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Sustaining Social and Sociomathematical Norms with Prospective Elementary Teachers in a Mathematics Content Course

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Abstract

As students contribute to social and sociomathematical norms, they re-organize their own understandings. As such, the purpose of this article is to describe the ways in which prospective elementary teachers contributed to the norms that were established and *re*-established throughout an entire semester in an undergraduate mathematics content course. Findings illustrate that norms must be renegotiated between an instructor and prospective teachers when the mathematics content changes.

Keywords: Whole Number concepts; Fractions concepts; Classroom Norms; Prospective Teachers

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Current reform efforts underscore shifts in the ways mathematics should be taught to students including those in elementary schools through university settings (Mathematical Association of America, n.d.; National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These recommendations portray classrooms in which students take a more active role in their learning. In these types of learning environments, the establishment of normative constructs dictates the interactions between all members of the classroom community and in turn determines the mathematics that is learned.

The ways in which students become accustomed to participating within a classroom community are described as a developmental process (Dixon, Andreasen, & Stephan, 2009). Students do not necessarily come into the classroom on the first day knowing how to make mathematical arguments or question one another. Rather these communal processes or social norms "are considered to be jointly established by the teacher and students as members of the classroom community" (Cobb & Yackel, 1996, p. 178). In the classroom setting, social norms define both the teacher's and students' roles. Even though the teacher initially introduces social norms at the beginning of a course they are continually negotiated and renegotiated throughout the course by both the teacher and students (Cobb, Wood, & Yackel, 1993; Dixon, Andreasen, & Stephan, 2009). Social norms are not specific to a content area and include: (a) explaining and justifying solution strategies; (b) making sense of another student's strategies; (c) questioning another students' solution strategies when misunderstandings occur; and (d) agreeing/disagreeing with other students (Cobb, Yackel, & Wood, 1989).

Although social norms provide a foundation for both the teacher's and students' roles in the classroom, norms also need to be established in order to attend to the mathematical activity that takes place in a mathematics classroom. The establishment of the social norms in the classroom fosters the sociomathematical norms, as students are expected to not only voice their solutions and solution processes, but also to analyze, critique, and make sense of one another's solutions in terms of their mathematical reasoning. Sociomathematical norms include determining what counts as: (a) an acceptable mathematical explanation, (b) a different mathematical solution, (c) a sophisticated mathematical solution, and (d) an efficient mathematical solution (Cobb & Yackel, 1996).

Research in which elementary-aged students (Dixon, Egendoerfer, & Clements, 2009; Lopez & Allal, 2007; Kazemi, 1998; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Stephan & Whitenack, 2003; Yackel, Cobb, & Wood, 1998), as well as undergraduate students studying mathematics (Ortiz-Robinson & Ellington, 2009; Stylianou & Blanton, 2002; Yackel, Rasmussen & King, 2000; Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011) participate in a classroom community have documented the importance and the manner in which social and sociomathematical norms allow students to take a more active role in their learning of mathematics. To extend the literature, this article emphasizes the ways in which a class of prospective elementary teachers and an instructor contributed to the social and sociomathematical norms that were established in an undergraduate mathematics content course designed specifically for prospective elementary teachers. The focus is to describe how these norms were jointly established and re-established specifically when the content focus shifted from whole numbers to rational numbers. As a result, the research questions were:

1. How are social and sociomathematical norms established in a mathematics content course for prospective elementary teachers?
2. In what ways do social and sociomathematical norms shift when the content focus changes from whole number concepts and operations to rational number concepts and operations?

Establishing Norms with Undergraduate Mathematics Students

Researchers have documented challenges and successes during the negotiation and establishment of norms between university instructors and undergraduate students. Yoon, Kensington-Miller, Sneddon, and Bartholomew (2011) identified the implicit and explicit norms that influence first or second year undergraduates during large group lectures. The researchers determined that undergraduate students studying mathematics felt that it was an instructor's duty to "get through" the mathematics material. As such, few students admitted to asking questions during a traditional lecture. The students were reluctant to ask questions for two reasons: (a) they did not want to hold up the lesson, and (b) they did not want their peers to perceive them as unintelligent. To reframe this passive approach, the researchers identified how two to three minute small group interactions where the undergraduates could only initiate contact with a peer by asking for help provides an effective way to engage and expect students to become active learners in the course content. Similarly, an instructor's commitment to learner-centered strategies (i.e., encouraged small group work on all homework, when studying for a test, and during in-class quizzes; requested participation during class; and provided prompt feedback) resulted in increased appreciation for the strategies while also decreasing the students' resistance to them (Ortiz-Robinson & Ellington, 2009). The students in the study conducted by Ortiz-Robinson and Ellington recognized the benefits of active learning in and out of a theoretical proof-intensive Real Analysis mathematics course. Furthermore, although surprised and initially resistant to following social norms requiring students to publicly explain their thinking as well as make sense of other students' thinking, undergraduates in a discrete mathematics course shifted from passively accepting an instructor's authority to becoming active participants who were responsible for shared mathematical understanding (Stylianou & Blanton, 2002). When explicit attention to the interactive establishment of social and sociomathematical norms were emphasized, students in a differential equations course freely offered alternative mathematical explanations (Yackel, Rasmussen, & King, 2000). As a result, these alternative explanations not only provided spontaneous mathematical connections for individual students but also served to further the mathematical development of the community.

As indicated above, norms are impactful in the mathematical learning of undergraduate mathematics students. However, it is unknown how many, if any, of the undergraduate students in these studies will become elementary school teachers. As such, a gap still exists in the research especially since research documents that many prospective teachers in elementary mathematics content courses often think they deeply understand the mathematics they will be responsible to teach when in fact many do not (e.g., Ball, 1990). Although exceptions exist (e.g., Andreasen, 2006; McClain, 2003; Szydlik, Szydlik, & Benson, 2003; Wheeldon, 2008), what is not readily evident in the research literature is the joint establishment of social and sociomathematical norms with undergraduate elementary education students as they are learning mathematics. It is vital that research with this subset of the undergraduate population participate in the development of these norms since these prospective teachers will be responsible for creating an active mathematics learning environment by negotiating these norms with their students as outlined in the K-12 process standards (National Council of Teachers of Mathematics, 2000) and the Standards for Mathematical Practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Theoretical Framework

Researchers have described students' learning using the emergent perspective, which coordinates the psychological and social aspects of a classroom (Cobb, 2000; Cobb & Yackel, 1996). As students contribute to social and sociomathematical norms, they reorganize their own understandings (Yackel & Cobb, 1996). Together an instructor and students jointly establish these norms.

