Prospective Primary Teachers' Beliefs about Mathematics

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Received: 28 October, 2013/ Revised: 16 March, 2014/ Accepted: 24 March, 2014
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Concerns about the negative mathematical beliefs of prospective primary teachers have arisen from evidence that such beliefs detrimentally impact future teaching practices. The aim of this study was to explore factors contributing to the effectiveness of a mathematics content-focused intervention designed to nurture positive mathematical beliefs. Utilising interviews, concept mapping, workshop observations and document analysis, shifts in the mathematical beliefs of prospective primary teachers were revealed. The role of the mathematics education tutors was critical in contributing to the development of positive beliefs. In particular, their abilities to develop a positive rapport with prospective teachers and to create learning environments conducive to increased understanding of mathematics concepts were most influential. Implications for primary mathematics teacher education courses are addressed.

Keywords · prospective primary teachers · teacher beliefs · teacher education · mathematics education

Introduction

Teacher quality is a growing concern for governments and educational stakeholders around the world (Australian Council of Deans of Education, 2012; Kuenzi, 2012; New Zealand Government, 2010; Organisation for Economic Cooperative Development, 2005). Particularly worrying is that primary teachers "are not being adequately prepared for teaching numeracy and mathematics" (Commonwealth of Australia, 2008, p. 21). Research has confirmed that a significant proportion of prospective primary teachers have inadequate understanding of the mathematical concepts they will be required to teach (Ball, 1990; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). This issue is compounded by the negative perception of mathematics that the majority of preservice primary teachers [PSTs] hold (Ambrose, 2004; Grootenboer, 2008; Lomas, Grootenboer, & Attard, 2012). Studies of teacher beliefs conducted across the Western world typically show that PSTs' beliefs about mathematics are quite narrow and rigid (Liljedahl, 2009; Szydlik, Szydlik, & Benson, 2003), have formed through many years of experience as students of mathematics, and are resistant to change (Forgasz & Leder, 2008; Lomas et al., 2012). Such beliefs are indicative of PSTs' future teaching practices and are likely to negatively impact their mathematics students (Rozelle & Wilson, 2012; White, Perry, Way, & Southwell, 2006). In fact, White et al. claim:

> … that negative beliefs may contribute to negative classroom teaching strategies, which may in turn contribute to negative pupil beliefs, attitudes and performance outcomes. If these pupils then go on to become teachers, a cycle of negativity may be created unless an appropriate intervention breaks the cycle (White, et al., 2006, p. 36).

The challenge faced by mathematics teacher educators, is how to break this cycle of negativity. Prospective teachers' beliefs about mathematics play a filtering role for new knowledge about teaching and learning the discipline; influencing the skills and knowledge they choose to develop and, ultimately, their classroom pedagogy (Rozelle & Wilson, 2012). Given the critical role beliefs play in perpetuating the cycle of negativity surrounding mathematics, there has been increased interest in exploring primary PSTs' beliefs and monitoring the changes to those beliefs as a result of deliberate intervention strategies throughout their teacher education programs (Grootenboer, 2008; Lomas, et al., 2012; McGinnis, et al., 2002). Studies have generally focused on the nature of PSTs' mathematical beliefs pre- and post-intervention (e.g., Philippou & Christou, 1998), with an increasing focus on the underlying structural or epistemological course elements considered responsible for shifts in PSTs' beliefs (e.g., Anhalt, Ward, & Vinson, 2006; Shilling-Traina & Styliandes, 2013).

Our aim was to examine the effectiveness of a mathematics intervention that was specifically designed to impact on the beliefs and mathematical content knowledge of primary PSTs. In particular, we sought to understand *why* shifts in PSTs' beliefs occurred (or not) so as to illuminate contributing factors to the success, or otherwise, of the intervention and further contribute to the evolving body of literature surrounding teacher education program development. Once identified, our intention is to conduct focused and systematic exploration of particularly salient aspects to aid our understanding of how best to modify our mathematics education courses. Knowing more about the existing mathematical beliefs of primary PSTs and how or why they are modified by the different professional education components and approaches adds vital information to improving the quality of our teacher education graduates and ultimately, the mathematical outcomes of our students. Therefore, it is not only important to explore and monitor shifts in PSTs' mathematical beliefs, but to understand the major underlying factors contributing to the effectiveness of the interventions designed to nurture positive beliefs towards mathematics and the teaching of the discipline.

Theoretical framework

The terms *beliefs, conceptions,* and *attitudes* are often used interchangeably. Similar to Philipp (2007), we accept the interrelatedness of the terms, considering *conceptions* of mathematics to encompass beliefs, understandings and views about the discipline; and *attitudes* to refer to the way an individual feels, acts and thinks about an idea or entity that may readily change given new knowledge or experiences. While there is no consensus on a definition of beliefs in the literature (Furinghetti & Pehkonen, 2002; Kim, Kim, Lee, Spector & DeMeester, 2013; Liljedakl, 2009), there is considerable agreement that beliefs are subjective, more resistant to change than attitudes, assumed to be true, organised in a systematic structured manner and can predispose people to act in particular ways (Lomas, et al., 2012). In this paper, *beliefs* refer to an individual's personal opinion on a specific issue or practice that is considered true. While the study of mathematical beliefs is classified in the literature into beliefs about the discipline, its teaching, student learning, and the like (Cross, 2009), researchers generally focus on a cluster of related beliefs or explore in depth a particular set of beliefs, such as teacher beliefs about technology (e.g., Kim et al., 2013) to give specificity to their research. In line with this research trend, *prospective teachers' mathematical beliefs* in this study, refers to the cluster of beliefs held by PSTs about the discipline of mathematics, mathematics teaching and learning, and about themselves as teachers of mathematics.

Green's (1971) theory of beliefs has been foundational to current research on teacher beliefs, with significant implications for the way beliefs are viewed. Exploring the systematic nature of beliefs, Green (1971) introduced three dimensions of belief systems. First, some beliefs may serve as the foundation for other beliefs in a quasi-logical relation and, in this sense, are referred to as either primary or derivative in nature. For instance, the belief that students 'should not use calculators' could be derived from the primary belief that students 'should be efficient at mental computation'. Secondly, some beliefs are more important for an individual than others, creating a centrality or set of core beliefs. Central beliefs are strongly held due to their connectedness and coherence to other sets of beliefs held by an individual and shared with those around them. The more central a belief, the less susceptible they are to change. Thirdly, beliefs are held in independent clusters—a property that explains possible inconsistencies found in an individual's belief systems, particularly when they are applied in different contexts (Green, 1971). For instance, a teacher may believe that working with technology is an integral part of mathematics. However, during mathematics examinations she may not permit the use of calculators because she believes its use might disguise a student's fluency of performing certain procedures. While the two beliefs appear contradictory, they may belong to different belief clusters—one relating to the effective use of technology in everyday contexts and another relating to assessment of mathematical procedures.

