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Primary School Teacher Candidates' Geometric Habits of Mind*

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Abstract

Geometric habits of mind are productive ways of thinking that support learning and using geometric concepts. Identifying primary school teacher candidates' geometric habits of mind is important as they affect the development of their future students' geometric thinking. Therefore, this study attempts to determine primary school teachers' geometric habits of mind. Participants were 57 primary school teacher candidates in their third year studying Primary School Education in a Faculty of Education at a state university in Turkey. The data were collected through four open-ended geometry problems on concepts of perimeter and area. The collected data were then analyzed based on the theoretical framework of the components of geometric habits of mind and in accordance with the stages of descriptive analysis. The results showed that the primary school teacher candidates did not possess different ways of thinking about the components indicating geometric habits of mind. The study also found that the candidates could not analyze the given problems appropriately and acted on the first idea they came up with, but they were unable to apply these actions on the problem and, therefore, their geometric habits of mind were not at the desired level.

Key Words

Geometric Habits of Mind, Mathematics Education, Perimeter and Area, Teacher Training.

Habits of mind are intellectual problem-solving skills necessary to promote reasoning, perseverance, creativity, and craftsmanship. Leikin (2007) suggested that "employing habits of mind means inclination and ability to choose effective patterns of intellectual behavior" (p. 2333). Cuoco, Goldenberg, and Mark (1996) identified two classes of habits of mind: general habits of mind that surpass every discipline and content-specific habits of mind specific to the discipline of mathematics. General habits of mind include basic qualities such as recognizing figures, exploring, describing, discovering, visualizing, conjecturing, and guessing. In contrast, mathematical habits of mind involve continuous reasoning, performing thinking experiments in extraordinary situations, and employing abstraction used by mathematicians in their work (Mark, Cuoco, Goldenberg, & Sword, 2009).

The main characteristics of mathematical habits of mind develop according to levels of learning (Cuoco, Goldenberg, & Mark, 2010; Goldenberg, Shteingold, & Feurzeig, 2003; Levasseur & Cuoco, 2003). In fact, the term "habits of mind"

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involves two major characteristics: "thinking" and "habituation." Harel (2007, 2008) explained the thinking dimension with ways of thinking and regarded habits of mind as internalized ways of thinking. Goldenberg (2009) described the habituation dimension as habits that "one acquires so well, makes so natural, and incorporates so fully into one's repertoire, that they become mental habits—one not only can draw upon them easily, but one is likely to do so" (p. 13). Mason and Spence (1999) explained the habituation character with their notion of "knowing-to act in the moment," while Lim (2008) explained it with the notion of "spontaneous anticipation" developing when a student instantly anticipates and performs an action in a problem case based on the first idea that comes to mind. Lim also regarded a habit of mind as a cognitive tendency to mentally act in a certain way in certain situations. By the habituation character, Lim (2009) referred to the tendency of "doing whatever first comes to mind" or "diving into the first approach that comes to mind" (Watson & Mason, 2007, p. 207). Mathematical habits of mind come to the fore in areas of mathematics such as geometry and algebra, and they are defined as geometric or algebraic habits of mind (Driscoll, 1999; Mark et al., 2009). A geometric habit of mind is a productive way of thinking that promotes learning and practicing geometry. This way of thinking involves exploring geometric relationships and reasoning with these relationships, generalizing geometric ideas, investigating variants and invariants in these relationships, and assessing a geometric figure with all these components (Driscoll, DiMatteo, Nikula, & Egan, 2007). In their study, conducted between 2004 and 2008, to explore how teachers can define productive ways of geometric thinking to foster it among students in grades 5–10, Driscoll et al. (2007) defined the ways of thinking needed by both teachers and students for becoming successful geometric problem solvers, presented the analyzes of the proofs of geometric thinking, and promoted four fundamental geometric habits of mind: reasoning with relationships, generalizing geometric ideas, investigating invariants, and balancing exploration and reflection. However, in Turkey, there have been only two studies conducted on this subject. Koç and Bozkurt (2012) conducted one on mathematics teacher candidates' knowledge about the volume of the cylinder, and Özen and Köse (2013) conducted a lesson study with mathematics teachers about geometric objects. Other similar studies in Turkey are mostly about van Hiele levels of geometric thinking. In their study, Koç and Bozkurt asked 172 mathematics teacher candidates in their first year to compare the volumes of two different cylinders that can be made with a rectangular (but non-square) paper and to identify and explain the geometric habits of mind that they used during the process. Approximately half the candidates generated incorrect comparisons of the two cylinders' volumes, and nearly half who provided correct answers were unable to provide any mathematical explanations to support their answers. In Özen and Köse's study, several sessions of lesson study were conducted with three mathematics teachers for eight weeks. During these sessions, the teachers developed, implemented, and assessed lesson plans about some subjects covered in the secondary school mathematics curriculum such as constructing geometric objects, identifying the basic components of geometric objects, and obtaining area and volume formulas. Özen and Köse found that the teachers benefited from each other's ways of geometric thinking during the planning, implementation, and assessment stages of the lesson study; they established increasingly stronger geometric relations and their geometric thinking improved. Few international and national studies have treated this subject, and most studies currently focus on mathematics teachers or mathematics teacher candidates.

