Misconceptions of Mexican Teachers in The Solution of Simple Pendulum

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Abstract
Solving the position of a simple pendulum at any time is apparently one of the most simple and basic problems to solve -in high school and college physics courses. However, because of this apparent simplicity, teachers and physics texts often assume that the solution is immediate -without pausing to reflect on the problem formulation or verifying that the solution obtained is indeed correct. This process causes conceptual errors to be carried out with them -students and even worse teachers. This paper presents some of the misconceptions found in teachers solving simple pendulum problems, moreover it presents proposals made in the texts, which generate the creation of such misconceptions in both teachers and students, and finally, it presents the proposal of a solution to correct this problem.
Keywords: Simple Pendulum, Misconceptions, Physics Teaching.

Introduction

In the teaching of physics, particularly mechanics, there are few traditional problems such as solving simple pendulum motion. In fact the vast majority of physics programs in high school and college-level introductory courses consider obligatory this problem. There is an extensive literature (Solaz-Portolés, Moreno-Cabo & Sanjosé López, 2008) about this problem and variants on the same topic. On the other side, from the hand of the current technologies, also pendulum problem has been solved by means of simulations (Torzo & Peranzoni, 2009), some of which are very illustrative and attractive to both students and teachers. However, this apparent abundance of resources to consult the pendulum problem comes in that teachers sometimes do not think about solving the problem, merely "playing" what the texts say, reaffirming their misconceptions and what is worse, creating misconceptions on students. In this paper, we point out some of the most common design errors found in several texts deemed essential for learning physics at high school and early university physics, then we show some mistakes made by physics teachers who retake models texts to solve the pendulum problem, and finally, we propose the right solution for the problem fashioned as on Tipler-Mosca text (2010).

Solutions to the simple pendulum problem

One justification to study the problem of the simple pendulum is that this may seem very basic but its scope is very broad, so all physics teacher must know correctly and properly solve because with it, you can illustrate Newton's laws, conservation energy, numerical solutions, approximate solutions and even effects of weightlessness.

As mentioned in the previous section, the pendulum problem is an icon for the learning of classical physics. It has a common relation and can be summarized as follows:

"Consider a simple pendulum motion, which is constituted by an ideal string and an object of mass m. Derive a mathematical expression to calculate the tensile force in the rope to the entire mass of the object position m."

Being a seemingly simple problem the vast majority of teachers deepen omit or consider whether the solution methods we use are correct. For the authors of this paper this reflection occurred when trying to do the numerical simulation of the problem based on the equations obtained from the solution to the tension rope thus inconsistent results were obtained, in which the trajectory of the object was not an arch in circumference, and the object began to descend even. This led to revise even the numerical integration code and finding no error, we concluded that the proposed equations were wrong. This was not immediate reflection since it was assumed for simplicity that the solution of the problem could not be mistaken.

In general, everyone solves the simple pendulum relatively well-finding an equation of motion. However, stress is not calculated explicitly, we arrive at the equation of motion without giving the value of the tensile force.

On the other hand, there are many studies about the solution of simple pendulum problem. However, some of them don't deduce the equation of motion. These give it for granted, and with the energy principle of conservation integrate this equation in terms of an expansion of power series, they focus in solve in the "exact form" the differential equation. They gives how "understood" all phenomenology of simple pendulum (Amore et al., 2007)

In other cases, it works about approximate methods to determine the period of a simple pendulum when is subjected to swing with large amplitudes making the experimental corroboration. However, it still is giving like well understood the differential equation that describes the simple pendulum (Amrani, Paradis & Beaduin, 2008)

In a last case, using Lagrangian, some authors deduce the equations of motion of a physical pendulum not forced (that is not a simple pendulum); the equations could reduce it to a simple pendulum equations putting the rotational inertia equal to ml^2 and making zero to the perturbative force, however, it is not our intention make the analysis from a Lagrangian perspective because we want make it to basic level (Quintero-Cabra & Silva-Valencia, 2009)

In the proposals of solution, almost under any methodology, the first step to solve the problem is making the "free body diagram". The majority of textbooks show this illustration, a sample is the next in the classical text of Resnick (1983):

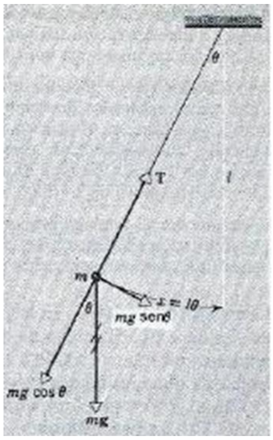

Fig. 1. Image shows the phenomenology of simple pendulum of Resnick´s text (1983)