Thought processes and instructional behaviours of teachers are influenced by existing knowledge and personal beliefs, affecting the quality of classroom teaching. Thus beliefs are viewed as a type of subjective knowledge operating as the primary regulator for professional behaviour in the classroom (Op'T Eynde, De Corte, & Verschaffell, 2002; Thompson, 1992). A number of studies in mathematics education indicate teachers' beliefs about mathematics and mathematics teaching play a significant role in shaping their instructional behaviour, suggesting that teachers often act upon their beliefs as if they are knowledge (Beswick, Callingham, & Watson, 2011). It is possible for two teachers to have the same knowledge, and teach very differently as a result of their beliefs (Ernest, 1989). Therefore, beliefs regarding the nature of teaching and learning mathematics are important because they can have a powerful impact on the way in which mathematics is taught. This means that teacher knowledge and beliefs are closely intertwined, and increasingly "viewed as complementary subsets of the set of things we believe" (Leatham, 2006, p. 92).

This theoretically grounded distinction served as the framework for the current investigation. While this research recognises the interconnected nature of beliefs and knowledge and the inevitability that both will arise in discussion, it focuses on beliefs. It is the mathematical beliefs of PSTs that influence knowledge acquisition and interpretation, task definition and selection, interpretation of content and comprehension monitoring (Pajares, 1992). These beliefs form the foundation from which teachers will eventually build their own practices as teachers of mathematics (Rozelle & Wilson, 2012).

Prospective Primary Teachers' Mathematical Beliefs

Educational beliefs of PSTs play a pivotal role in their acquisition and interpretation of knowledge and subsequent teaching behaviour (Llinares, 2002; Ng, Nicholas, & Williams, 2010; Wilkins, 2008). The process of becoming a primary teacher of mathematics involves the development of beliefs, knowledge, attitudes and skills. PSTs' beliefs about teaching are formed early, shaped by their own experiences as students and are often resistant to change (Ng et al., 2012). In their investigation of PSTs' beliefs about teaching, Stuart and Thurlow (2000) concluded that pre-service teachers "have erroneous and simplistic beliefs about what it takes to be a successful teacher" (p. 114). In mathematics education this issue can be exacerbated by PSTs' negative beliefs about the discipline of mathematics itself (Forgasz & Leder, 2008; Thompson, 1992). Grootenboer and Schuck (2004) concluded that primary PSTs generally hold beliefs about mathematics that "prevent them from teaching mathematics in ways that empower children" (p. 58).

The negative impact that prospective primary teachers' rigid beliefs about mathematics might have on their future teaching and students is a perennial problem recognised internationally (e.g., Philippou, & Christou, 1998; Shilling-Traina, & Stylianides, 2013). Within Australia, the review on *Education of Mathematics, Data Science and Quantitative Disciplines* (Brown, 2009) highlighted a 'gap' in current teacher education programs, indicating the need to address beliefs and attitudes in addition to content and pedagogical knowledge in primary mathematics education studies. It recommended that additional components should be designed to build the "mathematical confidence" (Brown, 2009, p. 10) of our future primary teachers and that these components would be taught by staff from Mathematics departments in universities. While the proviso was added that such staff needed to be appropriately chosen, no details about what constituted "appropriate" were provided. Nevertheless, PSTs' beliefs about mathematics are likely to be important determinants of their approaches to teaching and teacher education programs need to assist pre-service teachers in appreciating the role of their beliefs and understanding the ways their beliefs may influence them and their teaching (Scott, 2005). Moreover, negative mathematical beliefs are often attributed to prior school experiences, and specifically attributed to the teacher, rather than specific mathematical content (Uusimaki & Nason, 2004).

Prospective primary teachers' beliefs towards mathematics form the foundation upon which they will eventually build their own practice. Unfortunately, these deep-seated beliefs frequently contradict current research on what constitutes desirable beliefs about mathematics (Grootenboer, 2008). Hence it is "the role of teacher education programmes to reshape these beliefs and correct misconceptions" (Liljedahl, 2009, p. 27).

Changing Prospective Primary Teachers' Mathematical Beliefs

There has been an abundance of intervention studies focused on improving pre-service primary teachers' mathematical beliefs in the past few decades (e.g., Ambrose, 2004; Grootenboer, 2008; Lomas et al., 2012), including studies that have asked participants about course aspects considered most valuable for their learning (e.g., Beswick, 2006). Most have utilized pre- and post-data collection processes to help monitor the impact on teacher beliefs as a result of an intervention experience—usually involving aspects of reform-oriented teaching practices (Anhalt, Ward, & Vinson, 2006). While a growing number of practices are now linked to positive shifts in PSTs' mathematical beliefs, a combination of reflective practices and structured fieldwork emerge as more influential than others (Forgasz & Leder, 2008). Since PSTs' mathematical beliefs have developed over a long period of time via a variety of experiences, it is essential that existing beliefs be critically reflected upon and that opportunities for new teaching experiences be carefully structured to give rise to positive changes.

In line with these findings, Grootenboer (2008) aimed at explicating PSTs' mathematical beliefs through reflective processes intended to promote belief transformation. The study confirmed prior research that beliefs are grounded in past school experiences, which are not desirable for mathematics teaching. More importantly, it highlighted the complexity of trying to reshape the beliefs of PSTs through university courses focused on cognitive aspects of teacher education when prospective teachers' developing practices and identities are "significantly affective in nature and directed by personal beliefs" (Grootenboer, 2008, p. 495).

Conceptual Framework

Recognising the many factors impacting teacher beliefs and practices, Raymond (1997) proposed a model that highlighted the direct influence past school experiences, family experiences, immediate classroom situations and teacher education programs have on the mathematical beliefs of teachers. Similarly, after considering research exploring teachers' beliefs and practices, Anderson, White, and Sullivan (2005) proposed a model that represented the influences on, and relationships between, teachers' problem-solving beliefs and their practices. Their model placed teachers' beliefs between their knowledge and practices, suggesting beliefs filter the knowledge that is embraced by teachers.

Combining and adapting aspects of models proposed by Raymond (1997) and Anderson et al. (2005), Figure 1 provides a representation of the conceptual framework underpinning this study. It specifically addresses our focus on primary PSTs' mathematical beliefs and factors influencing these.

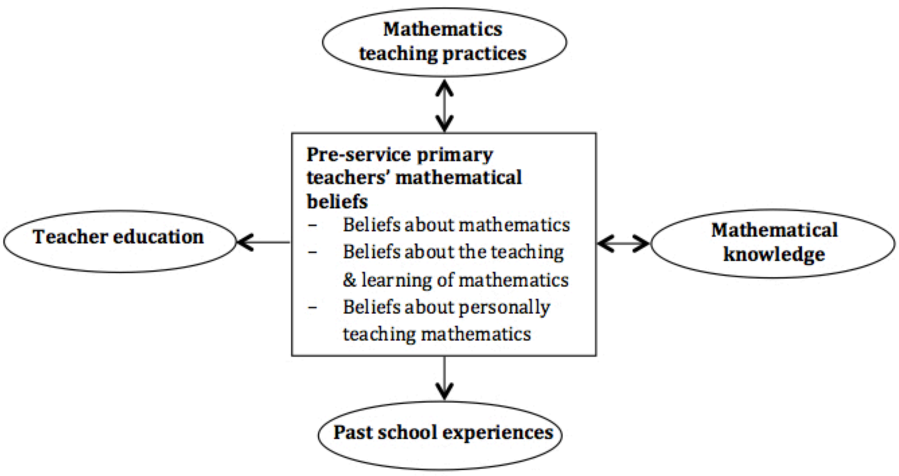

Figure 1. A model of factors contributing to mathematical beliefs of PSTs adapted from Anderson et al. (2005) and Raymond (1997).

