A Didactic Proposed for Teaching the Concepts of Electrons and Light in Secondary School Using Feynman´s Path Sum Method

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Abstract
This work comprises an investigation about basic Quantum Mechanics (QM) teaching in the high school. The organization of the concepts does not follow a historical line. The Path Integrals method of Feynman has been adopted as a Reference Conceptual Structure that is an alternative to the canonical formalism. We have designed a didactic sequence for teaching the basic ideas of Quantum Mechanics according to our theoretical frame (Otero, 2006), eluding the mathematical formalism and using simulation software. In the first part of this work we show our proposal for teach the quantum character of the electrons. At the end, we present the outlines of an approach the quantum character of the light with students of the secondary school, where also we use Feynman's technique as the most universal explanation of the light.
Keywords: Quantum mechanics, electron and light, college, teaching method.

Introduction

The need to teach Quantum physics in the secondary school comes rising in Argentina and many countries already. The scanty proposed, stocks in a boarding strictly historic, do not show satisfactory results in relation to the learning of the aspects fundamentals. We consider realizing a boarding alternative, adopting Feynman's method, adapting it to the knowledge of the students and proposing situations that allow them to give sense to the quantum concepts.

Other proposals (Taylor, (2003); Ogborn, Hanck, and Taylor (2006); Hanc, and Tuleja (2005)) have used the Feynman method too, but applied to photons to introduce quantum concepts. Nevertheless, we decided to start with electrons instead photons because with the experience of the double slit it is possible, controlling an alone variable - the mass - to make emerge the quantum character of the electrons, departing from the behaviour corpuscular classic from the matter, aspect known by the students. Also the classic studio of the quantum transition emerges naturally. On the other hand, in order the quantum aspect of the light to emerge, it is necessary to control the intensity and therefore two variables - the frequency or energy of the photons, and the flow of photons - we prefer leaving this boarding for hereinafter. Our proposal will use Feynman's method to model the quantum behaviour of the light, after establishing the quantum character of the electrons.

As object of analysis we propose the "unexpected" behaviour of electrons, from a classical point of view, in the double slit experience. We start by discussing with the students about the experimental results that would be observed in an idealized Double Slit Experience. With the assistance of software simulations, we focus on the changes observed as the objects masses are reduced from macroscopic to atomic values. After that we introduce the basis of quantum mechanics by means of the concepts of path integrals, which from now on we will call "Sum of All Alternatives". Finally it is shown how the transition from classical to quantum behaviour naturally emerges within the framework of Sum of All Alternatives.

Method

In the first moment, we analyzed and structured the concepts to teaching, based on Feynman's technique, and then we elaborated the didactic sequence. The material was used for first time in a senior-level high school physics course. The class was composed of 30 students who were 17-18 years old. There were two one-hour class periods per week. The students had already been exposed to trigonometric functions, vectors, and classical mechanics. The instructional sequence consisted of 13 lessons divided into four stages as follows in the next section. In this first implementation, the conversations about every situation were recorded in audio and then transcript, all the paper resolutions of the student was taking (problem solving, draws, personal synthesis, written test, etc.). From this registers we could to analyze the obstacles and how the sequence works.

The Didactic Sequence For The Electrons

1- Double Slit Experience with small balls and electrons

The students imagined and predicted the results of this experience where small balls were used. Afterwards, the Double Slit Experience with small balls was simulated using the software "Doppelspalt"[2]. This software allowed to appreciate the impacts in the screen to generate the histogram of frequencies and to visualize the theoretical curve of frequencies distribution, called I (x) generated by the software. The students compared their predictions about the results of the experiment with the simulation results. They solved a set of tasks to analyze the effect in the form of the curve when the distance between the slits and the slit widths were changed. This led the group to accept and to establish the following conclusion:

> *"When both slits are open, the resulting curve is the sum of the individual curves, i.e., one slit open and the other one closed and vice versa"*

After that the students analyzed the double-slit experiment with electrons instead of small balls. The simulation allowed assessing the shape of *I (x),* which turned on to be very different from the curve obtained with small balls. The result was inexplicable from the classical theory and the naive idea that electrons would be as small balls.

