Lesson Study with Mathematical Resources: A Sustainable Model for Locally-led Teacher Professional Learning

Catherine Lewis Rebecca Perry
Mills College

Received: March 30, 2013/ Accepted: September 3, 2013

Teams of educators conducted lesson study independently, supported by a resource kit that included mathematical tasks, curriculum materials, lesson videos and plans, and research articles, as well as protocols to support lesson study. The mathematical resources focused on linear measurement interpretation of fractions. This report examines the resource kit content, the changes in teachers' fractions knowledge, and the lesson study processes that enabled changes in teachers' knowledge. Quantitative findings show that teachers in the experimental condition (lesson study supported by resource kits) significantly improved three of the four facets of fractions knowledge studied, including understanding the whole, unit fractions, and fractions as numbers; whereas control group teachers did not. Qualitative data, including video and written reflections, illuminate activities that supported teachers' knowledge development, including solving and discussing mathematical tasks, studying curriculum and research, and observing students during research lessons.

Keywords: lesson study • fractions • professional learning • professional development

Overview

Lesson study is a common form of professional learning in Japan (National Institute for Educational Policy Research, 2011) and has spread to many other countries (World Association of Lesson Studies, 2012) since early English-language accounts of lesson study appeared (e.g., Lewis & Tsuchida, 1998a; Lewis & Tsuchida, 1998b; Stigler & Hiebert, 1999; Yoshida, 1999). A growing body of research suggests that lesson study can have an impact on teachers' knowledge, professional community, teaching practice and student learning (Hart, Alston, & Murata, 2011; Lewis, Perry, & Hurd, 2009; Lewis, Perry, Hurd, & O'Connell, 2006; Lo, Chik, & Pong, 2005; Meyer & Wilkerson, 2011; Olson, White, & Sparrow, 2011). Much existing research reports small-scale qualitative studies of lesson study facilitated by university-based educators. The data we report are drawn from groups in the experimental condition of a randomised controlled trial in which educators conducted lesson study supported by a resource kit for lesson study on fractions. Groups located across the United States operated without guidance from project researchers, other than the support provided by the resource kit. The larger randomised trial from which the data are drawn (Lewis & Perry, under review) found a significant impact of the experimental treatment on teachers' overall fractions knowledge, as well as on students' fraction knowledge. This paper examines the design of the resource kit, the impact of lesson study when using the resource kit on specific facets of teachers' fraction knowledge, and the processes by which teachers built their knowledge of fractions. By assembling a resource kit that provides mathematical and logistical support for each phase of the lesson study cycle, the research tests a potentially sustainable form of lesson study that can be conducted without outside facilitators.

Review of Literature

Lesson study is a nearly universal form of professional learning in Japan, practised in more than 98% of public elementary and junior high schools and more than 94% of public high schools (National Institute for Educational Policy Research, 2011). In lesson study, shown at the left of Figure 1, teachers conduct collaborative "study–plan–do–reflect" inquiry cycles designed to improve classroom instruction (Lewis & Hurd, 2011; Takahashi, 2014; Wang-Iverson & Yoshida, 2005). Typically, teachers begin the cycle by studying curriculum content and considering their long-term goals for students. Next, they plan a research lesson to be taught by one team member while other team members collect data on student learning. The research lesson provides an opportunity to enact and investigate the team's hypotheses about high-quality teaching and learning. During the post-lesson reflection, teachers present and discuss the data collected during the research lesson in order to draw out implications for teaching and learning of the particular topic as well as more broadly.

Diverse forms of lesson study are practised in Japan, sponsored by schools, districts, national professional organisations and other groups, and taking on somewhat different purposes in each setting: for example, to build teaching skill within a school, to build collective professional knowledge about how to implement a new mandate, or to improve curriculum and teaching methods for the future (Lewis, 2010; Lewis & Tsuchida, 1997; Takahashi, 2014). Honing a single lesson is not typically the primary goal of lesson study as practised in Japan (Isoda, Stephens, Ohara, & Miyakawa, 2007; Lewis, Akita, & Sato, 2010; Lewis & Hurd, 2011; Nihon Kyouiku Houhougakkai, 2009). Rather, as highlighted in the middle rectangle of Figure 1, lesson study is expected to improve instruction by developing knowledge, beliefs, norms, routines, and materials that contribute to continuing instructional improvement.

How does teachers' knowledge improve through lesson study? The striped rectangles in Figure 1 show features of effective professional learning identified by one major review (Desimone, 2009) and suggest how they relate to lesson study: teachers actively and collaboratively study content as they engage in lesson study, enabling them to build increasingly coherent knowledge, beliefs and routines. The hexagons in Figure 1 show some of the resource kit materials, and the arrows indicate their influence on the phases of lesson study.

The study of the academic content and teaching materials—called *kyozaikenkyu*—is integral to lesson study (especially the first two phases of the lesson study cycle) as practised in Japan (Takahashi, Watanabe, Yoshida, & Wang-Iverson, 2005). During *kyozaikenkyu*, teachers use documents such as teacher's manuals, content frameworks, and research reports to study both the subject matter and its teaching and learning (Doig, Groves, & Fujii, 2011; Shimizu, 1999; Takahashi et al., 2005). Japanese textbooks and accompanying teacher's manuals provide support for *kyozaikenkyu* by identifying and discussing the key mathematical ideas in each unit, providing likely student solution strategies and connecting them to the key mathematical ideas, and situating the current unit within a multi-year trajectory of mathematical learning (Doig, Groves, & Fujii, 2011; Lee & Zusho, 2002; Lewis, Perry, & Friedkin, 2011; Miyakawa, 2011). Teaching materials found outside Japan might not provide good support for *kyozaikenkyu*. For example, comparison of the treatment of quadrilateral area in Japanese and United States teachers' manuals revealed that 28% of the statements in the Japanese teachers' manual, but only 1% of the statements in the U.S. manual, focussed on student thinking (Lewis et al., 2011). As described below, we assembled mathematical resources designed to provide U.S. teachers with support for *kyozaikenkyu*.

