Changing Pre-service Elementary Teachers' Beliefs about Mathematical Knowledge

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Studies have reported that pre-service teachers often enter teacher preparation programs with beliefs and attitudes not conducive to teaching the subject conceptually. In the USA, the Common Core State Standards for Mathematics have brought a renewed focus on procedural and conceptual understanding. However, many U.S. pre-service teachers have developed a procedural focus from their own schooling experience. This study investigated the effect of a mathematics and pedagogy course focused on conceptual understanding on one class of U.S. preservice elementary teachers' beliefs about mathematical knowledge. The course used the Lesh Translation Model (Lesh, 1979) to build conceptual understanding through multiple representations. While the change in beliefs from the beginning to the end of the course was investigated, this study also specifically investigated the change in beliefs arising from session activities concerning division by fractions. The course combined difficulties that students can have when taught procedurally, shown with example video, and conceptual understanding that students can display when taught with well-structured activities. This proved to be a useful combination for changing pre-service teachers' beliefs by showing the need to learn fraction division differently and then providing conceptual ways to think about this concept.

Keywords · prospective primary teachers · teacher beliefs · fractions · attitudes · teaching styles

Teacher beliefs are of great importance for understanding mathematics teaching and learning (Philipp, 2007). A focus on teachers' beliefs is based on the assumption that beliefs are the best indicators of the decisions that individuals make throughout their lives, (Bandura, 1986; Dewey, 1933; Nisbett & Ross, 1980; Pajares, 1992) and there is evidence as well that teachers' beliefs about mathematics impact their teaching of mathematics (Hart, 2002; Philipp, 2007; Steele, 1994). Studies have reported that pre-service teachers often enter teacher preparation programs with beliefs and attitudes not conducive to teaching the subject conceptually and in ways that support making meaning (Ball, Lubienski, & Mewborn, 2001). Because of this, there is growing interest in exploring different aspects of teacher education programs that have the potential to affect changes in pre-service teachers' beliefs (Charalambous, Panaoura, & Philippou, 2009). An essential question to be explored is the extent to which teacher education programs affect change in pre-service teachers' beliefs about knowledge and knowing (Cheng, Chan, Tang, & Cheng, 2009).

One consideration to change beliefs about teaching and learning mathematics is to involve pre-service teachers in a context that allows them to look at the topics they will teach in a different manner (Furinghetti, 2007). Furinghetti (2007, p. 113) has called this "reorientation", providing pre-service teachers the opportunity to experience again the construction of mathematical ideas. One possible way to do this is to focus on topics that pre-service teachers have learned procedurally in the past and provide them opportunities to reconstruct this content in a more meaningful way. Fraction division has the potential to be such a context.

Ma's (1999) comparative study of Chinese and U.S. elementary teachers brought attention to the topic of fraction division. This topic can challenge the conceptual understanding of even those who have done well in mathematics, and is, therefore, a good example of the reliance on rote learning of procedures (Li & Kulm, 2008). In Ma's study, less than half of the U.S teachers were able to give a correct solution to a fraction division problem, while the Chinese teachers showed greater coherence and flexibility in their knowledge and explanations. The Chinese teachers had several explanations for fraction division, including the partitive model of division, while the U.S. teachers tried to remember procedures. The Chinese teachers also demonstrated a knowledge base that included the meaning of multiplication with fractions, concepts of unit, the meaning of division, and the meaning of fractions.

Tirosh (2000) also investigated fraction division, and found that in a class of pre-service elementary teachers from Israel most of the class knew how to divide fractions but could not explain why the procedure worked. Tirosh suggested that "a major goal in teacher education programs should be to promote development of prospective teachers' knowledge of common ways that children think about the mathematics topics the teachers will teach" (p. 5). Tirosh's study focused on making prospective teachers aware of major sources of students' incorrect responses. However, it was suggested that future research focus on additional questions: "Are prospective teachers (and in-service teachers) aware of children's informal knowledge of fractions? Do they know how to use this knowledge to develop an understanding of operations with fractions?" (p. 22). Sharing knowledge of children's thinking can not only increase pre-service teachers' content knowledge, but also can serve as a way to change beliefs about mathematics.

The purpose of this article is to share how one class of U.S. pre-service elementary teachers' beliefs about knowing mathematics changed from a mainly procedural focus to seeing the importance of conceptual understanding. The pre-service elementary teachers were enrolled in a mathematics and pedagogy content course that focused on algebraic functions and number and numeration. The class activities were intentionally structured for the pre-service teachers to demonstrate understanding of concepts in different representations. Specifically, we will discuss how models of students' thinking about fraction division contributed to this change in beliefs. These activities were designed to challenge pre-service teachers to reconstruct their own conceptual understanding of fraction division and in the process internalize the importance to learn procedures in meaningful ways using multiple representations. For this study, fraction division focused on a whole number or a mixed number divided by a fraction. In the U.S., fraction division is first included in the fifth grade standards for 10-11 year old students, making it an important topic for pre-service elementary teachers' content knowledge and pedagogical content knowledge.

This article is organized into four main sections. First relevant literature on pre-service elementary teachers' beliefs is discussed and the definition of conceptual knowledge is presented. Then, the methods of this study are detailed. Next, the results section includes a description of example session activities the pre-service teachers participated in and the change in beliefs that occurred. Finally, a discussion section provides a summary and recommendations based on this study for changing beliefs of pre-service elementary teachers.

Pre-service Elementary Teachers' Beliefs

Philipp (2007) notes, "the construct of *belief* is of great interest to those attempting to understand mathematics teaching and learning" (p. 265). Pre-service elementary teachers enter teacher education programs with preconceived beliefs and attitudes about various aspects of mathematics that they formed as students (Goodman, 1988; Lortie, 1975; Lubinski & Otto, 2004). However, what has been called the apprenticeship of observation is not likely to reveal the complexities of teaching (Lortie, 1975). In addition, beliefs that have been formed over time since elementary school can be difficult to change (Pajares, 1992). It is vital that teacher educators have a better understanding of not only what beliefs teachers hold but also how they hold them, because this affects the extent to which beliefs can be changed (Philipp, 2007). Because of this, teacher education programs should assess their effectiveness of how well they nurture beliefs that are consistent with their philosophy of teaching and learning (Hart, 2002).