Methodology

The study was designed to explore shifts in the mathematical beliefs of prospective primary teachers as they undertook a mathematics content-focused course. Since our aim was to identify salient aspects of the course that might contribute to such shifts for follow-up exploration, a qualitative study involving multiple case studies was considered most appropriate. While quantitative investigations involving whole cohorts have revealed shifts in the mathematical beliefs and attitudes of PSTs (White et al., 2006), they fail to enlighten researchers as to the qualitative reasons for such shifts.

A case study approach is of great value when there is little control over events and when qualitative understanding of why changes occur is sought, thus making such an approach viable for this study by focusing on the varying experiences of five participants (Cohen, Manion, & Morrison, 2007). Data were collected through multiple methods including some activities that were integral to the course implementation.

The research questions guiding the study were:

1. What impact does a mathematics intervention have on prospective primary teachers' mathematical beliefs?

2. What are the key factors contributing to shifts in prospective primary teachers' mathematical beliefs?

Participants and Setting

All 130 first-year PSTs enrolled in a four-year Bachelor of Education (BEd) program at an Australian metropolitan university were invited to participate in the study. Five PSTs (4 female and 1 male) volunteered to participate, ranging in age from 18 to 21 years. All five had completed their secondary education in New South Wales (NSW) high schools, having undertaken the NSW Higher School Certificate (HSC) in their final year of school (Year 12). All participants had studied the General Mathematics course—the lowest level of mathematics offered for study in the final two years of high school in NSW.

The mathematics content course at university for PSTs involved a one-hour lecture and a two-hour small-group workshop (approximately 25 students) per week over a 13-week semester during the first year of the BEd program. Three mathematics methods courses followed this course in subsequent years of the program. In line with course objectives, mathematics PSTs studied various mathematical content areas, including whole number, measurement, probability, statistics and algebra. The level of content covered was beyond that at which they would be required to teach at primary school, but sufficient to ensure they had a deeper understanding of the underlying mathematical concepts for each topic area. Students were asked to explore historical and cultural perspectives as well as real-life applications of the mathematics studied. The course implementation strategy was designed in accordance with research indicating features most likely to impact upon PSTs' beliefs (Forgasz & Leder, 2008). Namely, it specifically sought to challenge PSTs' existing beliefs about mathematics while increasing their mathematical content knowledge via collaborative/group work and inquiry-based learning experiences. Students were required to reflect actively on their own beliefs and experiences associated with learning mathematics at school and university via a range of activities. Lecturers and tutors in the course were mathematics educators who were experienced in both secondary and primary teacher education, who held university degrees in mathematics, and who had been selected based on their pedagogical affinity with reform-oriented teaching practices espoused in the course objectives. Neither researcher taught any of the participants when this study was conducted.

Data Sources and Collection

A range of data collection methods were used to provide multiple perspectives for identifying shifts in PSTs' beliefs and to help determine reasons for such shifts. Data were collected via four methods: semi-structured interviews; concept mapping; observations of participants during workshops; and assignments completed by participants as part of the course.

Interviews were held in two rounds, the first conducted prior to the course commencing and the second held towards its conclusion. A semi-structured method of interviewing was selected to account for the individual nature of beliefs. The interview questions focused on specific types of mathematical beliefs—beliefs about mathematics, as well as about the learning and the teaching of mathematics, including beliefs about themselves as teachers of mathematics. For example, interviewees were asked to comment on the importance of mathematics at school, their experiences of mathematics, and particularly influential teachers of mathematics.

Concept mapping was used to examine how individuals organise and change their beliefs and knowledge (Jones & Vesilind, 1996; Novak & Caña, 2006). During the first interview, participants received standardised instructions on how to create a concept map and were provided with an example on an unrelated topic. Participants were then asked to create a concept map focused on 'mathematics' and were encouraged to add to the concept map throughout the interview. The concept maps were revisited in the second interview, where participants were asked to study their previous map and make any desired modifications.

Observations were made of participants' involvement in the mathematics education workshops. Two workshops were observed for each participant, and field notes were completed for later analysis. The major purpose of these observations was to gain a grounded understanding of the nature, content, and expectations of the university course the participants were involved in. While it is acknowledged that beliefs are inherently 'unobservable', there are certain behaviours that may evince stated beliefs (such as "I'm not good at mathematics"). Hence, observations were focused on a participant's involvement in class-level activities (e.g., Do they initiate/contribute to discussions, listen attentively or appear disinterested?) and their on-task behaviour (e.g., What level of difficulty is being experienced as evident from their behaviour/completion times?). While it is acknowledged that a researcher's presence in workshops may have altered the behaviour of the participants, the observation field notes provided additional evidence of participants' engagement in the workshops and supplemented self-reported data.

Participants provided a copy of their assignment at the second interview. The assignment had required participants to select five mathematics tasks completed during workshops and reflect on their learning within each task. This material was used to assist with analysis of the interview data, and the reflective comments were used to provide further insight into the beliefs reported by participants.

Data Analysis

The qualitative nature of the data required an interpretive analysis. Digital recordings of the interviews were listened to multiple times in conjunction with several readings of the interview notes to assist with the analysis process. The researchers discussed themes as they emerged. Themes considered to be consistent with and to address the research questions were extracted for further analysis.

The concept maps were used to gain a snapshot of PSTs' beliefs about mathematics at two points in time (prior to the course intervention and towards its conclusion) and for noting any shifts in beliefs during that time. Maps were analysed using a qualitative, holistic approach. The basic units for analysis were *key nodes, links*, and *crosslinks* associated with PSTs' beliefs about mathematics. A *node* is a point on the concept map specifying a concept or piece of information. *Key nodes* occur when nodes are elaborated to become more specific, emphasised during the construction through circling or bolding of the word, or through the interview. *Links* are the lines connecting the nodes and indicate a relationship between nodes. Links made between different segments or domains of the map are referred to as *crosslinks* (Novak & Caña, 2006), and these provide information regarding the connectedness of concepts and beliefs.

Observation notes made during workshops were analysed in conjunction with the self-reported interview data and assignment samples. Analysis involved multiple readings of the notes to help identify examples (observable behaviour) of each participant's level of involvement in workshops. Additionally, assignment samples were read and used to supplement interpretations of concept maps and interviews. They provided an indication of any progressive transformation occurring, and concrete evidence of self-reported beliefs by the participants.

All collected data were used to form a case of each participant. Findings from the observations, interview data, and assignments were used to triangulate data and provided confirmation of the findings.

Results

In this paper, we present data from two participants—Peter and Lauren (pseudonyms)—to illustrate typical responses found in all five cases.

