Helping Teachers Connect Writing to Doing Mathematics

Sharon K. O'Kelley: Francis Marion University

Over the years, educators have noted how writing can support mathematical learning; however, little research has been done examining how mathematics teachers respond to writing. In this study, I followed the experiences of six teachers as they took a graduate class which required that they explore mathematical concepts with technology and post their written findings on the Internet. I found that the two participants who seemed to most favor the writing tended to engage in self-questioning. Based on this finding, I recommend mathematics teacher educators use explicit instruction that promotes self-questioning to guide mathematics teachers in their writing experiences.

Objective

As early as 1977, research in mathematics education indicated the benefit of using writing in the mathematics classroom "as a learning device for the student" (Geeslin, p. 113). Building on years of research and study, the National Council of the Teachers of Mathematics (NCTM) recommended in its 1989 Curriculum & Evaluation Standards that "all students need extensive experience…writing about… mathematical ideas" (p. 140). Since 1989, much research has been done citing the benefits of using writing as a tool of student learning in the mathematics classroom (Porter & Masingila, 2001). Despite the research and recommendations, however, 55% of secondary mathematics teachers surveyed nationally in 2000 indicated that they never use reflective writing in their lessons (Weiss, Banilower, McMahon, & Smith, 2001). Flores and Britain (2003) proposed that mathematics teachers will probably not use writing "unless they have had the experience themselves of writing in relation to mathematics" (p. 112). This statement, however, implies that the teachers should find the experience favorable and worthwhile. Before teacher educators can create activities that promote such favorable experiences with writing in mathematics, we must first understand how mathematics teachers respond to writing about mathematics. In this study, I sought to examine those responses.

Theoretical Framework

This examination was conducted from the perspective suggested by some scholars that writing and doing mathematics are related processes. In 1989, the NCTM described "writing as a process [that] emphasizes brainstorming, clarifying, and revising [which] can readily be applied to solving a mathematical problem" (p. 142). In Writing to Learn Mathematics, Countryman (1992) described a similar relationship between writing and learning mathematics. The author noted that students learn mathematics "by exploring, justifying, representing, discussing, using, describing, investigating, [and] predicting" (p. 2), and she maintained that "writing is an ideal activity" (p. 2) to support these endeavors. Inherent in these views is the notion that writing can be used to support learning mathematics because its processes mirror those of doing mathematics.

It has also been argued that the writing process and doing mathematics often invoke similar mental operations. Emig (1977) described "writing as heuristic" (p. 122) and stated that one of its strengths is that it can yield "self- provided feedback" (p. 128) during the writing process as well as in a review of the final product. Emig's description of writing is similar to Polya's description of problem solving in mathematics. Polya (1945/2004) described "modern heuristic" (p. 129) as an attempt "to understand the process of solving problems, especially the mental operations typically useful in this process" (pp. 129–130). He described how to navigate through the problem solving process and recommended questions for students to keep in mind as they take part in the process. Combining the ideas of both Emig and Polya, one can argue that writing may give the "self-provided feedback" (Emig, 1977, p. 128) that is beneficial to the development and use of questioning skills during the problem solving process.

Implied in this idea of the development of questioning skills is the notion that writers and doers of mathematics need to be aware of their mental operations in order to fully engage in the processes and learn from them. They need to be aware that thinking can be characterized as "a dialogical endeavor, where we inform ourselves, we argue, we ask questions, and we wait for our response" (Sfard, 2001, pp. 4-5). Essentially, they need to engage in metacognitive behavior. Schoenfeld (1992) noted that the term metacognition has many variations to its definition but that it basically involves "individuals' declarative knowledge about their cognitive processes . . . [and] self-regulatory procedures, including monitoring and 'on-line' decision-making" (p. 347). In essence, it involves a person's capacity to assess how he or she is processing information and the ability to make appropriate adjustments. As Schoenfeld suggested, these tools can play a crucial role in the development of students as productive problem-solvers. To assist in this development, he recommended that teachers use "explicit instruction that focuses on metacognitive aspects of mathematical thinking" (p. 356). It follows, therefore, that teachers need to use activities that help students become aware of their own thinking during the problem solving process. Research suggests that writing may be an activity well-suited for that purpose (Pugalee, 2001).

In 2001, Pugalee examined the writings of ninth-grade algebra students "to investigate whether students' written descriptions of their problem solving methods [showed] evidence of metacognitive behaviors, and if so to describe the types of behaviors that [were] evident" (p. 237). Over a 6 day period, twenty students solved one problem per day during a 10-minute session in which they recorded in writing what they were thinking as they worked through the problem. After analyzing the writings, Pugalee sorted the results into four categories specified by a metacognitive framework: orientation, organization, execution, and verification. He determined that "the data showed students' use of metacognitive behaviors in the orientation, organization, execution, and verification phases of problem solving" (p. 243) and that his "study supports reform efforts promoting writing in mathematics" (pp. 242–243).