Figure 1 which coming from the text of 1983, but in his 2001 text, Resnick (2001) is a bit more explicit about describing the phenomenology of the simple pendulum:

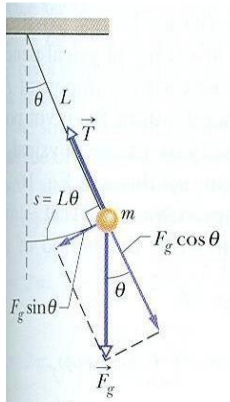

Fig. 2. Image shows the phenomenology of simple pendulum from Resnick's text of 2001

However, other texts follow the same path of Figure 1; the classic text of Sears (1975) is an example:

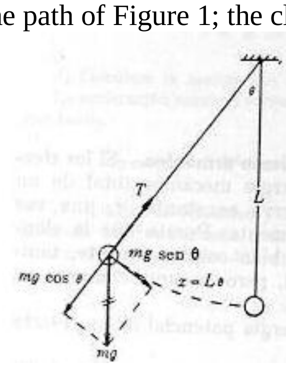

Fig. 3. Image shows the phenomenology of simple pendulum from Sears's text

Another classic text is from Alonso and Finn (1975), as given below:

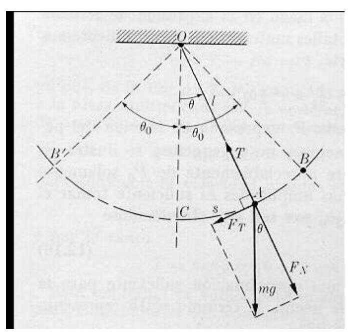

Fig. 4. Image shows the phenomenology of simple pendulum from Alonso & Finn's text

As mentioned earlier, the literature on the simple pendulum is wide, and the examples of Figs 1-4 are just a way of how to address the problem by classical physics texts. Other examples can be found in books Tipler (1977), Roller (1987), Gisbert (1998), Gertshen (1979), Gartenhaus (1979). All previous references illustrate how poor the phenomena associated with the dynamics of the simple pendulum is, and certainly, it should be one of the causes that lead to errors of interpretation in the solution of the simple pendulum. An example of this is shown in the next section where a group of teachers surveyed show deep conceptual errors when trying to solve this problem with first semester students in an engineering physics course.

This led us to propose a descriptive research to find out which are the misconceptions of teachers on the simple pendulum as well as explanatory way to find why such misconceptions. This research was conducted as a case study as described in the next section.

Misconceptions of The Teachers When Try to Solve The Simple Pendulum

The misconceptions in learning physics is a subject that has been studied extensively in students -a classic example is the work of Hammer (1996) focused on the subject of gravity but there are previous studies that illustrate preconceptions and influences in learning physics students from different grades (Champagne, Klopher, & Anderson, 1980; McCloskey, 1983). Around the world, it has also studied the misconceptions. They are sample of papers at conferences such as the ICPE, GIREP and WCPE (Mazzolini, Mann & Daniel, 2012; Choi & Kim, 2012); Houari & Benosman, 2007; Poling et al., 2008). In the particular case of Mexico, we have made some effort to study the misconceptions mainly in students -like in the case mentioned above- (Ramírez, González & Miranda). However, the case of physics teachers hasn't been explored in detail.

As shown in the previous section, the literature around the simple pendulum is wide. However, it doesn't always display appropriately the phenomenology that generates misconceptions. This is acute in the case of teachers since they are primarily responsible for giving students these matters so often. What they do is playing around their misconceptions. So in this way, they create misconceptions among students. As a small case study, physics teachers from the School of Computer Science at the National Polytechnic Institute of Mexico were asked to reply the pendulum problem using the wording indicated in the previous section, the following are some of the responses provided by teachers.