Previous research has addressed the importance of establishing norms in a classroom as well as the role the instructor plays in establishing them (Dixon, Andreasen, & Stephan, 2009). Initially, it is the role of the instructor to start establishing the norms whereas later this responsibility shifts to students sustaining the norms. As such, the instructor may have to continually ask students to adhere to a norm at the outset of the norm development. However, as the responsibility for maintaining the norm shifts, the students and instructor will negotiate what the norm means in the classroom environment. When the responsibility of maintaining the norms is sustained, students will reiterate the norm when violations of the perceived agreement of meaning occur. At this juncture, the norm is referred to as taken-as-shared implying that the norm is understood and maintained by the members of the class as a whole (Stephan & Cobb, 2003).

Methodology

Thirty-three prospective elementary teachers in a mathematics content course majoring in elementary or exceptional education participated in a semester-long classroom teaching experiment conducted at a large urban university in the southeastern United States. The class convened for 110-minute sessions twice per week. Since familiarity with whole number concepts and operations may mask mathematical reasoning, ten days of instruction focused on whole number concepts and operations tasks situated entirely in base-eight where prospective elementary teachers reasoned and operated entirely within that base (for further discussion of the instructional unit see Roy, 2008 and Safi, 2009). This instructional unit was followed by ten days of instruction on rational number concepts and operations in base-ten (for further discussion of the instructional unit see Tobias, 2009). Rational number concepts and operations were explored in base-ten rather than base-eight based on the assumption that familiarity with rational number concepts were not as strong as with whole numbers so pushing students to make sense of the content in another base was not warranted.

Throughout the semester the prospective elementary teachers were presented with mathematical tasks in three overarching phases. First, the instructor launched a task by presenting a mathematical scenario or context in the form of a word problem, picture, or both. Next, the prospective elementary teachers were encouraged to solve the problems in ways that made sense to them mathematically, often working individually or in small groups. Finally, after working on the problems, the instructor facilitated whole-class discussions where the verbalization of mathematical reasoning and strategies were used to establish and maintain norms. The choices of talk moves and strategies used to establish and maintain norms were based on work of Chapin, O'Connor, and Anderson (2003) and Stephan and Whitenack (2003), respectively.

Data Collection. The collected data included video recordings and transcripts from each whole-class discussion. Student work was collected from in-class activities, homework, and tests at the end of each instructional unit. The problems students were given were designed such that students could solve them using a method of their choice with the expectation that they would have to explain and justify their solution processes. In addition, field notes and reflective journals were gathered from each of the research team members after every class session and research team meeting following the procedures of Cobb and Gravemeijer (2008) for design research.

Data Analysis. The data were analyzed using Rasmussen and Stephan's (2008) method for documenting collective activity. First, each class session was videotaped and subsequently transcribed. Next, at least two members of the research team independently coded the transcripts determining where they saw instances of norms being introduced, negotiated, or sustained (Dixon, Andreasen, & Stephan, 2009). When a norm was introduced, the instructor explicitly stated expectations of what students were going to be required to do. As the norm was negotiated, both the instructor and students worked together to define what a particular norm meant. Sustained norms were those that students automatically followed without being prompted by the instructor. For example, if one of the norms required the students to explain their thinking the instructor could initiate this norm by saying, "You have to explain what you did." Furthermore, when negotiating, this could include defining explaining to mean "not only discuss what you did but to justify why your method works." When the norm was sustained, students would automatically provide an explanation and justification without being asked to do so by the instructor.

Finally, each of the coded norms was analyzed using Glaser and Strauss's (1967) constant comparative method to determine how the norm shifted during the whole number and rational number instructional units independently. These norms were then compared across both the whole number and rational number instructional units to determine when norms became established, and how norms needed to be renegotiated when a new content area was taught. The norms that became established overlapped throughout whole number and rational number instruction, but are discussed individually for the purposes of this paper.

In the following section, the introduction of each norm by the instructor, followed by the negotiation between the instructor and students and the sustaining of social norms (i.e., explaining and justifying solutions as well as making sense of others solutions; and questioning) and sociomathematical norms (i.e., acceptable and different) during the whole number instructional unit is discussed. Then the renegotiation and re-sustaining of each of the already established norms during the instructional unit emphasizing rational numbers is described.

Social Norms

Explaining and justifying solutions. Together, the instructor and prospective elementary teachers created an environment where mathematical reasoning became a focal point of the whole-class dialogue. Prior to the instructional unit on whole number concepts and operations, the prospective elementary teachers discussed general problem solving strategies during two class sessions. During these sessions, the prospective elementary teachers were given problem-solving tasks that were intended to promote mathematical thinking such as those described by Van de Walle (2003). The tasks were not specific to the content of whole or rational numbers, but were rather general in nature. For example:

> A bucket of honey weighs 50 pounds. The same bucket with kerosene in it weighs 35 pounds. If honey is twice as heavy as kerosene, how much does the empty bucket weigh?

The instructor used these problem-solving tasks during the first two sessions of the semester to engage the prospective elementary teachers in problem-solving as suggested by Hiebert (2003) and to have them explain their solution strategies while examining the mathematical reasoning and analyzing the methods of other members of the class.

After working with the problem solving activities the instructor introduced the whole number instructional unit by presenting a series of Double 10_8-frames. Each frame was displayed briefly and then hidden from view. After a few seconds the prospective elementary teachers were asked to determine the total number of dots relying, in part, on visualizing what they had been shown. Then the instructor redisplayed a picture of the Double 10_8-frames and asked the prospective elementary teachers to describe how they arrived at a total number of dots. A classroom discussion that occurred during the first day of the whole number instructional unit follows the Double 10_8-frames shown in Figure 1. Of particular emphasis was the instructor's expectation that the prospective elementary teachers explain their mathematical reasoning.

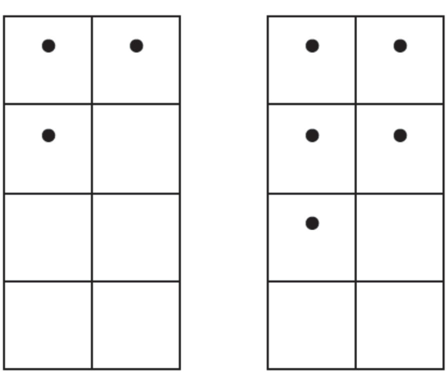


Figure 1: Double 10_8-Frame representing 10_8.

Instructor: How did you do it?

Claire: I moved the last dot over there and then knew the whole thing across is 10_8. No, over there [student motions from right to left], in the missing box and all across are 10_8.