Although some students could not identify the distribution with the interference pattern observed in experiments with mechanical waves, in general they showed themselves disturbed by the results of the simulation. This generated the need to seek for an explanation of the unexpected behaviour of electrons. The group accepted another key principle in the sequence:

"When both slits are open and although the electrons arrive in discreet units, the resulting curve can not be explained as if the electrons were small balls"

The distribution of electrons on the screen did not follow a pattern that can be produced by the separate contribution of particles emerging from each slit. It convinced the students that it is inadequate to consider electrons as particles, at least in a classical sense. This new way of considering the electrons was driving us to introduce the concept of "quantum system".

2- Analysis and application of Sum of all alternatives method for free electrons

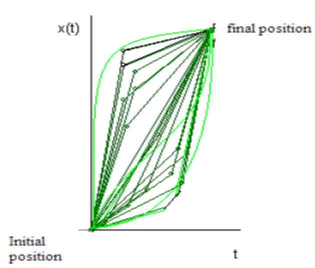
We started by declaring that there exists a set of laws that describe objects behaviour from macroscopic to atomic scale. They are called generically as *Quantum Mechanics laws.* They predict *only* the probability of an event. That is to say, given an initial state: which is the probability of arriving at a final state? In the case of the double slit experiment the question would be: what is the probability for an electron to arrive at a given point on the screen having started from the source?

Experimentally, this probability is measured as a ratio between the number of electrons that actually reach the point and the total number of electrons emitted by the source, when the latter is very large. It is with these types of measurements that quantum mechanics predictions are checked. We have designed a sequence that emphasizes the probabilistic character of the predictions as a central aspect of the quantum theory. We adopted the Feynman method for the QM and adapted it to student's mathematic level, calling it Sum of all alternatives formulation. We replaced complex numbers by two-dimensional vectors. Also integrals were approximated by sums and derivatives by finite increment ratios.

The sequence consists in the following steps,

1- Suppose as initial state (I) a particle at x (t=0)=0 and final state (F) the particle at x (T)=x_f. We consider here one-dimensional paths for simplicity.

Of course there are multiple forms (paths) to connect initial state I with the final state F, some of them are shown in the following figure with straight sections (the only functions that software used by students allows modelling).



Then, with each possible path x (t) we associate a numerical value called action, represented by "S". The action is the average difference between kinetic E_k and potential E_p energy times T.

$$S = \langle E_k - E_p \rangle\, T,$$

where <> denotes temporal average. If the particle is "free", thus it is not in the presence of forces and E_p=0. Then, in this case the action is simply $S = \langle E_k \rangle T$, i.e.

$$S = \tfrac{1}{2} m \langle v^2 \rangle\, T$$

Every path x(t) connecting I with F has a corresponding S, which is used to construct the Amplitude of probability vector associated to x(t), whose components are:

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$$(\cos\frac{S}{\hbar};\sin\frac{S}{\hbar})$$

3-All amplitude of probability vectors associated to different paths connecting I with F is added.

We call the resulting vector "total probability amplitude" ($\vec{A}_{tot}(x)$)

4- The square module of total probability amplitude gives the probability of arriving at final state F, having started at initial state I.

In the double slit experience electrons can be considered as free (except on screens). We can also suppose they are sent at time intervals as long as there is not interaction with each other. The analysis of the free particle allows: a) to validate the technique, b) predict the distribution pattern on the screen, obtained in the first simulation. To help the students to apply the technique Sum of all alternatives to the free electron, a simulation using *Modellus*^{MT} (Theodora, Duque y Costa, 2000) was developed. The Figure 1 shows an output screen of this simulation:

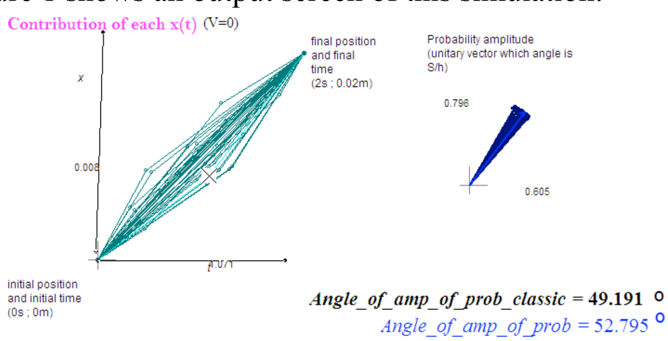

Figure 1. Screen about the first simulation. Selecting different functions x (t) that connect initial and final states.

