Acta Didactica Napocensia

Volume 6, Number 4, 2013

Pre-service Primary School Teachers' Logical Reasoning Skills

Iuliana Marchis

Abstract: Logical reasoning skills are important for a successful mathematical learning and in students' future career. These skills are essential for a primary school teacher, because they need to explain solving methods and solutions to their pupils. In this research we studied pre-service primary school teachers' logical reasoning skills. The results show that only one third of the students gave an argumentation for their answer: a very small percentage gave a complete, correct argumentation for the solution of the problem; one fifth of the students had an incomplete argumentation, missing important steps of the logical reasoning; and one tenth of the students made mistakes in their reasoning. These results highlight the importance of developing future primary school teachers' logical reasoning skills.

Keywords: mathematical problem solving, logical reasoning competency, pre-service primary school teacher

Introduction

The problem solving competency, which is "an individual's capacity to use cognitive processes to confront and resolve real, cross - disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricula areas that might be applicable are not within a single domain of mathematics, science or reading." (OECD, 2003, p. 156), is very important for every individual in their everyday life and workplace. When solving problems, in many cases, we have to explain our solution, thus good reasoning competency is also important. This is even more important for a teacher, thus developing pre-service teachers' logical reasoning competency is essential.

This paper presents a research on pre-service primary school teachers' logical reasoning skills.

1. Theoretical background

Mathematical reasoning competency is essential for a successful mathematical learning (Nunes et al., 2007). Also, reasoning could be important in many workspaces, having a high importance in collaborative work (Fiore et. al, 2010).

Mathematical reasoning competency includes the following skills: understanding others' explanations on their problem solving method; formulating mathematical arguments to justify a mathematical claim or to explain a solution; knowing what a mathematical proof is (Niss, 2011).

Reasoning could be deductive (drawing conclusions using rules from logic), inductive (making generalizations based on some particular cases), and transformational (mentally visualizing actions and results of actions (Simon, 1996)). This third type of reasoning produce creative thinking (Haylock, 1987), as require flexibility (Gray & Tall, 1994).

Pupils' reasoning competence can be influenced by teacher's competence (Brown & Campione, 1994). Pupils' reasoning could be developed by the teacher selecting cognitively challenging mathematical problems (Henningsen & Stein, 1997), designing such questions which may help the children to think and justify their answers (Albert, 2000; Baig & Halai, 2006); and creating a collaborative work environment in which pupils could express their thinking (Albert, 2000). Communication has an

Received 6 December 2013.

important role in developing reasoning skills (Campbell, Adams, & Davis, 2007), and teachers should help pupils to express themselves more precisely building on the answers given by the children (Evens & Houssart, 2004).

Evaluating pupils' reasoning teachers could improve their instruction (Moska & Magone, 2000). If teachers understand how their pupils use to make argumentation, they can build on their methods to develop their reasoning skills (Flores, 2006).

2. Research

Research design

The research was made in November 2013 among Primary and Preschool Pedagogy specialization students at Babes-Bolyai University, Cluj-Napoca.

The research sample were 74 pre-service primary school teachers, from which 72 (97,30 %) women and 2 (2,70%) men. 46 students (62.16 %) are in their second year of studies and 28 students (37.84) in their third year.

The research tool was a problem sheet with five problems, which don't require any mathematical knowledge, only logical thinking and mathematical reasoning skills. This problem sheet was solved by the students during the Mathematics course for preschool and primary school specialization students.

We have selected one logical problem for a qualitative analysis. This problem was given at the final phase of the Zrínyi Ilona Mathematical Competetion in 2005. The question of the problem was formulated as a multiple choice question: pupils have to choose the person who is living in Szeged without giving any argumentation. 41% of 3rd grade pupils and 53% of 4th grade pupils have chosen the correct answer. But these percentages are not showing the percentage of those pupils who have correctly solved the problem. Problems formulated as multiple-choice items not always reflect real results, as the pupil could select the correct answer without solving correctly the problem. Marchis (2013) have given this problem to 3rd and 4th grade pupils from schools in Cluj-Napoca, and 24.59% of 3rd grade and 18.82% of 4th grade pupils have chosen the correct answer. Many pupils who have selected the correct answer gave incorrect argumentation, so they haven't solved the problem correctly.

Problem. (Csordás et al., 2006)**.** At the Zrínyi Ilona Mathematical Competetion five participants living in five different cities (Győr, Kecskemét, Miskolc, Pécs és Szeged) affirm the following:

Anna: Dezső is from Kecskemét. I am from Szeged.
Béla: I am from Szeged. Csilla is from Pécs.
Csilla: I live in Szeged. Dezső is from Miskolc.
Dezső: I am from Kecskemét. Ernő is from Győr.
Ernő: I am really from Győr. Anna is from Miskolc.

