Editor's Perspective Article: Problem Solving Abilities and Perceptions in Alternative Certification Mathematics Teachers

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Abstract

It is important for teacher educators to understand new alternative certification middle and high school teachers' mathematical problem solving abilities and perceptions. Teachers in an alternative certification program in New York were enrolled in a proof-based algebra course. At the beginning and end of a semester participants were given a problem solving examination and were required to reflect upon their students' and their own problem solving abilities. There was a significant improvement in problem solving abilities for the teachers over the course of the semester, and there was a direct correlation between standardized content knowledge test scores and problem solving examination scores. Furthermore, teachers perceived that although their students had improved somewhat in their problem solving over the course of the semester, student problem solving abilities were generally weak due to not understanding how to start a problem, lack of persistence, and poor literacy skills. Teachers perceived that they shared similar problems as their students, but the algebra course and their teaching of problem solving helped them with their own problem solving abilities.

Keywords: *alternative certification; mathematics; problem solving*

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The goal of this article is to help teacher educators understand mathematics teachers' problem solving abilities and perceptions and how this relates to student problem solving from one particular cohort of alternative certification mathematics teachers. Secondly, the goal is to advocate for strengthening teacher problem solving in teacher preparation. Understanding teacher problem solving is critical in supporting them to teach their students from a problem solving perspective. The National Council of Supervisors of Mathematics (NCSM) considered problem solving to be the principal reason for studying mathematics (NCSM, 1978), and it is recommended mathematics content be taught from a problem solving perspective (National Council of Teachers of Mathematics [NCTM], 2000; Schoenfeld, 1985). NCTM said, "Problem solving is not only a goal of learning mathematics but also a major means of doing so" (2000, p. 52).

The key aspect of authentic problem solving is the problem must be unfamiliar to the person solving it. If the individual has solved the problem before, or watched a teacher solve the problem previously, then the process is not authentic problem solving. Posamentier and Krulik (2008) said, "A problem is a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known" (p. 1). According to Krulik and Rudnick (1989), problem solving is a process in which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. In *How to Solve It*, Polya (1945) outlined a general problem solving strategy that consisted of (a) Understanding the problem; (b) Making a plan; (c) Carrying out the plan; and (d) Looking back. Polya's steps for problem solving can help guide students in solving unfamiliar mathematical tasks and is analogous to a scientific method for mathematical problem solving.

Problem solving is the manifestation of constructivist learning in mathematics, which is the idea that students learn best through constructing their own knowledge, as promoted by prominent thinkers such as Jean Piaget and John Dewey. In the process of problem solving, students are required to construct their own solutions to unfamiliar situations, which means they are engaging in the process of constructing their own knowledge. Authentic problem solving in mathematics is the basis of reform- and inquiry-based instruction in mathematics (Clark, 1997), and problem solving can be viewed as critical thinking for the mathematics classroom.

There has been a lack of publications in the literature on problem solving from teachers' perspectives (Chapman, 1997; Thompson, 1985; Xenofontos, 2007). However, it has been shown that teacher beliefs about student ability greatly influence instructional practices (Nathan & Koedinger, 2000), which may be true for teacher beliefs about student problem solving ability. Asman and Markovits (2009) found teachers who were unable to solve difficult non-routine problems were less likely to include these types of problems on student assessments, even if they were willing to address such problems in their instruction.

Alternative Certification: New York City Teaching Fellows

The New York City Teaching Fellows (NYCTF) program is an alternative certification program that began in 2000 and was created by The New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007). The objective was to quickly bring career-changers from other professions into education to fill the

large teacher shortages in New York City's public schools. It had been predicted that the city would have a shortage of about 7000 teachers in fall 2000 and a possible shortage of 25,000 teachers over the next several years (Stein, 2002).

