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With a Little Help from My Friends: Scaffolding Techniques in Problem Solving

Michelle L. Frederick
Kent State University
mfreder1@kent.edu

Scott Courtney
Kent State University

Joanne Caniglia
Kent State University

Abstract

The purpose of this study was to explore middle grade mathematics students' uses of scaffolding and its effectiveness in helping students solve non-routine problems. Students were given two different types of scaffolds to support their learning of sixth grade geometry concepts. First, students solved a math task by using a four square graphic organizer that included the identification of critical components, strategy selection, computation, and analyses of answers. The second type of scaffolding occurred with different grouping formats, alternating work led by the teacher, working in groups of four, in pairs, and finally working alone. Measures included extended response daily tasks, teacher-created unit test, state standardized assessments, and surveys gauging student's satisfaction with types of scaffolding. Classroom use of a variety of scaffolds led to an increase in the number of correct responses and more detailed explanations. No significant differences occurred on the teacher-made tests, yet significant increases were found on students' state standardized tests. Students indicated that scaffolding by groups was helpful in initiating solution pathways.

Historically, the term "scaffolding" referred to bolted together tiers of boards upon which human workers stood to construct a building (Anghileri, 2006). This idea has been transformed over time to an analogy that relates learning to a hierarchical framework built upon firm foundations. According to Holton and Clarke (2006), "The analogy with construction of knowledge is that cognitive scaffolding allows learners to reach places that they would otherwise be unable to reach" (p. 129). Furthermore, when the "building is finished or the renovation complete, the scaffolding is removed, it is not seen in the final product" (Holton & Clarke, 2006, p. 129).

A variety of definitions have been established over the years to describe the term "scaffolding." For some researchers, scaffolding refers to many different aspects that occur within the classroom (Anghileri, Julia, 2006); whereas others have argued against the fact that not everything used in the classroom actually assists in learning and teaching (Holton & Clark, 2006). For example, simply providing students with facts or directly showing students how to do something would not be considered scaffolding. Holton and Clarke (2006) define scaffolding as "an act of teaching that (a) supports the immediate construction of knowledge by the learner; and (b) provides the basis for the future independent learning of the individual" (p. 131).
Although the term "scaffolding" was first introduced within an educational setting by Wood, Bruner, and Ross (1976) the concept of "scaffolded instruction" is linked to Vygotsky's idea of the zone of proximal development. For Vygotsky, the zone of proximal development (or ZPD) is the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Holton & Clarke, 2006, p. 128).

Background

Types of Scaffolds

Research has shown that scaffolds can help increase students' metacognition, thus, making it easier for students to relate to the material (Holton & Clarke, 2006). Such reflective cognition can help students process any complex information they come into contact with. In addition, the use of various instructional scaffolds can help students develop cognitive structures that support both metacognition and self-efficacy. Anghileri (2006) concluded that a flexible and moving scaffold is needed to create individual creativity to have more autonomous and independent, self-motivated learners.

There are many strategies that can be implemented in the classroom to help guide students to robust understandings of mathematical concepts. Holton and Clarke (2006) indicate that for some scaffolding strategies teachers and students do not even need to be present together. In such cases, scaffolds can be used for students to work individually, in pairs, or in groups outside of the general class instruction. According to Holton and Clarke (2006), in any such grouping, one or more students with strong self-metacognition and self-scaffolding abilities are able to help the other students scaffold to reach a solution.

Holton and Clarke (2006) broadened the idea of scaffolding to include four main elements, of which, heuristic and conceptual scaffolding constitute the domains of scaffolding. According to Holton and Clarke (2006), the aim of conceptual scaffolding "is the promotion of conceptual development" (p. 134); whereas, heuristic scaffolding "relates to the development of heuristics for learning or problem solving, that transcend specific content" (p. 134). In a mathematics context, a "concept" refers to the mathematics content, while "heuristic" is concerned with problem solving approaches that may be taken (Holton & Clarke, 2006). Holton and Clarke (2006) posit that both scaffolding domains can improve mathematics learning in the classroom.