Fostering students' geometric habits of mind is likely to promote academic achievement not only in geometry, but also in other mathematics courses. In fact, Cuoco (2008) argued that all habits of mind are an important part and an organizing principle of a mathematics curriculum. Moreover, it is essential that teachers help students internalize these habits. In fact, teachers must work on geometry problems with their students, help them express their ways of thinking, and provide learning experiences that will contribute to the development of their geometric thinking. Teachers can provide such learning environments through their own habits of mind, which are closely related to teacher training programs. Identifying the geometric habits of mind of primary school teacher candidates, who are supposed to provide students with a strong mathematical foundation, is extremely important because teacher candidates' geometric habits of mind inevitably affect their future students' habits of mind. In other words, this study is important because students' early acquisition of geometric habits of mind is likely to affect their future learning experiences.

Aim of the Study

This study aimed to determine primary school teacher candidates' geometric habits of mind about the concepts of perimeter and area.

Method

Participants

The study participants were 57 teacher candidates in their third year studying primary school education in a faculty of education at a state university in Turkey. This study used criterion sampling, a purposive sampling process that selects cases to satisfy a specific criterion (Yıldırım & Şimşek, 2003). The criterion here required the candidates to be in their third year in the department.

The data were collected using a test comprising four open-ended questions. Limited to the calculation of perimeter and area of geometric figures, the test questions were chosen from the problems about geometric measurement used by Driscoll et al. (2007) in their project on fostering geometric thinking among students in grades 5–10.

The data collected through the open-ended test were analyzed according to descriptive analysis stages, in line with components of the theoretical framework of geometric habits of mind. During the first phase of data analysis, the components developed by Driscoll et al. (2007) for each geometric habit of mind were taken as themes, and the indicators of each component were taken as sub-themes and sub-categories. First, the researcher and an expert in mathematics education each independently conducted the coding process. Then, by comparing their analyses, agreement and disagreement about the items were determined. After that, the themes and sub-themes were tested to ensure the reliability of the analysis results. The coding scheme was tested with Miles and Huberman's (1994) inter-coder reliability formula, which is the number of agreements between two independent coders divided by the number of possible agreements.

For explaining the cause–effect relationships in the results, the relationships between the themes and sub-themes were first illustrated through figures and were presented with direct quotations from the candidates' responses, with the teacher candidates' real names replaced by pseudonyms.

Results

This study aimed to determine primary school teacher candidates' geometric habits of mind; its results are presented below, separately for each of the four open-ended problems on the test taken by the candidates.

Pastures Problem

The teacher candidates were given a problem on finding the area and the amount of fencing required for a pasture's perimeter. As Figure 1 shows, for the reasoning with relationships component, the teacher candidates employed three ways of thinking: focusing on multiple figures, focusing on the pieces in a single figure, and using special reasoning skills.

The study found that seven teacher candidates who focused on multiple figures found the common properties, i.e., the circumference-diameter relationship, and compared the pastures.

Only one candidate expressed the circumference-diameter relationship and engaged in formula-oriented thinking while reasoning about the area, although this candidate actually focused on multiple figures. In contrast, the other six candidates considered the congruent and similar figures between the pastures while reasoning about both the circumference and the area relationships between the pastures. For example, a drawing made by one of the teacher candidates is given below.

Regarding reasoning with relationships, the majority of candidates focused on a single figure's parts and calculated the perimeter and area of each pasture separately, using formulas. However, they failed to perform a sufficient level of reasoning, particularly about the circumference-diameter relationship among all the pastures, but their thinking was based on operations and formulas. Some candidates formed new geometric figures by completing half circles and circles, and some established connections among the parts in these figures. Especially for the questions about calculating the area of a given pasture, the candidates calculated by making the semicircles into full circles. For questions about the amount of fencing required, they expressed the relationship between the fragments in a pasture via partial comparisons with the other pastures.