* The simulation shows the angles on the Cartesian plane and the angle value of this vector in sexagesimal degrees. The probability amplitude vectors are drawn simultaneously for each function $x(t)$ selected.

The use of the simulation software *Modellus*^{MT} (Teodoro, Duque y Costa, 2000) enabled the evaluation of the contributions of different paths to the probability amplitude. It allowed students to formulate the following conclusions:

• The classical path $x_{clas}(t)$ (a straight line from I to F) has the least action S.

• For atomic masses (e.g. electron mass), the angles of the amplitude vectors associated with those paths $x(t)$ near the classical path $x_{clas}(t)$ are very similar.

However, the angles of the vectors associated to paths $x(t)$ which are far from the classical path are different from each other. This means that only a set of paths "around" the classical path contributes to the sum. The vectors associated to the paths that are far from the classical one, have very different directions. They cancel each other in the sum. In this point it was emphasized that this is due to the fact that the electron is free, and that in general in a quantum context all paths contribute to the sum. The Figure 2 is a schematic representation of the sum for V=0.

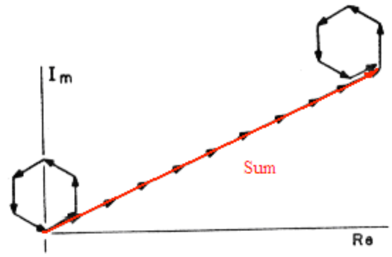

Figure 2. Schematic representation of the sum of amplitude of a finite set of x (t)

* That connect the initial and final points in the case V=0. Every represented vector is unitary and of angle $S/\hbar$, and individual corresponds to the amplitude associated with one x (t) of connecting the initial and final points.

As particle masses increase, the contribution to the total amplitude is reduced to paths extremely close to the classical path $x_{class}(t)$. In the limiting case of a macroscopic object only the classical path contributes to the sum. In this case quantum mechanics predicts the same results as classical physics, i.e. there is only one trajectory, the one of least action. The transition between quanta to classical behaviour can be understood in terms of the small value of Planck's constant in a macroscopic context.

3- Applying the Sum of all alternatives method to reconstruct the interference diagram with electrons

As mentioned in the previous Section, in the case of the free particle, the classical trajectory plays a central role (even at the quantum level). In fact, it can be shown that the sum over all paths can be performed analytically in this case (Shankar, 1980)

Applying these analytical results to the double slit experiment, the probability for a particle of mass m to arrive at a distance x of second screen centre, starting from the source, yields the following result

$$P(x) \propto 4\cos^2\left(\frac{md}{2\hbar T}x\right) \tag{1}$$

where d is the distance between screens. Of course we immediately recognize in the formula (1) the interference phenomenon. The derivation of this formula for *P(x)* was made on the basis on the mathematical level of students (see Appendix). However, it was emphasized that it is a direct consequence of quantum mechanics laws presented and the special role that classical paths play in the case of free particles, as they observed in the previous simulation.

The students discussed and analyzed in groups the functional form of the expression $P(x)$ above. Using this expression with typical values of the variables (provided by the teacher), they made approximate graphical representations of $P(x)$, and located maxima and minima. As a result of this analysis students recognized that these graphs adopt a similar form to graphs representing the distribution of electrons obtained in the first simulation.

4- Classic-quantum transition in the double slit experience

In order to show how the relationship between the mass and the Planck's constant determine the shape of the curve *P(x)*, an additional *Modellus* simulation was generated. Students observed that the distance between maxima and minima was reduced as the mass increased, keeping fixed all other parameters.

The following figures illustrate this behaviour, showing how the interference diagram disappears as the mass increases, making evident the transition between quantum and classical mechanics.

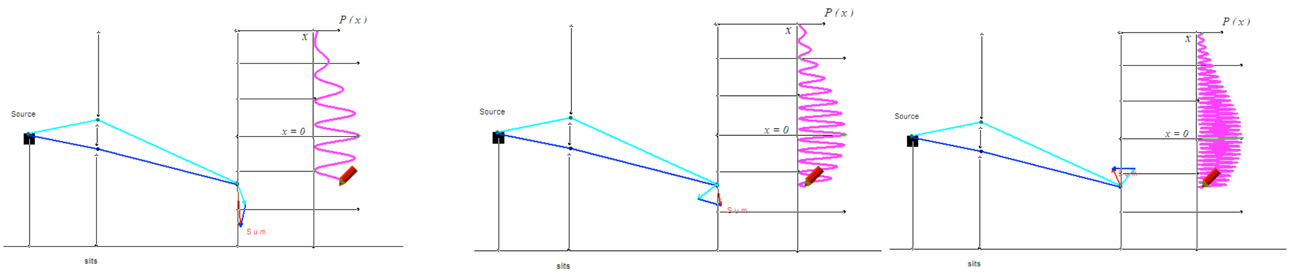

Figure 3. Transition between quantum (top) and classical behaviour (bottom) in the double slit experience.