Anghileri (2006) characterizes three levels of scaffolding practices as they relate to mathematical learning. According to Anghileri (2006), these levels "constitute a range of effective teaching strategies that may or may not be evident in the classroom" (p. 38). Level 1 scaffolding occurs prior to interacting with students and involves preparing classroom artifacts (e.g., wall displays, manipulatives) and classroom organization (e.g., seating arrangements, sequencing, and pacing). Such scaffolding does "not involve direct interactions between the teacher and students" (Anghileri, 2006, p. 40) and employs structured tasks (e.g., worksheets or directed activities) that may include a self-correcting element to provide feedback that supports students' autonomous learning. Furthermore, Level 1 scaffolding may include "emotive feedback that does not directly relate to the mathematics to be learned" (Anghileri, 2006, p. 40), including remarks and actions designed to gain attention, provide encouragement, or display approval.

Level 2 scaffolding involves "direct interactions between teacher and students related specifically to the mathematics being considered" (Anghileri, 2006, p. 41). Level 2 scaffolding can involve "showing and telling," constituted by traditional teacher-centered practices that make little use of students' contributions, or "explaining," constituted by ones-sided teacher dominated discussions (Anghileri, 2006). Preferred alternatives to these traditional Level 2 scaffolding practices include "reviewing and restructuring" which supports students' development of their own understanding of mathematics (Anghileri, 2006). According to Anghileri (2006), reviewing relates to "interactions where the teacher encourages experiences to focus students' attention on pertinent aspects of the mathematics involved . . ." (p. 41); whereas, restructuring involves "teachers making adaptations to modify the experiences and bring the mathematics involved closer to students' existing understanding" (p. 41).

Finally, Level 3 scaffolding involves students' development of "concepts through specialised processes such as generalisation, extrapolation and abstraction" (Anghileri, 2006, p. 47). Level 3 scaffolding "consists of teaching interactions that explicitly addresses developing conceptual thinking"

(p. 47). Such interactions provide support for students in making connections, developing a range of representational tools (i.e., systems of images, words and symbols), and generating conceptual discourse (Anghileri, 2006). Holton and Clarke's (2006) concept of heuristics is similar to the third level discussed by Anghileri (2006); both perspectives bring learning past the basic level, where students need to make deeper connections and promote higher-level thinking.

Metacognition and Problem Solving

Metacognition is often defined as "thinking about thinking." According to Livingston (1997), "Metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning" (para. 1). Furthermore, activities such as planning a strategy to implement as one engages in a learning task, monitoring one's own comprehension throughout engagement, and evaluating one's progress toward the completion of the task are all metacognitive in nature (Livingston, 1997). For Holton and Clarke (2006), "Metacognition mediates between the learner and their cognition. While cognition can be considered as the way learners' minds act on the real world, metacognition is the way that their minds act on their cognition" (p. 132). From this perspective, scaffolding can help make connections between the learners' cognition and their thought processes.

This progression in scaffolding is relevant and similar to processes used in problem solving. For instance, when a learner is solving problems individually they can self-regulate through the steps of the problem. As a teacher, the use of questioning and the types of questions a teacher selects to ask helps to guide students' metacognition, with an intent to improve their students' problem solving skills. Since scaffolds assist students in the process of learning and finding the solution to a problem, well-formulated questions can act as a metacognitive device.

Pape, Bell, and Yetkin (2003) conducted a study on a seventh-grade math teacher who used a variety of strategies designed to lead her math students (54 students in all) toward self-regulated learning (SRL). Strategies included multiple representations, rich tasks, classroom discourse, and environmental scaffolding. The teacher employed scaffolding through the use of probing questions and used a Strategy Observation Tool to help students identify which strategies work best for them. According to Pape et al. (2003), an essential component for developing students' SRL was "to provide a context to support their growing awareness of themselves as agents in the learning process by supporting their efforts to observe their strategic behaviors and to attribute outcomes to these behaviors" (p. 196).