Beliefs as disposition towards action. One of the main reasons for studying beliefs is that they can be viewed as disposition towards action (Ambrose et al., 2004; Pajares, 1992). "There is substantial evidence that teachers' beliefs about mathematics impact their teaching of mathematics" (Hart, 2002, p. 4). Teachers are constantly faced with uncertain situations that they are asked to interpret. Some of these challenging situations require quick thinking, where teachers' beliefs often compel them to act in certain ways (Ambrose, et al., 2004). The context of the situation, including a teachers' perceived ability level of their students, is an important consideration for how beliefs and practice are related. If a teacher does not believe that students can construct their own knowledge, he or she may have a more teacher directed classroom (Beswick, 2005). If teachers' beliefs about the nature of mathematics are not challenged or changed it may be difficult for them to change their practice (Nisbet & Warren, 2000).

Changing beliefs. Mathematics educators generally agree on what beliefs are; a greater challenge now is how to change teachers' beliefs (Philipp, 2007). Students' mathematical thinking can be one vehicle to impact pre-service teachers' beliefs in a positive way. Until pre-service teachers learn about children's mathematical thinking, they may fail to recognize that their own mathematical understanding is insufficient (Ambrose, 2004). Philipp et al. (2007) proposed a *circles of caring* model for how pre-service elementary teachers can move from caring about children to caring about mathematics. Pre-service elementary teachers care about children and by exposing them to children's mathematical thinking they may come to realize the benefits of conceptual understanding of mathematics.

Ambrose (2004) found that pre-service teachers who focused on children's mathematical thinking while working with a child underwent changes in their beliefs. The experiences were emotional and memorable for many of the pre-service teachers as they were excited when students learned, developed relationships with the children, and saw how difficult teaching can be when a child struggled. However, many of the children had difficulty with story problems and the use of manipulatives. This led the pre-service teachers to focus more on the children's difficulties rather than on their successes.

Philipp et al. (2007) conducted an experimental study with prospective elementary teachers (PSTs) enrolled in a mathematics content course who were randomly assigned to three conditions: learning about children's mathematical thinking, visiting elementary schools of teachers with constructivist teaching practices or visiting typical elementary schools. They concluded that "learning about children's mathematical thinking facilitated the learning of mathematics while supporting the development of the PSTs' beliefs" (p. 469). Classroom observations by pre-service teachers did not lead to much change in beliefs about mathematics. The vast majority of the pre-service teachers who learned about children's mathematical thinking had a positive change in beliefs about the benefits of conceptual understanding of mathematics.

We take the view that giving pre-service teachers the opportunity to see the difficulties that students can have with mathematics when they are only taught procedurally and later seeing examples of students' conceptual mathematical thinking can be an impactful experience for changing pre-service elementary teachers' beliefs. From their own mathematical experience, many pre-service elementary teachers have a view of mathematics that is based mainly on symbols and procedures. Such a view requires new experiences to change these beliefs to a focus on conceptual understanding. Robust conceptual understanding can build meaning for procedural knowledge.

Conceptual understanding. A widely agreed upon view of mathematical content knowledge is that it consists of conceptual and procedural knowledge. Hiebert and Lefevre (1986) define conceptual knowledge as a "connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (p. 3). If students have procedural knowledge it does not necessarily mean that they have understanding of the concepts underlying the procedures (Integrating Mathematics and Pedagogy, 2004). It is important that mathematics content courses for preservice teachers are structured with a focus on conceptual understanding to build meaning for procedures.

The Lesh Translation Model (Lesh, 1979, see Figure 1) was originally designed to represent understanding of conceptual mathematical knowledge. It consists of five categories of representation: (1) Representation through realistic, real-world, or experienced contexts, (2) Symbolic representation, (3) Language representation, (4) Pictorial representation, and (5) Representation with manipulatives (concrete, hands-on models).

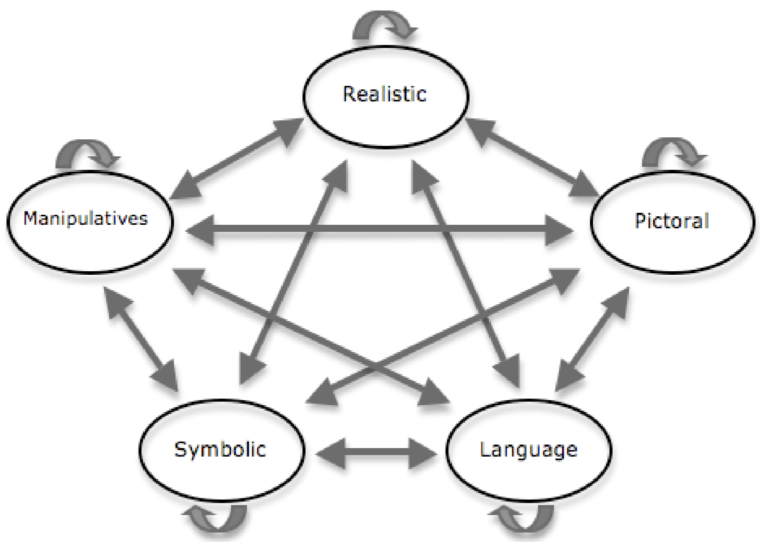


Figure 1. The Lesh Translation Model (Lesh, 1979).

The translation model emphasizes that the understanding of concepts lies in the ability of students to represent concepts through the five different categories of representation, as well as the ability to translate between and within representations (Lesh & Doerr, 2003). In order to teach effectively, teachers should have knowledge of how to understand concepts in different representations so that they can structure activities for students to develop conceptual understanding through different representations.