Case Study 1: Peter

Peter (aged 18) was a full time student in his first year of the BEd (Primary) program. He reported that the sole reason for him choosing to study Mathematics for his HSC was the belief that it was a requirement for entry into the teacher education program: "If it were not a requirement I wouldn't have done it". Peter had no formal experiences teaching mathematics in primary schools at this stage in his studies but had observed a few lessons in the previous semester during field visits.

Beliefs about mathematics. Data from the initial interview and concept map indicate that Peter held strong negative feelings towards mathematics. When asked, "What comes to mind when you hear the word 'mathematics'?" Peter immediately responded with "boring" and "tests" (see Figure 2). This is consistent with research findings suggesting primary PSTs generally hold negative beliefs about mathematics (e.g., White et al., 2005/2006), and was a common feeling amongst our subjects. Analysis of Peter's initial concept map shows that the majority of the nodes related to mathematics content such as multiplication, algebra, and division. There are no links drawn between these content areas, suggesting a utilitarian view of mathematics with various mathematics concepts isolated from each other. This also suggests that the content of mathematics is viewed as separate entities and perhaps Peter does not hold a holistic view of what mathematics entails.

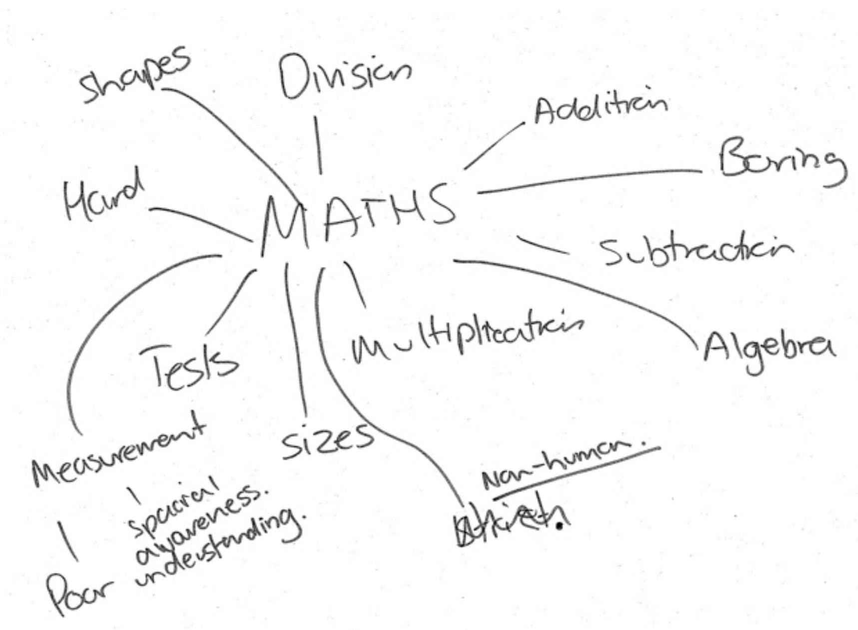

Figure 2. Peter's initial concept map.

Nodes or key nodes indicative of intense negative beliefs about mathematics include terms such as "hard", "boring", and "non-human". Peter explained his reasons for including these terms during the interview, saying that he regarded mathematics to be "pointless" as it never related to his life. He considered his beliefs about mathematics were formed mainly as a result of his experiences during high school. He related incidents heavily reliant upon textbooks that were compounded by a lack of confidence in his mathematics content knowledge, resulting in him "hating maths throughout high school" and possessing an intense "fear that [he] was going to keep failing".

Peter described mathematics teachers as, "callous, sadistic, mean-spirited, fun-killing and boring, un-empathetic, uncompassionate", causing him to change "strict" to "non-human" on his initial concept map. This is the only concept identified as a key node on the concept map, indicated by the underlining of the word—an important element to note as it identifies an intense emotional aspect in Peter's beliefs about mathematics.

The observational notes made during Peter's involvement in mathematics content-focused tutorials early in the semester indicated that he was behaviourally engaged with the activities, but was not concerned with the mathematics involved in the tasks. Peter participated and completed tutorial activities, but maintained high involvement in conversations unrelated to the task. He did not contribute to discussions about the tasks with peers or the tutor, and conveyed little cognitive interest in the mathematics.

Towards the end of the course, a second round of interview, concept map, and observational data were collected. Peter demonstrated a remarkable shift in his beliefs towards a more positive view of mathematics than previously held. He attributed this shift predominantly to the rapport he had developed with his tutor: "I think it is the first time in the history of the world that a mathematics teacher has a sense of humour". Nevertheless, throughout the interview a persistent negative belief towards mathematics remained. When faced with the original concept map, Peter added new words, including "creative" and "practical" indicating clear shifts in the way he viewed mathematics (see Figure 3). The words "boring" and "test" were removed, and he qualified this during the interview by stating that he no longer views mathematics as "universally a synonym for boring", but considered that if it was ineffectively taught it could be boring. Peter no longer made an association between mathematics and tests, stating "tests aren't necessary at all in the primary classroom".

The results indicate that Peter's experiences during the mathematics course contributed to an increasingly positive view of mathematics. However he continued to view mathematics as a "non-human" disciplinary area. While it is clear the mathematics intervention had a positive impact on Peter's beliefs about mathematics, much more needs to be done in order to ensure he does not maintain such beliefs as he enters the teaching profession and influence those of his students.

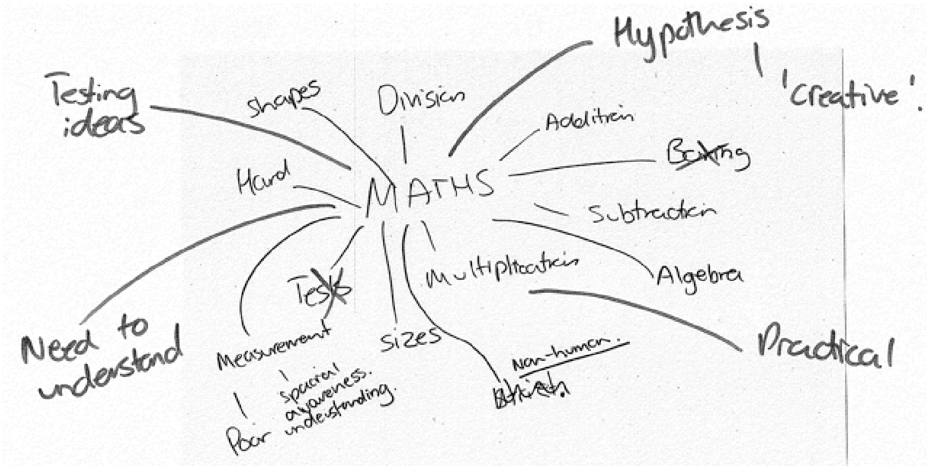

Figure 3. Peter's second concept map.