Instructor: So this right here. Huh? How many of you saw 10_8 that way? It's about five of you. How many of you probably start to see it that way now, because she shared her thinking strategy. This is why you're helping everyone else in addition to yourself by sharing your thinking strategy. If I had said to you, do this, you would have been okay, but you hear it from another student, and it's powerful. It is a great strategy in teaching, so we are going to do that a lot, ask you to share your thinking.

In this discussion, the instructor emphasized the strategy Claire used to arrive at a total of 10_8 dots, and introduced the expectation that students would be required to explain their thinking. As stated in previous research (Dixon, Andreasen, & Stephan, 2009), at first it is the instructor's responsibility to convey the expectation of a norm that is eventually negotiated. As such, the instructor encouraged the student to share her thinking strategy, a technique supported by Stephan and Whitenack (2003). This was accomplished by asking, "How did you do it?" The instructor also began to legitimize the norm by saying, "This is why you're helping everyone else in addition to yourself by sharing your thinking strategy." She highlighted the expectation that mathematical reasoning not just stating answers were valued by stating, "we are going to do that a lot, ask you to share your thinking."

During the second day of the instructional unit the instructor and the prospective elementary teachers were in the midst of negotiating this norm. In the following dialogue, the instructor focused the prospective elementary teachers to reflect on their beliefs and understandings regarding the importance of mathematical reasoning when explaining and justifying.

Instructor: Why am I going to make her say it? Why is that important?

Edith: Cause you are going to ask us on a test.

Instructor: That's why you guys would think that it is important. Why do I think it's important?

…

Claire: Because we're going to need to know how to explain this to our students when we are teachers in ways that they can comprehend.

Cordelia: Also, because when you say it, it's your solution.

Instructor: And that's helpful not only are you saying that when we are teachers we are going to want to explain it to our students. What are we going to want our students to do?

Student: Explain.
Instructor: Explain, so start practicing now.

As expected, in the above dialogue it was still necessary for the instructor to bring the norm of explaining and justifying to the forefront of whole-class conversation. However, when doing so the instructor also started to lay the foundation for the shift in responsibility to the prospective elementary teachers by questioning the importance of explanations in a mathematics class.

In the following conversation during day three of the instructional unit, the prospective elementary teachers were engaged with a *Candy Shop Activity* (for a description of these activities with elementary-aged students see Cobb, Boufi, McClain, & Whitenack, 1997 and Cobb, Yackel, & Wood, 1992). The prospective elementary teachers were asked to package candy in boxes, rolls, and pieces; during these tasks 10_8 pieces = 1_8 roll and 10_8 rolls = 1_8 box. For example, 27_8 individual candies could be packaged as 2_8 rolls and 7_8 pieces or 1_8 roll and 17_8 pieces. In the following dialogue, the instructor and the prospective elementary teachers were faced with negotiating what it meant to explain and justify one's mathematical reasoning?

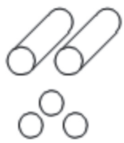
Nancy: What I did is I took apart one of the boxes, that's right, that's right, I took apart one of the boxes, so first I had, that's how many rolls that there were to start with. And I took apart, I took apart one of my boxes to show 10_8 rolls and those are my pieces that I didn't mess with. I left those alone. My question to you is how would I explain it so that way people could understand exactly what I did cause I feel like I am jibber-jabbing right now?
Instructor: What do you guys think jibber-jabber or makes sense?
Students: Makes sense.
Instructor: So I could ask anyone of you what she did and you could tell us? Jessica?
Jessica: Well I was going to say she said to write it down and convert it, she said she wrote it down and converted it.
Instructor: … So I know just what you did. We as a class agreed that we know just what you did. That's a great explanation then. Do we know why she did what she did? You're saying no you don't think so.
Nancy: I explained.
Instructor: You explained what you did.
Nancy: Yeah.
Instructor: But did you justify why what you did was okay? What do you guys think did she?
Edith: I think by saying she took it all out of the box she kind of like said why she got. I mean I understand why she did it.

The instructor used a talk move as described in Chapin, O'Connor, and Anderson (2003) by expecting the members of the class to restate someone else's reasoning when she stated, "So I could ask anyone of you what she did and you could tell us?" By asking a student to explain another student's solution the instructor continued to initiate the shift in responsibility of the norm. This technique of having one student explain another's solution is suggested in research conducted by Stephan and Whitenack (2003). Later in the conversation, the instructor and the prospective elementary teachers brought into focus the difference between a justification and an explanation.

Jackie:	Instead of just writing you took apart a box and ended up with that many you are kind of exploring how many rolls are in a box, how many pieces are in a roll.
Instructor:	Do you have to explain how many pieces are in a roll?
Jackie:	Well, for this one, no.
Instructor:	Okay.
Jackie:	But if you're just looking at the problem and explaining why, then you kind of have to explain that there are 10_8 rolls in a box. I'm just thinking like the outside person looking at the problem and trying to figure out how you take it apart, that you have to explain that's where that came from.
Nancy:	Every little piece or, or every part of it you would want, cause you are talking about not someone in this class, someone who is . . .
Jackie:	Yeah, someone who is outside the class trying to figure out what you did.
Nancy:	Okay, I understand.
Instructor:	You wouldn't have to say by the way we are in Eight World [base-8] but you need to …
Jackie:	Right, there is a lot more to explain than that, but just to explain that, to explain one box equals 10_8 rolls, you know.
Nancy:	Yeah, I understand.
Jackie:	So to take apart the box makes sense.
Instructor:	Barbara, what?
Barbara:	She is trying to explain what each thing is so people will understand it better.
Instructor:	So what did she need to explain here? To have it, not explain, justify, what, she explained well we knew just what she did, but what did she need to justify?
Barbara:	Justify what the box equals and what the roll equals and what pieces are.
Instructor:	Did she need to do all of that? What part of that did she need for this particular way she solved this problem?

Barbara:	How we understand how many were in the box, how many were in the roll because you could tell by the pieces there was just the one.

By the fifth day, the expectation and negotiation of what it means to explain and justify started to shift in responsibility from the instructor to the prospective elementary teachers. In Figure 2, the prospective elementary teachers were required to use mental mathematics to find the missing amount of candy when shown a picture of a part-part-whole relationship as defined by Carpenter, Fennema, Franke, Levi, and Empson, (1999).



There are 51_8 total candies.

Figure 2: Task: Given the following picture, how many candies are missing?

In following conversation, Beth, included an explanation without being prompted to do so.