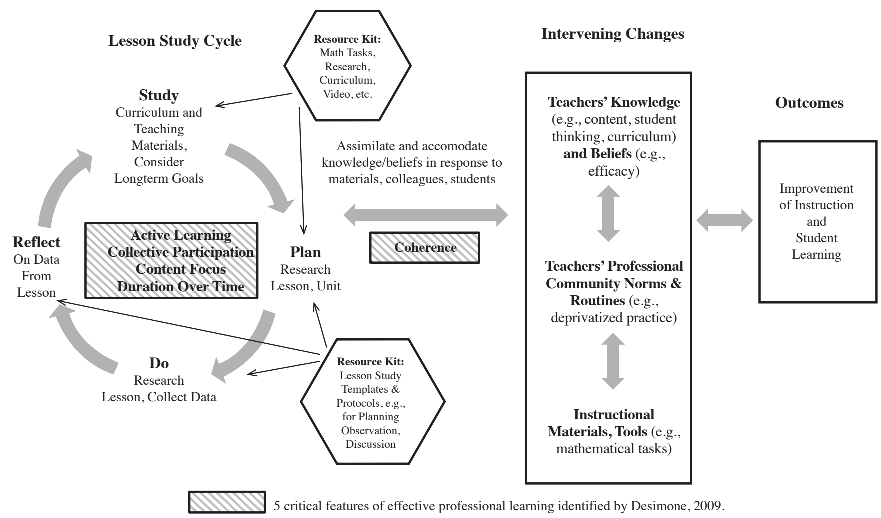


Figure 1. Lesson study supported by mathematics resource kits: Theoretical model of impact on instruction.

The topic of Fractions was chosen due to its fundamental nature, its difficulty for U.S. students (National Mathematics Advisory Panel, 2008), and frequent requests from teachers involved in lesson study for resources on this topic, perhaps reflecting the fact that as few as 20% of elementary teachers regard their fractions knowledge to be strong or very strong (Ward & Thomas, 2006). Our literature review documented a number of challenges in U.S. students' understanding of fractions (see Figure 2) and suggested that linear measurement representations of fractions might help alleviate some of these challenges (Davydov & Tsvetkovich, 1991; Olive & Steffe, 2002; Saxe, Diakow, & Gearhart, 2013). Although linear measurement representation of fractions is emphasised by some high-achieving countries and by the *State Standards* recently adopted by most states in the USA (*Common Core State Standards Initiative* of the National Governors Association Center for Best Practices, 2010), it is often neglected by U.S. textbooks (Lewis et al., 2011; Watanabe, 2007). Research on pre-service teachers' fraction knowledge reveals difficulties in common with elementary students (such as failure to grasp different denominators as different units that cannot be added without conversion) and also difficulties different from those of younger students (Newton, 2008).

The intervention (lesson study supported by a resource kit) allowed teachers to build fraction knowledge by solving and then discussing selected tasks, through study of students' solution strategies and (mis)understandings, and through study of curriculum materials and video of classroom lessons that employ a linear measurement representation of fractions.

Method

Volunteer teams of U.S. educators interested in improving fractions instruction in Grades 2 to 5 were recruited through personal and internet mathematics lesson study networks. In the interest of supporting naturally occurring collaborative groups, we did not specify local group membership except to require that each team include at least one elementary classroom teacher within the Grade 2 to 5 range. Teams of between four and nine educators from across the USA applied and were admitted to the study on a first-come first-served basis once they had signed a memorandum of understanding and obtained district permission to participate.

A total of 39 locally constituted groups of educators participated in the study; most participants were elementary teachers (87% of participants), with coaches, administrators, and middle-school teachers making up the remaining 13%. Of all participants, 41% were new to lesson study. After baseline assessments of fraction knowledge were completed (for teachers and for students in the classes expected to participate in the research lessons), the 39 groups of educators were randomly assigned to one of three professional learning conditions. This report focuses primarily on the 13 lesson study groups randomly assigned to the experimental condition.

Groups in the experimental condition were mailed mathematical and lesson study resources with written instructions to guide the group through the four phases of the lesson study cycle shown in Figure 1. (These resources are detailed in the later section *Fractions Resource Kit*). The teams independently conducted lesson study, guided by the resources, over an average period of 91 days.

Sites had no personal contact with the project investigators. They video-recorded their lesson study meetings and research lesson(s) and mailed the video data back to us, together with artefacts from the lesson study cycle (e.g., lesson plans, student work) and written reflections on each meeting and on their overall learning from the lesson study cycle. After completion of the lesson study cycle, participants once again took the fractions assessment (with items reordered).

Control groups were identical to the experimental group in all study assessments, but engaged in professional learning on self-chosen topics other than fractions, using lesson study or a locally selected form of professional learning other than lesson study. Requiring the control groups to focus their professional learning on fractions might have provided a stronger test of the intervention, but we chose not to do this because mathematics coaches (who recruited some groups to the study) felt that it was unethical to ask all groups to focus their professional learning on fractions and then withhold the fraction resources from some of the local groups. In addition, asking the control groups to focus on a topic other than fractions eliminated the worry of cross-condition contamination. Prior research indicates that simply participating in professional learning focused on fractions (or on various other mathematical topics) is *not* sufficient to improve teachers' mathematical knowledge of the target topic (Gearhart et al., 1999; Hill & Ball, 2004; Saxe, Gearhart, & Nasir, 2001; Timperley,

Timperley, Wilson, Barrar, & Fung, 2007) so the impact of the intervention cannot be attributed simply to the professional learning focus (fractions *versus* a self-chosen topic).

Data Sources and Coding

This study had three main sources of data, and each source required its own particular instrumentation, coding, and analysis.

Assessment of teachers' fraction knowledge. The assessment of teachers' knowledge of fractions included 47 items; with 21 of the items drawn from the *Learning Mathematics for Teaching* (LMT) study (Hill & Ball, 2004; Hill, Schilling, & Ball, 2004), and most of the remaining items drawn or adapted from other published assessments or research (Beckmann, 2005; Center for Research in Mathematics and Science Teacher Development, 2005a, 2005b; Norton & McCloskey, 2008; Ward & Thomas, 2006; Zhou, Peverly, & Xin, 2006).