Beliefs about the teaching and learning of mathematics. During the first interview Peter stated, "I think it's the role of a maths teacher in a primary school class to establish and monitor children and capitalise on their knowledge". He recalled a recent field experience spent observing a mathematics lesson that left him with the belief that mathematics "cannot be taught effectively through practical activities … it was the worst activity I've ever been involved in ... [it] made me question why I want to be an primary school teacher". As noted by others (e.g., Wilson, Floden, & Ferrini-Mundy, 2002), field experiences are so influential that they can determine whether teacher preparation is enhanced or hindered.

At the second interview, the assignment work sample was considered. The discussion highlighted a definite shift in Peter's views about how mathematics *should* be taught and its associated impact on how it is learnt. For example, he expressed the need for the development of a "hypothesis and testing of ideas" in mathematics, likening effective teaching to scientific inquiry through which students explore mathematical concepts. Peter also explained that students "need to see a practical application of the mathematics" that is being taught, an idea emphasised in his second concept map through nodes such as "practical" and "need to understand" (see Figure 3). This reflected a shift towards current reform-oriented views of teaching and learning mathematics that recommend teachers take a more problem solving and inquiry-based approach than they currently do (Common Core State Standards Initiative, 2010).

Despite the shift towards more positive beliefs about teaching and learning mathematics than his pre-intervention state, Peter continued to view "maths teachers, maths teaching, and maths as an illogical, absurdist, dysfunctional cycle of logical poverty", explaining that "maths has lost its spunk but refuses to die" in society. He also still questioned the importance of mathematics to the learner. The initial interview had highlighted that while Peter placed an importance on mathematics, he felt as though this was heavily dependent upon the effectiveness of the teaching. Such a view indicated that although Peter personally held negative views towards mathematics, he was aware of the need for it to be taught effectively. This belief remained largely unchanged by the second interview. However he then questioned the lack of time attributed to learning mathematics in schools, indicating a shift in his beliefs about its importance in the curriculum.

When asked how confident he felt about teaching mathematics during the first interview, Peter expressed "very little confidence". He explained, "My lack of confidence in my own maths ability means just the idea of having to teach maths is daunting". Peter's lack of confidence emanated from his perceived low level of content knowledge and clearly influenced his beliefs about his future teaching of mathematics.

A key difference seen in the second interview was the shift in Peter's thinking from being a *learner* of mathematics to becoming a *teacher* of mathematics. Through his experiences with the mathematics course, Peter explained that he now saw the way in which he was taught mathematics to be "boring" and "hard" and recognised that there were other ways to teach mathematics effectively. He noted that previously he was unable to see how mathematics could be taught without a textbook and had never experienced mathematics where there was not some form of textbook involved prior to participating in this course. Overall, it is apparent that Peter's beliefs about the teaching and learning of mathematics shifted to becoming more positive in nature than pre-intervention data indicated. However, he did not yet feel confident calling himself "a teacher of mathematics".

Case Study 2: Lauren

Lauren (aged 19) was a full time student in her first year of the BEd (Primary) program. During the first interview she explained, "If I wasn't choosing to do primary education I might have thought not to do maths". She believed mathematics looked "relevant" to primary teaching, and described it as "really helpful". Lauren had no formal experiences teaching mathematics in primary schools at this stage in her studies. She was in a different tutorial group from Peter and hence, had a different tutor.

Beliefs about mathematics. During the first interview, Lauren did not express negative beliefs about mathematics explicitly. However, insight concerning her level of interest and confidence in the subject was gained when she stated, "I much prefer to be doing art than having to sit down and do maths". When asked to record what comes to mind when "mathematics" is mentioned, via her concept map, Lauren initially responded with "important foundations" and "rote learning" (see Figure 4). She explained these nodes, stating "I struggled with maths in high school years. I was fine with the rote learning and basic functions ... but when it came to the extra stuff I really struggled". Lauren's concept map contained only two nodes relating to her personal beliefs about mathematics, "can be boring" and "difficult". It is clear that Lauren's views of mathematics were largely influenced by her experiences in high school and the difficulties she had as a learner of mathematics.

Although Lauren related a positive experience with mathematics in primary school, she recalled falling behind in high school because "there [were] all these new things I had to learn and apply". Like Peter, her negative beliefs about and confidence in mathematics were strongly linked to her high school experiences, particularly those regarding her difficulties understanding content. She found mathematics intimidating because she "didn't want to get it wrong". Her comments are indicative of a belief that mathematics is about getting one right answer—a belief also conveyed in her concept map (Figure 4).

The observational notes made during Lauren's involvement in mathematics tutorials indicated a high level of engagement with the activities. She regularly asked questions of both peers and the tutor in order to understand the content and she often contributed to class discussions. Such behaviour suggested a strong sense of willingness to engage with and understand the mathematics content and an overall appreciation of its value. These aspects were also reflected in her initial concept map and indicative of relatively positive beliefs about the nature of mathematics and about herself as a learner of mathematics: namely, that it is important to everyday life and that her ability to understand mathematics is influenced by the effort she applies to learning it. Overall, Lauren seemed to display a combination of both positive and negative beliefs about mathematics during the first round of data gathering.

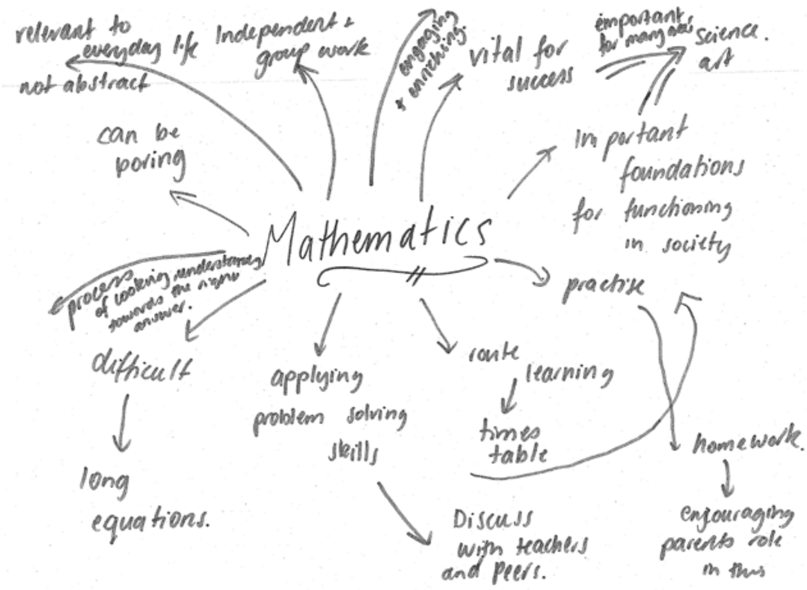

Figure 4. Lauren's initial concept map.

It was evident from the second interview and concept map that a shift in Lauren's beliefs had occurred. When faced with the original concept map, she added several new ideas and identified key nodes regarding her beliefs about mathematics (see Figure 5). The words "can be boring" and "difficult" were removed, with the explanation that mathematics is no longer viewed as "always being difficult".

