have already completed the problems then, I will do other things.

The IMTAs kept developing their materials and pedagogical practices since they believed that instructors' preparation is an important element for effective pedagogy.

The IMTAs provided more explanations based on their own experiences with the concepts and definitions, in order to assist students. According to the IMTAs' questionnaires and beliefs about

teaching, the IMTAs spent over 60% of their class time explaining the concepts and definitions. According to another IMTA's expanded field notes, even though the topic of a lesson was trigonometric equations, he reviewed characteristics and graphs of sine and cosine functions to help his students understand the concepts and solve the trigonometric equations.

Tom: Before solving problems of trigonometric equations, he asked simple questions to remind his students about definitions and properties about the sine and cosine functions while he drew the graphs of the functions. After he plotted a point  $\frac{\pi}{6}$  on the  $x$ -axis of the sine function, he asked the  $y$ -coordinate of  $\frac{\pi}{6}$  and then he plotted the ordered pair on the graph. He did the same for points  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$ . In addition, he showed the relationships to the trigonometric functions of angles using the unit circle and the sine and cosine functions.

In class, the IMTAs focused on detailed explanations of concepts, definitions, and rules instead of problem solving.

The IMTAs used easier problems as a complement to their explanations to help students' understanding more so than the DMTAs because of the IMTAs' beliefs about teaching. Through asking questions in the middle of solving the simple problems, the IMTAs encouraged their students to interact in the lessons and apply the concepts and definitions because of the IMTAs' roles and goals. From the expanded field notes, the two IMTAs used simple problems as a complement to improve their students' engagements even though the problems were not in their textbook.

Tom: After his explanations about the relationships to the trigonometric functions of angles using the unit circle and the sine and cosine functions, he asked the students to "find the solutions of  $\cos \theta = \frac{1}{2}$ ." Several students tried to answer the question. Using the graphs of sine and cosine functions and the graph of unit circle, he added explanations to the students' answers in order to teach "how to find all solutions of the equation."

Sam: He asked what quadratic functions were in order to explain the standard form of quadratic functions and to graph quadratic functions using the standard form. He wrote several examples on the board, and then he asked which one was a quadratic function. After several students answered his question, he explained the definition of a quadratic function and the standard form of quadratic functions. He wrote two equations " $f(x) = 2(x - 3)^2 + 5$  and  $f(x) = -2(x - 3)^2 + 5$ ." He asked, "What is the vertex of each standard form?" Several students answered the question. He asked, "Which parabola opens upward?"

Tom and Sam used their own problems in order to help their students confirm their understanding for the concepts and definitions.

In contrast, because of their beliefs about teaching, when the DMTAs taught new concepts and definitions, they focused on explaining the critical reasons why these concepts and definitions are important and valuable for problem solving and real-life problems in order to motivate their students to learn the lessons. After briefly introducing the law of sine, for example, Kelly solved real-life problems of the law of sine to encourage her students to realize how the law of sine was applied and why it was useful in the real world. According to the questionnaires, 80% of the DMTAs often used real-life problems in class, while 30% of the IMTAs used real-life problems.

Instead of spending time explaining new concepts and definitions, the DMTAs attached more weight to problem solving in class to facilitate understanding of concepts and definitions than the IMTAs did because of the DMTAs' beliefs about teaching and their beliefs about students' learning and students. From the questionnaires, the DMTAs spent 70% of their class time solving and providing as many problems as they could for repetition. In addition, the DMTAs' instructional

strategy, solving many mathematical problems, was also influenced by their instructional goals and roles. For example, one of the DMTAs taught strategies to solve problems before and after answering the problems in her expanded field notes.

Kelly: After solving a Law of sine problem, she introduced the strategy how to solve words problems of Law of sine.

Kelly: After introducing definition and concept of Law of cosines (about three minutes), she explained what kinds of Law of cosines problems were and what the best way was to solve the problems respectively. She began to solve the number three.

Kelly: She explained how the number three and four problems were different. In addition, she explained the strategy of the number four problem and then wrote the strategy on the board.

Kelly's students would prepare the exams and learn the lessons through explanations of strategies for solving problems by repetition.

According to the questionnaires, 80% of the DMTAs often provided the opportunities for their students to solve problems in class due to the DMTAs' instructional goals and roles, while 20% of the IMTAs taught thinking strategies. In addition, the DMTAs gave homework assignments and supplied information as much as their students wanted because of their instructional goals.