Additionally, seven candidates, who related the two figures according to the parts, used special skills by reasoning proportionally and using symmetry. While calculating the amount of fencing required for Pasture E, one candidate who used special

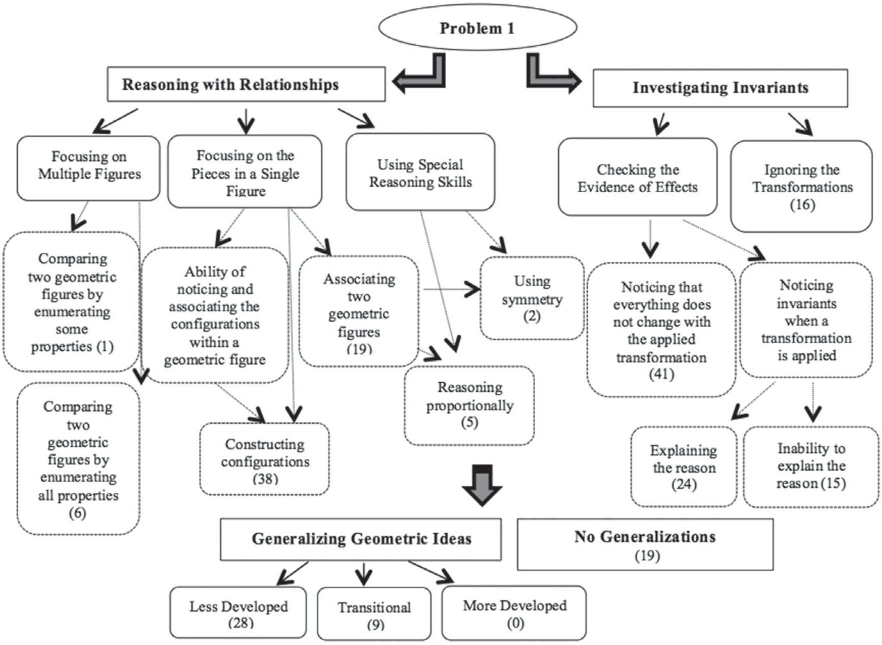


Figure 1.
The Geometric Habits of Mind Demonstrated by the Candidates for the Pasture Problem

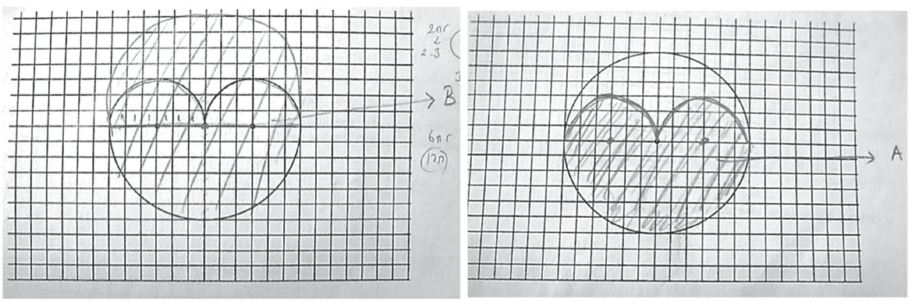


Figure 2.
A Drawing by a Teacher Candidate (T_{16})

reasoning skills created a geometric figure by using symmetry according to the line, established connections between the geometric parts in these figures, and stated that their perimeters were equal to those in the other pastures by saying, "*I drew the symmetry of the upper half of the circle with O_3 center on the lower side. I mean, in this way, Pastures A, B, C, D, and E were equal to each other.*" On the other hand, 28 candidates generalized by considering only special cases, but 19 candidates did not generalize at all. Moreover, only nine candidates at the transitional level of generalization verbally expressed the relationship between the diameters.

Another component of this problem, discussed in terms of reasoning with relationships and generalizing geometric ideas, was investigating invariants. In the latter part, the candidates investigated the area of Pasture E using two different methods. The majority of candidates (41) used rotation, a geometric transformation, in their calculations, but none used a geometric expression such as "rotational transformation," and only 24 candidates explained why the areas did not change during this transformation. Conversely, 16 candidates did not consider any transformation in their calculations, but they either preferred to complete the whole or to make separate calculations. In addition, only one candidate thought of symmetry according to the line while calculating the perimeter of Pasture E, and again, only one candidate recognized that Pasture E and Pasture B had similar figures.

Perimeter Problem

The candidates worked on solving the following problem: "*Two vertices of a triangle with a perimeter of 12 units are located at (4, 0) and (8, 0). What are all the possible positions for the third vertex? How do you know that you have them all?*" As Figure 3 illustrates, this problem emphasizes each component of geometric habits of mind. For reasoning with relationships and generalizing, nine candidates did not generalize at all, but 19 candidates were at the less developed level, 10 were at the transitional level, and 2 were at the more developed level of generalization. Finally, 17 candidates did not attempt to solve this problem.