Rittle-Johnson and Koedinger (2005) suggest three types of knowledge for problem solving: procedural, conceptual, and textual. In their study, they used scaffolds on each type of knowledge. Rittle-Johnson and Koedinger

(2005) conducted a pre- and post-test on 226 students in the sixth grade. Within the pre- and post-test, they implemented scaffolds for each type of problem solving strategy in mathematics. Rittle-Johnson and Koedinger's (2005) results suggested that improving students' conceptual knowledge can lead students to use better problem solving strategies.

This article presents results from a study whose underlying framework incorporates the research of Anghileri (2006), Holton and Clarke (2006), Pape et al. (2003), and Rittle-Johnson and Koedinger (2005). The study was designed to investigate the impact that multiple layers of instructional scaffolds has on student learning and problem solving. Applying Anghileri's (2006) scaffold levels and Holton and Clarke's (2006) categories of heuristic and conceptual scaffolding, the study employed two types of scaffolding concurrently (i.e., student groups and graphic organizers). In addition, as in Rittle-Johnson and Koedinger (2005), the study focused on enhancing students' problem solving abilities by improving their conceptual knowledge. Finally, the study attempted to motivate students to reflect on their own learning, and thus, in a manner consistent with Pape et al. (2003), identify those scaffolding strategies that were most effective in their own learning.

The study addressed the following two research questions:

1. How do a variety of scaffolding techniques support student achieve ment on non-routine mathematics tasks?
2. What forms of scaffolding do students perceive as being most effective in supporting their problem solving abilities?

Methodology

The study employed a quasi-experimental, mixed-method design (Creswell, 2002). Specifically, quantitative and qualitative data were collated simultaneously and results were used to best address the research questions. Results from multiple data sources were then used to confirm and test conclusions (Creswell, 2002).

Setting and Participants

The study involved approximately 50 middle school students (sixth grade) distributed between two mathematics classes within the same school building. Both classes were taught by the same teacher (the first author). Data collection occurred during students' regularly scheduled class times. Both classes involved students with similar academic abilities. One of the classes served as the control group receiving only the four-square graphic organizer (described below) scaffolding interventions, while the other class received both the four-square graphic organizer and a variety of grouping scaffolding interventions. Both classes covered the same instructional content which lasted for nine weeks.

In each class, students were assigned to groups of four based upon the students' state mathematics assessment scores (from the prior year). Each group consisted of one high, two average, and one low student in terms of cognitive ability.

Data

Data included daily tasks, a state standardized geometry subtest, a geometry unit test, and individual attitude and perception surveys. Tasks were selected from previously released state assessment items, provided that task topics were similar to one another and aligned to the daily objectives of the class. The daily tasks were taken from pre-existing state assessment questions and included non-routine problems requiring students to demonstrate "comprehension of mathematical concepts, operations, and relations" (Kilpatrick, Swafford, & Findell, 2001, p. 5); that is, the problems were designed to assess students' conceptual knowledge.

Table 1 illustrates a set of tasks (all involving geometry) that were administered during one week of class with the level of student support decreasing each day.

Table 1: Sample Tasks and Types of Student Groups (Ohio Department of Education, OAT Released Items, 2006-2012)

Task	Type of Student Group
Which measure of the toy chest should he use to determine how much paint to buy? Justify your answer. a) height b) perimeter c) surface area d) volume	Monday: Full class instruction.
The square and the rectangle have the following dimensions (8 cm x 8 cm) and (4 cm x 12 cm). What description is true about the figures? Justify your answer. a. same area, same perimeter b. same area, different perimeter c. different area, same perimeter d. different area, different perimeter	Tuesday: Small group of four students.
A can of tomato soup and its label are shown (a can with label of 300mL). Which statement is true? a. The label represents surface area and the perimeter is 300 mL. b. The label represents surface area and the volume is 300 mL. c. The label represents volume and the surface area is 300 mL. d. The label represents volume and the perimeter is 300 mL.	Wednesday: Partners.
Jeb wants to cover the floor of a sixth-grade classroom with square tiles. What information does Jeb need? a. the area of the floor and the area covered by each tile b. the volume of the classroom and the volume of each tile c. the perimeter of the classroom and the perimeter of each tile d. the height of the classroom and the height of each tile	Thursday: Individually.
Discussion and review of problems.	Friday

Scoring of Items

Each square was graded on a two-point scale. Scores for the daily tasks were determined for each student by summing the number of points obtained in each square. Four-square graphic organizers were used to help direct student thinking and encourage metacognition. A sample four-square graphic organizer is shown in Figure 1.