Which participant lives in Szeged, if every child had a true and a false affirmation?

All the solutions were qualitatively analyzed.

Results and discussion

64.86% of the students gave a correct answer for this problem, i.e. Anna is from Szeged; 31.08% of the students gave an incorrect answer; and 4.05% of the students didn't solve the problem (see Table 1).

In the following we analyze the argumentations given by students. In this analysis we write as quotes what students have written and without quote the author's comments on their logical reasoning. In many cases we interrupt students' argumentation with the author's comments, because we consider that in this case the comments are more understandable.

Table 1. Students' results for Problem 1

		Number of students/ percentage of students
Correct answer		48 / 64.86 %
	Correct logical reasoning	4 / 5.41 %
	Incomplete logical reasoning	15 / 20.27 %
	Incorrect logical reasoning	7 / 9.46 %
	Without logical reasoning	22 / 29.73 %
Incorrect answer		23 / 31.08 %
No answer		3 / 4.05 %

Analyzing those students' argumentation, who have given a correct answer; we could observe that only 5.41% of them gave a correct, **complete logical reasoning**.

One student had a very good remark: "Csilla can't be from Szeged, because if she is from Szeged, then both affirmations of Béla are false." Then she continues: "So, if Dezső is from Miskolc, then the following affirmations become false: Dezső's affirmation that he is from Kecskemét; Ernő's affirmation that Anna is from Miskolc; and Anna's affirmation that Dezső is from Kecskemét. Then it follows that Ernő is from Győr, Anna is from Szeged and Csilla is from Pécs. So Anna is from Szeged."

20.27% of the students gave a correct, but **incomplete argumentation**. They omit important steps in the logical reasoning.

"Anna is from Szeged. In the first affirmation she states that Dezső is from Kecskemét; Dezső states that he is from Kecskemét and Ernő is from Győr; Ernő says that he is from Győr and Anna is from Miskolc. Because only one affirmation of each child is true, it follows that Dezső's correct affirmation is that Ernő is from Győr." – This conclusion needs to be explained. "Then Dezső is not from Kecskemét, thus Anna's second affirmation is true, so she is from Szeged. Ernő's second affirmation, that Anna is from Miskolc, is false; and his first affirmation, that he is from Győr, is true."

9.46% of the students gave an **incorrect argumentation which leads to the correct answer**. We give three examples:

"If we suppose that Ernő is from Győr, then Anna is not from Miskolc and Dezső is not from Kecskemét. Then we arrive to the conclusion, that Dezső is from Miskolc." – This conclusion doesn't follow from the argumentation. We only know, that Dezső is not from Kecskemét, but he still can be from Szeged, Miskolc, and Pécs at this point of our argumentation, we have just excluded Kecskemét and Győr (as *Ernő is from Győr*).

"Ernő is from Győr, Anna is not from Miskolc, thus Ernő's statement is false." – The student should have said that she assumed that Ernő is from Győr. She said that Ernő's statement is false But Ernő has two statement, so which one is false? Maybe she reffered the second one, but it was useless to say that. "Thus Csilla's statement is false, so she is from Pécs." – Again, which statement is false, Csilla has two statements. From what we know so far, *Ernő is from Győr, Anna is not from Miskolc*, we can't conclude anything about Csilla. We should first conclude that Dezső is not from Kecskemét, as *Ernő is from Győr* is true; then *Anna is from Szeged,* as her first affirmation is false; and only after this we get that *Csilla is from Pécs,* as Béla's first affirmation is false (because we already know, that Anna is from Szeged). "Then Anna is from Szeged." – From the fact that *Csilla is from Pécs* doesn't follow, that Anna is from Szeged.

"We assume that Ernő's first affirmation is true. Then Dezső's second affirmation is also true, thus Csilla's second affirmation has to be true." - Ernő's first affirmation and Dezső's second affirmation is the same: *Ernő is from Győr*. Why from this affirmation follows that Dezső is from Miskolc? We could conclude that *Dezső is not from Kecskemét*, but he still can be from Szeged, Miskolc and Pécs.

31.08% of the students gave an **incorrect answer**. Some of them have written down also their argumentation. We give five such an argumentation, commenting on them.