Teachers from General Education Department of the School of Computing from IPN were interviewed; the department consists of more than 20 teachers -most with physics or mathematics teaching career with several years of experience. Physics teachers in general were taken as the significant sample. This sample consisted of 15 teachers, and the solutions proposed by the sample group were diverse. However, we can realize four general characteristics solutions which are presented below:

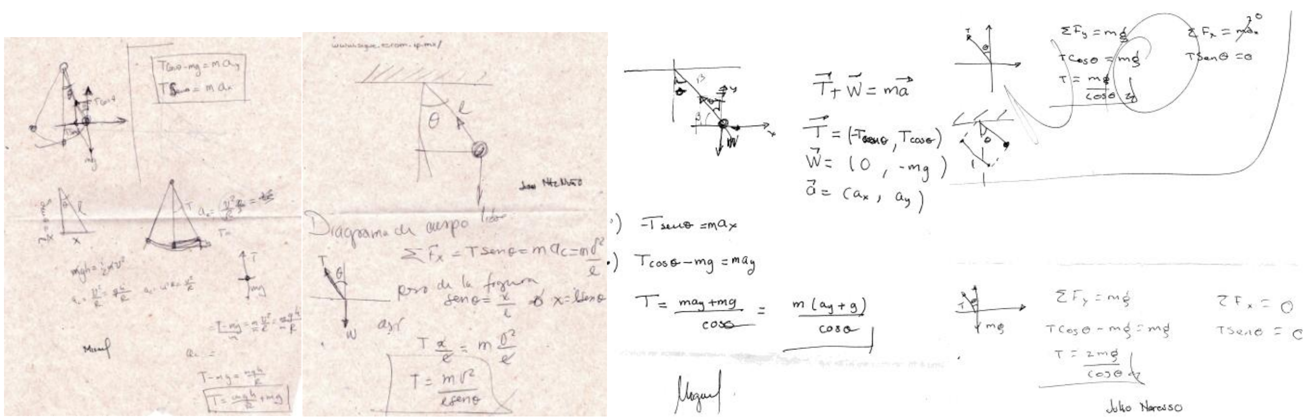

Fig. 5. Examples of proposals to solution of simple pendulum problem from some teachers

Above figure shows how teachers follow similar patterns - proposed by the texts from the previous section. However, it's known that they have different solutions due to a misinterpretation of the phenomena associated with the problem. The Interesting is to know the answer to a direct question: Why they don't corroborate with students whether the answer given is correct?

Usually teachers respond to this as follow: they do not think is necessary because they think they're not wrong or the interviewer were asked curiously, If the problem had a "trick"?

Proposal of Solution to The Problem

A study of this type would be incomplete but it's not a proposal for a solution to the proposed problem. Here is a proposal. Here, we present a proposal as working with students by the authors' -based on the same text for the problem as described in the previous sections.

Tension in The Rope of A Simple Pendulum

Consider a simple pendulum constituted by an object with a mass m suspended ideal rope length L, which at time t forms an angle θ with the vertical as shown in the following figure:

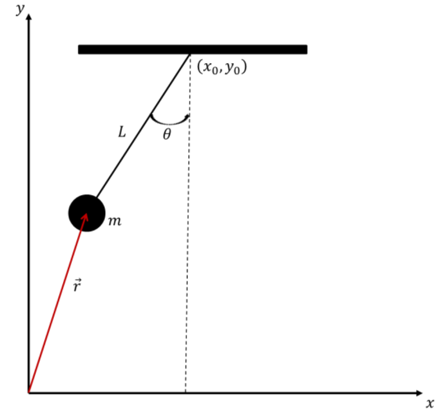

Fig. 6. Simple Pendulum

The above figure also shows a frame of reference from $\boldsymbol{xy}$ determining the position vector $\vec{\boldsymbol{r}} = (\boldsymbol{x}, \boldsymbol{y})$ of the object at time t, and the coordinates of the point of suspension of the rope$(\boldsymbol{x_0}, \boldsymbol{y_0})$ which remain fixed at all times.

Now, to calculate the tensile force felt by the object through the string for all time t, it is possible to proceed in two ways described below.

Newton's Laws and vectorial analysis

The following figure illustrates the free body diagram that shows the forces acting upon the object mass m at some instant of time t.

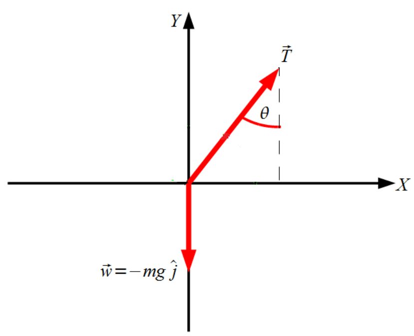

Fig. 7. Free body diagram that shows the forces acting on the object of mass m in anytime t