The NYCTF program grew quickly since its start in 2000. Boyd et al. (2007) said, "Fellows grew from about 1 percent of newly hired teachers in 2000 to 33 percent of all new teachers in 2005" (p. 10). Kane, Rockoff, and Staiger (2006) said the NYCTF program was the largest alternative certification program in New York. NYCTF teachers represent 11 percent of all New York public school teachers, and 24 percent of all mathematics teachers in the city (NYCTF, 2012).

NYCTF teachers begin their graduate courses in education in June while they are engaged in fieldwork in the schools. In September they become the teachers of record in their classrooms while continuing their studies at their partnering university over the next several years. Once they successfully complete their graduate program and certification examinations, NYCTF teachers can apply for initial certification.

Four Questions on Problem Solving and Alternative Certification Teachers

The following four questions guided the investigation of teacher problem solving abilities, content knowledge, and perceptions of student and their own problem solving abilities.

1. What differences were there in problem solving abilities between the beginning and end of the semester in a mathematics content course for NYCTF teachers?
2. Was there a relationship between teachers' content knowledge and problem solving abilities?
3. What were teacher perceptions of their students' problem solving abilities? Further, what differences in perceptions of student problem solving abilities existed between the beginning and end of the semester in a mathematics content course for NYCTF teachers?
4. What were teacher perceptions of their own problem solving abilities? Further, what differences in perceptions of their own problem solving abilities existed between the beginning and end of the semester in a mathematics content course for NYCTF teachers?

Algebra Class for Middle and High School NYCTF Teachers

A sample of 34 new teachers in the NYCTF program was selected from two sections of an algebra content mathematics course for middle and high school teachers that involved rigorous derivations and proofs. Teachers were given a problem solving examination at the beginning and end of the semester that consisted of five different problem situations, as adapted from the literature (Krulik & Rudnick, 1989; NCTM, 2000; Posamentier & Krulik, 2008; Posamentier, Smith, & Stepelman, 2008), which the students had not previously encountered. Each item was worth two points and possible scores ranged from zero to 10 points. Items differed on both the pre- and post- tests.

Teachers' scores on the New York State Content Specialty Test (CST) were recorded as a standardized measure of mathematical content knowledge. The CST assesses mathematical content knowledge for New York teacher certification and the scores range from 100 to 300 with

a minimum passing score of 220. It consists of multiple choice items and has six sub-areas including a written response: Mathematical Reasoning and Communication; Algebra; Trigonometry and Calculus; Measurement and Geometry; Data Analysis, Probability, Statistics and Discrete Mathematics; and Algebra Constructed Response.

Finally, teachers were required to reflect upon both their students' and their own problem solving at the beginning and end of the semester using four teacher journal entries over the course of the semester. Teachers were asked what problem solving challenges were faced by their students and themselves over the course of the semester, and what they could do to improve their students' and their own problem solving abilities.

Findings

Findings from Examinations

The first question was answered using scores from the pre- and post- problem solving examinations, and data were analyzed using paired samples *t*-test (see Table 1). The results of the paired samples *t*-test revealed a statistically significant difference between pre- and post- test scores for the problem solving examinations, and there was a very large effect size.

Table 1

Paired Samples t-Test Results for Problem Solving Ability

Problem Solving Examination	Mean	SD	*t*-value	*d*-value
Pretest	4.91	1.654	-8.679**	2.08
Posttest	8.35	1.649		

Note. $N = 34$; $df = 33$, two-tailed

** $p < 0.01$

The second question was answered using scores from the CST and posttest problem solving examination, and data were analyzed using a Pearson correlation (see Table 2). The results of the Pearson correlation revealed a statistically significant correlation between CST and problem solving examination scores.

Table 2

Pearson Correlations between CST and Problem Solving Scores

Assessment	Mean	SD	*r*-value
Content Specialty Test (CST)	257.12	19.445	0.431*
Problem Solving Examination	8.35	1.649	

Note. $N = 34$

* $p < 0.05$

Reflective Journals on Student Problem Solving: Beginning of Semester

Teacher reflections from their journals were analyzed to determine their perceptions of student problem solving, as well as their perceptions of their own problem solving, in order to answer the third and fourth questions. Furthermore, their reflections were analyzed to determine differences in perceptions of their students' problem solving abilities, as well as their own, both at the beginning and end of the semester.