Instructor:	What did you do Beth?
Beth:	I just subtracted 20_8 from 51_8, and minused 3_8 from that, to get 26_8.
Instructor:	That's a great example of what she did; she explained it. Can you say that again louder?
Beth:	I saw that there were 2_8 rolls, so I know that equaled 20_8 so I minused 20_8 from 51_8 and got 31_8, and then I subtracted 3_8 to get 26_8.
Instructor:	Okay. That time you included justification for part.

Beth automatically explained how she reasoned about the process she used to determine the missing candies. Although, at first she only provided a "great example of what she did," when prompted by the instructor to restate her explanation she maintained the norm that was being negotiated by also justifying why her solution was mathematically valid.

By the seventh day of instruction, the prospective elementary teachers sustained this norm. In the following dialogue, as students had just shared their thinking about the commutative property, Claire stated the taken-as-shared expectation that when an individual provides their mathematical thinking, one needs to be able to both explain and justify.

Instructor:	What do you guys think about that?
Claire:	You are going to have to explain and justify it.

To summarize the progression of the norm of explaining and justifying, during the first day of whole number instruction, the instructor introduced the norm that prospective elementary teachers would be asked to share their thinking. During the second and third days the instructor continued to emphasize the norm through negotiation with them. By the fifth day, the prospective elementary teachers were automatically providing explanations and justifications with their solutions. By day seven, they acknowledged independently that they would need to be able to explain and justify their mathematical reasoning.

When the class moved on to rational numbers, the expectation to explain and justify was already established. However, the research team had hypothesized as indicated in previous research that this expectation would have to be re-established due to the content area shifting from whole numbers to rational numbers (Dixon, Andreasen, & Stephan, 2009; Wheeldon, 2008).

The first problem the prospective elementary teachers were given was a picture representing an amount of leftover pizza shown in Figure 3 with the shaded region representing the part of the pizza with a mushroom topping.

When asked to find a fraction to represent the shaded amount, Jane automatically provided an explanation and justification for her solution of 1/3 and for another student's solution of 1/4.

Name a fraction that represents the shaded amount.

Figure 3: Leftover pizza on a table.

Jane: The question I have… which I think she tried to ask was is the empty space counted as pieces eaten or is it just not there? Because I did my answer to the fact that what I have is all that I'm counting and not as pizza eaten. But counting empty space. So my answer is 1/3 whereas theirs' is 1/4.

Though the answer came in the form of a question, Jane provided an explanation and justification for two different answers and did so without being prompted.

When the class moved on to the second problem where the prospective teachers were trying to determine the fraction to represent the shaded amount, shown in Figure 4. Edith was trying to explain and justify her answer of 2/8 and how that answer related to 1/4. Though she struggled to do this, she knew that she needed to provide an explanation and justification.

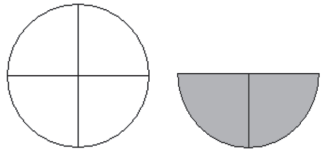


Figure 4: Leftover pizza at another table.

Edith:	Well I had two over eight and I thought about you know simplifying it to 1/4 because you could look at it as like. I'm trying to think of how to explain it in words.
Instructor:	I need more than this.
Edith:	I know.

Edith's response of "I know" meant that she understood the need to explain and justify her answer.

By the fourth day of rational numbers, the prospective elementary teachers initiated the idea of what needs to be explained and justified. This occurred when the class was given an activity that did not explicitly tell them to explain and justify. Evident from the following dialogue, the prospective elementary teachers believe they needed to either explain and justify or at least be able to explain and justify though they were not directly instructed to do so.

Cordelia:	I didn't catch what we were supposed to explain on this part because it doesn't say to.
…	
Jackie:	What was the question?
Instructor:	Do we need to explain and justify? It didn't directly say to. Jocelyn?
Jocelyn:	I'm sure we're supposed to.
Instructor:	Why are you sure we're supposed to? You're sure we're supposed to. So I think it's clear to me that you all realize there's this expectation that you need to be able to. That's consistent with each of your answers.

From each student's response, the class understood that they needed to explain and justify. Though the content area shifted from whole numbers to rational numbers, the expectation to explain and justify did not need to be re-established. Starting from the first day of rational numbers, the prospective elementary teachers were providing explanations and justifications within each of their answers without being prompted by the instructor to do so. The previous episode illustrated a departure and difference from previous research (Dixon, Andreasen, & Stephan, 2009) indicating that this norm would need to be re-established.

Make sense of others' explanations and justifications. Making sense of others included the need for the prospective elementary teachers to be able to understand someone else's explanation and justification as well as the ability to restate that method. During the second day of whole numbers, the instructor explicitly asked Jessica to restate what someone just said.

Jessica: I understand what she is saying.
Instructor: What is she saying?
Jessica: It's not that clear, but I know what she is saying.
Instructor: Tell me.
Jessica: She just explained it.
Instructor: Here is the thing; it doesn't count as knowing what she is saying unless you can explain it. So then, ask a question to help you explain it or explain it.

Jessica's inability to explain another's solution provided the instructor an opportunity to further explicate the norm. The instructor clearly stated that in order to make sense of a peer's solution, one must be able to explain that process as well. Later in the class the instructor was able to revisit the expectation.

Jessica: She was saying exactly what I was thinking.
Instructor: I know, but I am going to make you say it.
Jessica: I know you are.

In the presented dialogue, both the instructor and Jessica articulated the shared understanding that part of making sense of another student's solution is to describe their thinking.

By the third day of whole numbers, the responsibility of maintaining the norm, making sense of another's solution, started to shift. As Nancy explained her thinking when describing her representation of 246_8, she asked how she could explain and justify her thinking in a way that would make sense to everyone else. After she explained how she solved the problem, Nancy asked if her method made sense to others, as illustrated in the following dialogue. Importantly, Nancy's reasoning in the dialogue that follows supported how multiple norms developed simultaneously, since it also was evidence of the norm explaining and justifying.

Nancy: My question to you [*instructor*] is how would I explain it so that way people could understand exactly what I did because I feel like I am jibber-jabbing right now?

When Nancy exhibited concern with her explanation not making sense, this emphasized the importance of explaining in a way that makes sense to others. This was the beginning of the shift in responsibility of this norm from the instructor to the prospective elementary teachers.

This norm became taken-as-shared during the eighth day of instruction when the class was making sense of a hypothetical student's non-traditional algorithm [Figure 5] for the separate result-unknown problem as defined by Carpenter, Fennema, Franke, Levi, and Empson (1999).

There were 312_8 candies in a Candy Shop. 165_8 candies melted. How many candies were left?

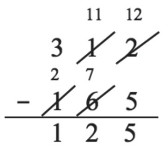


Figure 5: Non-traditional subtraction algorithm (note that calculations were determined in base-eight).