The way fraction division is presented to students varies in different countries, but the forms of representation that are used are an important consideration. When examining fraction division in Chinese and in U.S. textbooks Li, Chen, and An (2009) found that there is a focus on pictorial, verbal, and symbolic representations. When learning fraction division, Chinese students learn the meaning of fraction division through three related word problems, starting with a fraction multiplication problem. The standard invert and multiply procedure is then derived from a measurement interpretation of fraction division that is supported by pictures starting with a fraction divided by a whole number. However, while pictorial representations are presented, the Chinese textbook includes no requirements for students to explain how fraction division works through pictures and this has been suggested as an area that could be improved (Li, 2008). This can be a difficult task as Lo and Luo (2012) found that a majority of Taiwanese prospective elementary teachers that had developed the knowledge packet of fraction division, as described by Ma (1999), were unable to devise a correct word problem for a fraction division computation problem or develop an appropriate diagram to illustrate the solution process of their proposed word problems.

Sharp and Adams (2010) provide ideas for how to develop this understanding with elementary students. In their study that involved mixed ability U.S. fifth grade students, they emphasised the importance of students working with a variety of realistic problems over time before learning the standard invert and multiply procedure for fraction division. Also, they started with more familiar fractions like ½ and ¼ before moving to other fractions. They found that students initially verbalized their strategies well but needed encouragement to use pictures and written symbols. By the end of the eight-day unit, many of the students developed the common-denominator method of fraction division and were able to show their thinking with pictures. This understanding that the fifth grade students developed is similar to the models of students' thinking that were used in this study with preservice teachers that will be described further in the following sections.

The activities in the mathematics and pedagogy content class were intentionally structured for the preservice teachers to demonstrate understanding in different representations and to translate between different representations.

Methods

The study was conducted with one class of thirty pre-service elementary teachers enrolled in a mathematics and pedagogy content course. The pre-service teachers completed an online open-ended beliefs survey at the beginning and conclusion of the course. They also completed a reflection assignment after the class session based on the models of students' thinking about fraction division.

Setting

This study took place at a large Mid-western public university in the United States. The mathematics education requirements for the pre-service elementary teachers consist of two mathematics content courses and then a methods course before the student teaching experience. The mathematics content course in this study is the first of the two content courses. The class met twice a week for eighty minutes.

The mathematics course focuses on the content of algebraic functions and number and numeration. The functions and proportionality portion of the class is organised around the exploration of linear, quadratic, and exponential functions through multiple representations. The pre-service teachers then use their knowledge of functions, especially linear functions to understand the characteristics of proportional situations.

The number and numeration portion of the course has pre-service teachers explore different numeration systems (e.g., Egyptians, Chinese, Mayans) in order to better understand the Hindu-Arabic numeration system. The pre-service teachers then work through activities that explore different types of numbers starting with whole numbers, then integers, rational numbers and decimals, and finally irrational numbers.

The course was designed to be in-depth on a small number of topics, incorporate communication, reasoning, real world connections, and integration of technology through quality problem solving activities.

The Lesh Translation Model was designed explicitly as a framework for organizing instruction to involve pre-service teachers actively in building mathematical understandings, using multiple representations and connections within and between representations. Because mathematics is an abstraction, learners rely on embodiments or external representations of the constructs and conceptual systems to learn mathematics. From these embodiments, pre-service teachers are encouraged to abstract or generalize the relevant concepts without dependence on the representations used to foster their acquisition.

Representational fluency underlies some of the most important abilities associated with what it means to understand a given conceptual system. When building this course content, the instructor explicitly considered each of the representations and organised sessions around translations within and between different representations. Activities often involved pre-service teachers with concrete representations for the content taught. The course sessions involved pre-service teachers with contexts that engaged them and allowed them to translate to pictures and manipulatives to solve story-based tasks.

Sessions were organized to optimize pre-service teacher discourse in particular ways that allowed the pre-service teachers to describe their actions within other representations and translations between representations. Symbolic representations for mathematical ideas were introduced by connecting to other representations; symbols became a way of recording pre-service teachers' actions and verbal descriptions with pictures, contexts, and manipulatives. The questions and activities in the class enabled pre-service teachers to move from one representation to another to facilitate conceptual understanding.

Participants

One class of thirty pre-service elementary teachers—twenty-six women and four men—comprised the sample. For almost all of the subjects their only mathematics course since high school was college algebra. The instructor for the course was an Associate Professor in Mathematics Education with over twenty years' experience in teacher education.

Data Collection

Researcher field notes were taken during the course sessions for the whole class discussions and when circulating around the room observing the small group time. In addition, each group's work was checked throughout the activities to determine mathematical accuracy and solution strategies.

The pre-service teachers completed the online beliefs survey at the beginning and conclusion of the course. *The Integrating Mathematics and Pedagogy* (IMAP) online beliefs survey (2004) was completed by participants one week before the start of course and again one week before its completion. The IMAP survey was developed in part because of the limitations of Likert surveys to measure beliefs including that individuals usually cannot explain responses and that questions are not posed in contextual situations (Ambrose et al., 2004). The IMAP survey situates questions in contexts that include example student work and explanations, videos of children solving problems, and ranking example problems based on how difficult they would be for children. The survey measures seven beliefs about teaching and learning mathematics (see Table 1).

The pre-service teachers also completed a reflection assignment after the session that included activities regarding models of students' thinking about fraction division.

Table 1
Beliefs Measured by the IMAP Survey (Ambrose, Clement, Philipp, & Chauvot, 2004, p. 59)

Beliefs About Mathematics
Belief 1. Mathematics, including school mathematics, is a web of interrelated concepts and procedures.
Beliefs About Knowing/Learning Mathematics
Belief 2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts. That is, students or adults may know a procedure they do not understand.
Belief 3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
Belief 4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when learning them. If they learn the procedures first, they are less likely to learn concepts.
Beliefs About Children's Doing and Learning Mathematics
Belief 5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.
Belief 6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
Belief 7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

For the purpose of this study, just beliefs, 2, 3, and 4—which deal with conceptual understanding—were the focus. Based on the IMAP survey, scoring rubrics to show evidence of these beliefs participants have to demonstrate that conceptual understanding of mathematics involves real world situations, pictures, explanations, and hands-on manipulatives that can be used to develop meaning for the symbolic procedures.

Reflection Assignment

The pre-service teachers completed a reflection assignment after the session on fraction division. This was the 21st session in a series of 29 sessions for the semester. The pre-service teachers responded to one reflection question: "How would you know if students understood fraction division?"