**Interaction With Students.** Even though the majority of interactions with students for both groups occurred during problem solving and explanations of new concepts and definitions, there were differences in their question forms and responses to students' answers. The IMTAs used many closed-ended questions, which were directly related to concepts, definitions, rules, and formulas because the IMTAs believed that many questions motivate students to learn lessons and confirm understanding. In addition, the IMTAs checked understanding using closed-ended questions during problem solving. For example, the IMTAs asked "what is the definition?", "what are the rules?", "do you understand that?", "are you following me?", and "do you have any questions?" Compared with the DMTAs' waiting time for their students' answers, the IMTAs often tended to provide the answers instead of waiting for their students' responses because the questions directly asked the definitions, rules, and formulas. Thus, the IMTAs' lectures would be teacher-centered pedagogy since they spent more time explaining concepts, definitions, and solving problems through less interaction with their students.

On the other hand, the DMTAs spent more time interacting with their students through problem solving, which was consistent with the DMTAs' beliefs about teaching: emphasis on problem solving by repetition. The DMTAs often used open-ended and follow-up questions in order to improve the interaction with their students, whereas the IMTAs asked closed-ended questions. For example, the DMTAs asked "how would you apply this definition and formula?," "what does the problem want?," "how do you get it?," or "how come?" after listening to their students' answers. According to a DMTAs' expanded field notes, she asked several open-ended and follow-up questions in class.

After she drew two graphs with brief summary of last class in order to explain "one-to-one functions and their inverses", she asked "how are the graphs related?" and then she waited for a long time her students' answers. A student answered that two lines were symmetric then she asked "two graphs are symmetric with respect to what?" again. She drew a line  $y = x$  and then said "Two graphs are symmetric with respect to the line  $y = x$ ." She plotted a point (3,0) on a graph, she asked her students to find the symmetric point on the other graph.

Comparing the wait time of the IMTAs' responses to their students' answers, the DMTAs waited more time for students' answers. If the DMTAs' students could not answer the questions, the

students were prompted by other questions and hints because the DMTAs believed that thinking strategies were important for effective pedagogical practices. For example, Kelly's expanded field notes described how she encouraged her students to answer her questions.

Kelly asked "which angle is first to find out?" The students did not answer then Kelly asked "which angle is short?" The students answered "A" and then Kelly explained why the angle A was first to find.

Kelly made her students keep thinking to find the answers through hints and questions.

Because the high rate of interaction with their students through Socratic questioning during problem solving, the DMTAs' practices tended to be both instructor- and student- centered pedagogy instead of teacher-centered pedagogy.

In addition, the findings revealed that there were differences in methods of summary between the two groups because of the groups' different beliefs about teaching and student learning. The IMTAs spent less time summarizing the lessons and providing strategies for problem solving compared to the DMTAs. If the IMTAs needed, they showed the brief definitions, theories, and rules on the board with simple examples, which were used to explain the definitions and concepts before.

By contrast, the DMTAs often summarized concepts and definitions, procedures of problems, and strategies how to solve problems through interaction with their students by asking questions in the middle of and after problem solving because of their beliefs about teaching: learning by repetition and pattern recognition. For example, Kelly explained how to summarize strategies for problem solving through interaction with her students by asking questions and repetition.

Kelly: She explained the strategies of the number four problem and then wrote the strategies on the board. While she solved the problem by the strategies, she interacted with the students by asking next step. After listening the students' answers, she moved on the next step. When they solved the problem, Kelly summarized how to solve these problems and the strategies again. She asked "do you have any questions?" The students answered they did not have questions and then she moved to next problem and asked what the differences were between the number four and five.

The DMTAs' methods of summary were connected with their beliefs about teaching, and this was represented in their questionnaire responses in which 80% of the DMTAs taught thinking strategies because many questions during summaries afforded their students the opportunity to think how to apply the definitions, rules, and strategies of problems.

### **Conclusion**

Although there are consistencies or inconsistencies between beliefs and pedagogical practices in literature (Cohen, 1990; Speer, 2005; Thompson, 1984), the results of this study support that the MTAs' beliefs and pedagogical practices were consistent (Speer, 2005; Thompson, 1985). From this study, cultural differences in class and different mathematics learning experiences as undergraduate students were critical in building the different beliefs between the IMTAs and DMTAs (Chae, Lim, & Fisher, 2009; McGivney-Burrelle, DeFranco, Vinsonhaler, & Santucci, 2001; Twale, Shannon, & Moore 1997). In addition, the IMTAs' and DMTAs' different beliefs about teaching and students' learning and students significantly affected their different pedagogical practices regarding how to approach new concepts and definitions for students' conceptual understanding and how to interact with students in terms of the MTAs' question forms, responses to students' answers, and methods of summary.