In the following discussion, Edith verbally explained her understanding of the subtraction method; she continually asked the other prospective elementary teachers if what she was explaining made sense mathematically.

Edith: In a way, I think of it like the number line, like you add on to it and you subtract that distance. That is how I think of it, like the distance. Does that make better sense? Beause you add on to the distance between the two numbers so then you have to subtract it again.

Claire: The distance.

Edith: Not the actual numbers, but just the distance between the numbers, do you want me to draw it?

At this point of the conversation Edith realized that some of the other members of the class did not understand her understanding of the algorithm. As a result she drew the empty number line as described by Gravemeijer (1994) and replicated in Figure 6 on the whiteboard to help the other prospective elementary teachers make sense of the algorithm.

Figure 6: Edith's thinking on an empty number line.

She then continued to describe her mathematical reasoning while simultaneously filling the open number line.

Edith: Okay so you start off by adding 10_8, like you're moving it, like you're shifting the distance from here all the way out to here; so then you add a 10_8 down here.

Caroline:	I thought you said subtract though that's why, you were saying subtract.
Edith:	Yeah, because …
Claire:	You are not subtracting you're adding. You're making it larger.
Edith:	…but, this is how I am thinking of it. I am trying to justify why it makes sense to me. I'm not saying to subtract the number, you add the number, but you're subtracting like from the distance. Okay so this, okay, because this is the original difference [student gesturing with hands] then you are adding on to the distance, so you have to subtract from that distance by adding on. Does that make sense?
Nancy:	Um-hum.
Instructor:	Nancy, you're saying yes; what did she say to help you?
Nancy:	Her hands helped me, it's the shift you're changing the numbers, like you said, but the distance is staying the same, you're just adding on and adding on, but it is still the same distance from right here as it was from right here, where we shifted 10_8 and we shifted 10_8.
Edith:	Is anyone still confused?

At this point in the conversation, the members of the class were prompted by one of their peers, Edith, and not by the instructor to indicate if they understood her mathematical reasoning. This shift illustrates that the prospective elementary teachers in the course assumed the responsibility of making sense of another's reasoning. The assumption of responsibility was further exemplified as the conversation continued.

Caroline:	I get that, I already got that but I just don't get where you're saying you're adding and subtracting.
Edith:	Oh yeah because you …
Caroline:	Now you're subtracting again.
Edith:	Well, don't think numbers, think the distance, if that will help because you're adding then you have to subtract from that difference to get to the original distance between the two numbers, because you added 10_8, so it's a greater distance, so you have to subtract 10_8 from that difference by increasing the number value.
Caroline:	All right.
Edith:	Does everybody understand?
Instructor:	Claudia.
Claudia:	I was just going to ask is that what Jackie was thinking?
Jackie:	Yes.

It is evident from the dialogue that the prospective elementary teachers sustained this norm when Edith asked if how she thought about the problem made sense to the rest of the class. Furthermore, by the end of the conversation, Claudia asked if her reasoning was similar to reasoning Edith shared during the whole-class discussion. The previous conversation was illustrative of the type of whole-class dialogue that continued to take place during the remainder of the whole number instructional unit.

During whole numbers, the norm of making sense of others was introduced during the second day of instruction. On the third day the responsibility for the norm started to shift to the point that on the eighth day the prospective elementary teachers were sustaining the norm.

During rational numbers, this norm was still sustained by the class. This was demonstrated from the conversation that occurred on day 1 of the instructional unit while naming the amount of pizza leftover (see Figure 3).

While the class solved this problem, they were trying to make sense of the answers of 1/4 and 1/3. Although Laney concluded that there was 1/3 of a pizza leftover, she was able to provide an explanation and justification for the answer of 1/4. After explaining her solution, she went on to state, "If that makes sense."

Instructor:	How many of you got 1/4? Okay. Who wants to talk about it? Laney?
Laney:	I didn't really get 1/4 but I can see how she got 1/4. It all goes back to the confusion of taking it from the total amount to taking from what's left over.
Instructor:	Okay.
Laney:	Because if it's just what's left over, it's 1/3 because the other 1/4 isn't there. But if you're taking it from the whole it's 1/4 because there's four groups of two. If that makes sense.

Two noteworthy occurrences took place within this discussion. First, Laney explained the answer of 1/4 even though she stated that this was not the answer she got. At this point she was making sense of *another* prospective elementary teacher's answer. Second, when Laney was providing an explanation and justification for both answers, she asked if her explanation made sense. Within this discussion, the prospective elementary teachers initiated both aspects of this norm without being asked to do so. This illustrates that when the content shifted to rational numbers that the prospective elementary teachers already expected to make sense of others' explanations and justifications *and* to be able to explain and justify others' solutions.

Though the norm involving making sense of others did not have to be re-established, the instructor continued to sustain this norm throughout the rational number unit. Throughout the unit on rational numbers the instructor would ask for the prospective elementary teachers in the class to explain and justify someone else's solution process and then return the conversation back to the original prospective elementary teacher to verify the contribution.

Similar to the norm of explaining and justifying, the norm of making sense of others' also did not need to be re-established when the rational number unit started. Though the instructor sustained this norm by asking how and why someone did something, the prospective elementary teachers were the ones who initiated the conversations of explaining others' thinking and asking if what they were doing made sense.

Questioning. The social norm of questioning others was the first social norm not already established before the rational number unit started. This norm was introduced in the conversation that follows. The class was discussing how they arrived at their answers for the second rational number problem where the prospective elementary teachers were determining the amount of pizza leftover (see Figure 4). Claire brought up a new idea of solving the problem with "undividing." When Jocelyn responded that she did not understand what Claire meant by this, the instructor then prompted Jocelyn to ask Claire a clarifying question.

Claire:	To get the 1/3 I looked at it as sections. I kind of looked at the whole piece is a half. The top part would be one the bottom would be two, and then the shaded part would be three to get the 1/3. I divided them further into sections.
Instructor:	You divided them further into sections?
Claire:	Well, I guess I didn't divide further. I kind of undivided them.
Instructor:	So didn't you mean you can undivide to get one-third? What does she mean she undivided? Jocelyn, what did she mean when she said she undivided them?
Jocelyn:	I have no idea.
Instructor:	Ask her a question.
Jocelyn:	What do you mean [Claire]?

When Claire explained how she got the answer of 1/3, she introduced the idea of "undividing" to get the answer. When asked by the instructor what Claire meant, Jocelyn replied that she had no idea meaning that she could not make sense of Claire's method of solving the problem. The instructor then asked Jocelyn to ask a question.