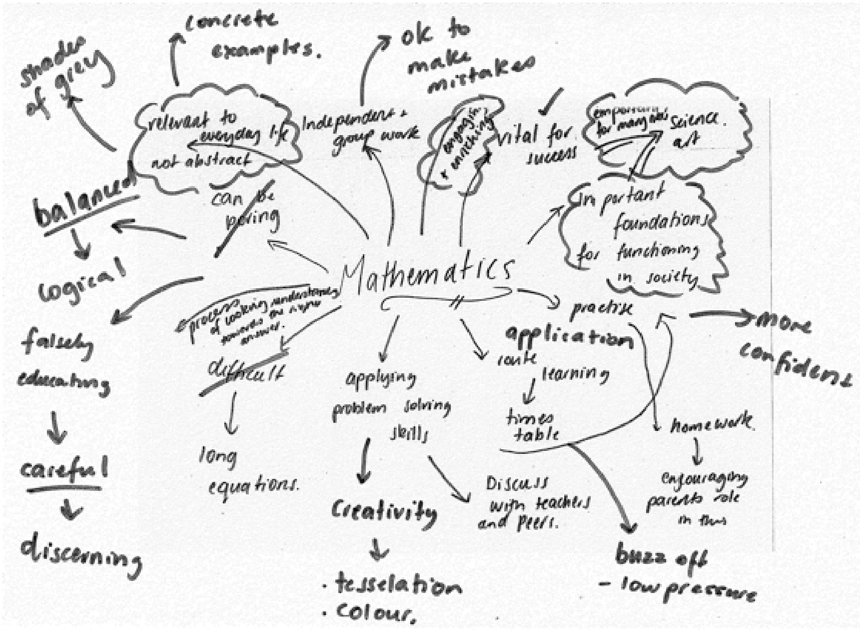

Figure 5. Lauren's second concept map.

Additionally, Lauren no longer viewed mathematics as "boring", but believed that "it's the teacher's fault for making it boring". Lauren considered the tutorial activities, combined with her tutor's personal interest in mathematics and his teaching practices helped her develop new perspectives of mathematics: "I never thought maths was creative ... I never thought maths was something you could share and enjoy".

When asked how she felt about mathematics in the second interview, Lauren said that she now views "mathematics" and "maths" differently—as two distinct ideas. She explained "mathematics is the application of maths in everyday life", a characteristic reflecting those commonly used to describe "numeracy" (Commonwealth of Australia, 2008). Lauren stated, "I enjoy mathematics because it is the applications ... now that I can understand the maths behind the mathematics I can say "Yes, I enjoy mathematics". This indicates that Lauren's beliefs about mathematics were linked largely to her perceived level of understanding of the relevance of content to everyday life. However, Lauren maintained that she still did not view mathematics so positively.

Lauren acknowledged that her beliefs about mathematics had changed as a direct result of the course, "I think I had a very set idea about what mathematics was; I now think maths is a lot more fluid and a lot more adaptable". She now realised that it was "Okay to make mistakes" in mathematics. Lauren attributed these changes in her views mostly to her tutor and the manner in which he taught, explaining that she felt more comfortable asking questions to clarify her understandings than she had in high school.

Beliefs about the teaching and learning of mathematics. During the first interview, Lauren stated that the role of a mathematics teacher was "first to model what you do in mathematics and then encourage [students] to apply it to different questions". Such a response indicates that Lauren saw the teaching and learning of mathematics as a process of transmitting knowledge from teacher to students. Lauren's initial concept map (see Figure 4) contained no key nodes but rather a collection of nodes that relate to her beliefs that mathematics should be taught in a way that is "relevant to everyday life", through the application of "problem solving skills", involving discussion and "independent and group work". While these views are aligned with progressive teaching practices advocated in current curricula, Lauren was clear that her responses were idealistic rather than indicative of her own experiences. Hence, her initial beliefs about the teaching of mathematics reflected a mix of traditional transmissive-style views—probably a result of her personal experiences learning mathematics—and reform-oriented views that reflected her personal aspirations as to how it should be taught.

The second interview, concept map and assignment sample highlight shifts in Lauren's views about mathematics teaching and learning, whereby her initial 'idealistic' views were strengthened and the conventional views were rarely mentioned. Lauren created a number of key nodes on her second concept map, emphasising the importance of "relevant to everyday life", "engaging and enriching", "important for many areas", and "important foundations". She also added "careful" and "balanced" as key nodes (see Figure 5). Lauren explained that she believed mathematics should be taught through the use of "carefully planned practical activities grounded in concrete mathematical ideas". She thought that mathematics should be "relevant" and "engaging" for the students. These sentiments were also evident when Lauren excitedly discussed her assignment involving an investigation of a ballerina's body proportions. She used her own investigation as an example of how mathematics should be taught through inquiry and made relevant to everyday life.

Shifts in Lauren's beliefs about the teaching and learning of mathematics were influenced largely by her realisation that mathematics could be creative. The assignment and tutorial tasks "gave [her] new perspectives on *how* to teach maths". Lauren clearly exhibited a positive energy towards the teaching and learning of mathematics at the second interview and considered that "maths should be entertaining, exciting, and adventurous".

Lauren was apprehensive when asked about her confidence in teaching mathematics at the initial interview. She explained, "I think I would definitely feel confident up to year four, but maybe with five and six I would have to brush up on my maths skills". While Lauren felt "happy with my understanding of maths", she considered "applying it and then teaching it … will be another step". Evidently Lauren recognised the complexities of teaching mathematics and while she felt confident with her current knowledge of primary-level mathematics, she was hesitant about her ability to teach it effectively.

During the second interview, Lauren explained that while her increased understanding of mathematics would make her a better teacher, she felt increased knowledge of the syllabus content was still required. Given that Lauren was only at the start of her teacher education program, it is understandable that she felt unprepared in terms of syllabus knowledge.

Discussion

These results confirm established research findings regarding the undesirable nature and origins of PSTs' mathematical beliefs (Beswick et al., 2011; Wilkins, 2008). Peter and Lauren's initial beliefs about mathematics and the teaching of mathematics were directly related to their past school experiences, which is consistent with the research model shown in Figure 1. Our findings further contribute to this body of research by highlighting the centrality to prospective teachers' mathematical belief formation of two salient experiences—their perceived lack of mathematical understanding and their interactions with mathematics teachers, particularly during the actual process of teaching/learning mathematics at school. More to the point, the purpose of this study was to identify key factors contributing to the effectiveness of an intervention designed to nurture positive beliefs towards mathematics and the teaching of the discipline in primary PSTs. Through the illustrative case studies of Peter and Lauren, our findings provide insight into why and how their mathematical beliefs began to shift during a course that focused on developing content knowledge and positive beliefs about mathematics.

The findings highlight substantial shifts in the beliefs of both prospective teachers. Prior to the mathematics course, Peter's difficulties understanding content, his experiences of "boring" lessons, a fear of failure during both primary and high school, and a recent traumatic experience during an observational classroom visit as part of his teacher education program, all fuelled intense negative beliefs about mathematics. This was compounded by his strong belief that mathematics teachers are "non-human". Similar difficulties were noted in Lauren's case, with a perceived lack of content knowledge and experiences of "maths anxiety" being most influential in the formation of her negative beliefs. However, unlike Peter, she expressed many positive, albeit idealistic, beliefs about the way mathematics should be taught and learnt.