Four scales tapped specific facets of teachers' fraction knowledge: equality of parts; fraction as a number; understanding of the whole; and unit fractions. Another scale counted overall errors. Two additional scales tracked teachers' attention to particular fraction representations: linear measurement and circle area.

Scores on the scales were produced by coding responses to open-response items like those shown in Figure 2.

Complete the following item by drawing additional parts or shading on the diagram.

If this rectangle is $\frac{4}{3}$, draw a shape that could be the whole.

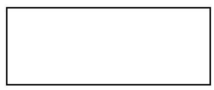


Figure 2. Sample item (adapted from Norton & McCloskey, 2008).

Teachers' responses, blinded as to experimental condition and whether they were from the pre-test or post-test, were coded by two researchers. A reliability estimate of 90% or higher was achieved for each item. (Coding protocols are available on request.) For example, the item shown in Figure 2 was coded as correct if the respondent drew a shape with an area approximately three-quarters of the area of the shown rectangle or divided the shown rectangle into four approximately equal pieces and shaded three of them. The item shown in Figure 2 was used in the scale "Understanding of Whole" and could contribute one point to the "Fraction Errors" scale if answered incorrectly.

The Appendix provides additional item examples and scale information, including reliability estimates (Cronbach's alpha). Since most of the scales do not reach conventional standards for reliability, we regard them not as stable constructs but as item collections that tell us about the particular clusters of ideas teachers gained (or failed to gain) during the lesson study reported here.

Written reflections. In addition to the assessment data, we collected written reflections at the end of the lesson study cycle, using the following prompt:

> Describe in some detail two or three things you learned from this lesson study cycle that you want to remember, and that you think will affect your future practice. These might be things about fractions or mathematics, about teaching, about student learning, or about working with colleagues. (If you don't feel you learned anything from this cycle of lesson study, please note that and identify changes that might have made the lesson study work more productively for you.)

Lesson study video and artefacts. Lesson study groups sent video data and artefacts from their lesson study meetings and research lessons by mail. To date, video recordings of meetings from four of the thirteen lesson study groups have been coded, using Studiocode software, for the extent of the use of the resource kit materials and selected aspects of discussion content, such as mention of linear measurement representations and focus on student thinking.

Using data from the coded groups, we identified segments of video related to the issues addressed in this paper and chose some for transcription, in order to examine the lesson study process. We cannot yet make any claims about how well the selected instances represent the overall lesson study process across the thirteen groups.

Design of the Mathematical Resource Kit

We began the design of the mathematical resource kit by reviewing research and developing a set of research-based conjectures about elementary students' fraction challenges and the learning experiences that help students overcome these challenges. Our ideas are roughly summarised in Table 1, which is taken from the Mathematical Resource Kit provided to the experimental group teachers. Table 1 is the final version of a table that appears in three progressively more complete versions in the Mathematical Resource Kit. The initial version of the table omits the right-most column and the next version includes a blank right column where groups are asked to record their ideas about "the tasks and experiences that build this understanding [of fractions]".

Table 2 below describes the content of the resource kit, which contains research and curriculum materials on fractions as well as tools to support lesson study.

As shown in these tables, the resource kit emphasises the potential of linear measurement representations to build students' understanding of equality of fractional parts (through iteration of the same unit); fractions as numbers (through relatively easy connection to the number line); attention to the whole (through use of a stable, familiar standard measurement unit, such as a metre); and understanding of non-unit fractions as composed of unit fractions (through iteration of a length unit).

The resource kit was provided in a binder, with a main section that included tasks and discussion questions to guide team members as they proceeded through each phase of the lesson study cycle. The four groups whose video records have been analysed to date used the binder as designed (Perry, Roth, & Friedkin, 2013).

Table 1
An Excerpt From the Mathematical Resource Kit (What's Hard About Fraction Number Sense?) (Revisited II)

Type of Understanding or Knowledge	Example of Student Difficulty or Understanding	How Might Linear Measurement Context Help?
A Fraction is a Number A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).	When asked to put the fraction $\frac{2}{3}$ on a number line, a student said "you can't put it on a number line, because it's two pieces out of three pieces, it's not a number." Or "$\frac{2}{3}$ is not a number, it's two numbers". (Kerslake, 1986; Behr & Post, 1992)	Linear measurement may lead students to think about "how much" or "how long" (bringing in images of relative size) not just "how many pieces"(which may focus on counting). Students may be asked to partition a whole for themselves, helping them understand the whole in a way other than just counting pieces.
Partitioning Fractions • A whole can be divided into smaller and smaller equal parts. • The same fractional quantity can be represented by different fractions.	• Difficulty seeing how to divide a whole into *equal* parts. • Difficulty seeing that $\frac{1}{2}$ is equal to $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots$	A number line (or ruler) may make it easy to see that the same point can be described by different fractions.
The Meaning of the Denominator • Different units (such as $\frac{1}{3}$ and $\frac{1}{5}$) are different sizes. • The more units a whole is partitioned into, the smaller each one is. • $\frac{1}{n}$ fits exactly n times into the whole.	• Students add $\frac{1}{3}+\frac{1}{5}$ and get $\frac{2}{8}$, without realising they are adding two different things (thirds and fifths) — a bit like adding apples and hammers. • Students may think "$\frac{1}{5}$ is bigger than $\frac{1}{4}$ because 5 is bigger than 4". • Difficulty seeing that as $\frac{1}{3}$ fits in the whole 3 times, $\frac{1}{4}$ fits in the whole 4 times. Has trouble seeing that $\frac{3}{3}, \frac{4}{4}$, etc. equal 1.	Compared to fractional parts of area (which can be rearranged in many ways), length may provide a clear image of what is $\frac{1}{3}$ m, $\frac{1}{2}$ m, $\frac{10}{11}$ m, etc. Linear measurement may help provide a strong image that the unit that fits in 3 times is longer than the one that fits in 4 times, that $\frac{1}{6}$ is half the length of $\frac{1}{3}$, etc.