On the second day of rational numbers, the instructor again prompted the class to ask a question. However, this time the prompt was in response to another member of the class getting an answer that was different from what was presented. The class was asked to determine how much pizza each person would get when four pizzas are shared equally among five people. Kassie got a solution of 4/20. Mary then was asked to question Kassie because she got a different solution.

Kassie: I got 4/20, because together it was 20 pieces and four for each person. Questions?

Instructor: So raise your hand if you got exactly the same thing. Okay raise your hand if you got something different. You've got a question.

Within this conversation, the idea of the prospective elementary teachers having questions was brought to the forefront when they indicated that they got a different answer from someone else. The idea of asking a question when an answer is different was introduced by the instructor.

On the third day of the rational number instructional unit, there was a shift from this norm being initiated by the instructor to being initiated by a prospective elementary teacher. Claudia was in front of the class explaining how she solved a problem. When she was finished, she left a pause in the conversation for the rest of the prospective elementary teachers to ask her questions.

Instructor: Are there questions?

Claudia: Do you guys have questions? I was waiting.

Instructor: You were waiting because you were expecting them to do it. Good.

By the third day of rational numbers the norm of questioning others became taken-as-shared. This occurred when there was a shift from the instructor initiating the responsibility to ask questions to the prospective elementary teachers taking the initiative in sustaining this norm. Claudia waiting after giving an explanation and justification indicated that she did so with the expectation that others in the class would ask her questions if they needed clarification about or disagreed with something she had said.

During the rational number unit, there were two types of instances where prospective elementary teachers needed to question. The first was when a prospective teacher did not understand another's thinking. The second was when a prospective teacher arrived at a different solution from someone else. Throughout the remainder of the rational number unit both the prospective elementary teachers and the instructor sustained this norm by asking if there were questions.

By the end of the semester, the three social norms that became taken-as-shared during the class were: (a) explaining and justifying, (b) making sense of others, and (c) questioning others. Cobb and Yackel (1996) discuss a fourth norm of indicating agreement/disagreement. While indicating agreement/disagreement was part of conversations as evident in the following discussion, it was provided in conjunction with the expectation to question others.

Instructor: Raise your hand if you agree. … Do we agree?

Caroline: Yeah.

Instructor: Okay. Are we okay with that?

There was never a shift from the instructor initiating this conversation to the prospective elementary teachers sustaining it; there were not enough instances during the classroom discussions to determine if agreeing/disagreeing became taken-as-shared.

Sociomathematical Norms

Acceptable solution. In the dialogue that follows, the sociomathematical norm of determining what constitutes an acceptable solution was introduced to the class during the second day of the whole number unit. Acceptable solutions were identified to be those that included both explanations and justifications. The instructor stated that it is not enough to know what you did, one also needs to be able to justify why the method was used.

Instructor:	Very good point, which is why, one of the many reasons I have you explain and justify what you did is so I can actually know your thought process. … What was your thought process and why is what you did okay?

By the seventh class day, a prior knowledge argument was used by some of the prospective elementary teachers. In the following example, Suzy states that she knows the commutative property for multiplication because she has previously learned it. The instructor then asks what Suzy should do if she knows something, to which Suzy replied that she needs to explain and justify.

Suzy:	Tell you how to justify it? Sure, get me a math book. I don't know the exact definition, but I do know from prior knowledge.
Instructor:	You know from prior memorization?
Suzy:	Yeah, from memorization that it doesn't matter which order.
Instructor:	So if you know that, what should she be able to do – if she knows that?
Suzy:	Explain and justify.

This conversation illustrates that by the seventh day of instruction, the prospective elementary teachers were sustaining the norm that acceptable solutions include both an explanation and justification. However, this conversation also introduced the idea that "prior knowledge" does not suffice to be an acceptable explanation or justification.

When the prospective elementary teachers moved from the whole number to the rational number unit, their discussions took on a markedly different tone. As such, when the rational number unit started, this norm had to be re-established. During the whole number instructional unit, the prospective elementary teachers knew for an acceptable solution they needed to explain and justify; however, the idea of what it means to explain and justify in *mathematically meaningful* ways had to be re-established.

During the first day of rational numbers, the prospective elementary teachers were quick to provide explanations that were reiterations of the procedures they had learned as children. Mary used a "prior knowledge" argument in her explanation of how she went from 2/6 to 1/3.

Mary: I got 2/6 but from my prior knowledge I know that I can divide that to make it a smaller fraction. So that would be 1/3.
Instructor: You divided it?
Mary: I knew you were going to do this to me. Oh you can break down two. I don't know how to explain that.

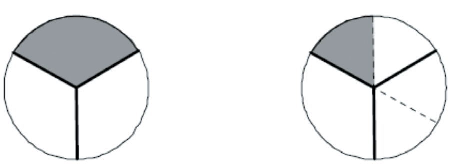
Mary's use of "prior knowledge" referred to the fraction knowledge she learned before taking this class. Apparent from this conversation, Mary knew that her answer would not be acceptable; however she still could not provide a more conceptual explanation. A few minutes later she attempted again to provide an explanation and justification.

Mary: …I knew that from prior knowledge.
Instructor: So now here we are with this prior knowledge business. Right? The prior knowledge is only okay if you can explain and justify it in mathematically meaningful ways.

The instructor once again reiterates that a "prior knowledge" argument does not suffice for constituting as an acceptable explanation or justification.

The negotiation of this norm continued on the third day of instruction when the members of the class were discussing their answers to the problem presented in Figure 7. This problem provided a fictitious student's solution to sharing two pizzas among four people.

A student was given the following problem:
Share two pizzas equally with four people. The student did the following:



The shaded amount in the diagram represents the amount of pizza one person got.

Figure 7: A student example of sharing two pizzas with four people.

Each small group of prospective elementary teachers was asked to come up with an explanation and a justification that could explicate what the student did to solve the problem. In this conversation, the instructor noted that an argument based on “a picture looks like it” is not an acceptable justification.

Instructor: We can’t say because the picture looks like it. That’s not an acceptable explanation or justification. It helps you solve the problem, but you need to explain and justify in ways that aren’t because it looks like the picture.

When the class was using pictures, the instructor said that using the argument that a picture “looks like the answer” is not acceptable.

On the fifth day, the prospective elementary teachers still reverted to procedures to explain how they solved a problem. In the following discussion, Jane used a procedure to multiply three and four to find out how much 1/3 of 1/4 is.

Jane: I just multiplied three times four equals 12. So then both of them would be the same…

Instructor: And since that’s not acceptable, what would you do? This is, you took this piece. How do you know? Just multiplying three times four, you’re pulling things out of the sky here.