The following argumentation lead to an incorrect answer. The student observed that in case of three children there is an affirmation related with Szeged, and she assumed that the first, the second, respectively the third affirmation is true:

- "If we assume that Anna is from Szeged, then based on Bélá's second affirmation Csilla is from Pécs. Thus Csilla's first affirmation is false, thus Dezső is from Miskolc. It follows that Dezső's first affirmation is false, thus Ernő is from Győr. Then Ernő's first affirmation is true, thus Anna is from Miskolc." – This student made a very good argumentation until the last sentence. If Ernő's first affirmation is true, then his second affirmation must be false, so *Anna is not from Miskolc*.
- "If Béla is from Szeged, then Csilla is not from Pécs, thus Csilla can't be from Szeged, but Dezső can live in Miskolc. Based on Dezső's second affirmation, Ernő is from Győr. Then Anna is not from Miskolc, thus Béla is not from Szeged." – Again, the student made a good reasoning up to a point. From *Anna is not from Miskolc* doesn't follow directly that *Béla is not from Szeged.* But if Béla is from Szeged, then Anna can't be from Szeged, thus Dezső is from Kecskemét, which is in contradiction with the conclusion that Dezső is from Miskolc. From this we can conclude that our assumption, that Béla is from Szeged, is false.
- "Then Csilla is from Szeged. Then Dezső is not from Miskolc, thus his first affirmation has to be true, so Ernő is from Győr. Ernő's first affirmation is true. Thus Csilla is from Szeged." – From the fact that *Dezső is not from Miskolc* doesn't follow that he is from Kecskemét.

We present another two argumentations which leads to the solution that *Csilla is from Szeged.*

"If Dezső lives in Kecskemét, then he lies regarding Ernő, so Ernő is not from Győr and Anna is from Miskolc. Then dezső is from Kecskemét, thus Csilla is from Szeged." – Until now the argumentation is correct, only redundant in some point. But if Csilla is from Szeged, then Béla's second affirmation is false, thus his first affirmation has to be true, so Béla is also from Szeged, which is a contradiction, as there are not two children from the same city.

„If we assume that Anna's and Dezső's first affirmaion is true, then Dezső is from Kecskemét. Thus Ernő is not from Győr, as Dezső's second affirmation is false. Thus Ernő's second affirmation is false, so Anna is not from Miskolc. Based on this Csilla's first affirmation is treu, so she is from Szeged." – Until this point the argumentation is correct. But the student didn't take in consideration Béla's affirmations. If Csilla is from Szeged, then Béla's second affirmation is false, thus his first affirmation has to be true, so Béla is also from Szeged, which is a contradiction.

In the following two argumentations students try to interpret the text of the problem differently:

"Csilla lives in Szeged, because she doesn't say only that *she is from Szeged,* like Béla, she states that *she lives in Szeged.*" – This student makes a difference between *being from a place* and *living in a place*. And these two things could be different for some people. But she didn't consider the case that Csilla's first affirmation could be false.

"Csilla lives in Szeged, because Béla's name is Béla Szegedi." – The reader should know that in Hungarian *Szegedi* means *from Szeged.* This argumentation suppose that Béla's first affirmation (thinking that this affirmation is related with Béla's name, and not with his origin) and Csilla's first affirmations are both true.

The following argumentation leads to the conclusion that *Béla is from Szeged*.

"If Béla is from Szeged, then Csilla is not from Pécs and Anna is not from Szeged. Thus Dezső is from Kecskemét, then Ernő is not from Győr, so Anna is from Miskolc." – But in this case Csilla then Béla's second affirmation is false, thus his first affirmation has to be true, so Béla is also from Szeged, which is a contradiction s both affirmations are false, as Csilla is not from Szeged (because Béla is from Szeged) and Dezső is not from Miskolc (as Anna is from Miskolc).

Students tried different visualizations for supporting their reasoning. The most frequently used is similar ones with the one presented in Figure 2. Here the student write down all the affirmations and cut out the false ones.

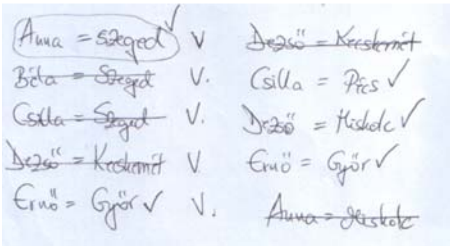

Figure 2. Visualization using text

Only two students used for visualization tables (Figure 3 and 4).

In Figure 3 each row of the table contains the affirmations of the child written in the first column of the row. The column headers are the cities. I.e. in Anna's row is a *Dezső* and *Anna* in column *Kecskemét* and *Szeged*, as her two affirmations are that She is from Szeged and Dezső is from Kecskemét.