Table 1
(continued)

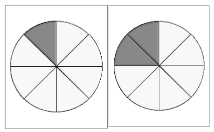
Type of Understanding or Knowledge	Example of Student Difficulty or Understanding	How Might Linear Measurement Context Help?
Knowing What is the Whole • Constructing the whole when given a fractional part. • Keeping track of the whole.	• Difficulty making the whole when you give them a fractional part like: "This paper is $\frac{2}{3}$, show me the whole". • Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g., half of a small cookie is not the same as half of a large cookie). • Confusion about whether the drawings together represent $\frac{3}{8}$ of a pie or $\frac{3}{16}$ of a pie. 	Using a standard measurement unit may be clearer, more familiar, and more stable than an ad hoc unit (such as pie pieces), making it easier to keep track of the whole.
Fraction Size • Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator — not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator.	• May think $\frac{4}{9}$ is bigger than $\frac{3}{4}$ because 4 is bigger than 3 (comparing numerators), or $\frac{4}{9}$ is bigger than $\frac{3}{4}$, because 9 is bigger than 4 (comparing denominators), or $\frac{3}{5}$ is the same size as $\frac{5}{7}$ because the difference between the top and the bottom in both fractions is 2.	Length measurement may transfer to the number line more easily than some other models, so that students see the relative size of fractions. A familiar standard measurement unit (a metre, foot, etc.) may make it relatively easy to see $\frac{1}{3}$ as a length that goes in 3 times, $\frac{1}{4}$ as a length that goes in 4 times, etc.

Table 1
(continued)

Type of Understanding or Knowledge	Example of Student Difficulty or Understanding	How Might Linear Measurement Context Help?
Fraction Size (continued) • Sees non-unit fraction as an accumulation of unit fractions. [A unit fraction has a numerator of 1. A non-unit fraction has a numerator other than 1.]	• Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In $\frac{2}{4}$, $\frac{4}{8}$, $\frac{3}{6}$, etc., denominator is two times numerator. • Sees $\frac{5}{8}$ is made up of 5 eighths or 5 times $\frac{1}{8}$, that $\frac{9}{8}$ is made up of 9 eighths or 9 times $\frac{1}{8}$, etc.	When students think about a turtle that travels in a straight line $\frac{1}{5}$ mile a day for 4 days, they may easily develop an image of $\frac{4}{5}$ as $\frac{1}{5}$ repeated four times. In contrast, $\frac{4}{5}$ of a rectangle or circle may not provide the same strong image of repetition of $\frac{1}{5}$ since the area can be split in many different ways.
Fractions Can Represent Quantities Greater Than One May be difficult for students who have a strong image of a fraction as a *piece* of something.	• "You can't have $\frac{6}{5}$, because there's only $\frac{5}{5}$ in a whole	When students measure an object that is longer than 1 foot (metre, etc.), it may be relatively easy to visualise something as a whole plus an additional fractional part and to understand fractions greater than 1.

Table 2
Fractions Resource Kit: Overview of Contents, With Examples

Section 1: Mathematics Tasks to Solve and Discuss

Participants individually solve three mathematics tasks, anticipate student approaches, discuss solutions with the group, and examine sample student work.

Tasks:

- Estimate the answer to $\frac{12}{13} + \frac{7}{8}$ (NAEP, reproduced in *Post*, 1981)
- Find two fractions between $\frac{1}{2}$ and 1 (Dougherty & Fillingim, 2009).
- Find the number of $\frac{1}{8}$ yard pieces that can be made from $\frac{3}{8}$ yard of string. (U.S. Department of Education, Institute of Educational Sciences, National Center for Education Statistics, 2003)

At the end of this section, groups examine the two left-most columns of Table 1 and discuss the connections between the two columns and the student work examined.

Section 2: Curriculum Inquiry: Different Models of Fractions

Groups examine eight common fraction representations and consider how they might shape students' understanding of fractions. Participants individually solve and then discuss a more difficult version of the task used to introduce fractions in the Japanese textbook: describe the length of a mystery piece in metres, using an un-ruled metre strip as reference. (The mystery piece was $\frac{2}{7}$ metres for teachers, $\frac{1}{3}$ metre in the textbook.) Groups then examine the Japanese textbook and fractions curriculum trajectory, as well as classroom video of fractions lessons taught by an experienced Japanese teacher to U.S. students. Discussion prompts call attention to issues such as how the textbook task helps students see non-unit fractions as accumulations of unit fractions and why the classroom teacher selects particular student misunderstandings for discussion.

Section 3: Choosing a Focus for Your Lesson Study Work

Groups choose a focus for their lesson study, focusing on either Path A (introduce fractions using a linear measurement context) or Path B (focus on another aspect of students' fraction number sense, such as connecting fractions to the number line). Path A groups study additional materials based on the Japanese curriculum (e.g., lesson plans; teaching manuals). Path B groups study other resources depending on the issue investigated (Saxe, Diakow, & Gearhart, 2013; Saxe, Shaughnessy, Shannon, Langer-Osuna, Chinn, & Gearhart, 2007; Van de Walle, 2007).

Section 4: Planning, Conducting, and Discussing the Research Lesson

This section includes a blank research lesson template to support groups as they plan, observe, and reflect on a research lesson. The template prompts groups to write a rationale for their lesson, consider how the lesson fits within the larger trajectory of student learning, design a data collection plan, and write up what they learned, among other activities. Protocols for observation and discussion of a research lesson are also included.

Section 5: Lesson Study Refresher: Overview and Suggestions for Getting Started

For groups new to lesson study or in need of a refresher, this section provides guidelines for norm setting, a sample meeting agenda, an overview of the whole lesson study cycle, and so forth.

What Was the Process of Change in Teachers' Knowledge of Fractions?

Teachers' end-of-cycle reflections provide insight into how the lesson study processes shown at the left of Figure 1 interacted with the resources kit (shown in the hexagons) to enable teachers' development of knowledge about fractions. In their written reflections, teachers mention experiences from every phase of the lesson study cycle, as the following excerpts illustrate.

