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FUTURE PRIMARY AND PRESCHOOL PEDAGOGY SPECIALIZATION STUDENTS' MATHEMATICAL PROBLEM SOLVING COMPETENCY

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Abstract: Developing the problem solving competency is one of the main goals of school education, as it is a very important competency in someone's everyday life and career as well. Mathematics is highly appropriate for developing this competence. This research studies future Primary and Preschool Pedagogy specialization students' mathematical problem solving and reasoning competency. These competencies are very important for a primary school teacher. The results show that most of the students easily can solve routine problems, but only half of them has adequate problem solving competency (i.e. they can successfully solve non-routine problems), and a quarter of them has good reasoning competency.

Keywords: Mathematical problem solving, Mathematical reasoning competency, primary school teacher

Introduction

One of the main goals of school education is developing pupils' problem solving competency. This competency is "an individual's capacity to use cognitive processes to confront and resolve real, cross - disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricula areas that might be applicable are not within a single domain of mathematics, science or reading." (OECD, 2003, p. 156) Mathematics is one of the subjects highly appropriate for developing this competency.

This research studies future Primary and Preschool Pedagogy specialization students' Mathematical problem solving and reasoning competency. These two competencies are very important for a future primary school teacher.

1. Theoretical background

Mathematics learning is based on problem solving. Mathematical tasks can be categorized in routine and non-routine tasks. Routine tasks are usually called exercises, as solving them is only required using already know methods. Non-routine tasks are called problems or, to be more specific, non-routine problems. According to TIMSS 2011, "non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned." (Mullis et al, 2009, p. 45). So "a problem differs from an exercise in that the problem solver does not have an algorithm that, when applied, will certainly lead to a solution." (Katowski, 1977, p. 163) Sometimes is difficult to decide if a Mathematical task is an exercise or a problem, because this depends on each student individually: the same task could be a problem for a student and an exercise for another student (Zhu & Fan, 2006) and it is not any more considered a problem if he/she solved it (Selden et al, 1999).

Mathematical problem solving needs application of multiple skills (De Corte, Verschaffel, & Op't Eynde, 2000). A successful problem solver needs to deeply understand Mathematical notions; to

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master problem solving strategies and techniques; to have good decision making abilities and to have a positive attitude towards Mathematics (Schoenfeld, 1985). We could observe that problem solving skills include domain specific knowledge, cognitive processes and motivational factors. Lester (1994) also includes the social-cultural context. Carlson and Bloom (2005) developed a taxonomy to characterize the attributes of problem solving, which has the following dimensions: resources, control, methods, heuristics, and affect. These dimensions are quite similar with those identified in previous research, for example of Schoenfeld (1985). Mathematical reasoning competency is also important for a successful mathematical learning (Nunes et al., 2007). Mathematical reasoning competency includes the following skills: understanding others' explanations on their problem solving method; devising mathematical arguments to justify a mathematical claim or to explain a solution; knowing what a mathematical proof is (Niss, 2011). Reasoning competency has a high importance in collaborative work, as one of major factor that contributes to success is effective communication among team members (Fiore et. al, 2010). PISA 2015 plan to include collaborative problem solving skill assessment as it "is a critical and necessary skill across educational settings and in the workforce" (OECD, 2013b, p. 9), thus it is important to teach and assess this skill (Griffin et. Al, 2011; Rosen & Rimon, 2012). According to PISA 2015 draft framework, "collaborative problem solving competency is the capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and efforts to reach that solution." (OECD, 2013b, p. 10)

As regarding cognitive processes, PISA 2012 framework identified the following four process: exploring and understanding; representing and formulating; planning and executing; monitoring and reflecting (OECS, 2013a).

2. Research

Research design

The research was made in July 2013, at the admission examination for Primary and Preschool Pedagogy specialization at Babes-Bolyai University, Cluj-Napoca.

The research sample were 54 candidates, from which 52 (96,30%) women and 2 (3,70%) men. 42 (77.78%) of the candidates have recently finished high-school.