From Figure 6 it is possible to note that the position coordinates of the object are always given in accordance with the equations:

$$x = x_0 - L\sin\theta, \qquad (1)$$

$$y = y_0 - L\cos\theta, \qquad (2)$$

Where the angle θ to establish the convention considered positive when it lies to the left of vertical, while it is considered negative when it lies to the right of vertical. Moreover, to obtain object velocity components, we derive the object's velocity relative to the previous time previous equation system, now obtaining:

$$v_x = \frac{dx}{dt} = -L\frac{d\theta}{dt}\cos\theta, \qquad (3)$$

$$v_y = \frac{dy}{dt} = L\frac{d\theta}{dt}\sin\theta, \qquad (4)$$

And deriving once more with respect to time obtain the acceleration components:

$$a_x = \frac{d^2x}{dt^2} = L\left(\frac{d\theta}{dt}\right)^2 \sin\theta - L\frac{d^2\theta}{dt^2}\cos\theta, \qquad (5)$$

$$a_y = \frac{d^2y}{dt^2} = L\left(\frac{d\theta}{dt}\right)^2 \cos\theta + L\frac{d^2\theta}{dt^2}\sin\theta. \qquad (6)$$

Acceleration and force are related through Newton's second law, which is to be written for each Cartesian component takes the form:

$$F_x = ma_x, \qquad (7)$$

$$F_y = ma_y. \qquad (8)$$

Using now the free body diagram shown in Figure 7, we can identify the Cartesian components of the total force exerted on the object:

$$F_x = T\sin\theta, \qquad (9)$$

$$F_y = T\cos\theta - mg. \qquad (10)$$

Substituting now the equations (5), (6), (9) and (10) in the Cartesian components of Newton's second law given by equations (7) and (8), we obtain the following system of differential equations:

$$mL\left[\left(\frac{d\theta}{dt}\right)^2 \sin\theta - \frac{d^2\theta}{dt^2}\cos\theta\right] = T\sin\theta, \qquad (11)$$

$$mL\left[\left(\frac{d\theta}{dt}\right)^2 \cos\theta + \frac{d^2\theta}{dt^2}\sin\theta\right] = T\cos\theta - mg. \qquad (12)$$

Note that in this system of differential equations, the only unknown quantities are the angle θ and the string tension T, and solve for these variables is a simple algebraic exercise. First multiply the equation (11) by cos θ and equation (12) for sin θ to obtain:

$$mL\left[\left(\frac{d\theta}{dt}\right)^2 \sin\theta\cos\theta - \frac{d^2\theta}{dt^2}(\cos\theta)^2\right] = T\sin\theta\cos\theta, \qquad (13)$$

$$mL\left[\left(\frac{d\theta}{dt}\right)^2 \cos\theta\sin\theta + \frac{d^2\theta}{dt^2}(\sin\theta)^2\right] = T\cos\theta\sin\theta - mg\sin\theta. \qquad (14)$$

Then equation (13) is subtracted from the equation (14) resulting in the expression:

$$mL\frac{d^2\theta}{dt^2}[(\sin\theta)^2 + (\cos\theta)^2] = -mg\sin\theta, \quad (15)$$

Which to be simplified leads to the equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta, \qquad (16)$$

Which happens to be the differential equation, which determines the angle θ for all time t, and usually follows but alternatively in all textbooks undergraduate, Physics.

If we now substitute[1] the equation (16) into equation (12) we obtain the expression:

$$mL\left[\left(\frac{d\theta}{dt}\right)^2\cos\theta - \frac{g}{L}(\sin\theta)^2\right] = T\cos\theta - mg, \quad (17)$$

Which can be rewritten as:

$$mL\left(\frac{d\theta}{dt}\right)^2\cos\theta = T\cos\theta - mg[1-(\sin\theta)^2], \quad (18)$$

And that to be simplified and solving for the tension T also leads to the equation:

$$T = mL\left(\frac{d\theta}{dt}\right)^2 + mg\cos\theta. \qquad (19)$$

Equations (16) and (19) completely determine the movement of the object mass m so that through them when determining the angle θ then we will be possible knowing its position every moment of time t. Although these equations are apparently not possible to give simple analytical solutions in terms of elementary functions[2], to obtain their solutions is necessary to use numerical integration methods.