> We clarified each other's misunderstanding as we read the material on fractions and discussed how ideas could be utilised in our classrooms. As teachers we enriched our own understanding of fraction content and student perceptions as we … [tried] to find more effective approaches to math instruction. I will introduce fractions using the linear model identified in this research, as I believe students can more clearly see the splitting of a whole into equal parts than they can in the area model of dividing brownies that I have used in the past. I appreciate the clean connection from the strips to the number line and expect my fourth graders to develop a clearer understanding and visualisation of fractions. Iterating the unit fraction to create other fractions is a logical and sound approach for students to manipulate and build fraction understanding. This requires a deeper understanding and more thought than my previous strategies of providing examples of various fractions and asking them to identify them. This lesson study has profoundly affected the activities I use to teach fractions. (#584)

> I had never considered giving a student a part of the whole such as 2/5 or 2/3 and asking the student to figure out the whole. The other really helpful part of the lesson study was watching my colleagues struggle with their own misconceptions. Math tends to come easily and, as a result, I need to watch others identify and correct their misunderstandings in order to fully anticipate student misunderstandings. (#4-1-655)

> Sharing ideas, listening to positive feedback and push back from colleagues, taking in critical feedback and de-privatising my practices helped me snag the … weak spots in my practice. I am extremely grateful. My students are all performing better, enjoying math more now. (#525)

Seven of the thirteen lesson study groups in the experimental condition chose to teach one or more of the lessons, which were provided on video in the resource kit, together with lesson plans, the textbook (Hironaka & Sugiyama, 2006), and excerpts from the associated Teacher's Edition. One teacher reflected:

> … the videos of Dr Takahashi's lessons were used as our model for our master lesson. Before we began, we were interested in how our students would react to such a lesson. We felt that the population of Dr Takahashi's students was quite different than ours. We weren't sure if our students would be as flexible in their thinking; however, we were very pleasantly surprised!

The video-recorded lesson study meetings offer an even more fine-grained opportunity to see how the activities and interactions within the lesson study cycle sparked changes in teachers' thinking about fractions. For example, in the following transcript, a group of teachers help each other unpack what it means that the size of the fraction depends on the size of the whole, as they together make sense of a section of the resource kit on understanding the whole.

Teacher 4: And the importance of the whole itself.

Teacher 2: So what would that be? Just understanding the fraction?

Teacher 4: That really has something to do with the whole of it doesn't it? A third can be bigger than a half depending on the size of the whole. I don't think I ever thought about that until I was teaching.

Teacher 5: I never thought about it until I read something today, actually about the third of a cookie versus half of the cookie. It depends on the size of the cookie and I never considered that until today.

Teacher 1: The books that we have … give you two fractions and [you] write less, greater, or equal, they would never say "half of a something" … "half of another", they would just say "half" and "half" and the kids end up putting "equal".

Teacher 2: There's one question in here [resource kit] … one kid said he could be correct because it's trying to get them to think that you don't know what size the original object was that we can have halves of different sizes, depending. And there was a question in there where I was like "Ohhh" —

Teacher 5: That's where I got it.

Teacher 4: Well think, would you rather have half of an individual pizza or a third of an extra-large? (Group 23, 11 16 09E 21:25.37)

How Did Teachers' Knowledge of Fractions Change?

Table 3 shows the specific changes in teachers' knowledge of fractions from pre-test to post-test. Teachers who participated in lesson study with mathematical resources showed significant increases in three of the four facets of fractions knowledge, and in use of linear measurement representations. Both experimental and control group teachers showed significant reduction in fractions errors, with greater reduction in the experimental group.

Teachers' end-of-cycle written reflections confirm their learning about the facets of fractions knowledge emphasised in the resource kit and suggest that the linear measurement representation offered support for teachers' knowledge development.

> Teaching fractions in a linear manner was a real aha moment for all of us on the team, especially me. Watching the students try to figure out how long a piece of ribbon was using linear models was wonderful!!! It just made so much more sense! I am left asking why fractions haven't always been introduced and taught in this way? Using linear fractions helped our children to clearly see fractional parts as equal in size and recognize how to build a new fraction from a unit fraction. (#578)

> I acquired a deeper understanding of teaching the meaning of fractions. Linear manipulations were effective in helping the children understand fractional parts as equal in size and how to build a new unit from a unit fraction. (#575)

> I think the most important idea I took away from this lesson study is the way we approach teaching fractions in the USA. We have such a variety of examples for the students that they don't seem to be able to truly understand what a fraction is. The research article entitled *Initial Treatment of Fractions in Japanese Textbooks* was very interesting. I think it was very helpful for us to focus on the linear method of looking at and learning about fractions as well as focusing on understanding fractions instead of relating them to multiplication, division, ratios etc. ... which is what our 3rd grade textbook does. (#561)

Table 3
Teachers' Fractions Knowledge and Representation Use, by Assessment Time and Treatment Group

	Lesson Study with Resource Kit (N = 73)		Control Groups (N = 140)	
	Pre-Test (SD)	Post-Test** (SD)	Pre-Test (SD)	Post-Test** (SD)
Knowledge Facet				
Equal parts	0.86 (0.87)	0.84 (0.85)	0.68 (0.83)	0.81 (0.89)
Fraction as number	1.18 (1.11)	2.08[1] (1.46)	1.49 (1.19)	1.38 (1.35)
Whole	4.88 (1.60)	5.27[2] (1.26)	4.80 (1.65)	4.9 (1.53)
Maths error*	2.63 (2.31)	1.67[3] (1.78)	2.20 (1.87)	1.76[4] (2.13)
Unit fraction	2.11 (0.97)	2.52[5] (0.99)	2.01 (1.00)	2.12 (1.12)
Representations				
Linear representation	1.33 (1.20)	2.95[6] (2.05)	1.61 (1.45)	1.88 (1.61)
Circle representation	0.77 (1.07)	0.58 (0.76)	0.81 (p.93)	0.82 (1.00)

* Higher maths error score indicates more errors.