At the beginning of the semester teachers categorized their students as having weak problem solving abilities. The most commonly reported problems were knowing how to get started and persistence. If a problem required too much time many students would not persist on finding a solution, which some teachers attributed to previous teachers giving students the solutions too early in the process. Teachers categorized their students as confused and not motivated, and teachers reported that their students had trouble understanding the problems, which was primarily due to weak literacy skills, with a few attributing this to students speaking languages other than English as their primary language. Teachers indicated that students lacked the critical thinking skills necessary to solve problems contextualized in unfamiliar situations. Teachers also found their students lacked the confidence needed to perform well on problem solving tasks. The teachers said that selecting the appropriate strategy was often difficult for their students, and many teachers found their students resistant to the challenge of an unfamiliar situation and constantly looked to the teacher for guidance. Several teachers discussed their students' lack of organization while problem solving, and a few teachers found their students lacked basic mathematics skills necessary to engage in problem solving. Several thought mathematics anxiety hindered their students' problem solving abilities.

Interestingly, many teachers indicated there were vast differences between students of varying mathematics ability. While most teachers indicated weak problem solving abilities among their students, several cited the strong skills held by the top students in their classes, as consistent with the findings by Ford (1994).

Reflective Journals on Student Problem Solving: End of Semester

At the end of the semester teachers found many of the problems they encountered with their students in the beginning of the semester still persisted. One teacher said he saw no difference in his students over the course of the semester. However, one teacher felt that by scaffolding his students, he was able to help them vastly improve their problem solving abilities.

Teachers felt there were several things they could do to help improve their students' problem solving abilities. Most commonly mentioned were the steps to problem solving as outlined by Polya (1945). Interestingly, no teacher explicitly mentioned Polya by name, but rather alluded to some variation of his steps to problem solving. Teachers also did not explicitly mention the Socratic Method, but alluded to this technique through using questioning as a type of scaffolding. Teachers commonly said scaffolding and differentiated instruction could be used to help improve problem solving in their students, and using group work and manipulatives were often cited as useful. Many teachers expressed frequently modeling good problem solving for their students. One teacher suggested using problem solving journals/logs to help students focus and reflect upon their thinking. Another teacher expressed his despair at not being able to give students more individualized attention, and said that working with one student individually produced great improvement in the student's problem solving abilities.

Reflective Journals on Teacher Problem Solving: Beginning of Semester

At the beginning of the semester teachers reported they shared many of the issues that their students had, such as difficulty with knowing how to start, persistence, understanding what the problem is asking, and what the problem requires. Many teachers implied they experienced the Einstellung effect, which is the tendency to approach a problem in the same manner even if it is not the best or even correct method to use. No teacher used the word "Einstellung," but rather described the thinking process that is inherent in the Einstellung effect. Several teachers indicated they lacked advanced mathematical knowledge to be good problem solvers.

Reflective Journals on Teacher Problem Solving: End of Semester

At the end of the semester, most teachers said having the algebra content class that focused on derivations and proofs had improved their problem solving abilities greatly. Several used the phase, "I have come a long way," referring to their problem solving abilities. Many said that it was the analytic nature of derivations and developing proofs that helped improve their problem solving abilities. Additionally, many found understanding how mathematics "works" furthered their analytic skills. Many teachers stated that teaching their students how to problem solve in their own classes had helped their problem solving abilities as well, which recalls Aristotle's notion that teaching is the highest form of understanding. However, even though many teachers said they had improved in their problem solving abilities, some indicated they could further improve.

Teachers stated they could best problem solve by drawing pictures to represent the problem, using the guess and test method, and searching for patterns. Teachers also said they could best improve their problem solving abilities through practice. Many teachers used the cliché, "thinking outside the box," to stress the necessary framework for effective problem solving. Many referred to improving problem solving in both their students and themselves by adding tools to their "toolbox." This is interpreted as adding basic mathematics skills to their skill sets.