Candidates at the less developed level of generalization (19) constructed only special triangles, for instance, a 3-4-5 triangle with a base of 4 units or an isosceles triangle. The candidates at the transitional level of generalization (10) recognized that the sum of the lengths of the two sides must add up to 8 and that the difference between the two sides must be less than 4; they constructed triangles based on triangle inequality by assigning several values to determine whether a triangle could be generated in a range. Nonetheless, few candidates at the more developed level of generalization (2) took triangle inequality into consideration and determined the set of possible points as a circle. These candidates generated a more developed example of generalization, but their generalization was still faulty. In terms of investigating invariants, nearly half the teacher candidates thought in a dynamic way by considering the idea of movement for finding the position of the third vertex. However, most of these candidates did not go beyond the starting point, constructing a few triangles only for a few points. On the other hand, six candidates thought that the perimeters of these triangles must be 12 units and that their vertices must be located, for example, between (4, 3) and (5, 4), considering the continuous movement of this point. Further, for the third vertex, six other candidates took the limit/extreme points that can be used in a triangle construction. Half the teacher candidates did not take dynamism into consideration and, most of their generalizations were at the less developed level. This result indicates that these candidates had difficulty visualizing the continuous movement of a point. In terms of balancing exploration and reflection, the teacher candidates demonstrated examples of two approaches: exploration in foreground and end goals in foreground. In terms of exploration in foreground, the majority of candidates (32) intuitively constructed random

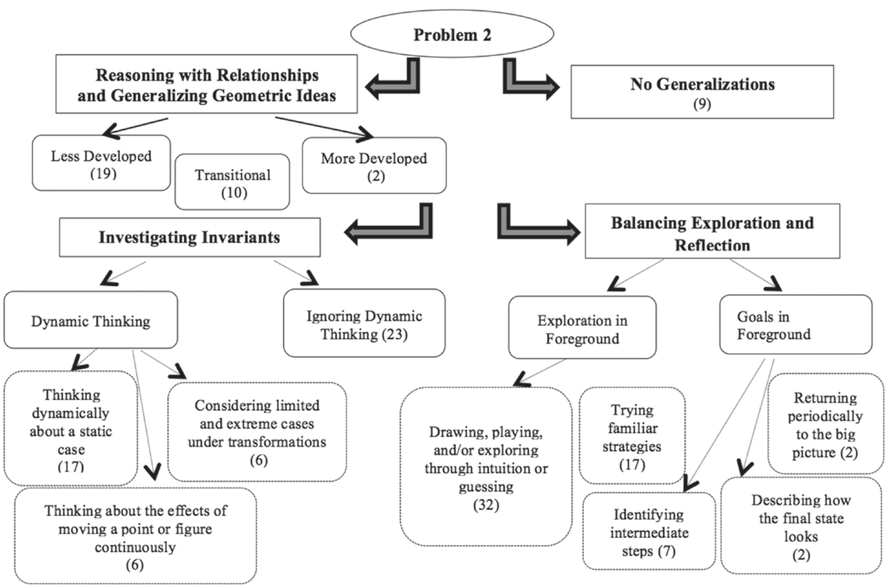


Figure 3.
The Geometric Habits of Mind Demonstrated by the Candidates for the Perimeter Problem

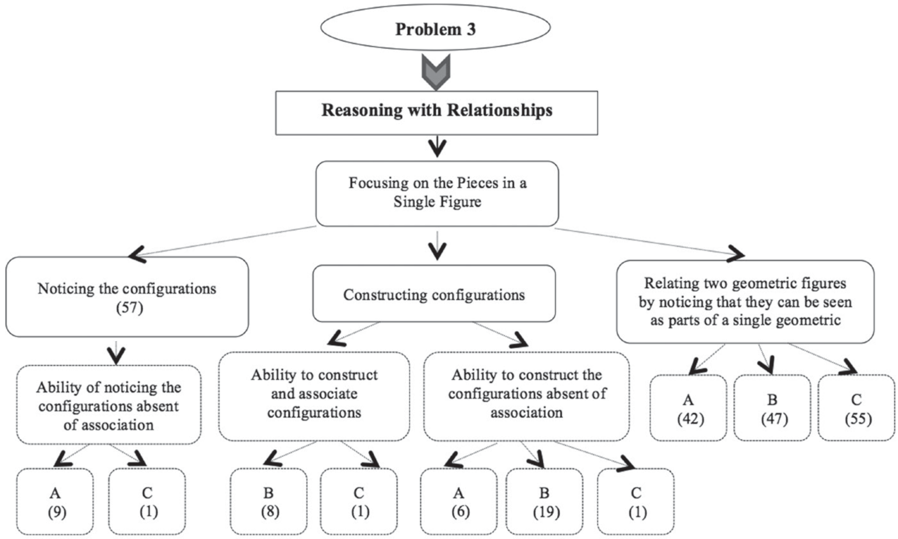


Figure 4.
The Geometric Habits of Mind Demonstrated by the Candidates for the Finding Areas in Different Ways Problem

triangles and 17 candidates used familiar strategies by considering triangles such as 3-4-5, equilateral, or isosceles triangles. In terms of end goals in foreground, two candidates returned to the big picture for a stocktaking, mentally visualized the set of possible points for the third vertex, i.e., the final state, and discovered that this set followed a curvilinear line (arc of circle).