** Significant pre- to post-test changes as indicated below:

[1] Paired difference $t = 4.802$, 72 df, $p < 0.001$

[2] Paired difference $t = 2.704$, 72 df, $p < 0.001$

[3] Paired difference $t = 5.113$, 72 df, $p < 0.001$

[4] Paired difference $t = 2.568$, 139 df, $p < 0.05$

[5] Paired difference $t = 2.84$, 72 df, $p < 0.01$

[6] Paired difference $t = 7.25$, 72 df, $p < 0.001$

Discussion

The assessment data indicate significant improvements in selected facets of fractions knowledge by educators who participated in lesson study on fractions supported by a mathematical resource kit. The qualitative data confirm and illuminate this picture, providing insight into how teachers' knowledge development occurred through experiences including solving and discussing mathematical tasks, analysing student work, examining curriculum materials, planning, and observing and reflecting on the research lesson.

The design of this study does not allow us to discern whether the mathematical resource kits themselves, without the lesson study process, would have resulted in similar changes. The written reflections quoted above and video coding conducted to date suggest, however, that lesson study was central to the changes that occurred for teachers. For example, collegial discussion during the "study" phase helped teachers unpack the mathematical content of the materials, the "plan" phase required teachers to negotiate a shared view of good instruction, and the research lesson and post-lesson discussion provided an opportunity to see the impact of the instructional approach on students and draw out implications for their own future instruction.

In their written reflections, teachers mentioned specific ideas from the resource kit and also mentioned elements of the lesson study experience that was likely to have made it possible to incorporate those ideas into practice: elements such as the "push back from colleagues" and "critical feedback" noted by the participants quoted above. Colleagues' ideas, including their "push back" and "feedback" may be critical in helping educators to assimilate knowledge and accommodate their existing knowledge and beliefs, building stronger "coherence" (Desimone, 2009; arrow near the centre of Figure 1) and in enabling teachers to take advantage of the research-based knowledge found in the resource kit and use it in the classroom. By studying, discussing, and enacting elements of fractions teaching and learning with colleagues—including using unit fractions to compose and decompose fractions, seeing fractions as numbers, looking at the fraction-whole relationship, and using linear measurement representations—teachers in this study were able to significantly improve their own understanding of these elements. As laid out by Desimone (2009), these experiences included strong emphasis on content, ongoing (rather than "one-shot") learning, and collective work with colleagues. Joint planning, enactment, and reflection on actual instruction was likely to have provided a powerful push to understand and use the resource kit contents well, since the lesson would be taught to students in front of colleagues.

The lesson video, plans and teacher's edition material included in the resource kit supported direct translation of the mathematical ideas into classroom practice, and although not required, most groups chose to base their research lesson on the one found in the video and the supporting lesson plans, textbook, and teacher's edition materials. Since 87% of the participants in this study were elementary teachers, and 41% of participants were new to lesson study, this study suggests the power of *kyozaikenkyu* (study of curriculum materials and content; Takahashi et al., 2005) for U.S. teachers, if supported by the types of resources available to Japanese teachers. In this study, these resources included a Japanese textbook series and teachers' edition (Hironaka & Sugiyama, 2006), video of the textbook content as taught during U.S. classroom lessons, and mathematical tasks and research enabling exploration of the linear measurement representation of fractions and its relationship to student thinking.

One timely feature of this study is the emphasis on a linear measurement representation of fractions, which appeared to be new to many of the U.S. teachers in the study, and which is emphasised by the new *Common Core State Standards* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Lesson study supported by mathematical resource kits may be a promising approach for supporting implementation of the fractions content of the Standards.

The current study pushes us to think in fundamentally new ways about the scaling-up of educational improvement. The model of lesson study with resource kits provided no support to local educators, beyond that included in the resource kits and that inherent in the lesson study process. Local teams managed their lesson study work, connecting the materials to their own local context—for example, observing their own students for the fraction (mis)understandings they read about in the mathematical resources. The intrinsic rewards of learning about content and student thinking pointed out in the following reflection bode well for sustainability of this form of professional learning:

> Each year when I sign up to be part of our school's lesson study team, I am always nervous and worried that I do not have enough knowledge to be beneficial to a team. As a first grade teacher I am always worried that my understanding of the teaching of some topics will be too simplistic. Yet, each year I feel as if I learn so much and grow so much as an educator. Even though the lesson that our team presents may not be on my grade level instruction, the process … helps me to be a better teacher. I find that I am more aware of each question that I ask. I am able to thoroughly think through the possible outcomes of a question that I may present in class. (#578)

Acknowledgements

This work was supported by the Institute for Education Sciences, U.S. Department of Education, under Grant no. R305A070237. Any opinions, findings, conclusions, or recommendations expressed in this article are those of the authors and do not necessarily reflect the views of the Institute for Education Sciences. We wish to acknowledge Drs Mengli Song and Motoko Akiba for their advice related to the statistical analyses, and Jane Gorman for her assistance in developing the coding protocols for the open-ended teacher assessment items. Melissa Crockett, Shelley Friedkin, and Elizabeth Baker contributed to the organization and rollout of all phases of the research. A version of this paper was presented at the September 2011 meeting of the Society for Research on Educational Effectiveness in Washington, DC, and the April 2012 meeting of the American Educational Research Association in Vancouver, BC.

References

Beckmann, S. (2005). *Mathematics for elementary teachers*. Boston: Pearson Education.

Behr, M. J., & Post, T. R. (1992). Teaching rational numbers and decimal concepts. In T. R. Post (Ed.), *Teaching mathematics in grades K–8: Research-based methods* (pp. 201–248). Boston: Allyn and Bacon.

Center for Research in Mathematics and Science Teacher Development. (2005a). *Diagnostic teacher assessment in mathematics and science, elementary mathematics, rational numbers assessment* (Version 1.2). Louisville, KY: Author. Retrieved from http://louisville.edu/education/research/centers/crmstd/diag_math_assess_elem_teachers.html

Center for Research in Mathematics and Science Teacher Development. (2005b). *Diagnostic teacher assessment in mathematics and science, Elementary mathematics, Number and computation assessment* (Version 2). Louisville, KY: Author. Retrieved from http://louisville.edu/education/research/centers/crmstd/diag_math_assess_elem_teachers.html

Common Core State Standards Initiative. (2012). *In the states.* Retrieved from http://www.corestandards.org/in-the-states

Davydov, V. V., & Tsvetkovich, Z. H. (1991). On the objective origin of the concept of fractions. *Focus on Learning Problems in Mathematics, 13*(1), 13–64.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181–189.