Math Daily Tasks	
1.) What is being asked by this problem?	**2.) What are the facts of the problem?**
4.) Check your Math? _____ Yes Have you answered the question? ___ Yes Reflect (What do you notice about this problem in relation to others solved?)	**3.) Pick a strategy and show your work:**

Figure 1: Four-Square Graphic Organizer. This figure illustrates a sample graphic organizer used by students to direct their thinking and promote metacognition.

The first and second authors independently scored each task in order to determine inter-scorer reliability. The scores that each student earned by each rater were compared for consistency. Any inconsistent scores were resolved through discussion. Inter-rater correlations were computed by dividing the number of item agreements by the sum of the number of item agreements and item disagreements. The class inter-rater correlations were 0.89 for the control group and 0.95 for the experimental group.

The geometry unit test was administered at the end of the nine weeks and consisted of a comprehensive assessment of geometric concepts (area and perimeter, dilation, symmetry, nets, 2-3 dimensional geometry). After the unit test, students completed a three-item survey. The survey consisted of questions measuring student attitudes toward the types of scaffolding students preferred and asked students to explain their preferences. One final measure, taken at the end of the academic year, included the state standardized grade level exam with a focus on geometric reasoning and spatial sense.

Results

Daily Tasks

All statistical tests were conducted using an alpha of 0.05. Table 2 displays the mean correct scores of the two classes with similar mathematical abilities on daily tasks with various grouping configurations. TL, SG, P, I represent group configurations **T**ota**L** class participation**, S**mall **G**roup, **P**artners, and **I**ndividual, respectively.

Table 2: Mean Scores of Experimental and Control Groups

Experimental Group																
Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Grouping	TL	SG	P	I	TL	SG	P	I	TL	SG	P	I	TL	SG	P	I
Mean	4.99	3.90	3.2	5.21	3.98	5.81	4.98	5.31	4.94	5.48	6.08	5.5	4.17	5.35	5.55	5.66
Control Group (No group scaffolding)																
Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Mean	4.8	3.74	3.0	5.03	3.64	5.51	4.74	5.26	4.84	5.62	6.03	5.8	4.18	5.11	5.33	5.07

An independent-samples t-test was conducted to compare the daily task scores of students with and without grouping scaffolds. There was a significant difference in the scores for scaffolding (M = 5.036, SD = 0.838) and no scaffolding (M = 4.861, SD = 0.825) conditions; $p < 0.01$. These results suggest that students in the classroom that promoted collaboration (e.g., small groups, partners) performed significantly better than the classroom that worked independently. In addition, students commented that they were able to more effectively start problems and gained greater confidence when working in small groups or with a partner.

Once researchers ascertained that the use of a variety of grouping sizes could produce significantly better results on daily tasks compared with a traditional lecture-discussion of the tasks, the authors examined the differences between the control and experimental groups with respect to the unit test.

Unit Test on Geometry

The teacher-created unit test was administered at the end of a comprehensive unit on geometric concepts, but prior to the state standardized assessment. The comparison showed no significant differences. Box-and-whisker plots help to visualize the similarity of these results (Figure 2).

State Standardized Assessment Results

Although the state standardized assessment covers a sample of all mathematics topics for sixth grade, the scores on the geometry section of this assessment showed increases for both classes. The control group showed an increase of 4.27%, while the experimental class averaged an increase of 10.96%.