Now, if we introduce the mathematical expression which defines the tangential velocity of an object ($\boldsymbol{v_t = d(L\theta)/dt}$), we can rewrite Equations (16) and (19) as follows:

$$a_t = \frac{dv_t}{dt} = -g\sin\theta, \qquad (20)$$

$$T = m\frac{v_t^2}{L} + mg\cos\theta. \qquad (21)$$

The interpretation of equation (20) is that along the simple pendulum motion its tangential acceleration $\boldsymbol{a_t}$ is not constant but varies from the value[3] $\boldsymbol{-g\sin\theta_0}$ to be identically zero at the point where the rope hangs so perfectly vertical, the latter being due to precisely this position θ = 0.0 rad. Moreover, the minus sign in this equation indicates that the power supplied by the tangential acceleration of the object is a restoring force. From this, it follows that if the object starts from rest -from the position where $\boldsymbol{\theta = \theta_0}$- then this will reach a maximum tangential velocity at the point where the rope hangs in a perfectly vertical, and then, the object

[1] The replacement can also be done in equation (11).

[2] For the case of little angles ($\theta \le 5^o$) the equations takes a form that is possible solve in terms of elementary functions

[3] Where θ_0 is the angle since the simple pendulum starting its move.

continue moving slowing down its tangential acceleration until it again reaches the rest position where $\boldsymbol{\theta} = -\boldsymbol{\theta_0}$.

For its part, the interpretation of equation (21) is that along the motion of a simple pendulum, rope tension force is not constant, but depends on the position where the object of mass m is furthermore; it depends on the value of the tangential speed at that position. Equation (21) shows us that at all times the tension of the string must be sufficient to overcome the pull that the rope gives to the component object's weight that acts radially, while the tension must be sufficient to achieve divert the direction of the tangential velocity of the object which will be more difficult with the increase in the magnitude of the rate as established by equation (20). It is very important to note that the only points where the tension is reduced to $\boldsymbol{T = mg\cos\theta}$ are those in which the object is at rest, and in any other point -where the object has a tangential velocity equal to zero- the tension of the rope will be given by equation (21); the reason of doing special emphasis in this situation is that both, students and teachers, often make the mistake of proposing without doing the minimum analysis about a simple pendulum string tension has always the value $\boldsymbol{T = mg\cos\theta}$. It's clear that it is not in this fashion but only in points where the pendulum is momentarily at rest.

Newton's Laws and Energy Conservation

The following figure shows again our simple pendulum, but this time illustrates two positions $\overrightarrow{\boldsymbol{r_1}} = (\boldsymbol{x_1}, \boldsymbol{y_1})$ **and** $\overrightarrow{\boldsymbol{r_2}} = (\boldsymbol{x_2}, \boldsymbol{y_2})$ that the object of mass m would be in two t_0 different time instants t respectively:

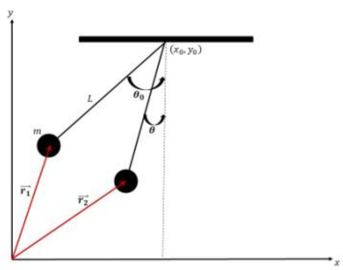

Fig. 8. Simple pendulum with two positions

In this analysis considering t_0 as the instant of time when the pendulum begins its movement, so that $\theta_0$will be the angle of the rope with the vertical at that instant of time. Below, it is the free body diagram of the object of mass m, which is part of our simple pendulum, but this time, the axis "y" was placed parallel to the string:

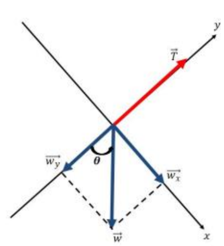

Fig. 9. The pendulum's free body diagram

Taking into consideration that a simple pendulum moves forming a circular arc of radius L, and also, according to the direction in which the tension $\vec{\boldsymbol{T}}$ acting upon the rope and the component $\overrightarrow{\boldsymbol{w_y}} = \boldsymbol{mg\cos\theta}$ of the weight from the perspective of the framework shown in Figure 9, we conclude that these two forces must provide the necessary centripetal acceleration along the pendulum motion. Mathematically this is:

$$T - mg\cos\theta = ma_c, \qquad (22)$$

Where $\boldsymbol{a_c}$ denotes the centripetal acceleration which is defined as:

$$a_c = \frac{v_t^2}{L}, \qquad (23)$$

Where $\boldsymbol{v_t}$ denotes the tangential velocity of the object of mass m. In this way, the tension of the rope will be expressed in the form:

$$T = m\frac{v_t^2}{L} + mg\cos\theta, \qquad (24)$$

As we can see, it is the same expression that we obtained in the previous section, and it was labeled as the equation (21).