Fraction Division Session Activities

The activities on fraction division focused on different representations and effective sequences for teaching these concepts.

Sixth grade children's thinking was used to demonstrate to the pre-service teachers the types of thinking that students can display. The models of students' thinking used were from a teaching experiment that examined how students can construct the common denominator procedure for fraction division; using realistic contexts, pictures, explanations, and connections to symbols as the major representational tools (Cramer, et al., 2010).

A measurement model for fraction division was used because it lends itself more naturally to drawing pictures. Figure 2 includes an example story problem that the pre-service teachers worked on and an example picture from a 6th grader's work (Cramer et al., 2010, p. 342).

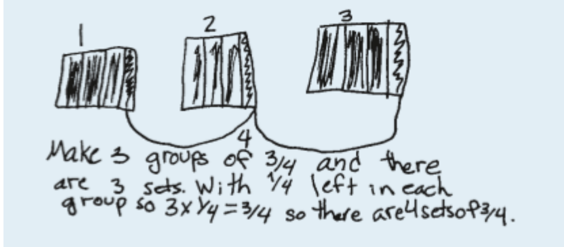
A scoop holds $\frac{3}{4}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3 cups of birdseed?	

Figure 2. Student solution to a fraction division story problem.

The Grade 6 child's picture shows the three cups of birdseed needed to fill the bird feeder. The student divided up each cup into fourths in order to mark off the number of scoops that held $\frac{3}{4}$ of a cup. Based on partitioning each rectangle into fourths, the student has now shown that 3 cups of birdseed is the same as $\frac{12}{4}$ cups of birdseed. This could then lead to writing the division problem, $3 \div \frac{3}{4}$ with common denominators, as $\frac{12}{4} \div \frac{3}{4} = 4$.

The activities attempt to demonstrate to pre-service teachers the benefits of conceptually understanding these concepts by seeing how students can work with the concepts in different representations. The class activities also incorporated estimation strategies and explanations to describe the use of the different representations.

One researcher took field notes of the whole class discussion and circulated around the groups during the small group time. The researcher also checked the groups' written work on the mathematics problems they solved. The class activities will be described in more detail in the Results section below.

Data analysis

The data from the reflection assignment were analysed using open coding and then axial coding to determine the main categories in the reflections (Strauss & Corbin, 1998). Based on the field notes, narratives of the session activities were written. The data from the IMAP survey segment were analysed using categories that had been previously developed during the development of the survey (IMAP, 2004). The coding of these data was done by two of the researchers. The Cohen's K coefficient of inter-rater agreement was .84, and thus within an acceptable range (Fleiss, 1981; Landis & Koch, 1977). Once coding differences were calculated, the raters came to agreement on the discrepancies so that full agreement was reached.

Previous studies (Ambrose et al., 2004; Philipp et al., 2007) have used the IMAP online beliefs survey and the analysis of the survey responses in this study was done in the same manner so the analysis procedures will be described briefly. The IMAP online beliefs survey uses rubrics to determine belief scores and each belief score is determined by using multiple parts of the survey. Each rubric contains sample participant responses for each coding and a description of the type of responses that will fit each coding.

Table 2, below, provides a summary of the parts of the survey that are used to determine the belief scores for Beliefs 2, 3, and 4. Some questions on the survey have multiple parts so they are used for multiple beliefs. Using the rubrics provided in the IMAP belief manual each participant that completed the survey was given a single score for each of the three beliefs. Pre- and post-test scores for each belief of the preservice teachers in this study were compared using descriptive statistics (IMAP, 2004).

Table 2

Description of Questions for Beliefs 2, 3, and 4 of the IMAP Survey

Belief	Description of the survey questions
Belief 2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts. That is, students or adults may know a procedure they do not understand.	Three students' strategies for adding 149 plus 286 are given including the standard algorithm. The participants are asked if the students would be able to use and explain the other students' algorithms. Two student strategies, the standard algorithm and a partial difference strategy, for solving 635-482 are given. Participants are asked which student shows the greater mathematical understanding and also which strategy students would choose more often to be more successful. Participants are asked to rank four fraction operation questions, including "Understanding $\frac{1}{5} \times \frac{1}{8}$", in terms of their relative difficulties and to explain their ranking. The focus of this segment is on the explanation for the fraction multiplication ranking and a follow up question that asks the participants by *understand* if they were thinking of the ability to get the right answer?
Belief 3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.	The two strategies for solving 635-482 are used again for this question with a focus this time on the responses indicating which student's strategy would be used with greater success and give rise to future conceptual development. The focus of this question is on what the participants suggest so a child will be successful with division of fractions in the future.

Belief 4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.	Five students' strategies, including the standard algorithm for adding 149 plus 286, are given. The question asks if you were a teacher of these students which of these solutions would you want to share with the whole class and why. A follow up question asks in what order the solutions should be presented. Participants predict if a child could solve a fraction division problem three days after receiving procedural instruction. They also comment on how to help this child learn division of fractions.

Results

At the beginning of the course, the majority of the pre-service teachers showed little or no evidence in the belief that conceptual understanding of mathematics is more powerful or generative than remembering mathematical procedures. Initially, the pre-service teachers appeared to be focused on mathematics as procedural fluency while the course activities throughout the semester focused on the importance of conceptual understanding through multiple representations. This information will be discussed in the following sections.

IMAP survey

The pre-service teachers completed the online open-ended survey one week before the beginning of the class. (See Table 3 for results.)

Table 3
IMAP Pre-Class Survey Results

Belief	No evidence N (%)	Weak evidence N (%)	Evidence N (%)	Strong evidence N (%)
2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.	12 (43%)	13 (46%)	1 (4%)	2 (7%)
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.	11 (42%)	9 (35%)	3 (12%)	3 (12%)
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn procedures first, they are	12 (46%)	5 (19%)	8 (31%)	1 (4%)

less likely ever to learn the concepts.

Note. There are a total of 28 pre-service teachers for beliefs 2 and 26 pre-service teachers for the other beliefs. Due to rounding, not all rows' percentages will add to 100.