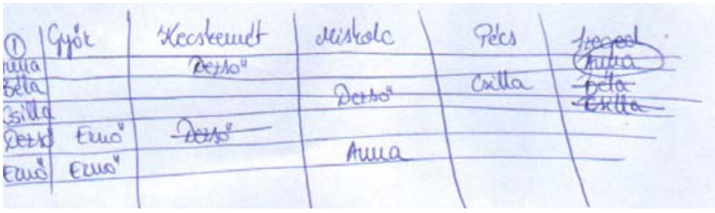

Figure 3. Visualization using table

In Figure 4 we see two tables. In the first table each row contains the affirmations of the child written in the first column of the row. The column headers are the children names. I.e. in Anna's row is an *Sz* and a *K* at row *Anna* respectively *Dezső*, as her two affirmations are that She is from Szeged and Dezső is from Kecskemét. In Figure 4 there is also a second table, which has a similar logic as the visualization from Figure 2: it records which affirmation is true and which is false in case of each pupil.

Figure 4. Visualization using two tables

Conclusions

The results show that only one third of the students gave an argumentation for their answer: a very small percentage gave a complete, correct argumentation for the solution of the problem; one fifth of the students had an incomplete argumentation, missing important steps of the logical reasoning; and one tenth of the students made mistakes in their reasoning.

Very few students tried different visualizations of their logical reasoning. Visualization is also very important in primary school, so teachers have to get used with a variety of visualizations.

These results highlight the importance of developing future primary school teachers' logical reasoning skills.

References

[1] Albert, L. R. (2000). Outside-in inside-out: Seventh grade students' mathematical thought processes. *Educational Studies in Mathematics,* 41(2), 109-141.

[2] Baig, S. & Halai, A. (2006). Learning Mathematical Rules with Reasoning, *Eurasia Journal of Mathematics, Science and Technology Education*, 2(2), 15-39

[3] Brown, A. L., & Campione, J. C., (1994). Guided discovery in a community of learners. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 229-270). Cambridge, MA: Bradford.

[4] Campbell, A. E. Adams, V. M, & Davis, G. E. (2007). Cognitive demands and second language learners: A framework for analyzing mathematical instructional contexts. *Mathematical Thinking and Learning,* 9(1), 3-30.

[5] Csordás M.; Csordásné Szécsi J.; Dobozi D.; Kovácsné Szipán A.; Nagy T. & Szabó I. (2006). Zrínyi 2005. A 2005. évi Zrínyi Ilona Matematikaverseny feladatai, megoldásai és eredményei. MATEGYE Alapítvány.

[6] Evens, H. & Houssart, J. (2004). Categorizing pupils' written answers to a mathematics test question: 'I know but I can't explain'. *Educational Research,* 46(3), 272-282.

[7] Fiore, S., Rosen, M., Smith-Jentsch, K., Salas, E., Letsky, M. & Warner, N. (2010). Toward an understanding of macrocognition in teams: Predicting process in complex collaborative contexts. *The Journal of the Human Factors and Ergonomics Society*, 53, 203-224.

[8] Flores, A. (2006). How do students know what they learn in middle school mathematics is true? *School Science and Mathematics,* 106(3), 124-132.

[9] Gray, E. & Tall, D. (1994). Duality, ambiguity and flexibility: A proceptual view of simple arithmetic, *Journal for Research in Mathematics Education*, 26, 115-141.

[10] Haylock, D.W. (1987). A framework for assessing mathematical creativity in schoolchildren, *Educational Studies in Mathematics*, 30, 197-210.

[11] Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28(5), 524-549.

[12] Marchis, I. (2013). Primary School Pupils' Problem Solving Competency and Reasoning Skills. *PedActa,* 3(1), 25-32.

[13] Moskal, B. M. & Magone, M. E. (2000). Making sense of what students know: Examining the referents, relationships and modes students displayed in response to a decimal task. *Educational Studies in Mathematics,* 43, 313-335.

[14] Nunes, T.; Bryant, P.; Evans, D.; Bell, B.; Gardner, S.; Gardner, A. & Carraher, J. (2007). The contribution of logical reasoning to the learning of mathematics in primary school, *British Journal of Developmental Psychology*, 25(1),147–166.

[15] Niss, M. (2011). Competencies in mathematics education – potentials and challenges. What's the point? What's new? What do we gain? What are the pitfalls? *XIII. Conferencia Interamericana de Education Matematica*, 26-30 June 2011, Recife (Brasil).

[16] OECD (2003). *The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills*, PISA, OECD Publishing.

[17] Simon, M. A. (1996). Beyond inductive and deductive reasoning: The search for a sense of knowing. *Educational Studies in Mathematics*, 30(2), 197-210.

Author

Iuliana Marchis, Babes-Bolyai University, Cluj-Napoca (Romania). E-mail: iuliana.marchis@ubbcluj.ro