However, according to the principle of conservation of energy applied to the two positions of the simple pendulum illustrated in Figure 3, we have:

$$mgy_1 = \frac{1}{2}mv_2^2 + mgy_2, \quad (25)$$

In this expression, the kinetic energy in the position $\overrightarrow{\boldsymbol{r_1}} = (\boldsymbol{x_1}, \boldsymbol{y_1})$ is identically zero because, in this position, the pendulum begins its motion from rest. In the other hand, the velocity $\overrightarrow{\boldsymbol{v_2}}$ of the object of mass m at position $\overrightarrow{\boldsymbol{r_2}} = (\boldsymbol{x_2}, \boldsymbol{y_2})$ is just its tangential speed $\overrightarrow{\boldsymbol{v_t}}$ for this position. At this way, it's possible to express the square of this tangential speed as:

$$v_t^2 = 2g(y_1 - y_2), \quad (26)$$

Looking now at Figure 8, it is evident that $\boldsymbol{y_1 = y_0 - L\cos\theta_0}$ and $\boldsymbol{y_2 = y_0 - L\cos\theta}$ which are substituted in the above expression to write the square of the tangential velocity as:

$$v_t^2 = 2gL(\cos\theta - \cos\theta_0), \qquad (27)$$

Finally, replacing the equation (27) into equation (24) to express the string tension as:

$$T = mg(3\cos\theta - 2\cos\theta_0), \qquad (28)$$

It is important to note that equation (28) is not different from the equation (21), but that equation (28) is an extension of equation (21) after applying the principle of conservation of energy. Consequently, the equation (28) reveals more clearly that the tension in the string of a simple pendulum depends on the initial conditions[4] and the dynamic conditions in the position[5] in which we want to know the tension of the rope. Moreover, equation (28) also illustrates that the only points where the tension of the rope is reduced to the expression $\boldsymbol{T = mg\cos\theta}$ **are those where** $\boldsymbol{\theta = \pm\theta_0}$ and with support from the equation (27), we see that such points in the object mass m is in a state of momentary rest.

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[4] The angle θ_0 and the velocity $v_1 = 0.0$ in the position $\overrightarrow{r_1} = (x_1, y_1)$.

[5] The angle θ and the velocity $v_2 = v_t$ in the position $\overrightarrow{r_2} = (x_2, y_2)$.

To conclude this section, we shall mention that to know the precise value of the tension of the rope at any arbitrary point on the path traced by the simple pendulum, the angle θ that the rope form with respect to the vertical must be known. This is possible to achieve, when the equation (18) is integrated numerically, which was derived in the previous section, and as previously we mentioned this equation is usually derived in textbooks of undergraduate physics.

Misconceptions of Mexican's Teachers

As mentioned in section 3, teachers' results can be classified into four solutions: Teachers, who were surveyed, got the following four types of results:

$$T = \frac{mgh}{R+mg}, \qquad (29)$$

$$T = \frac{mv^2}{l\sin\theta}, \qquad (30)$$

$$T = \frac{m(a_y+g)}{\cos\theta}, \qquad (31)$$

$$T = \frac{2mg}{\cos\theta}. \qquad (32)$$

From these results, the only one that is correct it's the type (31). But unfortunately, it isn't a complete result because teachers only expressed the tension T of the string in terms of the vertical component of the acceleration a_y, which is another unknown quantity to be determined in this problem.

The type (29), (30) and (32) results are fundamentally incorrect. This is due to the misapplication of Newton's laws –by professors. Also, the presented calculations always showed the characteristic of being worked in Cartesian coordinates. This action didn't permit them to determine the rectangular components of the acceleration in an explicitly way. Moreover, this approach led them to make incorrect conclusions such as (a) the direction in which operates centripetal acceleration, and (b) the mathematical form of the Cartesian components of acceleration. Finally, another important feature that should be mentioned: teachers never realized that they labored a system of two equations with two unknowns, and mistakenly, they considered that in order to solve the problem, it was enough to manipulate algebraically one of the given equations.

Conclusion

As we can see, the models of traditional texts -as those authored by physics teachers- not properly describe fully the phenomena associated with the dynamics of the simple pendulum. For teachers, this situation is even more worrisome because it is not only show strong misconceptions they have, but also, it shows the way in which they create misconceptions among the students.

The solution shown in Section III in this report is one of the possible solutions, and it explained in depth to correct this problem. Another valid solution to the problem can be obtained using energy conservation laws, but this was omitted (due to lack of space) due to the scope of this paper.

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