Teachers emphasized the importance of problem solving in developing student critical thinking, and many stressed that problem solving is important in real world applications. Consistent with the literature, several teachers indicated mathematics should be taught with a problem solving approach (NCTM, 2000; Schoenfeld, 1985), and that problem solving is the goal of mathematics instruction (NCSM, 1978).

Conclusion

It was found that there was a significant improvement in problem solving abilities for the teachers over the course of the semester, and there was a direct correlation between the CST and problem solving examination scores. Further, teachers perceived that although their students had improved somewhat in their problem solving abilities over the course of the semester, their abilities were generally weak due to not understanding how to start a problem, lack of persistence, and poor literacy skills. Teachers perceived that they had shared similar problems as

their students, but the proof-based algebra course and their teaching of problem solving had helped their own problem solving abilities.

Since there was an increase in teachers' problem solving scores over the course of the semester it can be argued that a strong mathematics requirement for alternative certification mathematics teachers, combined with their own teaching experiences, can lead to stronger problem solving abilities, which is important given the emphasis of teaching mathematics from a problem solving perspective (Clark, 1997; NCSM, 1978; NCTM, 2000; Posamentier et al., 2008). It would be interesting to examine how much of this improvement has to do with content classes for teachers or their teaching experience, which should be examined for alternative certification teachers.

It came as no surprise that there was a relationship between the content scores from the CST and the problem solving examination scores. This indicates that general content ability is related to problem solving ability. If good mathematics teaching is reliant on strong problem solving ability, it is important to consider teacher content knowledge as well.

Teachers perceived students did not persevere in their problem solving because they were reliant on the teacher giving them the solutions in previous years. While reliance on teachers providing solutions may be partially due to negative attitudes toward problem solving held by the students (Arslan & Altun, 2007), it also could be an issue with teachers not providing enough time for students to engage in problem solving. This finding supports the need to give students more time for problem solving, and to resist the temptation to simply "give" the solutions to the students.

Teachers reported their students had trouble understanding the problems, which was primarily due to weak literacy skills among the students. This provides evidence that it is increasingly important to improve student literacy. Given the importance of literacy in problem solving and other subject areas, it is recommended literacy be incorporated into the teaching of mathematics, as well as all other subjects. Students who have English as a second language need additional support. One teacher in this study suggested the following for improving student literacy, "I incorporate reading with math. I have them draw comic book scenarios of the problem, and this helps them break the problem down."

Many teachers said in order to improve their students' problem solving abilities, they often model good problem solving techniques for their students. While this may be helpful, teachers should be cautioned that too much modeling may not be good for their students. If teachers model various types of problem situations for their students, and demonstrate their own thought processes and steps taken to find solutions, overall student problem solving is likely to improve. However, if teachers model one type of problem, and then have their students work on similar problems, the teachers have created a situation in which authentic problem solving is not taking place. It is recommended teacher educators continue to stress the importance of good problem solving modeling, but that this should be done with various types of problems so that students gain understanding on how to satisfy problems with unfamiliar situations.

One teacher said he had time to work with one student individually and found great improvement in the student's problem solving abilities as a result. Individual student attention is important to improving student learning (Foote, 2009; Himley & Carini, 2000). In recent years state and local governments have been reducing school budgets, which can lead to high student to teacher ratios. It is recommended that funding be allocated in the schools to ensure students receive the proper individualized attention in their mathematics classes, which could improve their problem solving abilities and improve their mathematics achievement in general. Future work should examine the impact of more individualized attention on problem solving.

Strong problem solving abilities are essential not just in mathematics, but in other subject areas and life in general. Teachers should teach mathematics through a problem solving approach (NCTM, 2000; Schoenfeld, 1985), and it is important that teacher educators be aware of their teachers' problem solving abilities and perceptions for the sake of the students they teach.

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