The research tool was a problem sheet with two problems. The first problem requires only simple calculations, so it could be considered a routine problem:

> **Problem 1.** In a chocolate factory for one lot of chocolate they use 1251 kg of cacao, 918 kg milk powder and 512 kg of sugar. The management want to change the recipe in the following way: increase the amount of cacao with 125 kg and decrease the amount of sugar with 53 kg. How many kg of chocolate is made based on the old respectively new recipe?

The second problem doesn't require any mathematical knowledge, only logical thinking and mathematical reasoning skills:

> **Problem 2.** Peter has four balls, which have different colours (blue, yellow, red and green) and sizes. He ordered increasingly the balls according to their size. He told Ann the following hints:
>
> - The red ball is near the blue ball.
> - The blue ball is smaller than the red ball.
> - The green ball is in the one end of the queue.
> - One of the neighbors of the yellow ball is the blue ball.
>
> Ann guessed the order of the balls. What is the order of the balls and how did she guess it?

We have composed these problems in order to measure the cognitive processes essential for problem solving. Thus these problems don't need application of mathematical knowledge or methods, they require only some basic calculation skills, as it is recommended by OECD (2013a).

All the solutions were qualitatively analyzed.

Results and discussion

Problem 1. The results are summarized in Table 1. This problem was simple for the students, almost all of them (52 students; 96,30%) could solve it, 43 students (79,63%) obtained a correct solution, 9 students (16,67%) had some minor miscalculations. One student has an incomplete solution: she has calculated the amount of cacao and sugar in the new recipe, but she didn’t calculate the obtained amount of chocolate according to the old and new recipe. One student has calculated the amount of chocolate according to the old and new recipe, but than she divided by three the obtained results.

Table 1. Results of Problem 1.

	Number of students
Correct solution and results.	43
Correct solution with miscalculations.	9
Incomplete solution.	2

Almost all of the students first calculated the amount of cacao and sugar according the new recipe, then the amount of chocolate according this recipe. Only one student had a different approach: as in the new recipe the amount of the cacao was increasing with 125 kg and the amount of sugar was decreasing with 53 kg, she calculated the difference of these values: 125 – 53 = 72, thus according to the new recipe the obtained amount of chocolate is bigger with 72 kg than based on the old recipe.

9 (16.67%) students made miscalculations, which is a quite high percentage as they have to make only addition and subtraction in the concern 0-1000. This could be explained by the fact that usually they use a calculator instead or calculating in mind or on paper.

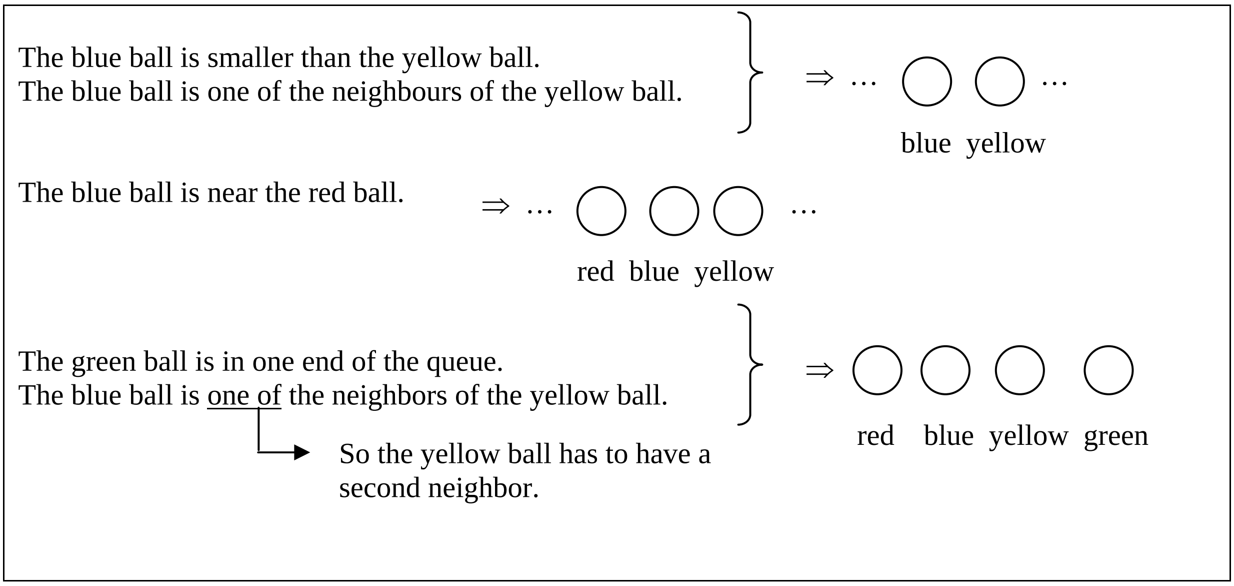
Problem 2. This problem was more difficult for students. Only 14 (25,93%) students have solved it correctly with right argumentation, 3 (5,56%) students gave a correct answer without any argumentation (Table 2). Only 2 (3,70%) students left this problem without a solution, the rest gave some solution (even if that solution was partially or totally wrong).