Finding Area in Different Ways Problem

For this problem, the candidates were given three polygons and asked to calculate the area of each in order to develop at least three methods to calculate the area of a given polygon (Polygon B) and to discuss whether it was possible to calculate the areas of the other polygons using the same method. All the teacher candidates focused on parts given in the problem and recognized the patterns and figures in a polygon. However, some failed to effectively use these figures, especially while calculating the area of Polygon A, although they recognized these figures. Among these candidates, five tried to find the area by calculating the side lengths of the given polygon, and six candidates failed to calculate the area of Polygon A because they were unable to determine the given triangle's height. As Figure 4 illustrates, a few candidates (8) could appropriately dissect Polygon B and find the area by using the geometric shapes that they constructed such as square, rectangle, and triangle. One candidate, however, dissected Polygon C appropriately and obtained a quadrilateral and a triangle, and then calculated the shaded spaces' area by making the quadrilateral into a trapezoid. Figure 5 shows some samples of the candidates' applications.

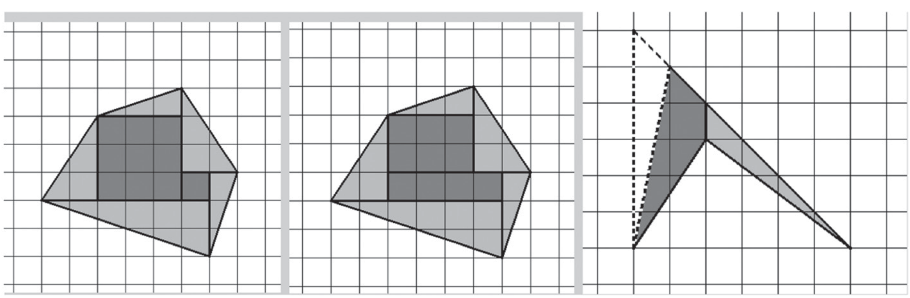


Figure 5.
Samples of the Candidates' Constructions for Polygon B (T_{33} and T_{52}) and Polygon C (T_{24})

In addition, 19 candidates dissected Polygon B for area calculation, but they could not arrive at a conclusion because they failed to dissect it appropriately for calculation. On the other hand, it was difficult to calculate the areas of Polygons A and C by constituting shapes or by dissecting. Nevertheless, six candidates tried to dissect Polygon A, and one candidate tried to dissect Polygon C, but they were unable to arrive at any conclusion. However, the majority of the teacher candidates preferred to make the given polygon into a familiar one for area calculation.

Area Problem

As a continuation of the problem about finding area in different ways, the candidates were asked to solve the following problem: "*Two vertices of a triangle with an area of 12 units² are located at (0, 4) and (0, 10). What are all the possible positions for the coordinates of the third vertex? Explain how you construct them.*" The candidates were also asked to determine the number of the right or isosceles triangles among the constructed triangles and the possible coordinates of the third vertex yielding these triangles.

For reasoning with relationships, the majority of teacher candidates focused on the parts in a single figure and stated that the height of the triangle must be 4 units so that the coordinates of the third vertex of the triangle with an area of 12 units² could be determined, and added that this point could be on $x = 4$ line. However, among these solvers, 10 candidates were unable to relate the area and the coordinates although they recognized the figure. After determining the $x = 4$ line, the majority of candidates also indicated the $x = -4$ line for the third vertex based on the symmetric relationships. On the other hand, one candidate suggested that symmetric relationships should not be used for this question and explained that "*the length cannot be negative.*" For investigating invariants, 14 candidates thinking in a dynamic way considered triangle inequality, stated that the third vertex could be on every point on $x = 4$ and $x = -4$ lines and, therefore, there could be an infinite number of triangles. However, some candidates (37) did not consider the movement of the third vertex and thought in a more limited way, although they found more than one triangle with an area of 12 units². Among these solvers, some candidates stated that the triangle's third vertex could be on $x = 4$ and $x = -4$ lines, but it could take values of $4 \leq y \leq 10$. The teacher candidates were asked to determine the number of right and isosceles triangles among those constructed to determine whether they considered limited and extreme cases through dynamic thinking. Although nearly all the candidates found the four right triangles, they stated that there were just two isosceles triangles. However, it was possible to construct 10 isosceles triangles with two vertices on coordinates (0, 4) and (0,10) with an area of 12 units.