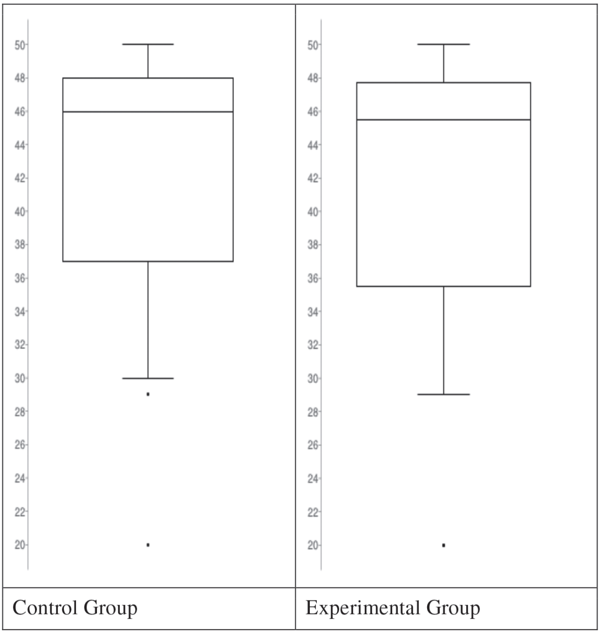

Figure 2: Mean Scores of Experimental and Control Groups

Student Surveys

Students completed a three-item survey containing questions that measured student attitudes toward the types of scaffolding students preferred and the rationale for their choices. Survey questions and responses are found in Table 4. For example, three students from the experimental groups judged that "whole class-teacher directed" instruction was the best form of grouping (Question 1).

Although the survey responses discussing students' rationale for their choices were qualitative in nature, a few patterns emerged. Students indicated an appreciation for the support they received from group members when solving non-routine problems. In particular, students stated that they appreciated working with partners. Overall, students indicated that the "daily task" four-square format was helpful in analyzing and solving the problem.

Discussion

The results of this study are consistent with those found in Rittle-Johnson and Koedinger (2005), which indicated that students can improve their problem solving skills with an increased focus on conceptual knowledge.

Table 4: Survey of Student Responses

Question 1 and Options	Control	Experimental
Which type of grouping do you feel is best for your learning?		
a. whole class-teacher directed	2	3
b. within a small group	7	2
c. with a partner	7	11
d. by yourself	6	3
e. does not matter	1	0

Question 2 and Options	Control	Experimental
What would you change in the problem of the day?		
a. Make it harder	2	2
b. Nothing	11	4
c. Make it easier	2	2
d. Change the type of question	3	5
e. Choose partners	2	0

Question 3 and Options	Control	Experimental
What else could help you to learn?		
a. Calculators	2	3
b. Visual/pictures	7	2
c. Groups	7	11
d. Being quiet	6	3
e. Alone	1	0
f. Showing work	0	0
g. Boxes for your answer	0	1
h. The teacher explaining in small groups	0	2
i. More time	0	0

The current study focused on improving students' problem solving skills by providing students with scaffolds as they engaged in tasks designed to support their development of conceptual understandings in geometry. Students showed an increase in mathematical problem solving skills over time with the use of peer grouping and a graphic organizer. In addition, consistent with results from Pape et al. (2003), students indicated—through their daily reflections and final survey—that they were developing an understanding of their role in their own learning process. In short, students began to understand what helps them become successful as a learner of mathematics. Although the majority of students in this study showed positive gains in their abilities to initiate problem-solving, there were some situational changes within the school that could have affected the study's findings. Specifically, the school was in their first year of implementing a block schedule, which doubled the time spent in mathematics courses. As such, students were in math class for 80 minutes daily instead of only 40 minutes. In addition, this was the first year that the school district switched over to full inclusion in all courses.

Future research should examine which particular scaffolding methods (e.g., particular group-sizes or graphic organizers) best support student learning. Furthermore, future research should examine how computer-based modes of scaffolding compare with traditional paper/pencil methods. Many variations on the research discussed in this paper can be used to further the collection of knowledge around scaffolding and its impact on learning in the mathematics classroom with a little help from their friends.

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