* The screens show the simulation of the Double Slit Experience using Modellus. Observe how the distances between maxima and minima are reduced, making the interference pattern disappear, as the mass is increased (left top, right top and bottom, respectively).

From previous observations students identified a wavelength (distance between successive maxima) to be dependent on the ratio h / (mv), where $v \approx d / T$ is necessary in order to give the correct units. Now, this wavelength depends only on the properties of the particle, so it makes sense to associate this wavelength to the particle itself.

In this way we arrive at the concept on wavelength λ associated to the particle. It is called De Broglie wavelength, in honour to its discoverer and it is given by,

$$\lambda = h / p, \quad (2)$$

where $p = mv$ is the momentum of the particle. From the formula it is clear that it is the radio between m (more precisely p) and h what determines the quantum or classical behaviour of the system. For macroscopic objects λ is so small that ondulatory characteristics are imperceptible. On the other hand for electrons, for instance, where p is of the order of h, λ is large enough to exhibit interference effects. The sequence ends with a projection on a real double slit experience with electrons, showing how the interference pattern on the screen is made up from the individual impacts of electrons.

Outlines For To Teach The Quantum Character Of The Light Using Feynman´s Approach

We depart from the base that in a set of experimental phenomena and under certain conditions, is possible to assign to the light characteristics that are associated with the particles, and in others to the waves (Arlego, Fanaro, Otero, Elgue, 2011). We refer exclusively to properties of the light, avoiding to raise the question "*What is the light?*" usually in the schoolbooks of text (Elgue, Fanaro, Arlego, Otero, 2011). In Physics this type of questions lacks sense, to the being a question of ontological and epistemological character. We assume explicitly that in Physics is sought to construct models which realize of the observed phenomena and predict new others. The principal aim of the proposed didactic is to depart from the Fig. 4, questioning to the students the experimental result[1].

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[1] The movie consists of 200 frames, with exposure times ranging between 0,025 milliseconds and 6,000 milliseconds. It shows how individual photons, transmitted through a double slit, form an interference pattern. Available online: http://www.youtube.com/watch?v=MbLzh1Y9POQ

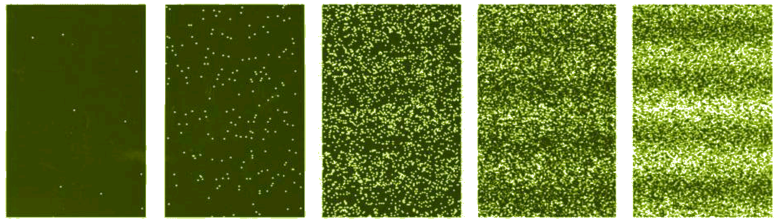

Figure 4. Sequence of photos of the screen of detection to increasing times in an experiment of double split with light of very low intensity.

* Initially the events of detection of light individuals seem at random, analogous to the impacts of "particles" (left). At the end, it glimpses a distribution of maximums and minimums typically of wave behaviour (right).

It is possible to observe, individual events of detection are produced, and the light comes to the screen in granular form, result that would seem to associate with the impact of "particles". Initially, the individual events seem to be distributed randomly on the screen. But as it passes the time, the formation of a pattern of maximums and minimums is appreciated, own of the wave behaviour. In this case the light exhibits a behaviour different from the previous experiences: some aspects corpuscular as the individual detection of events on the screen (also the process of emission of the light for the source), and wave others as the final pattern of maximums and alternated minimums formed in the screen. This example shows that it is not possible to assign to the light a purely wave character or purely corpuscular and therefore we cannot reconcile completely with the daily concept of wave or particle. This peculiar character of the light names "quantum behaviour", and it is not only for the case of the light (electromagnetic radiation), but also in what ordinarily we consider to be "matter". (Fanaro, Otero, Arlego, 2008; 2009)

Two approximations that might enrich the meaning of the presented concepts are:

1-The law of reflection reconsidered, that is to say to explain the reflection from probabilistic considerations from the Feynman approach, and
2-The pattern of interference of the experience of the double split with light, without using ondulatory concepts.