Conclusion and Discussion

Concerning the geometric habits of mind framework, the study found that the teacher candidates did not possess multiple ways of thinking, and they could not make generalizations at the desired level, but the majority could make generalizations at the less developed level. These results are comparable to those reported by Driscoll et al. (2007) and Koç and Bozkurt

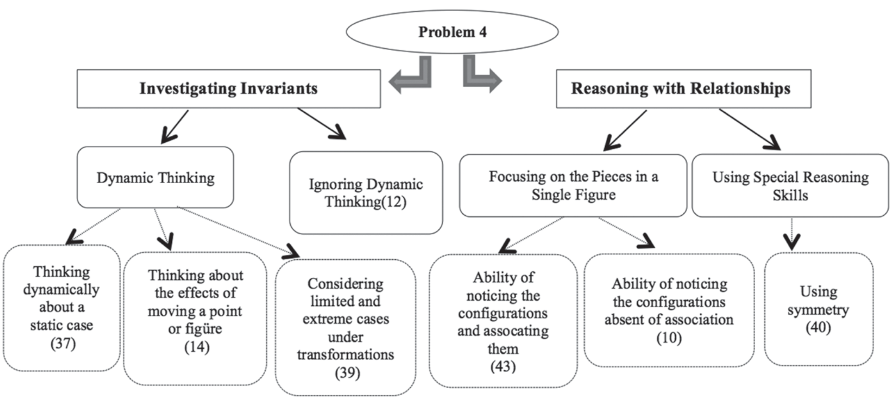


Figure 6.
The Geometric Habits of Mind Demonstrated by the Candidates for the Problem

(2012) in their studies on students and teacher candidates. Furthermore, regarding reasoning with relationships, the study found that the majority of teacher candidates recognized the patterns–figures in a single figure in the given problems and related them to each other, but they failed to construct an appropriate geometric figure. In contrast, when presented a problem of more than one geometric figure, the majority of the candidates analyzed each figure independently instead of comparing the figures and, therefore, thought in a uniform and formula-oriented way. However, it is essential that candidates think of multiple solutions so that they can acquire geometric habits of mind and become successful problem solvers. Unfortunately, few of these teacher candidates compared the geometric figures. The fact that few candidates engaged in reasoning based on unit squares especially for area calculation indicates that these candidates were aware that area measure is not a product based solely on formula (Driscoll et al., 2007). The study also found that, with respect to reasoning with relationships, the candidates failed to perform special reasoning skills, such as symmetry and reasoning proportionally, at the desired level. This result is similar to that of Akkuş Çıkla and Duatepe (2002), who found that teacher candidates failed to use clear, appropriate language while reasoning quantitatively about proportional cases and that they did not possess the conceptual knowledge underlying proportional reasoning. In addition, with respect to generalizing geometric ideas, the study found that nearly all the teacher candidates needed improvement in recognizing all solution sets of the given problems, determining a correct rule for a category of geometric figures, and generalizing rules and problem cases with the whole; therefore, they were unable to make generalizations at the more developed level. This situation could be attributed to the teacher candidates' knowledge of perimeter and area calculations. In fact, some studies (Menon, 1998; Reinke, 1997) conducted with primary school teacher candidates on perimeter and area calculations of geometric figures found that the candidates used inappropriate methods and strategies and approached the given problems operationally, rather than achieving conceptual and relational comprehension. Similarly, this study's candidates tended to engage in formula-oriented reasoning in area and perimeter calculations and failed to make generalizations at the more developed level. As mentioned previously, teacher candidates' need for improvement in exploring, seeking relationships, and reasoning about geometric problems could be another reason that they could not generalize at the desired level—a higher-order skill such as abstraction, holistic thinking, reasoning, and visualization (Greenes, 1981; Sriraman, 2003; Sternberg, 1979 as cited in Amit & Neria, 2008).