Overall, the results of the survey demonstrated that at the beginning of the class the majority of the pre-service teachers showed weak or no evidence in Beliefs 2, 3, and 4 (Table 3). Further, for Belief 2, only 11% of the pre-service teachers showed evidence or strong evidence that if a child knows procedures they may not understand the underlying concepts.

Fraction division survey segment

Here, examples of the pre-service teachers' responses to the questions on the fraction division survey segment to illustrate the level of evidence of Beliefs 2, 3 and 4 measured by the IMAP survey will be discussed.

Over 80% of the pre-service teachers were satisfied with the procedural focus the teacher used in the video in explaining fraction division. One of the pre-service teachers remarked that, "In my head I was thinking how I would teach this problem, and I would do the same."

After seeing the child unable to solve a fraction division problem successfully, these teachers focused on the need for more practice of the procedure.

> "I would say run through this problem more than just one day. Things are learn[ed] when you do them many times for many days because it gets ingrained into your brain."

After watching the child unsuccessfully remembering how to solve a fraction division problem, six of these pre-service teachers did also mention that, for example:

> "The teacher should also provide an explanation of why the problem is solved that way and why it works."

Five of the pre-service teachers focused on the need to teach for understanding. They felt that the child would not remember the procedure three days later. For example:

> "The teacher didn't explain WHY she was doing what she was doing. The student simply saw what she did and completed the same steps, but based on just this exercise, if I were her I wouldn't be able to explain what I'd just done."

While they did not provide details on how to promote understanding, they emphasised that the students should play an active role in their learning.

The sessions on fraction division later in the semester were designed for the pre-service teachers to see how conceptual understanding could be developed through the use of different representations.

Fraction Division Session Activities

In the session on fraction division, the pre-service teachers activated their prior knowledge with fraction division and division with whole numbers before exploring the sixth graders' models of thinking. First the pre-service teachers were told that people seem to have different approaches to solving problems involving division by fractions.

How do you solve this one? $3\frac{1}{4} \div \frac{3}{8}$

The pre-service teachers shared a few ideas including the standard invert and multiply rule and a method that used common denominators. The pre-service teachers were then asked to reflect on three questions that students could struggle with if they began working with division of fractions solely procedurally:

(a) Can you look at $8\frac{2}{3}$ and judge that it is indeed a reasonable answer?

(b) Isn't $8\frac{2}{3}$ greater than $3\frac{1}{4}$? Doesn't division "make smaller"?

(c) Why is the answer greater than the number you started with?

Next, the pre-service teachers were given a whole number multiplication problem and asked to write a division story problem using the same information in the problem. The pre-service teachers were asked to come up with two possible division problems for the following multiplication problem "Hamdi can earn $4 an hour babysitting. If she sits for 6 hours, how much money can she earn? ($24)". They decided on:

(a) "Hamdi earned $24 for babysitting. If she earns $4 an hour, how many hours did she baby sit?" (A measurement model)

(b) "Hamdi earned $24 for babysitting. If she sat for 6 hours, how much did she earn each hour?" (A partitive model)

The instructor told the class the names of the two types of problems and that the measurement model for division of fractions was chosen to model fraction division because it lends itself more naturally to drawing pictures to solve the problems.

The pre-service teachers were then presented with a problem to solve by drawing a picture: "A scoop holds $\frac{3}{4}$ cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3 cups of seed?"

After sharing a few solutions, the instructor showed an example of a 6th grade student's solution (Figure 2, above). Then the pre-service teachers were asked to explain a Grade 6 student's solution for 4 divided by $\frac{2}{5}$ (Figure 3).

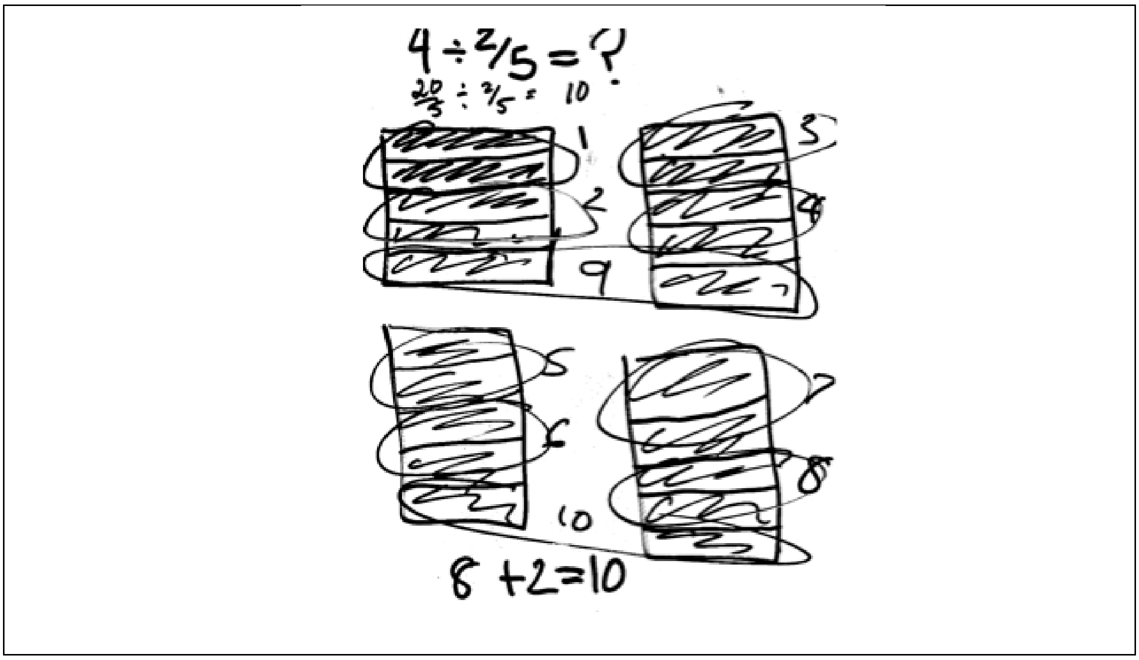


Figure 3. Example Grade 6 child's solution for $4 \div \frac{2}{5}$.