Doig, B., Groves, S., & Fujii, T. (2011). The critical role of task development in Lesson Study. In L. C. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education: Learning together* (pp. 181–199). Dordrecht, The Netherlands: Springer.

Dougherty, B. J. & Fillingim, J. (April, 2009). *Fractions, models and contexts: Impact on student learning*. Paper presented to the Research Pre-session of the Annual Meeting of the National Council of Teachers of Mathematics, Washington, DC.

Gearhart, M., Saxe, G. B., Seltzer, M., Schlackman, J., Ching, C. C., Nasir, N., … Sloan, T. F. (1999). Opportunities to learn fractions in elementary mathematics classrooms. *Journal for Research in Mathematics Education, 30*(3), 286–315.
Hart, L., Alston, A., & Murata, A. (Eds.). (2011). *Lesson study research and practice in mathematics education: Learning together*. Dordrecht, The Netherlands: Springer.
Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education 35*, 330–351.
Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11–30.
Hironaka, H., & Sugiyama, Y. (2006). *Mathematics for elementary school, Grades 1–6*. Tokyo: Tokyo Shoseki Co. (English language version available at http://www.globaledresources.com)
Isoda, M., Stephens, M., Ohara, Y., & Miyakawa, T. (Eds.) (2007). *Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement*. Singapore: World Scientific Publishing.
Kerslake, D. (1986). *Fractions: Children's strategies and errors. A report of the strategies and errors in Concepts in Secondary Mathematics and Science Project*. Windsor, UK: NFER-Nelson.
Lee, S., & Zusho, A. (1999). Comparing Japanese and US teachers' manuals: Implications for mathematics teaching and learning. In G. DeCoker (Ed.), *National standards in school reform in Japan and the United States* (pp. 67–91). New York, NY: Teachers College Press.
Lewis, C. (2010). A public proving ground for standards-based practice. *Education Week, 30*(3), 28–30.
Lewis, C., Akita, K., & Sato, M. (2010). Lesson study as a human science. In W. R. Penuel & K. O'Connor (Eds.), *Learning research as a human science. National Society for the Study of Education Yearbook* (Vol. 109, pp. 222–237). New York, NY: Teachers College, Columbia University.
Lewis, C., & Hurd, J. (2011). *Lesson study step by step: How teacher learning communities improve instruction*. Portsmouth, NH: Heinneman.
Lewis, C., & Perry, R. (2013). *Scale-up of instructional improvement through lesson study.* Manuscript submitted for publication.
Lewis, C., Perry, R., & Friedkin, S. (2011). Using Japanese curriculum materials to support lesson study outside Japan: Toward coherent curriculum. *Educational Studies in Japan: International Yearbook*, 6, 5–19.
Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285–304.
Lewis, C., Perry, R., Hurd, J., & O'Connell, M. P. (2006). Lesson study comes of age in North America. *Phi Delta Kappan, 88*(4), 273–281.
Lewis, C., & Tsuchida, I. (1997). Planned educational change in Japan: The case of elementary science instruction. *Journal of Educational Policy, 12*(5), 313–331.
Lewis, C., & Tsuchida, I. (1998a). The basics in Japan: The three C's. *Educational Leadership, 55*(6), 32–37.
Lewis, C., & Tsuchida, I. (1998b). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator* (Winter), 14-17; 50-52.
Lo, M., Chik, P., & Pong, W. (2005). *For each and everyone: Catering for individual differences through learning studies*. Hong Kong: Hong Kong University Press.
Meyer, R. D., & Wilkerson, T. L. (2011). Lesson study: The impact on teachers' knowedge for teaching mathematics. In A. Alston, L. Hart & A. Murata (Eds.), *Lesson-study research and practice in mathematics: Learning together* (pp. 15–26). Dordrecht, The Netherlands: Springer.
Mills College Lesson Study Group. (2009). *Improving the mathematical content base of lesson study: Survey of teachers of mathematics, Fall 2009*. Oakland, CA: Mills College.
Miyakawa, T. (2011). Textbooks and teaching guides. In M. Isoda, M. Stephens, Y. Ohara & T. Miyakawa (Eds.), *Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement* (pp. 48–51). Singapore: World Scientific Publishing.

National Institute for Educational Policy Research (NIEP) (2011). *Report of survey research on improvement of teacher quality [Kyouin no Shitsu no Koujou ni Kansuru Chosa Kenkyuu]*. Tokyo: National Institute for Educational Policy Research.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core Standards for Mathematics* (2010). Washington, DC: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U. S. Department of Education.

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Journal, 45*(4), 1080–1110.

Nihon Kyouiku Houhougakkai (2009). *Lesson study in Japan* (Vol. 1). Tokyo: Gakubunsha.

Norton, A. H., & McCloskey, A. V. (2008). Modeling students' mathematics using Steffe's fraction schemes. *Teaching Children Mathematics, 15*(1), 48–54.

Olive, J., & Steffe, L. (2002). The construction of an iterative fraction scheme: The case of Joe. *Journal of Mathematical Behavior, 20*, 413-437.

Olson, J., White, P., & Sparrow, L. (2011). Influence of lesson study on teachers' mathematics pedagogy. In L. Hart, Alston, A., & Murata, A. (Eds.), *Lesson-study research and practice in mathematics: Learning together* (pp. 39–57). Dordrecht, The Netherlands: Springer.

Perry, R., Roth, J., & Friedkin, S. (2013, April). *Using high-quality mathematical resources in lesson study.* Paper presented at the Annual Meeting of the American Educational Research Association in San Francisco, CA.

Post, T. (1981). Fractions: Results and implications from National Assessment. *The Arithmetic Teacher, 28*(9), 26-31.

Saxe, G. B., Shaughnessy, M. M., Shannon, A., Langer-Osuna, J. M., Chinn, R., & Gearhart, M. (2007). Learning about fractions as points on a number line. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The Learning of Mathematics: The 69th Yearbook of the National Council of Teachers of Mathematics* (pp. 221–237). Reston, VA: The National Council of Teachers of Mathematics.

Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *ZDM—The International Journal on Mathematics Education*, 45(3), 343–364.

Saxe, G. B., Gearhart, M., & Nasir, N. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. . *Journal of Mathematics Teacher Education, 4*, 55–79.

Shimizu, Y. (1999). Aspects of mathematics teacher education in Japan: Focusing on teachers' roles. *Journal of Mathematics Teacher Education,* 2(1), 107–116.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York, NY: Summit Books.

Takahashi, A. (2014). Supporting the effective implementation of a new mathematics curriculum: A case study of school-based lesson study at a Japanese public elementary school. In I. Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 417–442). Dordrecht, The Netherlands: Springer.

Takahashi, A., Watanabe, T., Yoshida, M., & Wang-Iverson, P. (2005). Improving content and pedagogical knowledge through kyozaikenkyu. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 77-84). Philadelphia: Research for Better Schools.

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration*. Retrieved from http://education counts.edcentre.govt.nz/goto/BES

U.S. Department of Education, Institute of Educational Sciences, National Center for Education Statistics (2003). *Mathematics assessment: Sample questions from the National Assessment of Educational Progress* (NAEP). Retrieved from http://www.nces.ed.gov/nationsreportcard/itmrls/startsearch.asp

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (Sixth ed.). Boston, MA: Pearson Education.

Wang-Iverson, P., & Yoshida, M. (2005). *Building our understanding of lesson study*. Philadelphia: Research for Better Schools.

Ward, J., & Thomas, G. (2006). *What do teachers know about fractions? Findings from the New Zealand Numeracy Development Projects 2006*. Wellington, New Zealand: Ministry of Education.
Ward, J., & Thomas, G. (2009). Presentation on teaching scenarios. Retrieved from http://www.nzmaths.co.nz/fractions
Watanabe, T. (2007). Initial treatment of fractions in Japanese textbooks. *Focus on Learning Problems in Mathematics, 29*(2), 41–60.
World Association of Lesson Studies (2012). *The World Association of Lesson Studies (WALS) International Conference 2012: Programme and abstracts*. Singapore. Retrieved from http://www.walsnet.org/2012/programme.html
Yoshida, M. (1999). *Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development* (Unpublished Doctoral dissertation). University of Chicago, Chicago, IL.
Zhou, Z., Peverly, S. T., & Xin, T. (2006). Knowing and teaching fractions: A cross-cultural study of American and Chinese mathematics teachers. *Contemporary Educational Psychology, 31*(4), 438–457.

Catherine C. Lewis
Mills College School of Education, 5000 MacArthur Boulevard, Oakland, CA 94613
email: clewis@mills.edu

Rebecca R. Perry
Mills College School of Education, 5000 MacArthur Boulevard, Oakland, CA 94613
email: rperry@mills.edu

Published online: 4 June 2014

Appendix

Scales Measuring Specific Facets of Teachers' Fraction Knowledge

Attention to Equality of Parts (Equal parts). (Range 0 – 5, alpha = 0.19 on pre- and 0.18 on post-test). A point was awarded for mention of equality of parts, in responses to five open-ended items. Two sample items follow:

1. Suppose students had just had a basic introductory unit on fractions. If you gave them the fraction $\frac{5}{8}$, what are all the things you hope they would understand and tell you/show you about this fraction? (Mills College Lesson Study Group, 2009)
2. Please explain what you see as the important mathematical connections between measurement and fractions. (Mills College Lesson Study Group, 2009)

Fractions as Numbers (Fraction as number). (Range 0 – 9, alpha = 0.27 on pre- and 0.46 on post-test). In responses to eight open-ended items, teachers noted that fractions are numbers or suggested using a number line to help students understand fractions. Two sample items follow:

3. What similarities and differences do you hope students will notice between fractions and whole numbers? (Mills College Lesson Study Group, 2009)
4. Anna says $\frac{7}{3}$ is not possible as a fraction.
   a) Is $\frac{7}{3}$ possible as a fraction? Yes No (Circle one.)
   b) What action, if any, do you take as a teacher to respond to Anna? (Ward & Thomas, 2009)

Understanding of the Whole (Whole). (Range 0 – 7, alpha = 0.64 on pre- and 0.60 on post-test). Five LMT items concerned with fraction-whole awarded 1 point if correct; following item (Ward & Thomas, 2009) awarded 1 point for correct answer to a), and 1 point for mention of whole in b).

5. A group of students are investigating the books they have in their homes. Steve notices that $\frac{1}{2}$ of the books in his house are fiction books, while Andrew finds that $\frac{1}{5}$ of the books his family owns are fiction. Steve states that his family has more fiction books than Andrew's.
   a) Is Steve necessarily correct? Yes No (Circle one.)
   b) Why/Why not?
   c) What action, if any, do you take as a teacher to respond to Steve? (Added to original item)

Unit Fractions (Unit fraction). (Range 0 – 7, alpha = 0.0 on pre- and post-test). In responses to five open-ended items, teachers mentioned unit fractions. For example, in response to problem 1, one teacher wrote, "I would want students to understand that 5/8 is five 1/8ths".

Mathematical Errors in Open-Ended Problems (Maths error). (Range 0 – 15, alpha = 0.63 on pre- and 0.73 on post-test). One point awarded for each of 15 qualitatively coded items in which error was identified — e.g., in response to 5b), respondent writes that ½ is always greater than 1/5. Skipped items were scored as errors.

Use of Linear Measurement Representation (Linear representation). (Range 0 – 11, alpha = 0.38 on pre- and 0.58 on post-test**)**. Respondents suggested linear representation (e.g., ruler, fraction strip) or number line in responses to eleven open-ended items (1 point for each item in which it is mentioned). For example, mentions linear representation in response to 6b), below:

6. When asked to order fractions from smallest to largest, Robin orders them: $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.
   a) What understanding does Robin need to develop?
   b) What action, if any, do you take as a teacher to respond to Robin?

Use of Circle Area Representation (Circle representation). (Range 0 – 11, alpha = 0.31 on pre- and 0.28 on post-test). Respondents suggested circle area representation (pizza, cookie, etc.) in responses to eleven open-ended items (such as item 6, above).