Which properties remain invariant and which vary is important in terms of investigating invariants. Driscoll et al. (2007) suggested that a solver's decision to try a transformation of figures in a problem and consideration of the changing and invariant attributes indicate a basic-level geometric habit of mind. This study found that nearly half the candidates tried dynamism in a static case based on actions such as dissection, reflection, translation, and rotation, but almost all the candidates did not consider continuous movement of a point or figure undergoing transformations; in other words, they failed to think in a dynamic way. For example, as Driscoll et al. (2007) also showed, for finding the coordinates of the third vertex of a triangle with an area of 12 units^2, some candidates found a finite set of points within certain intervals or in the form of whole numbers. However, few candidates stated that the vertex could be at any point on $x = 4$ and $x = -4$ lines and, therefore, an indefinite number of triangles could be constructed. This way of thinking is called "reasoning by continuity" (Goldenberg, Cuoco, & Mark, 1998), which is a powerful mathematical thinking and a basic indicator of the investigating invariants component of geometric habits of mind. In this study, balancing exploration and reflection appeared explicitly in the perimeter problem. The majority of candidates intuitively constructed random triangles for the desired triangle and tried familiar strategies. Only two went beyond spontaneous thinking and trials, and they engaged in holistic thinking by returning to the big picture and visualizing the final state—these strategies indicate geometric habits of mind. Similarly, the component balancing exploration and reflection was the least used habit among the teacher candidates in Koç and Bozkurt's (2012) study. In addition, in their study about primary school teacher candidates' problem-posing skills on semi-structured situations, Işık and Kar (2012) found that participants could pose very few problems regarding the desired situation, that most of the problems they posed could be solved through simple calculations, and that these were related to a limited number of different mathematical concepts.

This study investigated primary school teachers' ways of geometric thinking in perimeter and area problems. It found that the participants did not possess different ways of thinking with respect to the components of geometric habits of mind, that they could not analyze the given problems appropriately, that they acted on the first idea they came up with, but could not apply these actions on the whole of a problem and, therefore, their geometric habits of mind were not at the desired level. On the other hand, development of geometric habits of mind cannot be limited only to university education and, thus, teacher candidates' previous learning experiences could have played a key role in this result. Systematic problem-solving activities and classroom discussions are needed for developing geometric habits of mind.

References/Kaynakça

Akkuş Çıkla, O. ve Duatepe, A. (2002). İlköğretim matematik öğretmen adaylarının orantısal akıl yürütme becerileri üzerine niteliksel bir çalışma. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 23*, 32-40.

Amit, M., & Neria, D. (2008). "Rising to the challenge": using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *Zentralbatt für Didaktik der Mathematics (ZDM), 40*, 111-129.

Cuoco, A. (2008, January). *Mathematical habits of mind: An organizing principle for curriculum design*. Paper presented at a Project Next Session on Helping Students Develop Mathematical Habits on Mind, Joint Mathematics Meetings, San Diego, CA. Retrieved from http://www2.edc.org/CME/showcase.html.

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of minds: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior, 15*, 375-402.

Cuoco, A., Goldenberg, E. P., & Mark, J. (2010). Contemporary curriculum issues: Organizing a curriculum around mathematical habits of mind. *Mathematics Teacher*, *103*(9), 682-688

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6-10*. Portsmouth, NH: Heinemann.

Driscoll, M., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking. A guide for teachers, grades 5-10*. Portsmouth: Heinemann.

Goldenberg, P. (2009). *Mathematical habits of mind and the language-learning brain: Algebra as a second language*. Paper presented at an AMS-MAA-MER Special Session on Mathematics and Education Reform, Joint Mathematics Meetings, Washington, DC. Retrieved from http://www.math.utep.edu/Faculty/kienlim/hom.html

Goldenberg, E. P., Cuoco, A. A., & Mark, J. (1998). A role for geometry in general education. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 3-44). Mahwh, NJ: Lavrance Erlbaum Associates.

Goldenberg, E. P., Shteingold, N., & Feurzeig, N. (2003). Mathematical habits of mind for young children. In F. K. Lester & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-Grade 6* (pp. 15-29). Reston, VA: National Council of Teachers of Mathematics.

Harel, G. (2007). The DNR system as a conceptual framework for curriculum development an instruction. In R. Lesh, J. Kaput & E. Hamilton (Eds.), *Foundations for the future in mathematics education* (pp. 263-280). Mahwah, NJ: Lawrence Erlbaum Associates.

Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In B. Gold & R. Simons (Eds.), *Current issues in the philosophy of mathematics from the perspective of mathematicians* (pp. 265-290). Washington, DC: Mathematical American Association.

Işık, C. ve Kar, T. (2012). Sınıf öğretmeni adaylarının problem kurma becerileri. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi, 23*, 190-214.

Koç, Y., & Bozkurt, A. (2012). Investigating prospective mathematics teachers' knowledge of volume of cylinders. *Energy Education Science and Technology Part B: Social and Educational Studies* [Special Issue I]*, 4,* 148-153.

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2330-2339), Larnaca, Cyprus.

Levasseur, K., & Cuoco, A. (2003). Mathematical habits of mind. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grade 6-12* (pp. 23-37). Reston, VA: National Council of Teachers of Mathematics.