Towards the end of the intervention, data collected through interviews, concept maps, observations, and discussions around assignments, demonstrated that both Peter and Lauren experienced substantial shifts in their mathematical beliefs. Furthermore, they indicated a growing detachment from their negative experiences of mathematics during school; both identifying positively charged, salient experiences in the mathematics course that triggered a transformation of their beliefs. In Peter's case, he enjoyed the tutor's "humour" and no longer viewed mathematics as "universally a synonym for boring". On the other hand, shifts in Lauren's beliefs were triggered by increased confidence in understanding content that also resulted from salient interactions with her tutor and peers during class discussions and inquiry-based contexts. She experienced "excitement" by revelations that mathematics could be relevant, shared and enjoyed. Importantly, Peter and Lauren's changed beliefs about the nature of mathematics, fuelled by the positive interactions with their tutors, influenced not only the teaching approaches they aspire to implement in their mathematics classrooms of the future, but also how they started to see themselves as teachers of mathematics. In both cases, engagement with the course fostered the desire to use "practical" and "creative" approaches to teaching mathematics. The most substantial shift was seen in Peter's beliefs, he now considered mathematics teaching possible without textbooks, and emphasized the need for a clear practical application of content.

As mentioned earlier, beliefs are resistant to change mainly because they develop over many years of experience (Forgasz & Leder; 2008; Kim et al., 2013; Pajares, 1992). Hence, to affect desirable changes in primary PST's mathematical beliefs it is critical that they not only reflect on existing beliefs, but observe and experience many positively charged teaching and learning experiences involving mathematics throughout their teacher education program. It is unlikely that a one-off, semester-long intervention aimed at nurturing positive mathematical beliefs can change entire belief systems that have formed over many years and almost definitely will not transfer to classroom practice without further targeted support. Obviously, gradual and sustained change needs to be nurtured via repeated exposure to positive experiences that match PSTs' progression throughout their education program. Our findings suggest that the interactions between lecturers, tutors and PSTs are another crucial factor to be considered in the mix of elements intent on fostering positive beliefs about mathematics in teacher education programs.

The initial research model provided the framework for this study, but the research findings suggest that this model does not expose all factors influencing belief formation/transformation. A revised model is presented in Figure 6.

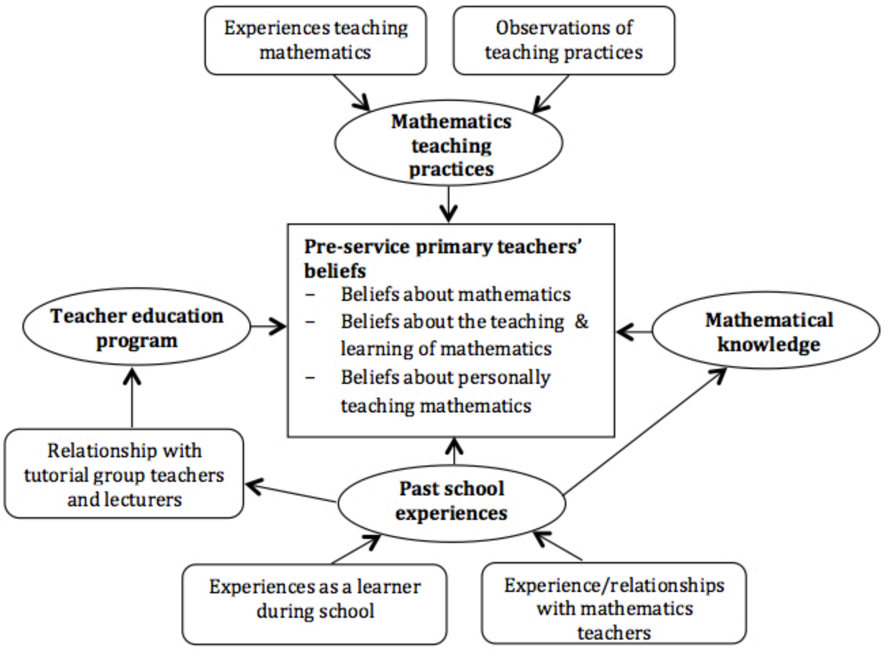

Figure 6. A revised model of factors contributing to mathematical beliefs of prospective primary teachers.

This new model refers to significant experiences at school, in particular the impact that past mathematics teachers have on mathematical beliefs. Additionally, the model now also recognises the potential influence of interactions with mathematics lecturers/tutors on PSTs' mathematical beliefs during their teacher education program.

Limitations and Future Research

Limitations need to be noted when considering the results and implications of our study. First, with such a small sample, the findings cannot be viewed as indicative of shifts occurring in the mathematical beliefs of all PSTs involved in the intervention. The selection of a qualitative multiple case study methodology was based on our aim to identify and better understand the salient factors contributing to shifts in mathematical beliefs of PSTs. Hence, we needed to know *why* changes occurred as well as what and how beliefs changed. While fairly typical of all five cases, only two were selected for reporting here due to the necessity of providing rich data, revealing important qualitative details. In short, our research methodology was well suited to our goal.

Second, while predominantly focused on the student teachers, it became obvious that salient interactions between PSTs and their mathematics tutors were particularly relevant to our investigation. As far as we are aware, the potential influence of such interactions has received little recognition in the literature on PSTs' mathematical beliefs and has certainly not been a consideration in the recommendations of reviews surrounding mathematics teacher education (e.g., Brown, 2009). This finding provides a clear direction for our on-going research that seeks to explore the factors impacting on PSTs' mathematical beliefs in greater detail than provided here. Future research should involve video recordings of lectures and tutorials and subsequent stimulated recall interviews to assist the detailed analysis of salient interactions and their potential for nurturing positive mathematical beliefs in prospective teachers.

Conclusion

Literature reports the need to address not only PSTs' content knowledge but also their beliefs about mathematics; as together these form the foundation on which primary teachers eventually build their own teaching practices (Brown, 2009; Grootenboer, 2008; Kim et al., 2013). However, simplistic strategies such as the introduction of additional content-focused courses intent on building the "mathematical confidence" (Brown, 2009, p. 10) of our future primary teachers, may be insufficient if the lecturers/tutors implementing the strategies lack desirable qualities, including the ability to develop a positive rapport with PSTs and create secure learning environments conducive to increased understanding of mathematics concepts. By identifying the potential importance of such qualities, our findings bring into question the benefits of primary teacher mathematics courses being taught by academics solely because they work in university departments of mathematics, as recommended by Brown (2009). Provisions regarding 'appropriateness' of staff should include consideration of their ability to create significant mathematical experiences capable of sparking positively charged responses in primary PSTs. In the current study, such positive experiences seemed to emerge from personal interactions over a semester between PSTs and their tutors.

The results of this study also suggest that primary PSTs' mathematical beliefs influence how they see themselves as teachers as well as the teaching approaches they aspire to implementing in future classrooms. Increasing PSTs' understandings of their beliefs might assist them see that possessing positive mathematical beliefs will impact upon how they manage the mathematical learning environment, including their personal interactions with students.