Methods and Data Sources

In this article, I provide a snapshot of a larger study. My overall goal for the study was to examine in an exploratory manner how teachers respond to writing about mathematics in terms of what they can do, what they experience, and what they believe. In this article, I focus on those responses that highlight how the process of writing might help teachers and students explore mathematics in a way they find useful. As a setting for my study, I chose to observe a graduate class in secondary mathematics education in which writing was heavily used but was not the primary focus of instruction. Because I was not the instructor of the class and writing was not an

instructional focus, the risk of biasing participant response in favor of writing about mathematics was greatly diminished and the participants could respond more freely.

In this class, the instructor asked the students to explore mathematics with technology and write 11 reports on their findings for Internet publication. The class met once a week for three hours. At the beginning of each class, the instructor would introduce a topic and then allow students to work on their reports at individual computer stations for the remainder of the class. These reports focused on activities that covered topics in algebra, geometry, data analysis, precalculus, and calculus and presented a variety of tasks that students could investigate using software such as Geometer's Sketchpad (Version 4.07) and Graphing Calculator (Version 3.5). Within each activity, students could choose a task to explore and about which they could write a report. For example, under the topic of Quadratics they could choose to use Graphing Calculator (Version 3.5) to explore how varying a, b, and c in the general quadratic equation, y=ax^2+bx+c, affects the behavior of the graph and then report their findings. The instructor referred to these reports as "write-ups" and they could be posted on the Internet at any time during the semester. Students were also free to revise and repost their findings as the semester progressed.

I asked participants to volunteer for this study based upon responses to a questionnaire that was administered to the class at the beginning of the semester. The participants selected were five preservice teachers, Grace, Lisa, Gwen, Amy, and Claire, and one inservice teacher, Kim. My objective was to select participants who expressed differing opinions about mathematics and about writing in mathematics. For example, I chose Amy because she indicated that she had struggled with writing in college and would only consider using reflections in her own classroom someday. In contrast to Amy, I chose Claire because she thought writing in mathematics was a "wonderful idea" (questionnaire) and stated that she "fully intends" (questionnaire) to use it in her own classroom.

I structured this study to gain as much insight into the writing process and the written product as possible. Therefore, I collected data from multiple sources: initial and final questionnaires, three interviews of each participant spread out over the semester, field notes, notes the students compiled while doing their explorations, and the write-ups. In addition, I had students complete, after finishing each write-up, what I called a "Post Write-Up Reflection Guide" in which they responded to prompts asking them to describe their experiences with exploring the mathematics and with writing the reports. The particular outcome discussed in this article does not focus on the write-ups but rather on the process of writing the reports.

To analyze the data, I studied all write-ups, interview transcripts, questionnaire responses, student notes and reflections, and my field notes based on classroom conversations. As I examined these documents, I categorized participant responses according to topics such as background, experiences with the course, and beliefs. This process enabled me to situate the participants according to their various experiences. I then compiled a report for each participant in outline form which addressed these topics. After I completed a report for each participant, I carefully examined each one in comparison to the others to make note of emerging themes across the reports. Once I identified these themes, I reexamined all the data noting any new evidence which supported or challenged these major ideas.

This was an exploratory, qualitative study by design. My goal was not to validate my own notions of how the participants should respond but rather to identify themes in how they did respond. In this paper, I focus on a theme that emerged in the responses of two participants who seemed to react most favorably to the writing.

Results

The majority of the participant responses to the writing tended to fall along a scale defined by how closely related the participants thought the writing process was to the exploration of the mathematics. Of the six participants, Amy and Kim clearly demonstrated and articulated that they believed writing and doing mathematics were unrelated. In the middle of the scale, Grace and Lisa found some relationship between the two processes but were somewhat equivocal in their beliefs about the use of writing in the mathematics classroom. At the top of the scale, however, the remaining two participants, Gwen and Claire, tended to view writing as a process that supported and enhanced their exploration of the mathematics. What set these two participants apart from the rest was their ability to use writing to engage in metacognitive behavior.

Gwen and Claire, both preservice teachers, noted that preparing the write-ups helped them to develop self-questioning skills which in turn seemed to push them further into the exploration of the mathematics. Claire described her experience in the final interview:

> I started asking myself…questions as I was writing, "Do I understand everything? What if something equaled zero? How would that affect it?" Internalizing those questions, I think, is the most valuable part of the writing experience.

Gwen freely admitted that she had made an effort to be more inquisitive because of instructor recommendations, but she also made the realization that "as you're writing up [your results], you sometimes realize that there's things that you didn't think to ask when you were just investigating" (Final Interview).