Table 2. Results of Problem 2.

	Number of students
Correct answer and argumentation	14
Correct answer, but without any argumentation	3
Correct answer, but without argumentation on the position of the green ball	9
Correct position of the red – blue – yellow balls, but incorrect position of the green ball	12
Decreasing order instead of increasing order	8
Without any solution	2

In the following we present two correct argumentations. The following one was the most frequently given (not necessary word by word): “We put the blue ball next to de red ball, then the yellow ball next to the blue ball, as the blue ball is smaller than the yellow and the blue ball is one of the neighbors of the yellow ball. The last ball in the queue is the green ball as it is in one end of the queue and the yellow ball has to have another neighbor (as the blue ball is one of the neighbors of the yellow ball).”

One of the students made the argumentation clearly understandable organizing the data and using drawings:



It is remarkable that this student started her argumentation by positioning the blue and the yellow ball, because the position of these balls is clearly defined. If we start the argumentation by positioning the red and the blue ball we have two cases: the red is smaller than the blue or the blue is smaller than the red. Only after positioning the yellow ball we can eliminate the case when the blue is smaller than the red.

Some argumentations look to be correct, but they are missing some explanations.

On of the students starts her argumentation by “the red ball is the smallest”. This statement is true, but it follows from all the statements of the problem, so can’t be used as an initial statement on which we build all our argumentation. Also this student states that “the green ball is the biggest, as the blue ball is one of the neighbors of the yellow ball and the green ball is in one end of the queue”. This is also a correct statement, but he doesn’t highlight the fact that from “the blue ball is one of the neighbors of the yellow ball” follows that the green ball has to be near the yellow ball.

Another student gave the following argumentation for positioning the red, blue and yellow balls: “The blue ball is smaller than the yellow ball, thus next to the blue ball is the yellow ball. The red ball is near the blue ball, but the yellow ball is in the right side of the blue ball, so the red ball has to be in the left side of the blue ball.” This statement is correct, but in the argumentation is missing the reference to the fact that the blue ball is one of the neighbors of the yellow ball. Without this fact we can’t conclude that to the blue ball is the yellow ball.

There were two frequent mistakes: wrong order of the balls and incorrect position of the green ball.

8 (14,81%) students ordered the balls decreasing instead of increasing. For example: “The order is the following: yellow – blue – red – green. This is because the red ball and the yellow ball are neighbors of the blue ball; the yellow ball is bigger than the blue ball, so it is in the beginning of the queue. The green ball is in the end of the queue, because we know that it is in one end, thus it has to be the last one as the yellow ball is the biggest.”

For 12 (22,22%) students determining the position of the green ball was a problem. One student gave the following argumentation: “Because the green ball is in the one end of the queue, we could think, that it is the first ball.” Other students weren’t attentive on the fact, that the blue ball is *one of the neighbors* of the yellow ball, so the yellow ball has to have another neighbor.

Conclusions

Students could easily solve routine problems, but almost one fifth of them make miscalculations in additions or subtractions. As one of the main goals of primary school mathematics education is to develop pupils' calculation skills, teachers have to master mental and writing calculations.
As regarding non-routine problems, only half of the students gave a correct answer, but only quarter of the students could give a correct argumentation for their answer. The results show the necessity of increasing the number of non-routine problems in textbooks, as for future primary school teachers is important to have a good problem solving competency. Also it is important to develop pre-service teachers' mathematical reasoning competency, as this is important when they explain mathematics for their pupils.

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