Our findings indicate the need to select tutors and lecturers for mathematics content and methods-focused courses in primary teacher education programs carefully, to help break down the stereotyped images of mathematics, mathematics teachers, and mathematics pedagogy. In particular, while the mathematics content knowledge of the tutors is critical, they must also be able to develop a positive rapport with PSTs.

It is important for teacher education programs to continue to nurture the transformation of mathematical beliefs; ensuring PSTs possess positive beliefs about mathematics when entering the teaching profession.

References

Ambrose, R. (2004). Initiating change in prospective primary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7*, 91–119.

Anderson, J., White, P., & Sullivan, P. (2005). Using a schematic model to represent influences on, and relationships between, teachers' problem-solving beliefs and practices. *Mathematics Education Research Journal, 17*(2), 9–38.

Anhalt, C., Ward, R., & Vinson, K. (2006). Teacher candidates' growth in designing mathematical tasks as exhibited in their lesson planning. *Teacher Educator, 41*(3), 172–186.

Australian Council of Deans of Education (2012). *ACDE report to AITSL on Standards 3.1 and 3.2*. Retrieved March 20, 2013 from http://www.acde.edu.au/pages/page15.asp

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *Primary School Journal, 90*(4), 449–466.

Beswick, K. (2006). Changes in preservice teachers' attitudes and beliefs: The net impact of two mathematics education units and intervening experiences. *School Science and Mathematics, 106*(1), 36–47.

Beswick, K., Callingham, R., & Watson, J. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 14*(1), 1–27.

Brown, G. (2009). *Review of education in mathematics, data science and quantitative disciplines.* Turner, ACT: The Group of Eight.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. New York: Routledge.

Common Core State Standards Initiative (2010). *Common Core State Standards for mathematics*. Retrieved September 20, 2013 from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Commonwealth of Australia (2008). *National numeracy review report*. Canberra: Author.

Cross, D. (2009). Alignment, cohesions, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education, 12*, 325–346.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching: International Research and Pedagogy, 15*(1), 13–33.

Forgasz, H., & Leder, G. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan and T. Wood (Eds.), *The international handbook of mathematics teacher education: Volume 1, Knowledge and beliefs in mathematics teaching and teaching development* (pp. 173–192), Rotterdam, The Netherlands: Sense Publishers.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39–57). Dordrecht, The Netherlands: Kluwer Academic.

Green, T. F. (1971). Teaching and the formation of beliefs. In T. F. Green (Ed.), *The activities of teaching*. New York: McGraw Hill.

Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*(6), 479–497.

Grootenboer, P., & Schuck, S. (2004). Affective issues in mathematics education. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Review of mathematics education in Australasia 2000-2003* (pp. 53–74). Sydney: MERGA.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Jones, M. G., & Vesilind, E. M. (1996). Putting practice into theory: Changes in the organization of preservice teachers' pedagogical knowledge. *American Educational Research Journal, 33*(1), 91–117.

Kim, C., Kim, M. K., Lee, C., Spector, M., & DeMeester, K. (2013). Teacher beliefs and technology integration. *Teaching and Teacher Education, 29*, 76–85.

Kuenzi, J. J. (2012). *Teacher quality issues in the Primary and Secondary Education Act*, Congressional Research Service. Retrieved February 15, 2013 from://www.fas.org/sgp/crs/misc/R42127.pdf

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*, 91102.

Liljedahl, P. (2009). Components of mathematics teacher training. In D. L. Ball & R. Even (Eds.), *The professional education and development of teachers of mathematics* (pp. 25-31). New York: Springer.

Llinares, S. (2002). Participation and reification in learning to teach: The role of knowledge and beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 195–209). Dordrecht, The Netherlands: Kluwer.

Lomas, G., Grootenboer, P., & Attard, C. (2012). The affective domain and mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Research in mathematics education in Australasia: 2008-2011* (pp. 23–37). Rotterdam, The Netherlands: Sense Publishers.

McGinnis, J. R., Kramer, S., Shama, G., Graeber, A. O., Parker, C. A., & Watanabe, T. (2002). Undergraduates' attitudes and beliefs about subject matter and pedagogy measured periodically in a reform-based mathematics and science teacher preparation program. *Journal of Research in Science Teaching*, *39*(8), 713–737.

Ng, W., Nicholas, H., & Williams, A. (2010). School experience influences on preservice teachers' evolving beliefs about effective teaching. *Teaching and Teacher Education, 26*, 278–289.

Novak, J., & Caña, A. (2006). The origins of the concept mapping tool and the continuing evolution of the tool, *Information Visualisation Journal, 5*(3), 175-184. Retrieved February 3, 2013 from://ivi.sagepub.com.
New Zealand Government (2010). *A vision for the teaching profession: Education workforce advisory group report to the Minister of Education*. Wellington, NZ: Author.
Organisation for Economic & Cooperative Development [OECD] (2005). *Teachers matter: Attracting, developing and retaining effective teachers*. Paris: Author.
Op'T Eynde, P., De Corte, E., & Verschaffell, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen & G. Turner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 13–37). Dordrecht, The Netherlands: Kluwer Academic Publishers.
Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.
Philipp, R. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age Publishing.
Philippou, G. N., & Christou, C. (1998). The effects of a preparatory mathematics program in changing prospective teachers' attitudes towards mathematics. *Educational Studies in Mathematics, 35*,189–206.
Raymond, A. M. (1997). Inconsistency between a beginning primary school teachers' mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550–576.
Rozelle, J., & Wilson, S. (2012). Opening the black box of field experiences: How cooperating teachers' beliefs and practices shape student teachers' beliefs and practices. *Teaching and Teacher Education, 28*, 1196–1205.
Scott, A. L. (2005). Pre-service teachers' experiences and the influences on their intentions for teaching primary school mathematics. *Mathematics Education Research Journal, 17*(3), 62–90.
Shilling-Traina, L., & Stylianides, G. (2013). Impacting prospective teachers' beliefs about mathematics. *ZDM Mathematics Education, 45*, 393–407.
Stuart, C., & Thurlow, D. (2000). Making it their own: Preservice teachers' experiences, beliefs, and classroom practices. *Journal of Teacher Education, 51*(2), 113–121.
Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre service primary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*(3), 253–279.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 390–419). New York, NY: Macmillan.
Uusimaki, L., & Nason, R. (2004). Causes underlying pre-service teachers' negative beliefs and anxieties about mathematics. *Proceedings of the 28th Annual Conference for the Psychology of Mathematics Education* (Vol. 4, pp. 369–376). Melbourne: IGPME.
White, A. L., Perry, B., Way, J., & Southwell, B. (2005/2006). Mathematical attitudes, beliefs and achievement in primary pre-service mathematics teacher education. *Mathematics Teacher Education and Development, 7*, 33–52.
Wilkins, J. L. M. (2008). The relationship among primary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139–164.
Wilson, S., Floden, R., & Ferrini-Mundy, J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education, 53*(3), 190–204.

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