For both Gwen and Claire, writing had become a way to capture and strengthen their intrapersonal communication. They seemed to be aware that their thinking about the mathematics was "a dialogical endeavor" (Sfard, p. 4, 2001) that could be supported by the process of writing. Using writing "to dig a little deeper" (Gwen, Post Write-Up Reflection Guide) into the mathematics through the use of self-questioning, Gwen and Claire both engaged in metacognitive behavior. Essentially, they recognized that the writing helped them to ask questions about the mathematics, and they found value in that process. In addition, Gwen used the writing to measure how well she understood the mathematics. She stated that "when my writing is vague or unclear, I tend to realize this and discover more about the graph so I can better explain the behavior" (Post Write-Up Reflection Guide).

Some of the other participants, however, did not share Gwen and Claire's awareness of the questioning they used while writing the reports. For example, Kim used questioning on three sets of her notes but she did not make reference to the questions during the study as Gwen and Claire did. Therefore, the questions did not seem to play a conscious role in how she processed the mathematics or prepared her write-ups. On eight of ten sets of notes, Lisa indicated questions that arose for her during the exploration process and implied in the second interview that the process of exploring the mathematics and taking notes sometimes created questions in her mind about the mathematics. Although Lisa was aware she was asking questions, she did not seem aware of any connections among the questioning, the note-taking, and the investigation of the mathematics. Essentially, self-questioning did not appear to be a metacognitive behavior for Lisa. She was aware she was creating questions for herself, but she did not indicate that she monitored or regulated the process nor did she express an appreciation for the questioning process.

Of the six participants, Gwen and Claire were also the most enthusiastic about using writing in their future classrooms. Gwen noted on the final

questionnaire that "I absolutely would have my students write in my mathematics class because it demonstrates that they have a true understanding of the curriculum." Claire expressed a similar view during the final interview and indicated that she thought writing would be useful in ascertaining whether or not students conceptually understand the mathematics.

Discussion

This study tends to show that writing can promote the metacognitive behavior of self-questioning that students may find useful in mathematical explorations. Specifically, it shows that when teachers and students use writing to promote self-questioning they tend to engage in a cognitive "'on-line' decision-making" (Schoenfeld, 1992, p. 347) process that deepens their exploration of the mathematics and that they find beneficial to their work. However, the results also tend to indicate that this engagement may not occur for all students. Some students may not engage in self-questioning while writing about mathematics or others may engage in self-questioning but be unaware of it.

To help develop metacognitive techniques, Schoenfeld (1992) suggested that teachers need to use "explicit instruction that focuses on metacognitive aspects of mathematical thinking" (p. 356). The participant responses in this study offer insight into how we can assist students in developing the technique of self-questioning while writing about mathematics. Like the instructor in this study who inspired Gwen to be more inquisitive in her work, we need to offer initial guidance to inspire the inquisitive nature in our students while writing about their mathematical explorations. We should encourage them not only to think about and describe the how of the mathematics in their writing but also to think about and explain the why.

From some of the participants, we also learned that the role of note-taking can provide an avenue for capturing questions that may arise when students are writing about their mathematical explorations. Therefore, when students are engaged in the writing process, we should explicitly ask them to write down any questions that may arise for them as they set about to explain their work. Writing down their mental questions provides a record of their thinking upon which they can later reflect.

As some of the participants demonstrated, however, it is simply not enough to write down questions that arise during the writing process. As indicated, reflecting on those questions is a critical piece to raising student awareness of how the writing process can assist in mathematical explorations. There are several methods of reflection that teachers can use to help students ponder their questions. For example, after students have made note of their questions during writing sessions, teachers can then lead discussions in which students share the questions that arose for them as they wrote about the mathematics and how those questions influenced their explorations. Such sharing can help students see examples of various ways in which questions may arise during writing about mathematics and how those questions can guide mathematical exploration. Teachers can also use journal writing to help students reflect on how self-questioning may have guided students' experiences with writing about their mathematical explorations

Conclusion

Flores and Brittain (2003) advised that future teachers need to experience writing before they will use it in their own classrooms. But mathematics teacher educators need to understand how to guide that experience. My goal in this study was to explore how teachers respond to writing about mathematics in hopes of gaining insight into how the process of writing might help teachers and students explore mathematics in a way that they find productive. What I learned is that by engaging mathematics teachers and

students in writing experiences that explicitly promote self-questioning, we are setting the stage for them to see how the process of writing can enrich their mathematical explorations. We are providing them with the opportunity to discover that the processes of writing and doing mathematics are intertwined and that the process of writing can propel them deeper into the process of exploring mathematics. Essentially, we are laying the groundwork for them to have favorable experiences with writing in mathematics. In turn, by helping teachers to have favorable experiences with writing in mathematics, we increase the chances that they will take the value of writing about mathematics into their own classrooms.

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Authors Note

Sharon K. O’Kelley is an assistant professor of mathematics education at Francis Marion University in Florence, South Carolina. Dr. O’Kelley’s area of scholarship is studying the role of communication in mathematics education.