Following this, the pre-service teachers explored the following two problems to see how to name a remainder for a fraction division problem:

(a) You have $2\frac{1}{2}$ pounds of fish. Your serving size is $\frac{3}{4}$ pound. How many full servings can you make? How can you describe the amount left over?

(b) You bought $2\frac{1}{6}$ pints of ice cream. You plan on giving each friend $\frac{2}{6}$ of a pint. How many servings can be given?

The pre-service teachers were able to draw pictures, but about half struggled with how to name the remainders. The teacher showed examples of sixth graders' thinking on these two problems. Figure 4 has the example used for the fish problem. The three boxes on the right side show how the 6th grade student named the remainder as $\frac{1}{3}$ and not $\frac{1}{4}$. Since each serving size was $\frac{3}{4}$ pound, the student was able to reason that the part that was left was $\frac{1}{3}$ of the serving size.

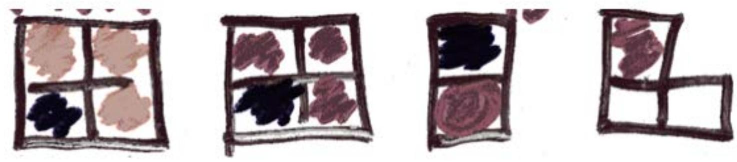


Figure 4. Example of a Grade 6 child's solution to the fish problem.

Figure 5 shows the two examples used for the ice-cream problem. Discussing the example solutions helped the pre-service teachers to explain how to name the remainders as the preservice teachers that struggled now stated that they understood. It also allowed them to see what mathematics students are capable of doing in well-structured activities. A few of the pre-service teachers commented in their groups that it was interesting to learn how to think through these problems from sixth graders and they wished they had learned mathematics this way in their own schooling.

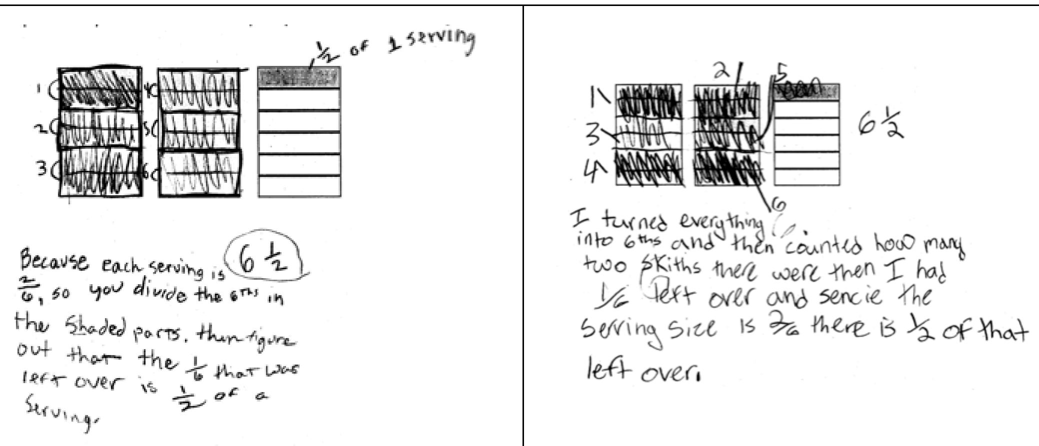


Figure 5. Two example Grade 6 children's solutions to the ice cream problem.

The pre-service teachers completed three more real world fraction division problems by drawing pictures, writing a division sentence, and then writing a division sentence with common denominators based on the pictures.

Next, the instructor facilitated a class discussion on four questions:

(a) What is the role of the unit in each of your picture solutions?

(b) How can you determine the fractional part of the answer when the answer is not a whole number?

(c) Why is flexibility of the unit important in fraction division?

(d) Does division always make smaller?

In concluding the session, the pre-service teachers used the mental images that they had developed through the session to complete nine division estimation problems. Two examples of the types of questions they completed are provided below:

(a) You have 4 cups of flour. The recipe you are making calls for $\frac{2}{3}$ cup of flour. Estimate: About how many full recipes can you make? At least 2? At least 4? At least 6?

(b) You know that $4 \div \frac{1}{2} = 8$. About how much would $4 \div \frac{1}{3}$ be? Is it more or less than 8?

Throughout the session the pre-service teachers appeared to be engaged with the activities.

The session for fraction division provided pre-service teachers opportunities to reconstruct their own understanding. They were able to do this by working through lessons, using models found to support Grade 6 students' understanding and by examining student work that demonstrates how children themselves made sense of fraction division.

Reflection Assignment

After the pre-service teachers completed the session activities on fraction division they responded to a reflection question: "How would you know if students understood fraction division?" The majority of pre-service teachers had only learned the standard procedures for solving multiplication and division of fractions and after the session activities they came to realise why a focus solely on procedures was insufficient.

In their responses, the pre-service teachers focused on students being able to use multiple methods, and most specifically they mentioned the different representations that were discussed in class.

Table 4, below, includes a summary of responses from the 27 pre-service teachers who responded to this question. (One pre-service teacher was not in attendance for the session and two other pre-service teachers did not complete this assignment.)

They mentioned several ideas, so the total number of responses is more than 27. Language and pictorial representations were most often discussed as a way for students to show understanding.

In summary, the pre-service teachers were impacted by the fraction division activities that went beyond procedural knowledge to show how conceptual knowledge can be developed to add meaning to the procedures.

It seems that the pre-service teachers were able to see throughout the session how well-structured classroom activities can develop powerful understanding and mathematical reasoning in students. The fraction division activities helped the pre-service teachers see how to go beyond procedural knowledge.