Taking these ideas as a base, we are elaborating a set of situations for the students. We will start by presenting diverse experiences with light where is possible to observe characteristics that ordinarily we associate with phenomena of particles and others where the results are associated with experiences realized with waves. Then, we will show the experience on that we comment before, for which the light would show a new character, impossible to deal from a point of view of particles or waves. To describe this variety of experimental results we will do it taking the most universal of point of view: the quantum mechanics.

Discussion

For the first implementation we analysed in depth all the protocols in their entirety about the six class groups and the synthesizing activities where the teacher and the students are interacting. In

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general terms the results were satisfactory, students expressed on many occasions that they had made an intense but possible effort. They were not surpassed by the proposed situations and they accepted the challenges. The results of the final test were satisfactory (see Fanaro, Otero, Arlego, (2007) for details). Regarding software aspects, students recognized their advantages to support the understanding of new concepts, but they also emphasized the effort required, since it was a new tool unknown for them. Although the designed tools try to lighten certain difficult aspects of calculations, they do not suppose a passive use. They represent an indispensable part of situations conceptualization.

Nevertheless, one of the problems was the classical concept of path in space. It was an obstacle to the students to understanding the path concept established in this didactic sequence. This restricted the interpretation and its consequences when all contributions of different paths x(t) were considered, calculating the probability. This difficulty could be minimized if in the first physics courses the association between the physical path and the image of the single and deterministic path of the instrumental and functional viewpoint is avoided. Quantum mechanics teaching requires emphasising the idea that physics does not involve "reality", but builds abstract models, and within them our old and crystallized images are inappropriate.

We have repeated the sequence twice, with adjustments and improvements to the proposed situations. To know more over the viability of our proposed it is necessary to continue implementing and to analyse new obstacles in the learning.

Conclusions

We applied our alternative method for teaching the fundaments of Quantum Mechanics for secondary school students based on Feynman´s sum of all paths in three courses of similar characteristics. Our work has focused on the emergence of quantum behaviour in the double-slit experiment.

On the other hand our plan to address the quantum aspects of light, in particular the emergence of the concept of photon, is raised. Our next step is to implement it in the school and to analyze its results. With this structure we try to offer the idea to the students that quantum behaviour is unique; neither wave nor particle behaviour.

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Appendix

The calculus of the probability using the Sum of all alternatives method in the Double slit experience presented and discussed with the students.

Lets start with a scheme of the Double slit experience:

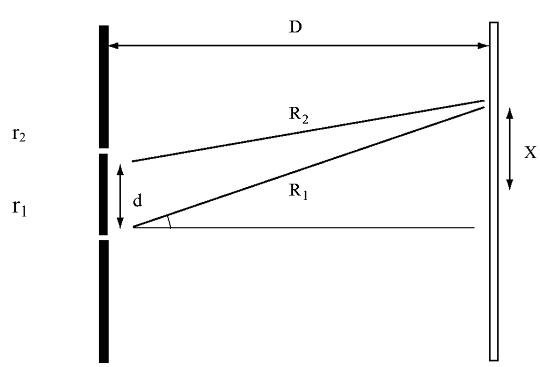

Figure I : Schematic illustration of the Double slit experience

The vector associated to each a form of arrive at the screen via slit r_1 or slit r_2 is

$\overrightarrow{Amp}\,(r_1 \rightarrow x) = (\,N\cos(Scl[r_1 \rightarrow x]\ /\ \hbar\,)\,;\,N\sin(Scl[r_1 \rightarrow x]\ /\ \hbar\,)\,)$ and

$\overrightarrow{Amp}\,(r_2 \rightarrow x) = (N\cos(Scl[r_2 \rightarrow x]\ /\ \hbar\,)\,;\,N\sin(Scl[r_2 \rightarrow x]\ /\ \hbar\,)\,)$