The cultural gaps in mathematics learning and teaching encouraged both groups to construct different beliefs. Understanding concepts and definitions were fundamental to learning mathematics because of the IMTAs' beliefs about teaching and students' learning and students. The IMTAs believed that explicit explanations, which came from the instructors' preparation, knowledge of mathematics, and pedagogy, were the main cause for effective teaching of Mathematics, whereas the DMTAs' beliefs relied on problem solving by repetition. In class, the IMTAs spent more time explaining new concepts, theorems, and definitions than the DMTAs did because of the IMTAs' beliefs about teaching: clear explanations helped students not only improve their problem solving abilities but also intrinsically motivated to study mathematics. Because of experiencing the gap between the IMTAs' expectations for students and the actual level of students' mathematical abilities (Chae, Lim, & Fisher, 2009), the IMTAs strongly believed that recognition of students' mathematics abilities was an important aspect of teaching mathematics. Instead of using many intermediate or real-life problems in class, the IMTAs often used simple problems on their own as a complement in order to help their students understand concepts and definitions. Through the adjusted problems and lessons, the IMTAs encouraged their students to engage in the problem solving and the IMTAs' lectures. Instead of solving a number of intermediate problems in class, the IMTAs provided the problems as homework assignments.

On the other hand, the DMTAs' beliefs about teaching and students' learning and students were that understanding concepts and definitions came from solving problems by repetition. Compared to the IMTAs' pedagogical practices in class, majority of the DMTAs' class time was spent problem solving. Even though the DMTAs did not spend much time explaining new concepts and definitions, they often provided intermediate and real-life problems to show their students realistic mathematical practices and several reasons why mathematical ideas were valuable and how these theorems were applied to real-world problems. In addition, they solved many problems by repetition in class in order to improve students' pattern recognition because pattern recognition was a vital means for learning mathematics. The students could have opportunities to improve critical thinking because the DMTAs provided more time for their students to solve problems in class.

Even though there were three categories of beliefs from this study, the beliefs about teaching strongly influenced the IMTAs' and DMTAs' different practices regarding how to interact with their students. Since the IMTAs believed that clear explanations of concepts and definitions were more important aspects of teaching mathematics than problem solving, they did not pay much attention to interactions with their students nor did they summarize problem solving strategies in class. Even though the IMTAs asked many questions in order to motivate their students to learn lessons, the questions did not improve the IMTAs' interactions with their students because of the IMTAs' closed-ended question forms and few follow-up questions. Therefore, the IMTAs' lectures tended to archetypal teacher-centered pedagogy.

In contrast, the DMTAs actively interacted with their students in class because of their beliefs about teaching, open-ended question forms, and plenty of follow-up questions. In addition, the DMTAs had more interaction with their students than the IMTAs did. For example, the DMTAs guided their students to find correct answers prompted by more questions and hints. Because the DMTAs believed that learning by repetition and learning to critically think were important to learn mathematics, the DMTAs spent time summarizing lessons and strategies how to solve problems more so than the IMTAs did. In addition, the DMTAs often encouraged their students to engage in the summaries of lessons by asking questions. Even though the DMTAs' lectures were based on teacher-centered pedagogy, the DMTAs tried to approach student-centered pedagogy.

### Limitations and Implications

Although these findings answered the research questions, there were some limitations regarding the number of participants and the MTAs' lack of pedagogical knowledge during the interviews. Because of the limited number of IMTAs in the department, it was hard to find a sufficient number of IMTAs under the same nationality. During the interviewing, a few IMTAs were confused about the meanings of the interview questions. Because of a lack of pedagogical knowledge and practices, several MTAs spent more time answering the interview questions. Thus, future research is needed to examine the MTAs' beliefs and practices with the appropriate number of IMTAs under the same nationality versus DMTAs.

Even if there were limitations, I believe that this study contributes to the understanding of the different practices and beliefs between IMTAs and DMTAs and critical resources for the body of knowledge about MTAs. In addition, mathematics departments have the opportunity to have insight into the proper support for MTAs and to design professional development programs for MTAs.

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