Table 4
Summary of Pre-service Teachers' Responses for Understanding of Fraction Division

Response category	N
If students can show understanding visually or through pictures. *Example:* "In regards to division, finding common denominators to divide the fractions or using diagrams helps the students visualise what it means to divide fractions and also what the fractions would look like."	22
If students can explain their understanding through verbal or written language. *Example:* "You could look for pictures and ask the students to explain their understanding with the use of them. Ask for them to write out their understanding in written sentences to check for actual comprehension of the fractions."	21
Just doing the procedure is not enough. *Example:* "It has become apparent to me that does not mean you *understand* how to do it." *Example:* "I thought I understood multiplication and division until asked why in this class. I found that I was merely taught the how to do it method, such as dividing is flipping the fraction and multiplying."	12
If students can solve real world problems or develop their own story problems. *Example:* "As a teacher, you could show them fraction problems with just numbers and then put them in a story problem to show how the story makes them easier to solve."	11
Be able to "read" a fraction division problem *Example*: $\frac{3}{4} \div \frac{1}{8}$. How many times does $\frac{1}{8}$ go into $\frac{3}{4}$?	5

Post IMAP survey segment

The post IMAP survey was completed in the last week of the semester. Two of the pre-service teachers did not complete the survey and one had trouble viewing the videos in the survey. The post survey was identical to the survey at the beginning of the class.

Table 5, below, includes a summary of the results for the pre-service teachers that completed both the pre- and post-IMAP survey items. The post-survey results are in italics under the pre-survey results for each belief.

Table 5.
IMAP Pre and Post-Class Survey Results

Belief	No evidence N (%)	Weak evidence N (%)	Evidence N (%)	Strong evidence N (%)
2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.	11 (41%) *4* *(15%)*	13 (48%) *5* *(19%)*	1 (4%) *13* *(48%)*	2 (7%) *5* *(19%)*

3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.	10 (40%) *4* *(16%)*	9 (36%) *2* *(8%)*	3 (12%) *6* *(24%)*	3 (12%) *13* *(52%)*
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn procedures first, they are less likely ever to learn the concepts.	11 (44%) *3* *(12%)*	5 (20%) *4* *(16%)*	8 (32%) *7* *(28%)*	1 (4%) *11* *(44%)*

Note. There were a total of 27 pre-service teachers for Belief 2 and 25 pre-service teachers for the other beliefs that completed the pre and post-survey. Due to rounding not all rows' percentages will add to 100.

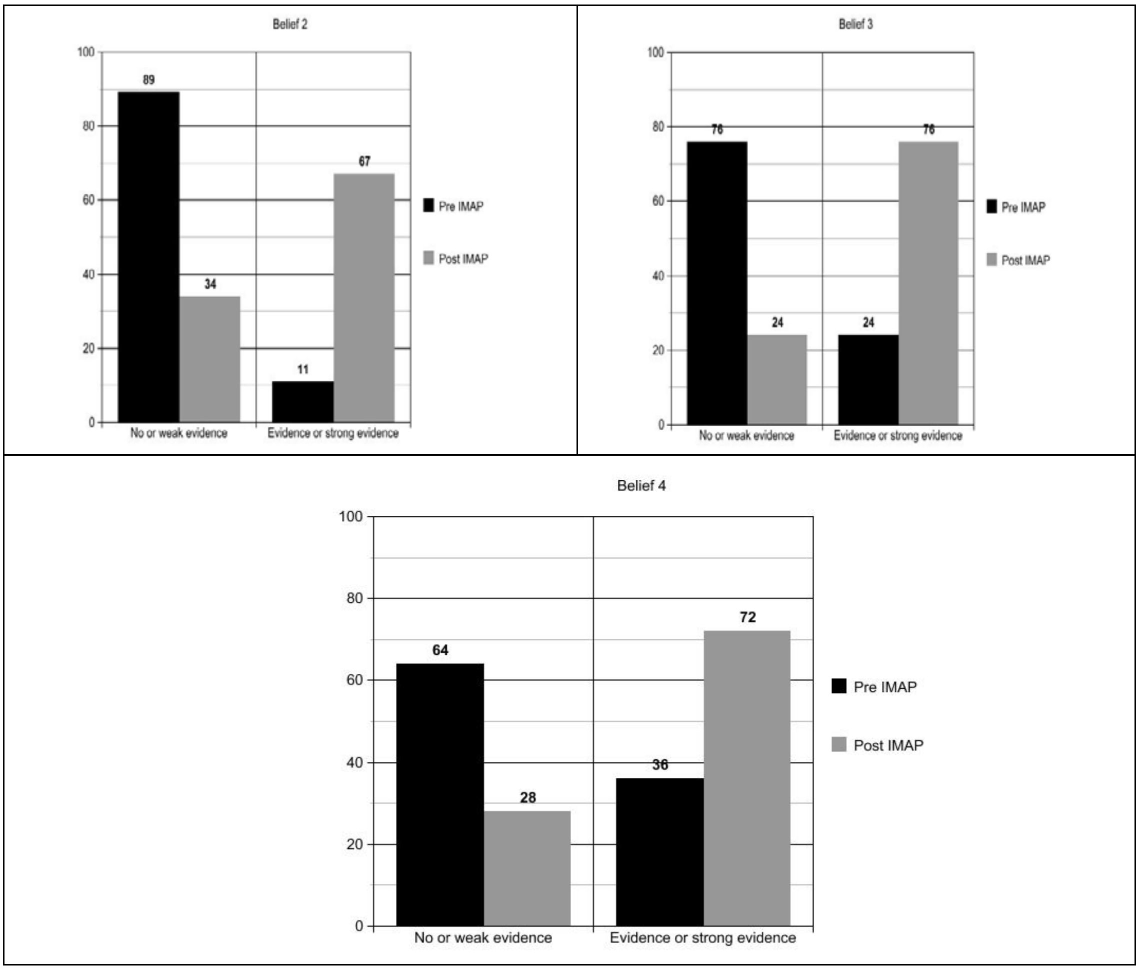


Figure 6. Change in Beliefs 2, 3, and 4.

The class as a whole showed greater evidence that they believed that it is important that students have conceptual understanding before learning standard algorithms (Figure 6). 67% of the class showed evidence or strong evidence of the belief that knowing how to do a procedure does not mean that students will necessarily understand why the procedure works. 76% of the class showed evidence or strong evidence of the belief that understanding concepts leads to greater retention and increased ability to learn new mathematical ideas. 72% of the teachers showed evidence or strong evidence of the importance of learning concepts before procedures. The pre-service teachers mentioned the importance of promoting understanding through different representations to build understanding for procedures.