Lim, K. H. (2008). *Students' mental acts of anticipating: Foreseeing and predicting while solving problems involving algebraic inequalities and equations*. Saarbrücken, Germany: VDM.

Mark, J., Cuoco, A., Goldenberg, E. P., & Sword, (2009). *Developing mathematical habits of mind in the middle grades*. Retrieved from http://www2.edc.org/cme/hom/hom-middle-grades.pdf

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics*, *38*, 135-161.

Menon, R. (1998). Pre-service teachers' understanding of perimeter and area. *School Science and Mathematics*, *98*(7), 361-368.

Miles, M., & Huberman, M. (1994). *An expanded sourcebook qualitative data analysis* (2nd ed.). California: Sage.

Özen, D. ve Köse, N. Y. (2013, Haziran). *Geometrik cisimler konusunda bir ders imecesi örneği*. 1. Türk Bilgisayar ve Matematik Eğitimi Sempozyumu'nda sunulan bildiri, Trabzon.

Reinke, K. S. (1997). Area and perimeter: Pre-service teachers' confusion. *School Science and Mathematics*, *97*(2), 75-77.

Watson, A., & Mason, J. (2007). Taken-as-shared: A review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education, 10*, 205-215.

Yıldırım, A. ve Şimşek, H. (2003). *Sosyal bilimlerde nitel araştırma yöntemleri.* Ankara: Sözkesen Matbaacılık.

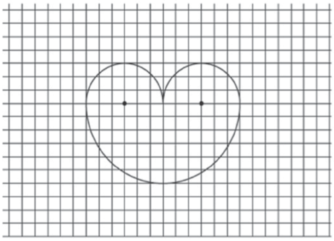
Ek 1.

Öğretmen Adaylarına Sorulan Problemlerden Bir Örnek

Ad-Soyad:

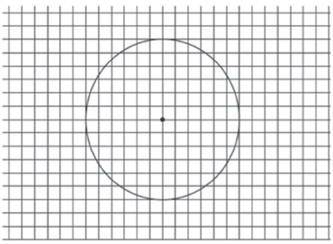
OTLAKLARI HESAPLAYALIM

Eskişehir'de tüm otlakların çemberler ve çember parçaları ile birbiriyle bağlandıklarını düşünelim. Örneğin otlak A şekilde görüldüğü gibi 3 yarım çemberden oluşturulur:



Otlak A

1. Otlak A ile aşağıda şekli verilen otlak B'yi karşılaştırınız.

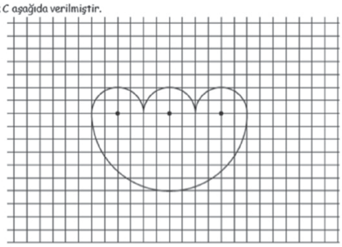


Otlak B

a. Bu otlaklardan hangisinin alanı büyüktür. Açıklayın (uygun matematiksel dil ile)

b. Her otlağın dış çeperindeki eğri çit ile çevrilirse, hangi otlak için daha fazla çite gereksinim vardır? Otlak A ve otlak B için ne kadar çit gerekir? Açıklayınız.

2. Otlak C aşağıda verilmiştir.



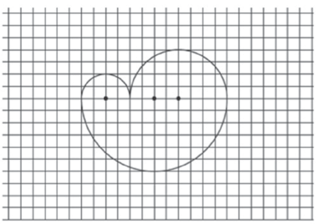
Otlak C

a. Otlak C için ne kadar çite gereksinim vardır? Açıklayınız.

b. Otlak A ve otlak B için gerekli olan çit sayısı ile Otlak C için gerekli olan çit sayısı karşılaştırın. Aralarındaki ilişkiyi açıklayınız.

c. Otlak C'nin alanı nedir? Açıklayınız.

3. Otlak D aşağıda verilmiştir.



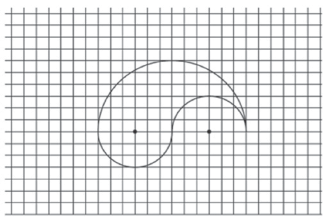
Otlak D

a. Hesaplamadan önce Otlak D için gerekli olan çit sayısını tahmin ediniz. Bu tahmininizi nasıl yaptığınızı açıklayınız.

b. Tahmininizi Otlak D için gerekli çit sayısını hesaplayarak yaparak kontrol ediniz.

Ek

Otlak E aşağıda verilmiştir:



Otlak E

1. Otlak E'nin tam alanını hesaplamak için iki yol tanımlayınız. Her bir yöntemi açıklayınız.

2. Otlak E için gerekli olan çit sayısını açıklayarak hesaplayınız.