As soon as Jane started explaining in terms of just multiplying to arrive at the answer the instructor immediately replied that that is not acceptable and asked for a different way to explain the problem.

The sixth day of rational numbers involved the idea of what constitutes an acceptable explanation and justification and the discussion shifted from being initiated by the instructor to being initiated by the prospective elementary teachers. Within the following problem, they were developing ways in which to compare two fractions shown in the following problem:

> At the party, the trapezoid table was decorated with 5/6 of a spool of ribbon. The rectangle table was decorated with 9/10 of a spool of ribbon. On which table was more ribbon used.

When comparing the fractions 5/6 and 9/10, several of the prospective elementary teachers solved the problem by finding a common denominator. Both Suzy and Caroline questioned the validity of just saying you multiply to get 30.

Suzy: So just by multiplying you found a common denominator. Is that acceptable?

Katherine: The way I wrote it, in my justification. I put I found the least common denominator which is 30. This is the number that both six and ten can be multiplied to make or into. Then I put six times five equals 30, so six and then I went into it.

Instructor:	So I heard somebody, I think Caroline, say it didn't seem like it's sufficient.
Caroline:	Yeah.

This episode represented the first instance where the prospective elementary teachers initiated the conversation of an answer not being acceptable and this was in response to someone using a procedure to solve a problem. Common denominators were an acceptable method; however, the prospective elementary teachers' ways of explaining and justifying them were not, thus they could not be used.

Towards the end of the rational number instructional unit, the prospective elementary teachers sustained the norm of what constitutes an acceptable solution. In the following dialogue, members of the class were discussing the use of pictures in explanations and justifications. Caitlyn used a picture to multiply 2/3 × 3/4. She then commented that it is unacceptable to say because a picture looks like the answer.

Caitlyn:	You can't just say because the picture looks like it. You have to provide what you did and why you did it on the picture.
Instructor:	I agree. Olympia?
Olympia:	I mean we look at your picture we can see what you did. But I guess if you want to be safe you can just in writing write what you did. And how you, added what you added
Caitlyn:	You just explain what you did
	…
Caitlyn:	And why.

This discussion occurred after Caitlyn used a picture to show how she got an answer to 2/3 x 3/4. A picture was used in conjunction with Caitlyn's explanation and justification, however she did not include the argument that, "the picture looked like it," which made her explanation acceptable.

There were two aspects of acceptable solutions that had to be negotiated by the prospective elementary teachers and instructor. The first was the idea that reiterating known procedures did not suffice as being an acceptable explanation and justification. This was introduced during whole numbers and revisited within rational numbers. The second type of an acceptable solution involved the use of a picture. The class had to negotiate that pictures could be used in solution strategies, however it was not acceptable to use the argument that, "the picture looks like it." One reason this may have occurred is that during the whole number instructional unit pictorial representation were provided to the prospective elementary teachers, whereas in the fraction instructional unit they had to develop their own pictorial representations.

*Different solution***.** Different mathematical solutions included finding a different solution, getting the same answer in a different way, and justifying how methods are different from one another. During the first session of the whole number unit, the instructor introduced this during the problem in Figure 1. After the prospective elementary teachers shared the number of dots they saw, the instructor asked them to raise their hand to indicate how they solved the problem. In response to the class having various methods, the instructor noted that it was interesting how many different methods were used.

Instructor: Isn't it interesting, how we are just saying how many and there are that many ways of getting there. I wonder if we just left it showing, if you would have done it differently than you have.

The instructor supported this norm by focusing on the methods the prospective elementary teachers used to solve the problem. This emphasized the expectation that if a problem is solved differently, it is important to share that with the class. This was reiterated on the sixth day and expanded to include the idea that understanding differences among solution strategies is also important.

Instructor: Okay, so that is the difference of these two. Excellent, because these are different and they're different because this one we counted by groups, this one you counted by ones. How many of you agree that makes them different solution strategies? Raise your hand if you think they are not different. I am interested in sharing those differences, so that's good.

This norm was sustained by the prospective elementary teachers during the eighth day. When finding how many eggs would fit in a carton that has 12_8 rows with 6_8 eggs in each row, Katherine commented that she counted the eggs one-by-one, and then tried to find a different way to solve the problem.

Instructor: What did you do with it? Katherine?

Katherine: Well, I did it and just counted them all every one. Then I was trying to figure out another way to do it.

Though Katherine had a correct method, she strived to find a different way to solve the problem.

On the first day of rational numbers, the instructor brought different solutions to the forefront of the conversation.

Instructor: So you've got some different answers it seems… Raise your hand if you're at a table that has different answers from each other at one table. Look at that. Four out of the seven tables have different answers at the same table.

Before discussing the first problem [Figure 3], the prospective elementary teachers were expecting that different solutions were going to be discussed because the instructor highlighted the fact that they responded to the question in different ways.

Name a fraction to represent the shaded amount.

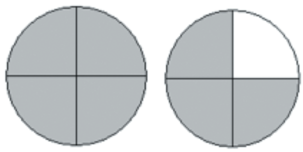


Figure 8: Restaurant table 3.

When the class moved on to discuss the problem shown in Figure 8, the prospective elementary teachers presented the idea of what constitutes a different solution. Within this discussion the prospective elementary teachers were determining if 1 3/4 is equal to 7/8 in the context of describing how much mushroom pizza was leftover with the shaded region representing the part of the pizza with a mushroom topping and Alex found that the different solutions depended on how you defined your group or whole.

Alex:	It's just a question of how you group your problem. I grouped mine into eight individual groups, so I have seven of the eight that are shaded.
Instructor:	Okay.
Alex:	Kassie did hers in fourths.
Instructor:	Okay.
Alex:	So what's hers, is one group of four, two groups of four. Her one group of four is an entire mushroom pizza and the 3/4 is the second group of four that she worked with. So she was looking at it, but just grouped it differently.

The idea presented by several members of the class, before this conversation started, was that 1 3/4 is equal to 7/8. Out of the prospective elementary teacher who disagreed with this statement, Alex noted that the difference was in the way the picture was grouped. The instructor did not have to prompt the class to determine how the two solutions were different.

At the end of the first day of rational numbers, the instructor asked the following questions.

Instructor:	Anyone have another way of describing that they want to share? Questions? Different answer?

The instructor frequently asked this question throughout the other eight days of instruction. From the beginning of the rational number unit, the class negotiated that different solutions tended to come in two forms. One involved a different answer and the other illustrated a different way to represent the same answer.

Midway through the rational numbers instructional unit, the prospective elementary teachers started to present different solutions without the instructor asking for someone who got something different.

Instructor: Claire?
Claire: Should I show how I did it? Because I got something different.