In the fraction division segment of the survey, more of the pre-service teachers wrote about the need for students to have conceptual understanding along with procedural understanding. Twelve of the pre-service teachers provided details on how to do this as well. For example a pre-service teacher wrote that,

"The teacher should use pictures and real life situations to show the children what dividing fractions really means. Then after this the teacher can show the students the formula."

Six additional pre-service teachers mentioned the need to promote understanding, but did not provide details on how to do this. For example,

"The teacher should explain the process with more reasoning. Why do we multiply by the reciprocal? What is another method to show this process rather than pencil and paper?"

Nine of the pre-service teachers were still focused on the procedure and the child needing to practice this more. For example, a pre-service teacher mentioned the teacher should,

"Model many problems and have them practice often, so they are comfortable to do it themselves."

Five of these pre-service teachers also felt that the teachers' explanations could be improved but appeared to focus on the procedure and not on building conceptual understanding through different representations.

Discussion

This study was conducted with one class of pre-service elementary teachers enrolled in a mathematics content course to show how models of children's conceptual understanding combined with seeing the drawbacks of children only having procedural understanding can change pre-service teachers' beliefs about mathematics. Classroom activities on fraction division (Cramer et al., 2010) showed the pre-service teachers the importance of developing conceptual understanding. For most of the pre-service teachers, these activities were impactful as they found they never really understood fraction division.

There were three main aspects of fraction division and the activities that the pre-service teachers experienced that appeared to contribute to the change in beliefs. First, fraction division can be taught by focusing on a procedure instead of conceptually. Second, the course sessions the pre-service teachers experienced focused on learning fraction division conceptually through the multiple representations of realistic, pictorial, language, and symbolic. Third, the pre-service teachers were able to see a child struggling to remember a procedure when she had not been taught conceptually.

Fraction division is a topic that often is taught procedurally and thus when taught in a different way can show the benefits of conceptual understanding of mathematics. In order to change beliefs about mathematical knowledge, Furinghetti (2007) suggested that pre-service teachers need a context allowing them to look at the topics they will teach in a different manner (Furinghetti, 2007). As shown in this study, focusing on topics that many pre-service teachers understand only procedurally has potential to be a useful way to change beliefs. Based on their previous schooling, many preservice elementary teachers believe that mathematical knowledge is based on procedures. In Ma's (1999) study, only 43% of U.S. teachers correctly solved a fraction division problem. In this study, similarly to Tirosh (2000), the pre-service elementary teachers could solve fraction division problems procedurally at the beginning of the course but did not have conceptual understanding of these topics.

The focus on conceptual understanding throughout the semester and in the fraction division sessions appeared to affect the teachers' beliefs even on topics that were not covered in the course. The *Integrating Mathematics and Pedagogy* (IMAP) online open-ended beliefs survey measured the beliefs about knowing mathematics through the contexts of multi-digit addition and multi-digit subtraction, which were not covered during the semester, as well as fraction operations. At the beginning of the semester, 11%, 24%, and 36% of the pre-service teachers showed evidence or strong evidence in Beliefs 2, 3, and 4 respectively. At the conclusion of the semester, 67%, 76%, and 72% of the pre-service teachers showed evidence or strong evidence in Beliefs 2, 3, and 4 respectively.

The specific example of fraction division discussed in this study showed that the pre-service teachers' beliefs changed to value multiple representations as a measure of understanding. On the online survey segment at the beginning of the course, only five of the pre-service teachers suggested promoting conceptual understanding to help a child that had been taught fraction division solely procedurally. After the course activities on fraction division, the pre-service teachers had an improved focus on conceptual understanding as twenty-two of the pre-service teachers mentioned that students would have to show understanding through pictorial representations, twenty-one through language representations, and eleven through realistic representations. For the end of course online survey, eighteen pre-service teachers mentioned the importance of providing meaning to the procedure to help students learn fraction division.

The videos used in the IMAP survey demonstrated to many of the pre-service teachers the importance of teaching fraction division in a different way than solely procedurally. The class activities during the semester then provided a different way for fraction division to be learned. As noted by Philipp et al. (2007), pre-service elementary teachers care about children and want to help them learn. Through seeing the child struggle in the video, the pre-service teachers came to realize that a more conceptual method of teaching and learning fraction division was needed. This then helps the pre-service teachers care about teaching mathematics as well. It has been argued that change in teachers' beliefs can take effect primarily after some change in student learning has been evidenced (Guskey, 1986). For pre-service teachers this change in student learning can be shown through videos of students solving problems and through example student work.

The U.S. Common Core Standards for Mathematics (CCSSM, 2010) extol the value of mathematical understanding and procedural skill:

> One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. (p. 4)

Similarly, the Australian Curriculum: Mathematics' (ACARA, 2012) proficiency strand of Understanding states that students must "develop an understanding of the relationship between the 'why' and the 'how' of mathematics" (p. 6). This is important because if students lack understanding of a topic they may rely too heavily on procedures. When mathematics content courses for pre-service teachers are structured for conceptual understanding, it allows pre-service teachers' beliefs about mathematics to be changed as well.

While the majority of the pre-service teachers demonstrated a change in beliefs about fraction division from the beginning to the conclusion of the class, it is important that their professed beliefs will match their future teaching practices.

A major reason for why teachers' beliefs may not match their practices is time constraints (Kennedy, 2005). The U.S. CCSSM (2010) were designed to focus on fewer topics in depth, similar to the curricula of many of the Asian countries that perform well on international mathematics tests, to allow for improved understanding. Fraction division first appears in fifth grade in the CCSSM. Previously, this topic was introduced at differing grade levels in the U.S., though most commonly in the middle grades (Li, Chen, & An, 2009). It is important that pre-service elementary teachers are given experiences to see how fraction division can be conceptually understood to develop this understanding in their students. The knowledge packet for fraction division that was described by Ma (1999) is essential content knowledge for elementary teachers. However, building understanding through different representations, including pictorial and realistic representations, is also important for conceptual understanding of fraction division. Future research can focus on longitudinal studies to see if the implementation of the CCSSM (2010) supports pre-service teachers' to translate these beliefs into practice.

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