$A_{total}(x)$ *is the sum:*

$$\overrightarrow{A_{total}}\,(x) \sim \overrightarrow{Amp\,(r_1} \rightarrow x) + \overrightarrow{Amp\,(r_2} \rightarrow x)$$
$$\sim (N\cos(S_{cl}[r_1 \rightarrow x]\ /\ \hbar\,)\,;\,N\sin(S_{cl}[r_1 \rightarrow x]\ /\ \hbar\,)\,)$$
$$+$$
$$(N\cos(S_{cl}[r_2 \rightarrow x]\ /\ \hbar\,)\,;\,N\sin(S_{cl}[r_2 \rightarrow x]\ /\ \hbar\,)\,)$$

The class action in this case is $\boldsymbol{S} = \ <Ek>*\,T$, *where T is the time of trip of electrons since source to the screen. Here,*

$$S_{class}[r_1 \rightarrow x] = \frac{1}{2} m \frac{(R_1)^2}{T}$$

and according to the scheme of the Figure I, $(R_1)^2 = \left(x + d/2\right)^2 + D^2$

$$S_{class}[r_2 \rightarrow x] = \frac{1}{2} m \frac{(R_2)^2}{T}$$

and according to the scheme of the Figure I, $(R_2)^2 = \left(x - d/2\right)^2 + D^2$

Then, lets do the sum of both vectors, from component to component:

$$\overrightarrow{A_{total}}(x) \approx N\left[\cos\frac{S_{class}[r_1 \rightarrow x]}{\hbar} + \cos\frac{S_{class}[r_2 \rightarrow x]}{\hbar};\sin\frac{S_{class}[r_1 \rightarrow x]}{\hbar} + \sin\frac{S_{class}[r_2 \rightarrow x]}{\hbar}\right]$$

Replacing the expression for the classic action S above, we can take common factor:

$$\overrightarrow{A_{total}}(x) \approx N\left[\cos\frac{mR_1^{\ 2}}{2\hbar T} + \cos\frac{mR_2^{\ 2}}{2\hbar T};\sin\frac{mR_1^{\ 2}}{2\hbar T} + \sin\frac{mR_2^{\ 2}}{2\hbar T}\right]$$

Lets call: $a = \frac{mR_1^{\ 2}}{2\hbar T}$ *and* $b = \frac{mR_2^{\ 2}}{2\hbar T}$

Then we have the expression of the total amplitude:

$$\overrightarrow{A_{total}}(x) \sim N\,(\,\cos[a] + \cos[b]\ \ ;\ \ \sin[a] + \sin[b]\ \,)$$

Using trigonometric identities:

$$\cos a + \cos b = 2\cos(\frac{a+b}{2}).\cos(\frac{a-b}{2}) \quad \textbf{and} \quad \sin a + \sin b = 2\cos(\frac{a+b}{2}).\cos(\frac{a-b}{2})$$

Then

$$\overrightarrow{A_{total}}(x) \approx N\left(2\cos\frac{a+b}{2}.\cos\frac{a-b}{2};2\sin\frac{a+b}{2}\right)$$

Now lets calculate the modulus of the total amplitude vector and then lets take the square:

$$\overrightarrow{A_{total}}(x) \approx N\left(2\cos\frac{a+b}{2}.\cos\frac{a-b}{2};2\sin\frac{a+b}{2}\right)$$

$$\left|\overrightarrow{A_{total}(x)}\right|^2 \approx 4\cos^2\left(\frac{a+b}{2}\right).\cos^2\left(\frac{a-b}{2}\right) + 4\sin^2\left(\frac{a+b}{2}\right).\sin^2\left(\frac{a-b}{2}\right))$$

Getting common factor $4\cos^2(\frac{a-b}{2})$

$$\left|\overrightarrow{A_{total}(x)}\right|^2 \approx 4\cos^2\left(\frac{a-b}{2}\right).\Bigg[\underbrace{\cos^2\left(\frac{a+b}{2}\right) + \sin^2\left(\frac{a+b}{2}\right)}_{=\ 1\ \text{(trigonometric identity)}}\Bigg]$$

Like $P(x) \sim |A_{total}(x)|^2$, *replacing the expressions of* ***a*** *and* ***b*** *previous and bearing in mind that*

$(R_1)^2 = (x + d/2)^2 + D^2$ *and* $(R_2)^2 = (x - d/2)^2 + D^2$

we come to

$$\boxed{P(x) \propto 4\cos^2\left(\frac{md}{2\hbar T}x\right)}$$