This conversation illustrated the first time a prospective elementary teacher indicated that they had gotten something different from what was presented. Claire not only said she got something different, but also offered to come to the board to show how she got a different solution without being asked to do so.

Throughout the rational number unit, the instructor did not have to re-establish what constitutes a different solution. Though some of the members of the class struggled with this idea on the first day, they generated the conversations on what makes solutions different. The instructor also kept this norm in the forefront of conversations and there was a shift to prospective elementary teachers sustaining this.

Sophisticated solution. The sociomathematical norm of what it means to have a more sophisticated solution and/or solution strategy was addressed for the first time the second to last day of the rational number unit. This discussion was initiated by Claudia when the class was finding a common denominator for the problem 5/6 + 5/8.

Claudia: I mean it just goes back to the fact of trying to find the more sophisticated way of solving things.
Instructor: So what does she mean by this trying to find the more sophisticated way? Which is more sophisticated? Finding 24 or finding 48?
Class: 24.
Instructor: 24? So is using 48 acceptable?
Class: Yes.
Instructor: But not completely sophisticated.
Class: Right.

Though the discussion was generated by the prospective elementary teachers, this was the only instance in the class of what it means to have a sophisticated solution. Thus, there were not enough classroom episodes to determine that prospective elementary teachers understood what it means to have a more sophisticated solution, though they correctly identified the sophisticated solution within this discussion. Similarly, there were also insufficient occurrences within the discussions to conclude what constitutes an efficient solution.

Table 1:

Whole Number Unit	Rational Number Unit
Social Norms • Explain and Justify • Make Sense of Others' Explanations and Justifications	Social Norms • Explain and Justify • Make Sense of Others' Explanations and Justifications

Sociomathematical Norms
• Acceptable Solution
• Different Solution
	Sociomathematical Norms
	• Acceptable Solution
	  o Without using prior knowledge
	  o Without using what a picture looks like
	• Different Solution
	  o Different answer
	  o Different process to obtain the same answer

Discussion

The norms that were established and/or sustained within the whole number and rational number units are summarized in Table 1.

Explaining and justifying, making sense of others, and determining what constitutes a different solution were established before the rational number unit started. Though these norms were established they continued to be sustained and negotiated by the instructor and prospective elementary teachers throughout the rational number unit.

The only norm that had to be re-established during the rational number unit was what constitutes an acceptable solution. When the class moved on to the rational number unit, the prospective elementary teachers were quick to revert to the procedures they learned as children. This may have been in part due to the decision to present that unit in base-ten rather than base-eight or it could have been due to the prospective elementary teachers' comfort using procedures that were poorly understood. In addition, the acceptable use of pictures in explanations and justifications had to be negotiated.

The social norm of questioning others was the only norm completely established within the rational number unit. This may have occurred because of the prospective elementary teachers' familiarity with fractions, and their inexperience with base-8. When the class was focused on whole numbers, questioning others was not needed. Although different strategies were focused on during instruction on whole numbers, due to the understandability of their solution procedures the prospective elementary teachers never questioned each other about them. Within the rational numbers unit, the prospective elementary teachers frequently questioned the solutions and solution strategies of others. Although this norm had to be established in rational numbers, it was found to be taken-as-shared by the third day of class in which the members of the class were expecting others to ask questions for what they had done.

When a social or sociomathematical norm was needed it was introduced within the first two days of instruction. All shifts in responsibility occurred somewhere between the third and sixth day of instruction. The norms that were sustained were not sustained until at least the seventh day of class. The results indicate that norms may not necessarily develop linearly throughout instruction. All of the norms that were established also overlapped throughout instruction.

Implications

This paper documented the social and sociomathematical norms introduced to, negotiated with, and sustained by prospective elementary teachers in a mathematics content course. Consistent with previous research with elementary-aged students (Dixon, Egendoerfer, & Clements, 2009; Lopez & Allal, 2007; Kazemi, 1998; Kazemi & Stipek, 2001; McClain & Cobb, 2001; Stephan & Whitenack, 2003; Yackel, Cobb, & Wood, 1998) and undergraduate students studying mathematics (Ortiz-Robinson & Ellington, 2009; Stylianou & Blanton, 2002; Yackel, Rasmussen, & King, 2000; Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011), the prospective elementary teachers were able to take a central role in their learning. Rasmussen and Stephan's (2008) method was shown to be effective in determining when social and sociomathematical norms were established and how norms need to be reinforced and not just introduced or discussed once. The results of this study, document the social and sociomathematical norms that are jointly established by the course instructor and prospective elementary teachers. In addition, the results identified that one norm had to be re-established when transitioning between whole number and rational number instructional units.

This finding is important since the prospective teachers will find themselves responsible for fostering the mathematical and process underpinnings identified in both the *Principles and Standards* (National Council of Teachers of Mathematics, 2000) and the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). For example, a mathematics teacher can provoke students to communicate their mathematical thinking, by asking them to explain and justify. However, if this teacher has not experienced this norm being validated in a mathematics classroom, this request may not elicit the foundational standard for mathematical practice in which students construct mathematical arguments and critique their own reasoning as well as the reasoning of others. Thus, further research is needed to determine if these experiences are transferred to practice as prospective elementary teachers foster their ability to establish social and sociomathematical norms once they enter their own classrooms.

The findings also document a break from previous research (Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011) where undergraduate students were reluctant to interact in whole-class settings because they did not want to hold up a lesson or did not want their peers to perceive them as unintelligent. Through whole-class dialogue the negotiation of both social and sociomathematical norms not only addresses the students' sentiments identified by Yoon et al., they refocus them and make them the central focus of the classroom. First, since mathematical thinking is a goal of the lesson, an individual cannot interrupt a lesson rather they can only add to development of the lesson. Furthermore, since sense making is foundational, the opportunity to explore both correct and incomplete lines of reasoning are supported during the negotiation of norms. As a result, all mathematical contributions add to mathematical development of the prospective elementary teachers. In the end, although the research was conducted with a specific subset of the undergraduate mathematics students, the findings of this study are consistent with research in which undergraduate mathematics students actively participate in their learning (Stylianou & Blanton, 2002; Yackel, Rasmussen, & King, 2000), and although it is not typical that in this type of course that ten sessions are devoted each to whole number concepts and to rational number concepts, the presented context provides insight for other mathematics educators who are teaching courses in which social and sociomathematical norms are essential to students' participation in the class. Furthermore, since the analysis was only completed with whole-class discussions, it may be that norms were established earlier and just not a topic of discussion until later. Though not evident from the data collected in this study, further research is needed to determine if this is the case.

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